3D Scintillation Positioning Method in a Breast-specific Gamma Camera

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Abstract

In modern clinical practice, gamma camera is one of the most important imaging modalities for tumour diagnosis. The standard technique uses scintillator-based gamma cameras equipped with parallel-hole collimator to detect the planar position of $\gamma$ photon interaction (scintillation). However, the positioning is of insufficient resolution and linearity for breast imaging. With the aim to improve spatial resolution and positioning linearity, a new gamma camera configuration was described specifically for breast-imaging. This breast-specific gamma camera was supposed to have the following technical features: variable angle slant-hole collimator; double SiPM arrays readout at the front and back sides of the scintillator; diffusive reflectors at the edges around the scintillator. Because slant-hole collimator was used, a new 3D scintillation positioning method was introduced and tested. The setup of the gamma detector was created in a Monte Carlo simulation toolkit, and a library of a number of light distributions from known positions was acquired through optical simulation. Two library-based positioning algorithms, similarity comparison and maximum likelihood, were developed to estimate the 3D scintillation position by comparing the responses from simulated gamma interactions and the responses from library. Results indicated that the planar spatial resolution and positioning linearity estimated with this gamma detector setup and positioning algorithm was higher than the conventional gamma detectors. The depth-of-interaction estimation was also of high linearity and resolution. With the results presented, the gamma detector setup and positioning method is promising in future breast cancer diagnosis.
Sammanfattning

3D Scintillationspositioneringsmetod i en gammakamera för Bröstdianostik

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Chapter 1

Background

1.1 Gamma camera

A gamma camera is a device to image radioactive sources which emit $\gamma$ photons. In medicine, gamma camera and gamma camera-based computed tomography are frequently used in image-based diagnosis. Radioactive nuclei with $\gamma$ decay can be bonded to pharmaceutical molecules, which are called tracers, and these tracers tempt to concentrate in particular target tissues once injected to human body. Therefore, acquiring the distribution of these tracers, the distribution of the target tissue will be subsequently known. If the target is a tumour, then the location of tumour in body is known. Gamma cameras nowadays are vastly used in clinical diagnosis of cancers [1].

The most important form of gamma camera is scintillator-based gamma camera. The scintillator converts $\gamma$ radiation signal to optical signal, and a light sensor array is applied to convert the optical signal to electric signal so that ordinary electronic device can process.

A typical scintillator-based gamma camera consists:

- A **collimator**. A collimator limits the directions of detectable incident $\gamma$ photons. The most common collimator is a piece of heavy metal with plenty small parallel holes. Radioactive sources emit $\gamma$ photons homogeneously which is similar to the scattered visible light. A collimator in a gamma camera functions similar to a lens set in an ordinary camera, which makes it possible to know from which direction these $\gamma$ photons come from. A collimator is often made of heavy metals, e.g. lead and tungsten, so that most $\gamma$ photons from unexpected directions would be absorbed by the collimator.

- A **scintillator**. A scintillator absorbs $\gamma$ photons and yields optical photons. The process of $\gamma$ photon absorption is called photoelectric effect, while the process of optical photon yielding is called scintillation. From the distribution for the yielded photons detected on the sensor array, the position of $\gamma$ photon absorption could be estimated. However, if the $\gamma$ photon is not absorbed
completely, but partly absorbed and scattered in another direction, there will be one scintillation at the first scattering point and a second scintillation at the position where the scattered photon is eventually absorbed. The process of scattering is called Compton scattering. Compton scattering in scintillators is always undesired, because it is very difficult to distinguish the two scintillation point, and thus hard to locate the first scintillation.

- **A set of light sensors.** Light sensors are devices that transfer optical signal to electric signal. In gamma detection, every $\gamma$ photon has to be analysed and the amount of photons generated per scintillation is quite limited which often gives a very weak signal. Therefore, the light sensor applied in gamma cameras has to be very sensitive and should be coupled to amplifiers with very high gain. In practice, the most commonly used light sensor is photomultiplier tube (PMT). PMTs detect even single optical photon, and have a gain as high as $10^6$. Nowadays, with the rapid development of semiconductor technology, photodiodes are widely used to detect photons. Silicon photomultiplier (SiPM) is a type of photodiode array that is used in gamma cameras. SiPMs also have relatively high gains (approximately $10^6$), and can be manufactured smaller than PMTs. But SiPMs cannot be manufactured as low-noise as PMTs today, so that PMTs are still the first choice of many gamma detector applications today.

- **Positioning** circuit or algorithm. With a light sensor set, it is possible to obtain the position information of the scintillation from the signal intensities among different sensor pixels. In conventional gamma cameras, PMTs are connected in parallel in a resistor/capacitor circuit. This circuit acts as an analog computer that computes the position of the scintillation. This method is called Anger logic, which is discussed in detail in section 2.1. In modern gamma cameras, the output of each light sensor may be converted to digital signal and read by a computer. Sophisticated algorithms can be applied to estimate the scintillation position from the light sensor readouts. But usually only the planar scintillation position is estimated. The depth of the scintillation is not estimated.

Figure 1.1 is a basic schematic of a scintillator-based gamma camera with PMTs is light sensors.

Gamma cameras can be used as an independent 2D imaging modality, but they can also be used in single photon emission computed tomography (SPECT) to obtain 3D image of the target volume. The most common tracer used in SPECT is $^{99m}$Tc, which has a half-life of about 6 hours and emits $\gamma$ photons of 140 keV [1].

A similar tomographic imaging modality of SPECT is positron emission tomography (PET). PET employs tracers with $\beta^+$ decay. An emitted positron from $\beta^+$ decay will interact with a surrounding electron within a very short time and distance, and emit two photons with a fixed energy 511 keV and opposite travelling directions [1], [2]. In PET, collimators are not necessary because the tracks of the
detected photons can be determined by coincident detection of two 511 keV photons. But in the gamma detectors used in PET are based on the same principle as SPECT.

1.2 Breast-specific gamma camera

Breast cancer is the most common cancer among the female population [3]. In breast cancer diagnosis, gamma camera is a powerful tool [4], [5]. An example clinic setup of breast-specific gamma imaging is like Figure 1.2. The breast to be imaged is usually compressed so that less scattering events will occur in the breast and the image will be of better quality.

However, to obtain 3D distribution of tracers in the breast, conventional SPECT and PET can hardly be applied. In both SPECT and PET, the γ detectors should be put around the breast and rotate to obtain projection image from 360°. However, the space in front of the chest does not allow the placement and rotation.

One possible solution is to apply tomosynthesis with variable slant-hole collimator to obtain 3D image by limited projection angles [6], [7]. Tomosynthesis is a quasi-tomography technique which aims to reconstruct 3D image with much less than 360° image projection angles. As in Figure 1.3, the two objects could be completely distinguished in vertical direction by projections of only 37.5°.

1.3 Depth-of-interaction estimation

Tomosynthesis with variable slant-hole collimator seems to be a solution to obtain 3D gamma images for breast cancer diagnosis. But conventional scintillation positioning method (Anger logic, see section 1.1 and section 2.1) is not appropriate in
gamma cameras with slant-hole collimator. In [Figure 1.4] it is clear that if only the XY-plane position of the scintillation is known, it is possible to locate the origin of \( \gamma \) photon wrongly. The third dimension of the scintillation position, the depth-of-interaction (DoI), is necessary information to correctly track the incident \( \gamma \) photon.

In traditional SPECT, DoI is not of much interest, because the parallel-hole collimator only allows the perpendicular incident \( \gamma \) photon to be detected. But in PET, DoI estimation has been vastly studied due to the fact that the absence of DoI information degrades the imaging accuracy in PET. In [Figure 1.5] it is clear that the source of the two 511 keV photons will be mislocated without DoI estimation [8].

DoI estimation for 140 keV \( \gamma \) photons has not been thoroughly investigated before. However, in order to apply gamma cameras with variable slant-hole collimator in breast gamma imaging, DoI estimation is necessary. Thus, there is a need of 3D scintillation positioning in the gamma detector.
1.3. **DEPTH-OF-INTERACTION ESTIMATION**

![Diagram showing object discrimination and interaction position](image)

**Figure 1.3** A simple example of vertical object discrimination achieved by gamma camera with variable angle slant-hole collimator. This is the basis of tomosynthesis.

![Diagram showing decay position and interaction depth](image)

**Figure 1.4** For parallel-hole collimator perpendicular to scintillator surface, DoI information is not important in the imaging system’s spatial resolution; for slant-hole collimator, DoI information is necessary in high resolution imaging.
Figure 1.5 An example of mislocating the source of 511 keV photons due to the absence of DoI information.
Chapter 2

Theory and introduction

2.1 Anger logic positioning

In 1950, Hal O. Anger from Donner Laboratory in Berkeley, California, developed a simple method to locate scintillation in scintillator-based gamma camera, which is known today as Anger logic [9]. Anger logic is the most commonly used positioning method in nuclear medicine in the past 60 year. Despite its simplicity, Anger logic gives satisfactory spatial resolution as well as linearity for basic clinical practice [1]. However, in more specific applications, e.g. breast imaging, Anger logic cannot achieve desirable spatial resolution and linearity.

A simple example of Anger logic positioning circuit is illustrated in Figure 2.1. The output of this circuit (2.1) (amplitude and polarity) depends on the X position of interaction. If all the impedance in the circuit is carefully chosen, with some additional circuits, this output should be a weighted average of all the outputs of the PMTs in the circuit and be proportional to the X position of scintillation. For the Y position, the circuit is similar [9].

\[ V_{\text{out}} = Z_0 \left( \frac{V_4}{Z_4} + \frac{V_5}{Z_5} + \frac{V_6}{Z_6} - \frac{V_1}{Z_1} - \frac{V_2}{Z_2} - \frac{V_3}{Z_3} \right) \] (2.1)

The weighted average relationship can also be expressed as (2.2). \( X_i \)'s and \( Y_i \)'s are the positions of the PMTs (decided by \( Z_i \)'s in the schematic Figure 2.1) and their responses (output voltage) are \( A_i \)'s. Then the location of scintillation can be calculated as a weighted average of the all \( X_i \)'s (\( M \) PMTs will give \( M \) \( X_i \)'s and \( Y_i \)'s):

\[
\bar{X} = \frac{\sum_{i=1}^{M} (X_i A_i)}{\sum_{i=1}^{M} A_i} \\
\bar{Y} = \frac{\sum_{i=1}^{M} (Y_i A_i)}{\sum_{i=1}^{M} A_i}
\] (2.2)

Anger logic positioning circuits often give good enough spatial resolution and linearity at the centre of the scintillator. But when it comes to the periphery, the
CHAPTER 2. THEORY AND INTRODUCTION

Figure 2.1 A simple example of Anger logic positioning circuit; where $V_i$ is the output voltage of the $i$th photomultiplier tube; $Z_i$s are resistors/capacitors; $V_{out}$ is the output voltage which should be proportional to the position of scintillation

linearity degrades a lot. To improve the spatial and linearity of Anger logic positioning, calibration has to be applied to gamma camera. However, a significant drawback of Anger logic is that no matter how fine the calibration is, severe non-linearity exists at the edges of scintillator [10]. In addition, Anger logic does not provide any depth of interaction (DoI) information; it determines only 2D position of scintillation. Nevertheless, Anger logic is still robust in many applications nowadays, and inspires engineers seeking new scintillation positioning method.

2.2 Concepts in gamma cameras

To evaluate the performance of a gamma camera, several concepts should be introduced:

Energy resolution and linearity

A gamma camera, though equipped with collimator, can be used as energy
spectrometer. In fact, most gamma cameras check the detected γ photon’s energy before applying its positioning algorithm. If the detected incoming γ photon has too low energy, it might be a Compton scattered photon, and thus should be ignored. If the detected incoming γ photon has too high energy, probably this is a coincident detection of two γ photons, and this detection should be ignored as well.

Figure 2.2A is a spectrum of a gamma detector with three γ sources (γ photon energies: 60 keV, 140 keV, and 511 keV). The spectrum is the number of γ photons detected of given energy (normalised value here), as a function of the output of all the light sensors (sum of detected optical photons). But an energy spectrum is supposed to be the number of detected γ photons of given energy, as a function of energy. To obtain the energy spectrum, Figure 2.2A need to be calibrated. The three abscissas of the peaks in Figure 2.2A is obtained. These abscissas are plotted against the three γ photon energies (60 keV, 140 keV, and 511 keV) and linearly fitted (like in Figure 2.2B). This process is called energy calibration. The goodness of the linear fitting (often the R^2 of fitting) is referred as the energy linearity. With this fitting, Figure 2.2A can be mapped into a curve of number of γ photon detection as a function of detected energies.

\[ ER = \frac{\Delta E_{0FWHM}}{E_0} \]
In a scintillator-based gamma detector, the detection of $\gamma$ photons involves two processes. The first one is scintillation which convert single $\gamma$ photon into a large number of optical photons. The second one is to convert optical signals into electric signals. Both of these two processes affect the energy resolution of the gamma detector. Their contribution to total $ER_{\text{tot}}$ can be denoted as:

$$ER_{\text{tot}} \simeq \sqrt{ER_{\text{scinti}}^2 + ER_{\text{conver.}}^2} \quad (2.4)$$

$ER_{\text{conver.}}$ is a quality of the light sensors and subsequent electronics, e.g. amplifiers, filters, analog-to-digital converters, et al.. $ER_{\text{scinti}}$, mainly depends on two factors: the intrinsic energy resolution of the scintillator ($ER_{\text{intrin.}}$), which is a intrinsic property of the crystal; and the statistical energy resolution ($ER_{\text{stat.}}$), which is determined by the total number of detectable photons $N_{\text{det.}}$. $N_{\text{det.}}$ is the product of the total scintillated number of photons in the crystal ($N_{\text{scinti.}}$) and the net photon detection efficiency ($\eta_{PD}$): $N_{\text{det.}} = N_{\text{scinti.}} \cdot \eta_{PD}$. Based on the fact that $N_{\text{det.}}$ in a specific gamma detector for a specific $\gamma$ photon energy is Poisson distributed, $ER_{\text{stat.}}$ can be expressed as:

$$ER_{\text{stat.}} \simeq \frac{2.35}{\sqrt{N_{\text{det.}}}} \quad (2.5)$$

and the total energy resolution of the detector would be:

$$ER_{\text{tot.}} \simeq \sqrt{ER_{\text{intrin.}}^2 + ER_{\text{stat.}}^2 + ER_{\text{conver.}}^2} \simeq \sqrt{ER_{\text{intrin.}}^2 + \frac{5.52}{N_{\text{det.}}} + ER_{\text{conver.}}^2} \quad (2.6)$$

From (2.6), it is clear that the more detectable scintillated photons there are, the better the energy resolution would be. In order to make more scintillated photons detectable, light sensors of high $\eta_{PD}$ is preferred. Moreover, crystals with high light yield are preferred to be used as scintillator. Furthermore, the edges of the scintillator in the gamma detector could be coated with reflective material (white edge), so that more scintillated photons travelling to the edges can be reflected and detected by the light sensors. On the contrary, if the edges of scintillator are coated with absorbing material (black edges), the energy resolution would be lowered. Additionally, when the scintillation is closed to the black edges, where more scintillated optical photons will be absorbed by the edges, the energy resolution is lower than scintillations occur in the centre of the crystal. Thus the energy resolution is not consistent in gamma detectors with black coated scintillators. However, if the scintillator is with white edges, a large fraction of the detected optical photons from a periphery scintillation position on the light sensors will be reflected photons. Reflection inside the scintillator will cause big problem in positioning linearity with Anger logic method.
In practice, the energy threshold and uphold of the $\gamma$ photons are determined by the energy resolution of the $\gamma$ energy to be detected and the energy linearity in the gamma detector.

**Spatial resolution and linearity**

As an imaging device, the spatial resolution and linearity might be the most important properties of a gamma camera. Figure 2.3A is a scanning of pencil beam 140 keV $\gamma$ photons from the left side of a gamma detector to the right side. The edges of the scintillator are treated white. The positioning is calculated through Anger logic. It is clear that the positioning is highly nonlinear at the periphery. Figure 2.3B shows the estimated scintillation positions (by Anger logic) against the true scintillation positions. Spatial linearity describes the difference between the mean estimated position and the true scintillation.

Spatial resolution describes the minimum distance between two infinitely small point that an imaging system can distinguish. In Figure 2.3B, the error bars in the linear area denote the spatial resolution. The spatial resolution of a gamma camera depends on positioning algorithm, the light sensors, and the collimator. It can be written as

$$SR_{tot.} \simeq \sqrt{SR_{algo.}^2 + SR_{sen.}^2 + SR_{colli.}^2} \tag{2.7}$$

For each scanned position in the linear area in Figure 2.3A, the spatial resolution is calculated as the FWHM of the profile of the spot, i.e. point spread function (PSF). For the DoI estimation, the spatial resolution and linearity in the depth direction are calculated in the same way.

In breast imaging, it is important to detect the tumours in the part of breast closed to the patient’s chest. Thus the gamma camera has to be placed closed to the chest (like in Figure 1.2). However, if the periphery of the scintillator is as nonlinear as in Figure 2.3, it is still difficult to obtain near-chest images. Therefore, the new scintillation positioning method should not only have DoI estimation, but also higher positioning linearity at the edges of the scintillator.

**Noise tolerance**

Gamma detectors, as electronic devices, contain noises from many different sources. Sensors, amplifiers, filters, and many other components in electronic circuits may contribute to noise in the electronic system. But among them, the noise from the light sensors is the most significant one in gamma detectors. The most common way of describing noise level is signal-to-noise ratio (SNR), which is the total detectable photon counts in the region-of-interest (RoI) on the light sensor array from scintillation divided by the total random counts in the same region.

$$\text{SNR}_0 = \frac{\sum_{m \in \text{RoI}} n_{m,\text{photon}}}{\sum_{m \in \text{RoI}} n_{m,\text{noise}}} \tag{2.8}$$
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\[ \text{(A) Estimated positions} \]

\[ \text{(B) Estimated positions as a function of true position} \]

\[ \text{(C) Estimation deviation as a function of true position} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure23.png}
\caption{The assessment of XY-plane positioning estimation by Anger logic. A $\gamma$ source emitting photons perpendicular to the gamma detector was scanned from left to right at $y = 13$ mm with $\Delta x = 2.15$ mm.}
\end{figure}

\[ \text{(2.8) gives the SNR for one recorded event. SNR for a whole measurement in time } T \text{ would be:} \]

\[ \text{SNR} = \frac{\int_0^T \sum_{m \in \text{RoI}} n_{m,\text{photon}}(t) \, dt}{\int_0^T \sum_{m \in \text{RoI}} n_{m,\text{noise}}(t) \, dt} \quad (2.9) \]

Light sensors with high SNR are preferred in application. PMTs often have higher SNR than SiPM. But SiPM can be manufactured much smaller than
most PMTs, and low noise semiconductor technology is ever developing. Therefore, SiPMs are of great interests in many applications today.

Gamma cameras are aimed to be manufactured with high spatial linearity, high spatial resolution, high energy linearity, high energy resolution, and high SNR. But sometimes there are trade-offs between some properties. For example, white edges enhances energy resolution of the gamma detector, but deteriorate the spatial linearity when positioning with Anger logic. This trade-off also calls for better scintillation positioning algorithms which can realise high positioning linearity in white edge scintillator.

2.3 Current solutions for DoI estimation

Nowadays, there are many positioning methods with DoI estimation, especially in PET. A very straightforward method is to design multilayer detector. S. Moehrs, et al. proposed a detector of three-layer scintillator and three-layer SiPM sensors [11]; with this design, scintillation position can be categorised into three depths; however, it is difficult to manufacture three layers of detectors. An alternative is to manufacture three layers of different scintillators and only one layer of PMTs. J. Seidel, et al. used three layers of scintillators of different materials [12]; by distinguishing the signal shapes from different scintillator materials, DoI can be also categorised into three clusters; this is much easier in electronic design, but the signals from different scintillators are not that easy to distinguish and the DoI resolution is still low. F. Pennazio, et al. proposed a scintillator double readout method to determine DoI [13]; instead of having only one SiPM array, they used two SiPM arrays on two sides of the scintillator, and estimated the DoI by comparing the size of light spread on the two SiPM arrays; however, the size of light spread is not easy to estimate since signals from SiPMs are often noisy.

The methods mentioned above require no prior knowledge of the light distribution inside the scintillator. The advantage is that the positioning procedure is simple and fast. But the shortcoming is that the DoI resolution and linearity of positioning is limited at the edges. Many other methods estimate DoI with prior knowledge of the light distribution from known positions in the scintillator. The basic procedure of all these methods is: 1) obtain the response of the sensors from a scintillation event; 2) compare this response with the responses from a set of known scintillation positions in the scintillator; 3) assign the scintillation position of this event to the known position which has most similar response with the measured event. Zhi Li, et al. built an analytical model of monolithic scintillator and used nonlinear least-square optimisation methods to find out the most probable scintillation position [14]; the best part of this model is that it included light spread considering reflection, refraction, and absorption in the scintillator; but the possible diffusive light reflection is not considered. D. Clément, et al. used artificial neural network to estimate scintillation positions; this neural network should be trained
with known irradiation positions \[15\]; this work was further studied by P. Bruyn- donckx, \textit{et al.} \[16\]; but it would take long time to train the neural network before it was possible to apply in practice and provided better results in XY-plane spatial resolution than conventional Anger logic. M. Korevaar, \textit{et al.} borrowed the multiscale algorithm commonly used in computer vision to acquire DoI \[17\]; in their work DoI is related to the scale concept in an analytical model; they also used a charge-coupled device (CCD) sensor array in optical photon measurement; this method could achieve very high spatial and DoI resolution; but CCD sensors were slow to read out, and costly to manufacture in large size. H. van Dam, \textit{et al.} computed Euclidean distance between the measured light distribution and the light distribution from known positions \[18\]; the known position of the smallest difference was considered the scintillation position; But the detector model in the simulation in this study was not realistic enough, so that the result was not ideal. H. Barrett and W. C. Hunter, \textit{et al.} proposed to use the maximum likelihood method to estimate DoI \[19\]–\[21\]; light distributions were considered to be Poisson distributed, and thus the likelihood between the measured light distribution and that from known positions could be calculated in a Poisson function; the known position that maximise the likelihood was regarded as the scintillation position; however, the results of H. Barrett, \textit{et al.} focused on gamma cameras with PMTs as readout; gamma cameras with SiPMs were not discussed.

### 2.4 Current treatment for positioning at edges

Traditional Anger camera has significant nonlinearity at the edges of the scintillator. To avoid nonlinearing positioning, the periphery of traditional Anger camera are not used in imaging. But this approach does not solve the problem. One way to make the edges of the gamma camera usable is to use pixellated scintillator instead of monolithic scintillator; the scintillated light from every scintillation will be reflected multiple times inside one scintillator pixel, and detected by only one or a few attached light sensors; however, the crystals used in scintillators cannot be manufactured in very small size, and thus makes the scintillator pixel size a limiting factor of the spatial resolution of the gamma camera; moreover, DoI estimation will be very difficult to apply in pixellated scintillator. Another possible solution is to locate the scintillation with prior knowledge of the light distribution inside the scintillator, as is described in section 2.3. But in most of the gamma cameras mentioned above, the edges of scintillators should be cover with black reflectors to avoid reflected light from the edges degrading the positioning linearity. This approach, on the other hand, degrades the energy resolution at the edge a lot. Because of this trade-off between positioning linearity and energy resolution, black edges plus prior knowledge of light distribution could not achieve high spatial resolution at the edge.
2.5 Proposed gamma detector configuration

The aim of this thesis is to introduce a scintillation positioning method which has DoI estimation and better positioning linearity at the edges of scintillator. The positioning method includes a new gamma detector configuration and new positioning algorithms. In order to obtain DoI estimation, the proposed gamma detector is designed with SiPM double readout, i.e. both the top and bottom of the scintillator are attached to SiPM arrays. In order to obtain high energy resolution, white reflectors around the scintillator should be coated. White edges could achieve consistent energy resolution, but the reflection will also degrade the spatial linearity. To overcome the reflection issue caused by white edges, a positioning algorithm different from Anger logic should be implemented.
Chapter 3

Methods

3.1 Gamma detector in simulation

GATE (GEANT4 Application for Tomographic Emission) is a Monte Carlo simulation toolkit for nuclear medicine simulation [22], [23]. In order to investigate the proposed gamma detector model, a gamma detector with SiPM sensors was set up in GATE, and scintillations from different known positions inside the monolithic scintillator were simulated. All the optical photons generated from scintillations were tracked and their distributions on SiPM sensors were analysed.

In GATE, a scintillator of LaBr$_3$:Ce crystal with dimension $43.4 \times 43.4 \times 8$ mm$^3$ was created. The physical properties of the crystal was taken from literature [24]–[27]. On both sides of the crystal, two SiPM arrays were placed symmetrically. Each of these two SiPM arrays had $9 \times 9$ quads with dimension $3.96 \times 4.44$ mm$^2$, and each quad contained $2 \times 2$ pixels with dimension $1.95 \times 2.2$ mm$^2$. The physical properties of the SiPMs were defined according to STMicroelectronics’ 4SPM20H6-60P SiPM light sensors [28]. A real SiPM array as described had been assembled and tested by ATOMKI, Debrecen, Hungary [29]. The thickness of the SiPM array was assumed to be $1.2$ mm thick silicon plates in simulation, which was $0.5$ mm thicker than the thickness of a single SiPM considering that supplementary electronics might be mounted [28]. Between the scintillator and each of these two SiPM arrays, an optical guide of $1.8$ mm quartz glass and $0.2$ mm epoxy was inserted. The edges of the scintillator were covered by reflectors. In the experiments with white reflectors, these reflectors were set to diffusive Teflon of reflective efficiency 0.98, so that most optical photons originated from scintillation could be detected by the SiPMs.

In order to compare this configuration with conventional design, a black-edge gamma detector and a single readout detector were also created in the simulation. In black reflector detector, the reflective efficiency of the edges were set to 0.05 reflective efficiency. In single readout simulations, the front SiPM array was changed to a mirror reflector with 0.98 reflective efficiency. Figure 3.1 is a schematic of the detector; Figure 3.2 shows the 3D morphology of the gamma detector in GATE simulation.
CHAPTER 3. METHODS

Table 3.1 lists the relevant physical properties of the materials assigned in simulation. More detailed description of settings in simulation is presented in Appendix A.

3.2 Library-based positioning

The idea of library-based positioning methods is that positioning is based on the prior knowledge of how the response of the light sensors would be when scintillations occur at different pre-known positions inside the scintillator. These reference responses are stored in a library. When a new scintillation event is detected, its response will be compared with references in the library. For each reference, its similarity with the detected response is computed. The scintillation is considered to occur at the reference position with the highest similarity.

Based on the idea above, the structure of the library and the definition of similarity should be explained in a library based method.

The interval between two neighbour reference positions in the library should be as small as possible. But the pixel size of the SiPMs was $1.95 \times 2.2 \text{mm}^2$, so that it was pointless to set the reference position interval orders smaller than this dimension. In addition, if the reference position interval is too small, the total number of references required to describe the whole scintillator would be too large, which, on the other hand, will slow down the positioning process, and make it impossible to apply the algorithm in real-time. Therefore, the interval between two neighbouring reference positions was chosen 1.6 mm in depth, and 1.2 mm in X and
3.3 SIMILARITY COMPARISON ALGORITHM

The similarity was defined as the reciprocal of the square-root distance (Euclidean distance) between two responses \([18]\). Assuming \(n_m\) photons are detected on the \(m\)th pixel of a sensor array. If this sensor array has 18 \(\times\) 18 pixels, the total response can be written as:

\[
N = [n_1, n_2, ..., n_{18\times18}] \tag{3.1}
\]

If a \(\gamma\) photon interacts with the scintillator at the position \(R = (x, y, z)\) and deposits energy \(E\) then the average number of photons detected by \(m\)th pixel can be written as \(\bar{n}_m(R, E)\). For a sensor array of 18 \(\times\) 18 pixels, the average response on this sensor array is:

\[
\bar{N}(R, E) = [\bar{n}_1, \bar{n}_2, ..., \bar{n}_{18\times18}] \tag{3.2}
\]

\(\bar{N}(R, E)\) was regarded as a reference in the library. It is a function of scintillation position and energy. The Euclidean distance between the measured response \(N\), and
### Table 3.1 List of most significant properties in GATE simulation

<table>
<thead>
<tr>
<th>component</th>
<th>property</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaBr₃:Ce</td>
<td>photon yield</td>
<td>63 000 MeV⁻¹</td>
</tr>
<tr>
<td></td>
<td>emission peak</td>
<td>380 nm</td>
</tr>
<tr>
<td></td>
<td>refractive index @ emission peak</td>
<td>2.00</td>
</tr>
<tr>
<td>quartz</td>
<td>refractive index @ emission peak</td>
<td>1.56</td>
</tr>
<tr>
<td>epoxy</td>
<td>refractive index @ emission peak</td>
<td>1.53</td>
</tr>
<tr>
<td>diffusive reflector</td>
<td>micro-facet angle σ</td>
<td>6°</td>
</tr>
<tr>
<td>mirror reflector</td>
<td>micro-facet angle σ</td>
<td>0.1°</td>
</tr>
<tr>
<td>SiPM</td>
<td>detection efficiency @ emission peak</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>photoelectric effect</td>
<td>On</td>
</tr>
<tr>
<td></td>
<td>Compton scattering</td>
<td>On</td>
</tr>
<tr>
<td></td>
<td>electron ionisation</td>
<td>On</td>
</tr>
<tr>
<td></td>
<td>scintillation</td>
<td>On</td>
</tr>
<tr>
<td></td>
<td>optical absorption</td>
<td>On</td>
</tr>
<tr>
<td></td>
<td>optical boundary</td>
<td>On</td>
</tr>
<tr>
<td></td>
<td>optical Rayleigh scattering</td>
<td>On</td>
</tr>
</tbody>
</table>

The reference \( \bar{N}(\mathbf{R}, E) \) is:

\[
D(\mathbf{R}, E) = \sqrt{\sum_{m=1}^{18 \times 18} (n_m - \bar{n}_m(\mathbf{R}, E))^2}
\]  \( (3.3) \)

The scintillation position and energy can be estimated by minimising the distance

\[
(\hat{\mathbf{R}}, \hat{E}) = \arg \min_{\mathbf{R}, E} D(\mathbf{R}, E)
\]  \( (3.4) \)

As an alternative, the reciprocal of the distance was defined as similarity \( S(\mathbf{R}, E) \). Thus, the scintillation position and energy could be given by maximising the similarity:

\[
S(\mathbf{R}, E) = 1/D(\mathbf{R}, E)
\]  \( (3.5) \)

\[
(\hat{\mathbf{R}}, \hat{E}) = \arg \max_{\mathbf{R}, E} S(\mathbf{R}, E)
\]  \( (3.6) \)

### 3.4 Maximum likelihood algorithm

Library-based similarity comparison algorithm employed the simple reciprocal of Euclidean distance as the similarity of two light distributions. In this algorithm, if the number of photons detected on one pixel \( n_m \) is 1, for example, and on the same pixel in the library \( \bar{n}_m \) is 10, the similarity would be 1/9; if \( n_m = 101 \) and \( \bar{n}_m = 110 \) in the corresponding pixel in the library, the similarity would still be
1/9. However, from common sense, this definition of similarity might be debatable. The average number of photons detected on each pixel (\( \bar{n}_m \)) can be a factor that strongly affects the estimation of similarity. To include the consideration of \( \bar{n}_m \), maximum likelihood estimation was introduced. A comparison between similarity described in section 3.2 and likelihood to be introduced in this section is shown in Figure 3.3.

Maximum likelihood estimation was first vastly studied by R. Fisher in 1910s [30]. In 1976, R. Gray and A. Macovski used it in scintillation position estimation [31]. In 1993, D. Gagnon proposed that it can be a possible means in 3D positioning in scintillator [32]. In the past decade, H. Barrett, et al. vastly studied it and applied it in more general gamma detectors, and obtained very high resolution 3D positioning results from PMT and scintillator-based gamma detectors [19]–[21]. C. Lerche, et al. evaluated the usage of maximum likelihood estimation in SiPM arrays as light sensors [33], but no DoI information was provided.

Maximum likelihood method is based on an assumption that for a given scintillation position in the scintillator and a given deposited energy, the number of photons detected by a given light sensor is Poisson distributed. This Poisson model is quite easy to be understood, and is discussed in H. Barrett’s work in detail [19], [21]. If this basic assumption is accepted, then the probability of detecting \( n_m \) photons on \( m \)th pixel of the sensor array from a scintillation position \( \mathbf{R} = (x, y, z) \) with energy \( E \) deposited is:

\[
Pr(n_m|\mathbf{R}, E) = \frac{[\bar{n}_m(\mathbf{R}, E)]^{n_m}}{n_m!}e^{-\bar{n}_m(\mathbf{R}, E)}
\]  

(3.7)

where \( \bar{n}_m \) is the average number of photons detected at pixel \( m \) as a function of scintillation position \( \mathbf{R} \) and energy \( E \).
The product of the probabilities in all pixels from the same scintillation event is given by

\[ L_0(R, E) = \prod_{m=1}^{18 \times 18} \Pr(n_m|R, E) = \prod_{m=1}^{18 \times 18} \frac{[\bar{n}_m(R, E)]^{n_m}}{n_m!} e^{-\bar{n}_m(R, E)} \] (3.8)

This product is called the likelihood of this event. This likelihood is a function of scintillation position and energy. Thus, we may estimate the scintillation position and energy by maximising this likelihood expression. This is called maximum likelihood estimation.

\[ (\hat{R}, \hat{E}) = \arg \max_{R, E} L_0(R, E) \] (3.9)

If \( \bar{n}_m \) is big enough, the Poisson function can be approximated by a Gaussian function [19]. The mean and variance of this Gaussian function are both \( \bar{n}_m \): 

\[ \Pr(n_m|R, E) = \frac{1}{\sqrt{2\pi \bar{n}_m(R, E)}} e^{-\left(\frac{n_m - \bar{n}_m(R, E)}{\sqrt{2\bar{n}_m(R, E)}}\right)^2} \] (3.10)

The logarithm of the likelihood was be taken in order to speed up calculation

\[ L(R, E) = \ln(L_0(R, E)) = -\sum_{m=1}^{18 \times 18} \left[ \frac{1}{2} \ln(2\pi \bar{n}_m(R, E)) + \left(\frac{n_m - \bar{n}_m(R, E)}{\sqrt{2\bar{n}_m(R, E)}}\right)^2 \right] \] (3.11)

\[ (\hat{R}, \hat{E}) = \arg \max_{R, E} L(R, E) \] (3.12)

Maximum likelihood method is also a library-based method. The \( \bar{n}_m \)s are also the references in the library.

### 3.5 Positioning Interpolation

If the similarities/likelihoods between a measured response and all the reference responses are calculated, it is possible to assign the scintillation position as the reference position that maximise the similarity/likelihood. However, it requires a large library to achieve accurate estimation and relatively long time to calculate all the similarities/likelihoods. Inspired by Anger logic, higher spatial resolution was achieved by applying weighted average-based interpolation with the data.

First, the similarity matrix was summed up along Z direction (the same XY position and different depths). After that, a set of most similar XY positions were selected. The scintillation XY positions were estimated by calculating the weighted average of these positions in the data set. The weights would be the similarity sums along each XY position.

\[ \bar{x} = \frac{\sum_{m=1}^{M} \left[ \sum_{z_m=1}^{6} S(x_m, y_m, z_m) - \sum_{z_{M+1}=1}^{6} S(x_{M+1}, y_{M+1}, z_{M+1}) \right] k x_m}{\sum_{m=1}^{M} \left[ \sum_{z_m=1}^{6} S(x_m, y_m, z_m) - \sum_{z_{M+1}=1}^{6} S(x_{M+1}, y_{M+1}, z_{M+1}) \right] k} \] (3.13)
3.6. DATA PROCESSING ACCELERATION

\( \bar{y} = \frac{\sum_{m=1}^{M} \sum_{z_m=1}^{6} S(x_m, y_m, z_m) - \sum_{z_{M+1}=1}^{6} S(x_{M+1}, y_{M+1}, z_{M+1})}{\sum_{m=1}^{M} \sum_{z_m=1}^{6} S(x_m, y_m, z_m) - \sum_{z_{M+1}=1}^{6} S(x_{M+1}, y_{M+1}, z_{M+1})} k y_m \)  

(3.14)

where \( \sum_{z_m=1}^{6} S(x_m, y_m, z_m) \)s are the biggest similarity sums from the reference position set; \( M \) most similar XY positions are involved in the interpolation; the similarity sums from these \( M \) most similar positions are subtracted by the \( M+1 \)th most similar sum; the subtraction is used as weights in the averaging process; the interpolated position \( \bar{x} \) and \( \bar{y} \) are the \( k \)th order weighted average of the reference positions in the data set. In practice, \( M \) was often set to 16 or 4 while \( k \) was often set to 1 or 2.

DoIs were also calculated from the XY positions of largest similarities. Along Z dimension at all relevant XY positions, a weighted average of the depths was calculated.

\( \bar{z} = \frac{\sum_{m=1}^{M} \sum_{d=1}^{D} [S(x_m, y_m, z_d) - S(x_m, y_m, z_{D+1})] k z_d}{\sum_{m=1}^{M} \sum_{d=1}^{D} [S(x_m, y_m, z_d) - S(x_m, y_m, z_{D+1})] k} \)  

(3.15)

where \( S(x_m, y_m, z_d) \) was the \( d \)th largest similarity value at \( (x_m, y_m) \) column; \( D \) largest reference depth similarities are involved at each XY position; all the similarity values are subtracted by the \( D+1 \)th largest similarity values in the respective XY position; the interpolation is obtained by \( k \)th order weighted average of \( D \) reference depths at \( M \) XY positions. In practice, \( M \) was often set to 4, \( k \) was often set to 2 to 4, and \( D \) was often set to 3 to 5.

It is worth noting that likelihood \( L \) can replace the similarity \( S \) in equations (3.13), (3.14), and (3.15).

3.6 Data processing acceleration

Four measures were taken to accelerate positioning procedures without deteriorating the spatial resolution so much.

First, the energy information of the scintillation might be ignored. The similarity (3.5) or likelihood expression (3.11), which were a function of energy could be simplified by normalisation, i.e. replace \( n_m \) in measured light distribution with \( n_m/N \) in the whole sensor plane. This normalisation was also applied to the light distributions in the library. In this way, only one library of light distributions from different scintillation positions is required. A library of different deposited scintillation energies were not necessary. The dimension of the library could be reduced from 4D \((x, y, z, E)\) to 3D \((x, y, z)\), as well as the time of computation. The energy of each interaction was calculated separately by (3.16) instead.

\( \hat{E} \propto N = \sum_{m=1}^{18 \times 18} n_m \)  

(3.16)

where \( n_m \) is the number of photons detected at the \( m \)th SiPM pixel.
CHAPTER 3. METHODS

For each interaction, the energy was calculated, and thus, the energy spectrum of a series of interactions could be plotted. Compton scattering could be sorted out from photoelectric effects by adding an energy window to the energy spectrum. The second approach was to use the projections of the light distribution on X and Y axes as the response instead of the total 2D light distribution. In this way, one can reduce the size of each sensor array readout from $18 \times 18$ to $2 \times 18$. Hence the data processing time was shortened. Replacing total 2D distribution with projection may not lose the most significant characteristics of responses. It was also easier to realise the electronics if only the projections are read out. Furthermore, if maximum likelihood estimation was applied, the summed up photon numbers along X and Y directions would make it more reasonable to replace Poisson function with Gaussian function, since the projection $\sum_{m\in X \text{ or } Y \text{ projection}} \bar{n}_m$ was always bigger than each single pixel $\bar{n}_m$.

The third approach was to estimate the scintillation XY position roughly before computing the similarity or likelihood of measured response with all the references in the library. With a rough estimation of scintillation position, the number of reference positions that need to be compared with the measured response was reduced, and thus speeded up computation. This rough estimation was done by a modified Anger logic algorithm. Instead of using the whole sensor array response for the weighted average, only the pixel with the maximum number of photons detected and its neighbours were used in the weighted average. When this rough estimation was done, the search loop for the maximum similarity or likelihood was limited to the surrounding reference positions of the rough estimation, e.g. $\pm 3$ neighbouring reference positions. In this way, the iteration of the searching loop could be reduced from $37 \times 37 \times 6$ to $6 \times 6 \times 6$. If the neighbouring position is outside the scintillator, it should be discarded.

The last approach was to use only a subset of the light distribution on the sensor array to calculate similarities or likelihood. Usually the subset was chosen from the part of the sensor array most light distributed, e.g. $\pm 2$ neighbouring SiPMs of the roughly estimated scintillation XY position. The similarity or likelihood could be calculated by only this subset:

$$S(R, E) = \frac{1}{\sqrt{\sum_{m\in \text{subset}} (n_m - \bar{n}_m(R, E))^2}}$$ (3.17)

$$L(R, E) = -\sum_{m\in \text{subset}} \left[ \frac{1}{2} \ln (2\pi \bar{n}_m(R, E)) + \left( \frac{n_m - \bar{n}_m(R, E)}{\sqrt{2\bar{n}_m(R, E)}} \right)^2 \right]$$ (3.18)

This simplification might make the similarity and likelihood less accurate. But the computational efficiency was largely enhanced. If using $\pm 2$ neighbouring pixels as subset, the similarity or likelihood between every two responses on each sensor array will be computation of $4 \times 2$ pixels instead of $18 \times 2$. Additionally, the readout of only a subset of the largest projections could be automatically implemented in the time-over-threshold (ToT) readout method [29].
3.7 Library construction

The library of the reference scintillation responses for the gamma detector can be built by Monte Carlo simulation or analytical models. In a Monte Carlo simulation, as in experiments, it is not possible to fix the interaction position (including DoI and planar position) of $\gamma$ photons in scintillators. However, in simulations, there is another simple way to imitate scintillation process. The scintillated optical photons are isotropically emitted. Therefore, an optical source in the scintillator emitting light isotropically could be placed inside the scintillator. In this way, the distribution of light on sensors should be the same as the real scintillated light distribution. The library was built by moving an optical source along a grid of $1.2 \times 1.2 \times 1.6 \text{mm}^3$ in the scintillator and recording the light distribution at the light sensor plane. In a double readout gamma detector, scintillations at only half of a quarter of the reference positions in the scintillator volume needed to be simulated to obtain the library. The rest was obtained by symmetry. In a single readout gamma detector, a quarter of the scintillator volume needed to be simulated because of the asymmetry along the depth direction.

3.8 Evaluation of gamma imaging quality

For the proposed gamma detector the energy resolution and linearity were checked. For each of the two positioning algorithms, the spatial resolution and linearity were investigated. Moreover, the impact of noise to the properties above was also studied.

3.8.1 Energy resolution and linearity

As was discussed in section 2.2, the energy resolution and linearity were important in evaluation of a gamma detector. In this study, the energy linearity and resolution was assessed in the following procedure: 1) localise the energy peak positions in the given spectrum; 2) plot the peak energies ($\hat{E}_{\text{peak}}$) or total detected number of photons $N_{\text{peak}}$ as a function of true interaction energies ($E_{\text{true}}$) (Figure 2.2A); 3) fit the plotting as a linear function and get the slope and coefficient of determination ($R^2$) of the fitting (Figure 2.2B); 3) calibrate the energy spectrum according to the fit and calculate the energy resolution. $R^2$ is defined as in equation (3.19), which is always between 0 and 1. The closer this value is to 1, the more linear the gamma detector is. Therefore the energy linearity of the gamma detector was simplified into two parameters, the slope and $R^2$.

\[
R^2 = 1 - \frac{\sum_i (\hat{N}_i - N_i)^2}{\sum_i (N_i - \bar{N}_i)^2}
\]  

(3.19)

where $N_i$ is the total detected number of photons in $i$th interaction, $\hat{N}_i$ is the estimated detected number of photons according to the fitting, and $\bar{N}_i$ is the average of $N_i$. 
In section 4.1, the energy spectra of three γ photon energies ($E_{true}$s were 60 keV, 140 keV, and 511 keV) from different sensor readouts, different scintillator edge treatments, different scintillation positions were investigated and compared; energy resolutions of these three energies were calculated.

3.8.2 Spatial resolution and linearity

Referring to section 2.2, the XY-plane spatial linearity was measured in the following way: 1) obtain the PSF of the gamma detector with the γ source placed at one side of the scintillator closed to the edge; 2) move the γ source by a certain distance towards the other side of the scintillator and do the same measurement; 3) continue the translation and do the same measurement until the γ source reaches the other side of the scintillator (Figure 2.3A); 4) plot the measured XY positions as a function of the true positions (Figure 2.3B) and fit it linear with a linear function; the next process could be to use the slope and $R^2$ to describe the spatial linearity; however, the spatial linearity might be much better in the centre of the scintillator than at the edge; therefore, a better way to illustrate this issue would be 5) subtract the estimated XY positions with the true positions and plot mean estimation deviation as a function of true positions (Figure 2.3C); 6) calculate the root-mean-square error (RMSE) of mean estimation deviations (centre and edge should be calculated separately $RMSE_c$ and $RMSE_e$). To sum, the spatial linearity could be simplified into three parameters, the slope, $RMSE_c$, and $RMSE_e$. RMSE is defined:

$$RMSE_e = \sqrt{\frac{\sum_{m}^{M} [\hat{x}(m) - x(m)]^2}{M}}$$

(3.20)

where $\hat{x}(m)$ is estimated XY position; $x(m)$ is the true XY positions; $M$ is the total number of sampling.

The FWHMs of the PSFs, as was described in section 2.2, were used to describe XY-plane spatial resolution. They were measured for PSFs during the scanning, and the average of the FWHMs were regarded as the XY-plane spatial resolution. Figure 2.3 shows the assessment of XY-plane positioning by Anger logic. The scanning was from one side of the scintillator to the other side, but the response was different. Apparently, the nonlinearity of the estimation was significant. The parameters are listed in Table 4.4.

DoI estimation assessment was accomplished in almost the same way. For DoI linearity, the evaluation was simpler than that for XY-plane positioning linearity: 1) use the data set obtained in XY-plane scanning to plot the estimated DoIs as a function of true DoI; 2) fit it with a linear function and obtain the slope and $R^2$. The DoI linearity was parametrised in the slope and $R^2$. For DoI resolution, only the resolution at the depth of 2 mm was considered. As is in XY-plane spatial resolution, FWHM was used in DoI resolution evaluation.

In section 4.2 and section 4.3, the spatial resolution and linearity of using different positioning algorithms, different gamma detector setups, and different scintilla-
tion positions were investigated and compared. The energy of $\gamma$ photons was only set to 140 keV.

### 3.8.3 Noise level tolerance

In SiPMs, the noise can be simplified as random photon counts. For each SiPM, if the noise is white noise, these random photon counts follow Poisson distribution. The mean value of this Poisson distribution is referred as the mean firing rate \( \text{MFR} \) of the SiPM array. MFR has the unit of number-of-counts per SiPM-pixel. The noise level is correlated with the power voltage of the SiPM array, the temperature of the sensors, and the sampling frequency. For every single SiPM, their MFRs can also be different.

The noise level of SiPMs affects the precision of positioning. If the SiPMs contain very high level of noise (high MFR), signals would be buried in the noise, and thus, less useful information can be extracted from the signal. Therefore, measures should be taken to suppress the noise from SiPMs, and the noise tolerance of the gamma detector and positioning algorithms should be analysed. In the study, a global SiPM photon count threshold was set. If the photon count on a particular SiPM was lower than the threshold, this SiPM output would be discarded and set to zero. For SiPMs, of which the photon counts were higher than the threshold, they were subtracted by the threshold. The threshold value should be set according to the noise level. In this study, it was set to the same value as MFR. MFR = 5 is a noise level that can be achieved with ordinary readout technique in SiPMs \[34\]. Figure 3.4 displays the light distribution with noise level MFR = 5, and the effect of thresholding. The positioning results from different noise levels are also discussed in the next chapter.

$$n_{\text{record}} = \begin{cases} n_{\text{detect}} - n_{\text{thres}}, & \text{if } n_{\text{detect}} > n_{\text{thres}}. \\ 0, & \text{if } n_{\text{detect}} < n_{\text{thres}}. \end{cases}$$  \hspace{1cm} (3.21)

The most common way to describe noise, SNR was discussed in section 2.2. But it was not proper to use SNR in this context. Because completely the same SiPM arrays would give different SNRs in different detection situations, e.g. double readout and single readout. Therefore, SNR was not used to describe noise level in this report. However, to bridge the straightforward concept of SNR with mean firing rate (MFR)\[34\], Table 3.2 was generated to illustrate the relationship between the SNR for 140 keV $\gamma$ photon (white edge) and the MFR. The SNR after thresholding is also listed.
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Table 3.2 The relationship between MFR and SNR

<table>
<thead>
<tr>
<th>Readout type</th>
<th>MFR (#/px)</th>
<th>SNR</th>
<th>SNR_{thres.}</th>
</tr>
</thead>
<tbody>
<tr>
<td>double</td>
<td>5</td>
<td>2.98</td>
<td>8.51</td>
</tr>
<tr>
<td></td>
<td>10</td>
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<tr>
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<td></td>
<td>25</td>
<td>0.89</td>
<td>5.57</td>
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</tbody>
</table>
Chapter 4

Results

4.1 Energy resolution and linearity

In this section, the energy spectra of the gamma detector with different setups described in Table 4.1 were compared. In every setup, three $\gamma$ sources with energies: 60 keV, 140 keV, 511 keV were placed in front of the gamma detector. Energy resolution and linearity were calculated as was described in subsection 3.8.1. It is worth noting that the energy spectra in this section are all measured without calibration. But the energy resolutions displayed were calculated from the calibrated spectra, as was described in section 2.2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Edge treat.</th>
<th>Readout type</th>
<th>Scinti. position</th>
<th>MFR (#/px)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black &amp; white</td>
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<td>Double</td>
<td>Centre</td>
<td>5</td>
</tr>
<tr>
<td>Black &amp; white</td>
<td>✓</td>
<td>Double</td>
<td>Edge</td>
<td>5</td>
</tr>
<tr>
<td>Double &amp; single</td>
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<td>✓</td>
<td>Centre</td>
<td>5</td>
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<tr>
<td>Noise tolerance</td>
<td>White</td>
<td>Double</td>
<td>Centre</td>
<td>✓</td>
</tr>
</tbody>
</table>

Edge treatment: black and white

The energy spectra of gamma detectors with black edges and white edges are displayed in Figure 4.1. Interactions at the centre of scintillator and at the edges are also compared.

Detector readout: single and double

The energy spectra of double and single readout detectors are shown in Figure 4.2.

Noise tolerance

The energy spectra of the measurement with different noise levels on SiPMs
CHAPTER 4. RESULTS

Figure 4.1 Energy spectra of scintillator with black edges and white edges (double readout, MFR=5). 'white centre' means the white edge detector with scintillations at the centre of the crystal, etc.

Figure 4.2 Energy spectra of double and single readout detectors (white edges, MFR=5) are shown in Figure 4.3.
4.2. **XY-PLANE SPATIAL RESOLUTION AND LINEARITY**

The energy linearity and resolutions were calculated and listed in **Table 4.2**.

**Table 4.2 Results of energy resolution and linearity measurement**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Measurement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Edge treat.</td>
<td>Readout</td>
</tr>
<tr>
<td>Black Single Centre</td>
<td>Black</td>
<td>Single</td>
</tr>
<tr>
<td>White Single Centre</td>
<td>White</td>
<td>Single</td>
</tr>
<tr>
<td>Black Double Centre</td>
<td>Black</td>
<td>Double</td>
</tr>
<tr>
<td>White Double Centre</td>
<td>White</td>
<td>Double</td>
</tr>
<tr>
<td>Black Double Edge</td>
<td>Black</td>
<td>Double</td>
</tr>
<tr>
<td>White Double Edge</td>
<td>White</td>
<td>Double</td>
</tr>
<tr>
<td>White Double Centre</td>
<td>White</td>
<td>Double</td>
</tr>
<tr>
<td>White Double Centre</td>
<td>White</td>
<td>Double</td>
</tr>
<tr>
<td>White Double Centre</td>
<td>White</td>
<td>Double</td>
</tr>
</tbody>
</table>

4.2 **XY-plane spatial resolution and linearity**

In this section, the results of the spatial resolution and linearity on XY-plane with different configurations (**Table 4.3**) are displayed. A scan of γ source from one side of the gamma detector to the other was done. XY-plane spatial resolution and linearity were acquired from the scanning and calculated according to subsection 3.8.2.

Comparing to **Figure 2.3** [Figure 4.4](#) is an illustration of the assessment of XY-plane positioning by similarity comparison algorithm. Unlike the results from Anger logic positioning, the linearity in [Figure 4.4B](#) was too good to evaluate. So the po-
Table 4.3 List of setup comparison in spatial resolution and linearity

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Algorithm</th>
<th>Readout type</th>
<th>Subset</th>
<th>MFR (#/px)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simi &amp; ML</td>
<td>✓</td>
<td>Double</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>Simi &amp; ML</td>
<td>✓</td>
<td>Single</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>Double &amp; single</td>
<td>Simi</td>
<td>✓</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>Double &amp; single</td>
<td>ML</td>
<td>✓</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>Subset performance</td>
<td>Simi</td>
<td>Double</td>
<td>✓</td>
<td>5</td>
</tr>
<tr>
<td>Subset performance</td>
<td>ML</td>
<td>Double</td>
<td>✓</td>
<td>5</td>
</tr>
<tr>
<td>Noise tolerance</td>
<td>Simi</td>
<td>Double</td>
<td>No</td>
<td>✓</td>
</tr>
<tr>
<td>Noise tolerance</td>
<td>Simi</td>
<td>Double</td>
<td>Yes</td>
<td>✓</td>
</tr>
</tbody>
</table>

Positioning deviation as a function of true positions was plotted (Figure 4.4C) and used in results comparison.

**Positioning algorithm**
The two positioning algorithms, similarity comparison and maximum likelihood, are compared in Figure 4.5. Figure 4.5A shows the results derived from double readout detector, and Figure 4.5B shows those from single readout detector.

**Detector readout: single and double**
The results from double readout and single readout detectors are displayed in Figure 4.6. Figure 4.6A shows the results from similarity comparison algorithm, and Figure 4.6B shows those from maximum likelihood algorithm.

**Performance of projection subset**
The influence of projection subset readout is shown in Figure 4.7. Figure 4.7A compares the full projection readout (with 18 channels) and projection subset readout (with only 4 channels with maximum output signal) with similarity comparison positioning algorithm; Figure 4.7B compares those with maximum likelihood positioning algorithm.

**Noise tolerance**
Figure 4.8 compares the results from different noise levels. Figure 4.8A shows the results from four noise levels with full projection readout, and Figure 4.8B displays those from projection subset readout.
4.2. **XY-PLANE SPATIAL RESOLUTION AND LINEARITY**

![XY-Plane Spatial Resolution and Linearity Illustration](image)

**Figure 4.4** Illustration of XY-plane positioning estimation

(A) Estimated positions

(B) Estimated positions as a function of true position

(C) Estimation deviation as a function of true position
CHAPTER 4. RESULTS

Figure 4.5 XY positioning with different positioning algorithm (MFR=5)

(A) With double readout
(B) With single readout

Figure 4.6 XY positioning with different readouts (MFR=5)

(A) With similarity comparison algorithm
(B) With maximum likelihood algorithm
4.2. XY-PLANE SPATIAL RESOLUTION AND LINEARITY

Figure 4.7 XY positioning with full projection readout and with subset readout (double readout detector, MFR=5)

(A) With similarity comparison algorithm  (B) With maximum likelihood algorithm

Figure 4.8 XY positioning with different noise levels (double readout detector, similarity comparison positioning algorithm)
The numerical XY-plane spatial resolution and linearity measurement results are displayed in Table 4.4.

Table 4.4 Results of XY-plane spatial resolution and linearity measurement

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Readout</th>
<th>Subset</th>
<th>MFR (#/px)</th>
<th>Configuration</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>SR FWHM (mm)</td>
<td>Slope RMSEc (mm)</td>
<td>RMSEe (mm)</td>
</tr>
<tr>
<td>Anger</td>
<td>Single</td>
<td>No</td>
<td>1.58</td>
<td>0.6840</td>
<td>2.20</td>
</tr>
<tr>
<td>Simi</td>
<td>Double</td>
<td>No</td>
<td>0.84</td>
<td>0.9939</td>
<td>0.39</td>
</tr>
<tr>
<td>ML</td>
<td>Double</td>
<td>No</td>
<td>0.95</td>
<td>0.9925</td>
<td>0.47</td>
</tr>
<tr>
<td>Simi</td>
<td>Single</td>
<td>No</td>
<td>1.40</td>
<td>0.9935</td>
<td>0.58</td>
</tr>
<tr>
<td>ML</td>
<td>Single</td>
<td>No</td>
<td>1.33</td>
<td>0.9926</td>
<td>0.61</td>
</tr>
<tr>
<td>Simi</td>
<td>Double</td>
<td>Yes</td>
<td>5</td>
<td>0.80</td>
<td>0.9944</td>
</tr>
<tr>
<td>ML</td>
<td>Double</td>
<td>Yes</td>
<td>5</td>
<td>0.97</td>
<td>0.9918</td>
</tr>
<tr>
<td>Simi</td>
<td>Double</td>
<td>No</td>
<td>0</td>
<td>0.61</td>
<td>0.9997</td>
</tr>
<tr>
<td>Simi</td>
<td>Double</td>
<td>No</td>
<td>10</td>
<td>1.02</td>
<td>0.9882</td>
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<tr>
<td>Simi</td>
<td>Double</td>
<td>No</td>
<td>25</td>
<td>1.51</td>
<td>0.9663</td>
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<tr>
<td>Simi</td>
<td>Double</td>
<td>Yes</td>
<td>0</td>
<td>0.63</td>
<td>1.0006</td>
</tr>
<tr>
<td>Simi</td>
<td>Double</td>
<td>Yes</td>
<td>10</td>
<td>0.96</td>
<td>0.9875</td>
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<tr>
<td>Simi</td>
<td>Double</td>
<td>Yes</td>
<td>25</td>
<td>1.40</td>
<td>0.9661</td>
</tr>
</tbody>
</table>

4.3 DoI resolution and linearity

In this section, the results of the DoI resolution and estimation linearity with different configurations (Table 4.3) are displayed. Since the DoI of every event is uncontrollable, only the DoI resolution at 2 mm depth is considered. The process to calculate the DoI resolution and linearity was described in subsection 3.8.2.

To assess DoI resolution and linearity, the estimated DoI as a function of true DoI was plotted and fitted with a linear function Figure 4.9A. But apparently it is not the best way in illustration. To show the DoI estimation in a better way, Figure 4.9A was digitised with pixel size $0.2 \times 0.2 \text{ mm}^2$. Figure 4.9C One vertical slice of Figure 4.9C at true DoI $2.0 \pm 0.1 \text{ mm}$, was extracted and fitted with Gaussian function Figure 4.9B. The FWHM of the fitting was considered the DoI resolution at this depth.

Positioning algorithm

Figure 4.10 shows the DoI estimation from the two positioning algorithms. Figure 4.10A is the DoI estimation with similarity comparison algorithm, and Figure 4.10B is the DoI estimation with maximum likelihood algorithm.

Detector readout: single and double

The DoI estimation results with single readout detector (Figure 4.11) are
4.3. DOI RESOLUTION AND LINEARITY

(A) DOI estimation vs. true DOI

(B) Fitting of DOI estimation @ true DOI is 2 mm

(C) DOI estimation vs. true DOI. The pixel size is 0.2 x 0.2 mm².

Figure 4.9 Illustration of DOI estimation

compared with those from double readout detector in Figure 4.10. Figure 4.11A displays the estimation with similarity comparison algorithm, and Figure 4.11B displays that with maximum likelihood algorithm.

Performance of projection subset

With projection subset of only 4 channels per projection as readout, the DOI estimation results are shown in Figure 4.12. Figure 4.12A displays the estimation with similarity comparison algorithm, and Figure 4.12B displays that
CHAPTER 4. RESULTS

Figure 4.10 DoI positioning with different algorithms (double readout detector, MFR=5)

(A) With similarity comparison algorithm
(B) With maximum likelihood algorithm

Figure 4.11 DoI positioning with single readout detectors (MFR=5)

(A) With similarity comparison algorithm
(B) With maximum likelihood algorithm

with maximum likelihood algorithm.

Noise tolerance
DoI estimation results with different sensor noise levels are compared in Figure 4.13 (with full projection readout) and Figure 4.14 (with projection subset readout of only 4 channels per projection).
(A) With similarity comparison algorithm  
(B) With maximum likelihood algorithm

**Figure 4.12** DOI positioning with projection subset readout (double readout detector, MFR=5)
CHAPTER 4. RESULTS

Figure 4.13 DoI positioning with double readout detector, and similarity comparison algorithm
4.3. DOI RESOLUTION AND LINEARITY

Figure 4.14 DOI positioning with projection subset readout (double readout detector, similarity comparison algorithm)
The numerical DoI resolution and linearity measurement results are displayed in Table 4.5.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Measurement</th>
<th>DoI.R_{FWHM}</th>
<th>Slope</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>Readout</td>
<td>Subset</td>
<td>MFR</td>
<td>@2mm</td>
</tr>
<tr>
<td>Simi Double No</td>
<td>No 5</td>
<td>1.07</td>
<td>0.9014</td>
<td>0.43</td>
</tr>
<tr>
<td>Simi Single No</td>
<td>5</td>
<td>1.25</td>
<td>0.8621</td>
<td>0.41</td>
</tr>
<tr>
<td>Simi Single Yes</td>
<td>Yes 5</td>
<td>2.66</td>
<td>0.8069</td>
<td>1.07</td>
</tr>
<tr>
<td>Simi Double Yes</td>
<td>Yes 5</td>
<td>2.86</td>
<td>0.7621</td>
<td>1.03</td>
</tr>
<tr>
<td>Simi Double No</td>
<td>No 0</td>
<td>1.19</td>
<td>0.9250</td>
<td>0.51</td>
</tr>
<tr>
<td>Simi Double No</td>
<td>No 10</td>
<td>1.19</td>
<td>0.8239</td>
<td>0.45</td>
</tr>
<tr>
<td>Simi Double No</td>
<td>No 25</td>
<td>1.30</td>
<td>0.6697</td>
<td>0.61</td>
</tr>
<tr>
<td>Simi Double Yes</td>
<td>Yes 0</td>
<td>0.77</td>
<td>0.9739</td>
<td>0.40</td>
</tr>
<tr>
<td>Simi Double Yes</td>
<td>Yes 10</td>
<td>1.57</td>
<td>0.8378</td>
<td>0.56</td>
</tr>
<tr>
<td>Simi Double Yes</td>
<td>Yes 25</td>
<td>1.55</td>
<td>0.6538</td>
<td>0.62</td>
</tr>
</tbody>
</table>
Chapter 5

Discussions and Conclusion

In this chapter, the results listed in Chapter 4 are discussed and the conclusions for different aspects of the study are given.

5.1 Library-based positioning algorithm and non-library-based algorithm

The positioning algorithms implemented in the study, similarity comparison and maximum likelihood, were both library based positioning algorithms. Comparing with non-library-based positioning algorithms, like Anger logic results in Figure 2.3, library-based algorithms provided better positioning linearity. One possible way to improve Anger logic’s linearity is to apply pixellated scintillator. However, with pixellated scintillator, DoI information would be very difficult to obtain. The best thing that high linearity could bring was that the linear region of the scintillator was significantly increased. There was only less than 2 mm nonlinear area within the edges. Therefore, in a breast-specific gamma camera, it should be possible to image the tumours close to the chest.

But the disadvantages of library-based positioning algorithms are that they are often difficult in implementation and slow in computation. Anger logic can be implemented directly by resistor/capacitor network, while library-based algorithms usually require processors to compute and memories to store the library. Moreover, how to search for the reference scintillation position with the maximum similarity or likelihood would be rather slow if the library is big. Comparing the two positioning algorithms, similarity comparison algorithm was relatively easier in implementation and faster in computation.

Another significant problem with library-based algorithms is how to obtain the library. Usually 2D libraries can be obtained through measurements. But this is not the case in 3D library measurement. DoI is not possible to be obtained precisely in real measurement. One common way to obtain DoI is to use well-collimated $\gamma$ source to irradiate the scintillator in a certain slant angle; the DoI could be calculated from the XY position of the scintillation and the slant angle [21]; however,
the precise XY positions of scintillations are difficult to obtain by Anger logic or any other non-library-based methods, especially at the edges where positioning is non-linear. In this study, the library is obtained by simulating source emitting optical photons homogeneously inside the scintillator. However, 3D calibration for the simulated scintillation should be applied, which is also difficult. Furthermore, an analytical model of light distribution considering scintillation, refraction, reflection, and scattering might be built to generate the library [14]. But currently, no model is available for the diffusive white edge scintillator. The precision of the library generated by analytical model is also based on the precision of the geometry setup. Calibration should be applied too if the model cannot completely define the geometry of the gamma detector.

5.2 Maximum likelihood algorithm and similarity comparison algorithm

In section 4.2, the results obtained from maximum likelihood estimation were mostly not as good as those from similarity comparison method. This is opposite to what was expected. However, this might be due to the implementation of interpolation. Figure 5.1 is a comparison between a typical similarity distribution and likelihood distribution in a local $6 \times 6$ reference XY positions. The XY position interpolation would be calculated by equation (3.13) and (3.14) with $M = 16$ and $k = 2$, so that the displayed similarity/likelihood distributions are already values subtracted by the $\sum_{z_{M+1}} S(x_{M+1}, y_{M+1}, z_{M+1})$ component. In likelihood distribution Figure 5.1B, the maximum four weights (likelihoods) are almost the same, and the weighted average position would be almost the average of the maximum four positions in this graph. On the contrary, the similarity distribution in Figure 5.1A contains a much more ‘prominent’ peak, which makes the weights at these reference positions differ from each other. The subtraction operation of $\sum_{z_{M+1}} S(x_{M+1}, y_{M+1}, z_{M+1})$ component as a thresholding process is a way to make the likelihood/similarity more peaked. If comparing the similarity and likelihood distributions without subtraction, it would be Figure 5.2. The fact that the likelihood distribution is ‘broader’ than similarity distribution would be more obvious.

The ‘shape’ of the likelihood distribution is independent of total number of scintillated photons if the likelihood is defined as equation (3.11). When the total scintillated photon number changes, one can assume $\bar{n}_m(R, E)$ and $n_m$ change simultaneously, and thus $\left(\frac{n_m - \bar{n}_m(R, E)}{\sqrt{2\bar{n}_m(R, E)}}\right)^2$ also changes with the same scale; for example, if $\bar{n}_m(R, E)$ and $n_m$ increase 10 times, $\left(\frac{n_m - \bar{n}_m(R, E)}{\sqrt{2\bar{n}_m(R, E)}}\right)^2$ will increase 10 times too. For $\sum_{m=1}^{18} \frac{1}{2} \ln (2\pi \bar{n}_m(R, E))$ in (3.11), it does not differ very much from one reference position to another nearby one; thus it only add a ‘shift’ to all likelihood values. In sum, if the total scintillated photon number changes, the likelihood value
5.2. MAXIMUM LIKELIHOOD ALGORITHM AND SIMILARITY COMPARISON ALGORITHM

Figure 5.1 A typical example of local similarity/likelihood distribution. The XY positions in the graph are the reference positions relative to the rough estimation position.

would have a scaling plus shifting change simultaneously, and the 'shape' of the distribution is preserved.

Therefore, the implementation of interpolation is the reason that maximum likelihood does not give as good positioning results as similarity comparison. But
if interpolation is not implemented, a library with much finer grid, e.g. voxel of $0.2 \times 0.2 \times 0.2 \text{mm}^3$, is needed. Thus the library will be much bigger and the positioning computation time will be largely increased.

For DoI estimation, maximum likelihood estimation gave more significant non-
linearity at the top and bottom of the scintillator, and lower resolution at 2 mm depth. This could also be attributed to the implementation of interpolation.

5.3 Double readout and single readout

In section 4.2, it was clear that double readout brought significant advantage in DoI estimation. Moreover, with the additional sensor array from the top of the crystal, XY position estimation and DoI estimation were improved in term of spatial resolution and linearity. This was partly due to the fact that for 140 keV energy $\gamma$ photons, the probability of interaction at the first three millimetre’s of scintillator at the top was much larger than that at the bottom five millimetre; so the light distribution at the front SiPM array was narrower than the back SiPM array, which was an advantage in both XY positioning and DoI estimation, even though double sensor array readout also introduced double noise.

For single readout, a problem in positioning was the reflected photons from the front reflector. The purpose of this mirror reflector at the front of the scintillator was to increase the energy resolution of the gamma detector by making more scintillated photons detectable, as well as to improve the XY positioning comparing with using black reflectors. However, the single readout showed much lower XY-plane spatial resolution than double readout. This phenomenon could be explained with the help of Figure 5.3. In Figure 5.3, the reflected photons from the top spread much broader than the forward projected photons; with limited number of scintillated photons, this reflected light distribution acted as a blurring factor and degraded the XY positioning.

However, when an additional SiPM array is inserted between the collimator and the scintillator, the collimator resolution is reduced. As is shown in (2.7), the overall spatial resolution of the gamma camera depends also on the collimator’s resolution $SR_{\text{colli}}$. If $\sqrt{SR_{\text{algo}}^2 + SR_{\text{det}}^2}$ is improved by this method while $SR_{\text{colli}}$ is undermined, the overall change of the spatial resolution $SR_{\text{tot}}$ is not certain. The spatial resolution of collimator at the top of the scintillator is calculated as:

$$SR_{\text{colli}} = \frac{d(h + l + t)}{l} = d + \frac{d(h + t)}{l}$$  \hspace{1cm} (5.1)
The variables $d$, $h$, $l$, $t$ are explained in Figure 5.4.

Figure 5.4 The trade-off between double sensor array readout and collimator resolution

With different collimators, the overall spatial resolutions of the gamma camera are listed in Table 5.1 and illustrated in Figure 5.5. From Table 5.1 and Figure 5.5, apparently the advantage of overall spatial resolution in double readout design no longer existed especially when collimator with high sensitivity is used. This is because the overall resolution would mostly be limited by the collimator resolution. However, it does not mean that double readout is meaningless. Double readout provides better positioning linearity and much more precise DoI information than single readout.

Table 5.1 The total spatial resolution of the gamma camera with different collimators; single readout detector with general collimator is compared with double readout detector with different collimators all the values have the unit millimetre (mm).

<table>
<thead>
<tr>
<th></th>
<th>$\sqrt{SR_{\text{algo.}}^2 + SR_{\text{det.}}^2}$</th>
<th>$d$</th>
<th>$h$</th>
<th>$l$</th>
<th>$t$</th>
<th>$SR_{\text{colli.}}$</th>
<th>$SR_{\text{tot.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single readout</td>
<td>1.4</td>
<td>1.5</td>
<td>15</td>
<td>25</td>
<td>1</td>
<td>2.46</td>
<td>2.83</td>
</tr>
<tr>
<td>High sensitivity</td>
<td>0.8</td>
<td>2.5</td>
<td>15</td>
<td>22</td>
<td>5</td>
<td>4.77</td>
<td>4.84</td>
</tr>
<tr>
<td>General</td>
<td>0.8</td>
<td>1.5</td>
<td>15</td>
<td>25</td>
<td>5</td>
<td>2.70</td>
<td>2.82</td>
</tr>
<tr>
<td>High resolution</td>
<td>0.8</td>
<td>1.3</td>
<td>15</td>
<td>30</td>
<td>5</td>
<td>1.67</td>
<td>1.85</td>
</tr>
</tbody>
</table>

5.4 White reflector and black reflector

From equation (2.6), it is clear that the more photons detected, the higher the energy resolution should be. From section 4.1 diffusive white reflectors, which let more scintillated photons be detected, provided higher energy resolution than black reflector. Besides, comparing the centre and peripheral scintillation positions, white reflector gave more consistent energy resolution. To obtain high energy resolution, diffusive white reflector should be the first choice. Only with consistent energy resolution and linearity, can one obtain high spatial resolution and linearity with a
The overall spatial resolution of double and single readouts with different collimator hole diameters; $h = 15\text{ mm}$, $l = 25\text{ mm}$, and $t = 5\text{ mm}$ for double readout

single energy threshold. Otherwise, the number of optical photons detected, which is proportional to the energy deposited, would always be smaller for scintillations at the edge of scintillator than that for scintillations in the centre. Significant less optical photons detected in one interaction would be regarded as a Compton scattering by a threshold, and discarded in the subsequent positioning procedure. If most scintillation events at the edge of scintillator was discarded in energy thresholding, the spatial linearity of the scintillator would be low.

It is worth noting that mirror white reflectors would not improve the energy resolution of the gamma detector as large as diffusive reflector. For scintillators with mirror reflectors at the edges (Figure 5.6A), considerable scintillated photons would be total reflected in the scintillator for many time before being absorbed eventually. But for diffusive reflectors, much more photons can be detected (Figure 5.6B).
5.5 Performance of the subset readout method

In section 4.2 and section 4.3, the positioning results of applying the full read-out channels and a subset of 4 readout channels per projection were displayed. Results from a double readout detector were compared. For both algorithm, the subset readout method did not degrade XY positioning resolution and linearity very much. This phenomenon could be understood as that the estimated position mainly depended on a few sensor array pixels where most photons distributed.

On the other hand, for DoI estimation, the positioning resolution and linearity were better for full readout channels. The light distribution over the whole sensor plane was more important in DoI determination than XY position determination. The SiPMs with only a few photons detected still played a vital role in DoI estimation; but in XY position estimation, these SiPMs were negligible. However, comparing with the complexity of the readout circuit and computation time needed for the full channel readout, the subset readout method was still a plausible choice in implementation with the advantage of fast.

Moreover, the subset readout method was less sensitive to noise, which was to be discussed in the next section.

5.6 Noise analysis

As expected, the higher the noise level, the worse the image quality would be. In Figure 4.3, it was clear that high noise level largely reduced the energy resolution. In addition, at the same MFR, single readout gamma detector had higher energy resolution than double readout. This was because that the total number of detectable photons remained the same in double readout while the noise was doubled by the additional SiPM array.

In Figure 4.8A and Table 4.4, high noise level reduced both the XY positioning resolution and linearity. The spatial resolution and linearity for MFR = 0 is the best result that this positioning method can achieve. It is worth noting that even at noise level as high as MFR = 25, the XY positioning linearity was still much higher than that with Anger logic and much lower noise level. The positioning results obtained with subset readout in Figure 4.8B is better than that with full readout. This could be explained as that the SNR in the subset projections with biggest signal magnitude was always higher than that in full projection readout.

In Figure 4.13 and Table 4.5, it was clear that high noise level degraded the DoI positioning resolution and linearity, especially the linearity. When the noise level was high, e.g. MFR = 25, the signal on the two sensor arrays was totally buried in Poisson distributed random counts; this random distribution was similar to the light distribution of scintillations occurred far away from the sensors; for a double readout detector, if the high-MFR Poisson noise spread over the two sensor arrays at the top and bottom of scintillator, the signal distributions on the two sensor arrays would be quite similar; thus, it would be likely that the algorithms
estimate the DoI to the middle of the scintillator (about 4 mm in depth), where the light distributions on the two sensor arrays were identical. This explained the phenomenon that the estimated DoI concentrated at the middle of the scintillator in Figure 4.13 and the slope of DoI fitting in Table 4.5 decreased as the noise level increased.

For DoI estimation with projection subset readout (Figure 4.14), the estimation linearity and resolution was even lower than that for full projection readout of the same noise level. The subset method brought little advantage in reducing noise’s impact on DoI estimation.

5.7 Validity of the simulation

GATE has been proved a powerful tool in Monte Carlo simulation in nuclear medicine [22]. The toolkit of optical simulation in GATE was also proved useful in many applications [23]. However, there were still some debatable physical configurations in the simulation. For example, the roughness of the reflectors may not be expressed as simple as the Gaussian distribution of micro-facade tilting angle as it was in the simulation; additionally, the noise on sensor arrays cannot be as ideal as white noise.

A vital problem was that the method of obtaining the library in the report was totally based on simulation. There is no rigorous way to calibrate the simulation-generated 3D library by experiment, which will be a difficulty in implementing this 3D positioning method in hardware.

5.8 Future investigation and application

The 3D positioning method introduced in this report was proved to be a promising method to obtain accurate 3D scintillation positions. Nevertheless, there are still a lot of investigations that can be done to improve current studies.

First of all, the gamma detector setup should be realised in hardware and real measurement should be taken to verify the simulation.

Second, a feasible calibration method is required to adjust the sensor array response from simulation to the hardware device.

Third, the gamma detector model built in the simulation was basically based on simple geometrical and physical laws. There is a possibility to come up with a mathematical model that involves all these relevant physics laws. With a mathematical model, the gamma detector and the scintillation process might be described in a simpler way, and thus, the library can be generated easier and faster.

Fourth, the noise analysed in this study was white noise with Poisson distribution in amplitude. However, white noise is just a mathematical model. Real noise on the SiPM arrays depends also on the amplitude of each SiPM, which is more complicated than the MFR noise level described here. Thus, the thresholding method may also
be adjusted to real noise. There is also possibility to estimate the noise distribution on the sensor arrays in the future.

Fifth, the 3D positioning method described in this report was specified for a breast-specific gamma camera. But 3D positioning is also vital in PET. Therefore, it might be interesting to adapt this 3D positioning method in PET.

Sixth, in current simulation, only point source of single energy was considered. However, in real breast imaging situation, the scattering γ photons from other organs like heart, liver, and thyroid would be interference factors. Therefore, there is a need to simulate the breast-specific gamma camera with phantoms of human body in order to test the positioning method in real gamma imaging situation.

5.9 Conclusion

In sum, the 3D scintillation positioning method introduced in the thesis report is a possible approach to improve the positioning in a scintillator-based gamma detector.

The white edge setup which made more scintillated photons detectable, succeeded in realising high energy resolution in the gamma detector. Moreover, the energy resolution was almost constant for different scintillation positions (centre or edge) in the crystal. Therefore, a single threshold on the energy could be set to remove most incident Compton scattered γ photons.

The double readout setup had the advantage of obtaining DoI information with high resolution and high linearity. The XY plane resolution and linearity was also higher in double readout than single readout. Even though the additional sensor array at the front of the scintillator would add to the complexity of the hardware, and degrade the collimator resolution of the entire imaging system, the overall spatial resolution of the gamma camera would still be improved if the collimator is well-designed, and the DoI information would be valuable.

The similarity comparison and maximum likelihood methods are powerful algorithms to get scintillation positions with high spatial resolution and linearity. At the same single readout setup, both these two library-based algorithms reduced the spatial nonlinearity (RMSE value) to less than 1/3 of the value in Anger logic. For double readout setup, the RMSEs of the library-based algorithms were about 0.7 of the single readout one, and the resolution was also higher. This improvement of linearity (and resolution) indicated that positioning is more precise. But the most important advantage of this improvement would be that the linear region in the scintillator could be significantly enlarged in the library-based positioning method. With the present gamma detector setup and interpolation method, similarity comparison algorithm was faster and provided more accurate results than maximum likelihood algorithm.

In cases with sufficient readout channels, the double readout detector with full readout projections and positioning with similarity comparison algorithm would be the combination to achieve the most desirable spatial resolution and linearity. If the number of readout channels is limited, or faster computation is preferred, the
subset readout method could be an alternative to achieve acceptable results.

In conclusion, the gamma detector with double readout and white edges surrounding the scintillator suggests a setup possible to achieve high resolution and linearity 3D positioning, which is promising technical feature in breast cancer diagnosis. The results also favours the similarity comparison algorithm applied to full channel readout comparing the maximum likelihood algorithm. However, since current study is simulation-based, further investigation is require to validate the current results.
Bibliography


Appendix A

Monte Carlo simulation scripts (GATE)

A.1 Main code

```plaintext
# VISUALIZATION
#/control/execute visual.mac

gate/geometry/setMaterialDatabase GateMaterials.db

# WORLD
/gate/world/geometry/setXLength 20. cm
/gate/world/geometry/setYLength 20. cm
/gate/world/geometry/setZLength 20. cm

#========================= # SYSTEM
/gate/world/daughters/name OpticalSystem
/gate/world/daughters/insert box
/gate/OpticalSystem/setMaterial Air
/gate/OpticalSystem/geometry/setXLength 15. cm
/gate/OpticalSystem/geometry/setYLength 15. cm
/gate/OpticalSystem/geometry/setZLength 8. cm
/gate/OpticalSystem/placement/setTranslation 0. 0. -1. cm
/gate/OpticalSystem/vis/setLineStyle dotted
/gate/OpticalSystem/vis/setColor red
/gate/OpticalSystem/vis/forceWireframe

# FRAME SCINTILLATOR
/gate/OpticalSystem/daughters/name frameS
/gate/OpticalSystem/daughters/insert box
/gate/frameS/setMaterial Lead
/gate/frameS/geometry/setXLength 4.34 cm
/gate/frameS/geometry/setYLength 0.33 cm
/gate/frameS/geometry/setZLength 0.8 cm
/gate/frameS/placement/setTranslation 0. 2.335 0.6 cm
/gate/frameS/vis/setLineStyle dotted
/gate/frameS/vis/setColor blue
/gate/frameS/vis/forceWireframe
```
# FRAME REPLICATION
/gate/frameS/repeaters/insert ring
/gate/frameS/ring/setRepeatNumber 4

# FRAME MATCHING
/gate/OpticalSystem/daughters/name frameM
/gate/OpticalSystem/daughters/insert box
/gate/frameM/setMaterial Lead
/gate/frameM/geometry/setXLength 4.6 cm
/gate/frameM/geometry/setYLength 0.2 cm
/gate/frameM/geometry/setZLength 0.2 cm
/gate/frameM/placement/setTranslation 0.2 4 0.1 cm
/gate/frameM/vis/setLineStyle dotted
/gate/frameM/vis/setColor blue
/gate/frameM/vis/forceWireframe

# FRAME REPLICATION
/gate/frameM/repeaters/insert ring
/gate/frameM/ring/setRepeatNumber 4
/gate/frameM/repeaters/insert linear
/gate/frameM/linear/setRepeatNumber 2
/gate/frameM/linear/setRepeatVector 0.0 10. mm
/gate/frameM/linear/autoCenter false

# SCINTILLATOR
/gate/OpticalSystem/daughters/name scintillator
/gate/OpticalSystem/daughters/insert box
/gate/scintillator/setMaterial LaBr3_Ce
/gate/scintillator/geometry/setXLength 4.34 cm
/gate/scintillator/geometry/setYLength 4.34 cm
/gate/scintillator/geometry/setZLength 0.8 cm
/gate/scintillator/placement/setTranslation 0.0 0.6 cm
/gate/scintillator/vis/setColor red
/gate/scintillator/vis/forceSolid

# MATCHING LAYER
/gate/OpticalSystem/daughters/name matching
/gate/OpticalSystem/daughters/insert box
/gate/matching/setMaterial Air
/gate/matching/geometry/setXLength 46 mm
/gate/matching/geometry/setYLength 46 mm
/gate/matching/geometry/setZLength 2. mm
/gate/matching/placement/setTranslation 0.0 1 mm
/gate/matching/vis/setColor white
/gate/matching/vis/forceWireframe

# EPOXY
/gate/matching/daughters/name epoxylayer
/gate/matching/daughters/insert box
/gate/epoxylayer/setMaterial Epoxy
/gate/epoxylayer/geometry/setXLength 46 mm
/gate/epoxylayer/geometry/setYLength 46 mm
/gate/epoxylayer/geometry/setZLength 0.2 mm
A.1. MAIN CODE

90 /gate/epoxylayer/placement/setTranslation 0 0 0.9 mm
91 /gate/epoxylayer/vis/setColor grey
92 /gate/epoxylayer/vis/forceSolid
93
94 # GLASS
95 /gate/matching/daughters/name glasslayer
96 /gate/matching/daughters/insert box
97 /gate/glasslayer/setMaterial Quartz
98 /gate/glasslayer/geometry/setXLength 46 mm
99 /gate/glasslayer/geometry/setYLength 46 mm
100 /gate/glasslayer/geometry/setZLength 1.8 mm
101 /gate/glasslayer/placement/setTranslation 0 0 -0.1 mm
102 /gate/glasslayer/vis/setColor white
103 /gate/glasslayer/vis/forceWireframe
104
105 # OPPOSITE MATCHING LAYER
106 /gate/matching/repeaters/insert ring
107 /gate/matching/ring/setRepeatNumber 2
108 /gate/matching/ring/setPoint1 0 1 6 mm
109 /gate/matching/ring/setPoint2 0 0 6 mm
110 /gate/matching/ring/enableAutoRotation
111
112 # ELECTRONICS
113 /gate/OpticalSystem/daughters/name electronics
114 /gate/OpticalSystem/daughters/insert box
115 /gate/electronics/setMaterial Air
116 /gate/electronics/geometry/setXLength 4.6 cm
117 /gate/electronics/geometry/setYLength 4.6 cm
118 /gate/electronics/geometry/setZLength 1.2 mm
119 /gate/electronics/placement/setTranslation 0 0 -0.6 mm
120 /gate/electronics/vis/setColor yellow
121 /gate/electronics/vis/forceWireframe
122
123 # MACRO-PIXELS
124 /gate/electronics/daughters/name macropixels
125 /gate/electronics/daughters/insert box
126 /gate/macropixels/setMaterial Air
127 /gate/macropixels/geometry/setXLength 4.6 mm
128 /gate/macropixels/geometry/setYLength 4.6 mm
129 /gate/macropixels/geometry/setZLength 1.2 mm
130 /gate/macropixels/placement/setTranslation 0 0 0 mm
131 /gate/macropixels/vis/setColor green
132 /gate/macropixels/vis/forceWireframe
133
134 # MICRO-PIXELS
135 /gate/macropixels/daughters/name micropixels
136 /gate/macropixels/daughters/insert box
137 /gate/micropixels/setMaterial Silicon
138 /gate/micropixels/geometry/setXLength 1.95 mm
139 /gate/micropixels/geometry/setYLength 2.22 mm
140 /gate/micropixels/geometry/setZLength 1.2 mm
141 /gate/micropixels/placement/setTranslation 0 0 0 mm
142 /gate/micropixels/vis/setColor yellow
143 /gate/micropixels/vis/forceWireframe
# Repeaters
/gate/micropixels/repeaters/insert cubicArray
/gate/micropixels/cubicArray/setRepeatNumberX 2
/gate/micropixels/cubicArray/setRepeatNumberY 2
/gate/micropixels/cubicArray/setRepeatNumberZ 1
/gate/micropixels/cubicArray/setRepeatVector -1.98 -2.22 0. mm
/
/gate/macropixels/repeaters/insert cubicArray
/gate/macropixels/cubicArray/setRepeatNumberX 9
/gate/macropixels/cubicArray/setRepeatNumberY 9
/gate/macropixels/cubicArray/setRepeatNumberZ 1
/gate/macropixels/cubicArray/setRepeatVector -5.1 -5.1 0. mm
/
# the other half of detector
/gate/electronics/repeaters/insert ring
/gate/electronics/ring/setRepeatNumber 2
/gate/electronics/ring/setPoint1 0 1 6 mm
/gate/electronics/ring/setPoint2 0 0 6 mm
/
# Attaching
/gate/glasslayer/attachCrystalSD
/gate/scintillator/attachCrystalSD
/gate/systems/OpticalSystem/crystal/attach electronics
/gate/systems/OpticalSystem/pixel/attach micropixels
#
# Physics
/gate/physics/addProcess PhotoElectric
/gate/physics/processes/PhotoElectric/setModel StandardModel
/gate/physics/addProcess Compton
/gate/physics/processes/Compton/setModel StandardModel
/gate/physics/addProcess RayleighScattering gamma
/gate/physics/processes/RayleighScattering/setModel PenelopeModel
/gate/physics/addProcess ElectronIonisation e-
/gate/physics/processes/ElectronIonisation/setModel StandardModel e-
/gate/physics/addProcess Bremsstrahlung e-
/gate/physics/processes/Bremsstrahlung/setModel StandardModel e-
/gate/physics/addProcess Scintillation
/gate/physics/addProcess OpticalAbsorption
/gate/physics/addProcess OpticalRayleigh
/gate/physics/addProcess OpticalBoundary
/
/gate/physics/processList Enabled
/gate/physics/processList Initialized
#
# Surface
# Detecting Surface
/gate/micropixels/surfaces/name glasslayer_micropixels
/gate/micropixels/surfaces/insert glasslayer
/gate/micropixels/surfaces/glasslayer_micropixels/setSurface perfect_apd
/gate/glasslayer/surfaces/name micropixels_glasslayer
A.1. MAIN CODE

198 /gate/glasslayer/surfaces/insert micropixels
199 /gate/glasslayer/surfaces/micropixels_glasslayer/setSurface perfect_apd
200
201 # REFLECTING SURFACE
202 /gate/frameM/surfaces/name matching_frameM
203 /gate/frameM/surfaces/insert matching
204 /gate/frameM/surfaces/matching_frameM/setSurface rough_teflon_wrapped
205 /gate/matching/surfaces/name frameM_matching
206 /gate/matching/surfaces/insert frameM
207 /gate/matching/surfaces/frameM_matching/setSurface rough_teflon_wrapped
208
209 /gate/frameS/surfaces/name scintillator_frameS
210 /gate/frameS/surfaces/insert scintillator
211 /gate/frameS/surfaces/scintillator_frameS/setSurface rough_teflon_wrapped
212 /gate/scintillator/surfaces/name frameS_scintillator
213 /gate/scintillator/surfaces/insert frameS
214 /gate/scintillator/surfaces/frameS_scintillator/setSurface rough_teflon_wrapped
215
216 # REFRACTING SURFACE
217 /gate/epoxylayer/surfaces/name glasslayer_epoxylayer
218 /gate/epoxylayer/surfaces/insert glasslayer
219 /gate/epoxylayer/surfaces/glasslayer_epoxylayer/setSurface smooth
220 /gate/glasslayer/surfaces/name epoxylayer_glasslayer
221 /gate/glasslayer/surfaces/insert epoxylayer
222 /gate/glasslayer/surfaces/epoxylayer_glasslayer/setSurface smooth
223
224 /gate/epoxylayer/surfaces/name scintillator_epoxylayer
225 /gate/epoxylayer/surfaces/insert scintillator
226 /gate/epoxylayer/surfaces/scintillator_epoxylayer/setSurface smooth
227 /gate/scintillator/surfaces/name epoxylayer_scintillator
228 /gate/scintillator/surfaces/insert epoxylayer
229 /gate/scintillator/surfaces/epoxylayer_scintillator/setSurface smooth
230
231 #---------------------------------------------------------------
232 # DIGITIZER
233 /gate/digitizer/Singles/insert opticaladder
234 /gate/digitizer/Singles/insert readout
235 /gate/digitizer/Singles/readout/setDepth 1
236
237 #---------------------------------------------------------------
238 # INITIALIZE
239 /gate/run/initialize
240
241 #---------------------------------------------------------------
242 # DEFINE THE SOURCE
243 #---------------------------------------------------------------
244 /gate/source/addSource pointsource
245 /gate/source/pointsource/gps/type Point
246 /gate/source/pointsource/gps/centre 1.4 -1.4 20 mm
247
248 /gate/source/pointsource/gps/particle gamma
A.1.1 Geometry setup with single readout

```plaintext
/* VISUALIZATION */
#control/execute visual.mac

/* WORLD */
gate/world/geometry/setXLength 20. cm
gate/world/geometry/setYLength 20. cm
gate/world/geometry/setZLength 20. cm

/* SYSTEM */
gate/world/daughters/name OpticalSystem
gate/world/daughters/insert box
gate/OpticalSystem/setMaterial Air
gate/OpticalSystem/geometry/setXLength 15. cm
gate/OpticalSystem/geometry/setYLength 15. cm
gate/OpticalSystem/geometry/setZLength 8. cm
gate/OpticalSystem/placement/setTranslation 0. 0. -1. cm
gate/OpticalSystem/vis/setLineStyle dotted
gate/OpticalSystem/vis/setColor red
gate/OpticalSystem/vis/forceWireframe
```
A.1. MAIN CODE

# FRAME SCINTILLATOR
/gate/OpticalSystem/daughters/name frameS
/gate/OpticalSystem/daughters/insert box
/gate/frameS/setMaterial Lead
/gate/frameS/geometry/setXLength 4.34 cm
/gate/frameS/geometry/setYLength 0.33 cm
/gate/frameS/geometry/setZLength 0.8 cm
/gate/frameS/placement/setTranslation 0. 2.335 0.6 cm
/gate/frameS/vis/setLineStyle dotted
/gate/frameS/vis/setColor blue
/gate/frameS/vis/forceWireframe

# FRAMES REpetition
/gate/frameS/repeaters/insert ring
/gate/frameS/ring/setRepeatNumber 4

# FRAME MATCHING
/gate/OpticalSystem/daughters/name frameM
/gate/OpticalSystem/daughters/insert box
/gate/frameM/setMaterial Lead
/gate/frameM/geometry/setXLength 4.6 cm
/gate/frameM/geometry/setYLength 0.2 cm
/gate/frameM/geometry/setZLength 0.2 cm
/gate/frameM/placement/setTranslation 0. 2.4 0.1 cm
/gate/frameM/vis/setLineStyle dotted
/gate/frameM/vis/setColor blue
/gate/frameM/vis/forceWireframe

# FRAMES REpetition
/gate/frameM/repeaters/insert ring
/gate/frameM/ring/setRepeatNumber 4
/gate/frameM/repeaters/insert linear

# SCINTILLATOR
/gate/OpticalSystem/daughters/name scintillator
/gate/OpticalSystem/daughters/insert box
/gate/scintillator/setMaterial LaBr3_Ce
/gate/scintillator/geometry/setXLength 4.34 cm
/gate/scintillator/geometry/setYLength 4.34 cm
/gate/scintillator/geometry/setZLength 0.8 cm
/gate/scintillator/placement/setTranslation 0. 0. 0.6 cm
/gate/scintillator/vis/setColor red
/gate/scintillator/vis/forceSolid

# MATCHING LAYER
/gate/OpticalSystem/daughters/name matching
/gate/OpticalSystem/daughters/insert box
/gate/matching/setMaterial Air
/gate/matching/geometry/setXLength 46 mm
/gate/matching/geometry/setYLength 46 mm
/gate/matching/geometry/setZLength 2. mm
APPENDIX A. MONTE CARLO SIMULATION SCRIPTS (GATE)

77 /gate/matching/placement/setTranslation 0. 0. 1 mm
78 /gate/matching/vis/setColor white
79 /gate/matching/vis/forceWireframe
80
81 # E P O X Y
82 /gate/matching/daughters/name epoxylayer
83 /gate/matching/daughters/insert box
84 /gate/epoxylayer/setMaterial Epoxy
85 /gate/epoxylayer/geometry/setXLength 46 mm
86 /gate/epoxylayer/geometry/setYLength 46 mm
87 /gate/epoxylayer/geometry/setZLength 0.2 mm
88 /gate/epoxylayer/placement/setTranslation 0 0 0.9 mm
89 /gate/epoxylayer/vis/setColor grey
90 /gate/epoxylayer/vis/forceSolid
91
92 # G L A S S
93 /gate/matching/daughters/name glasslayer
94 /gate/matching/daughters/insert box
95 /gate/glasslayer/setMaterial Quartz
96 /gate/glasslayer/geometry/setXLength 46 mm
97 /gate/glasslayer/geometry/setYLength 46 mm
98 /gate/glasslayer/geometry/setZLength 1.8 mm
99 /gate/glasslayer/placement/setTranslation 0 0 -0.1 mm
100 /gate/glasslayer/vis/setColor white
101 /gate/glasslayer/vis/forceWireframe
102
103 # E L E C T R O N I C S
104 /gate/OpticalSystem/daughters/name electronics
105 /gate/OpticalSystem/daughters/insert box
106 /gate/electronics/setMaterial Air
107 /gate/electronics/geometry/setXLength 4.6 cm
108 /gate/electronics/geometry/setYLength 4.6 cm
109 /gate/electronics/geometry/setZLength 1.2 mm
110 /gate/electronics/placement/setTranslation 0. 0. -0.6 mm
111 /gate/electronics/vis/setColor yellow
112 /gate/electronics/vis/forceWireframe
113
114 # M A C R O - P I X E L s
115 /gate/electronics/daughters/name macropixels
116 /gate/electronics/daughters/insert box
117 /gate/macropixels/setMaterial Air
118 /gate/macropixels/geometry/setXLength 4.6 mm
119 /gate/macropixels/geometry/setYLength 4.6 mm
120 /gate/macropixels/geometry/setZLength 1.2 mm
121 /gate/macropixels/placement/setTranslation 0. 0. 0. mm
122 /gate/macropixels/vis/setColor green
123 /gate/macropixels/vis/forceWireframe
124
125 # M I C R O - P I X E L s
126 /gate/macropixels/daughters/name micropixels
127 /gate/macropixels/daughters/insert box
128 /gate/micropixels/setMaterial Silicon
129 /gate/micropixels/geometry/setXLength 1.95 mm
A.1. MAIN CODE

111 /gate/micropixels/geometry/setYLength 2.22 mm
112 /gate/micropixels/geometry/setZLength 1.2 mm
113 /gate/micropixels/placement/setTranslation 0. 0. 0. mm
114 /gate/micropixels/vis/setColor yellow
115 /gate/micropixels/vis/forceWireframe
116
117 # REPEATERS
118 /gate/micropixels/repeaters/insert cubicArray
119 /gate/micropixels/cubicArray/setRepeatNumberX 2
120 /gate/micropixels/cubicArray/setRepeatNumberY 2
121 /gate/micropixels/cubicArray/setRepeatNumberZ 1
122 /gate/micropixels/cubicArray/setRepeatVector -1.98 -2.22 0. mm
123 #
124 /gate/macropixels/repeaters/insert cubicArray
125 /gate/macropixels/cubicArray/setRepeatNumberX 9
126 /gate/macropixels/cubicArray/setRepeatNumberY 9
127 /gate/macropixels/cubicArray/setRepeatNumberZ 1
128 /gate/macropixels/cubicArray/setRepeatVector -5.1 -5.1 0. mm
129 #

130 # FRONT REFLECTOR
131 /gate/OpticalSystem/daughters/name frontreflector
132 /gate/OpticalSystem/daughters/insert box
133 /gate/frontreflector/setMaterial PTFE
134 /gate/frontreflector/geometry/setXLength 46 mm
135 /gate/frontreflector/geometry/setYLength 46 mm
136 /gate/frontreflector/geometry/setZLength 1 mm
137 /gate/frontreflector/placement/setTranslation 0. 0. 10.5 mm
138 /gate/frontreflector/vis/setColor magenta
139 /gate/frontreflector/vis/forceSolid
140
141 # ATTACHING
142 /gate/glasslayer/attachCrystalSD
143 /gate/scintillator/attachCrystalSD
144 /gate/systems/OpticalSystem/crystal/attach electronics
145 /gate/systems/OpticalSystem/pixel/attach micropixels
146

147 #=====================================================================
148 # SURFACE
149 # DETECTING SURFACE
150 /gate/micropixels/surfaces/name glasslayer_micropixels
151 /gate/micropixels/surfaces/insert glasslayer
152 /gate/micropixels/surfaces/glasslayer_micropixels/setSurface perfect_apd
153 /gate/glasslayer/surfaces/name micropixels_glasslayer
154 /gate/glasslayer/surfaces/insert micropixels
155 /gate/glasslayer/surfaces/micropixels_glasslayer/setSurface perfect_apd
156
157 # REFLECTING SURFACE
158 /gate/frameS/surfaces/name matching_frameS
159 /gate/frameS/surfaces/insert matching
160 /gate/frameS/surfaces/matching_frameS/setSurface rough_teflon_wrapped
161 /gate/matching/surfaces/name frameS_matching
162 /gate/matching/surfaces/insert frameS
163 /gate/matching/surfaces/frameS_matching/setSurface rough_teflon_wrapped
A.2 Supplementary settings

Only modification of the default script is displayed

A.2.1 Materials' physical properties (GateMaterials.db)
A.2. SUPPLEMENTARY SETTINGS

LaBr₃_Ce: d=5.29g/cm³; n=3; state=Solid
+el: name=Lanthanum; f=0.348
+el: name=Bromine; f=0.602
+el: name=Cerium; f=0.05

A.2.2 Materials' optical properties (Materials.xml)

<?xml version="1.1"?>
<materials>
  <material name="Epoxy">
    <propertiestable>
      <propertyvector name="ABSLENGTH" unit="m" energyunit="eV">
        <ve energy="1.84" value="50"/>
        <ve energy="4.08" value="50"/>
      </propertyvector>
      <propertyvector name="RINDEX" energyunit="eV">
        <ve energy="1.84" value="1.532"/>
        <ve energy="4.08" value="1.532"/>
      </propertyvector>
    </propertiestable>
  </material>

  <material name="Quartz">
    <propertiestable>
      <propertyvector name="ABSLENGTH" unit="m" energyunit="eV">
        <ve energy="1.84" value="50"/>
        <ve energy="4.08" value="50"/>
      </propertyvector>
      <propertyvector name="RINDEX" energyunit="eV">
        <ve energy="1.55" value="1.538"/>
        <ve energy="2.07" value="1.544"/>
        <ve energy="2.49" value="1.549"/>
        <ve energy="2.76" value="1.552"/>
        <ve energy="3.11" value="1.558"/>
        <ve energy="3.55" value="1.566"/>
        <ve energy="4.14" value="1.578"/>
      </propertyvector>
    </propertiestable>
  </material>

  <material name="LaBr₃_Ce">
    <propertiestable>
      <property name="SCINTILLATIONYIELD" value="63000" unit="1/MeV"/>
      <property name="RESOLUTIONSCALE" value="1.8"/>
      <property name="FASTTIMECONSTANT" value="16" unit="ns"/>
      <property name="YIELDRATIO" value="1"/>
      <propertyvector name="FASTCOMPONENT" energyunit="eV">
        <ve energy="2.76" value="0"/>
        <ve energy="3.03" value="0.01"/>
        <ve energy="3.105" value="0.04"/>
        <ve energy="3.1846" value="0.14"/>
        <ve energy="3.27" value="0.19"/>
        <ve energy="3.3568" value="0.18"/>
        <ve energy="3.45" value="0.33"/>
      </propertyvector>
    </propertiestable>
  </material>
</materials>
APPENDIX A. MONTE CARLO SIMULATION SCRIPTS (GATE)

A.2.3 Surfaces’ treatment (Surfaces.xml)

<?xml version="1.0"?>
<surfaces>
  <surface name="polished_teflon_wrapped" type="dielectric_dielectric"
    sigmaalpha="0.1" finish="groundbackpainted">
    <propertytable>
      <propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
        <ve energy="1.84" value="0"/>
        <ve energy="4.08" value="0"/>
      </propertyvector>
      <propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
        <ve energy="1.84" value="0"/>
        <ve energy="4.08" value="0"/>
      </propertyvector>
      <propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
        <ve energy="1.84" value="0"/>
        <ve energy="4.08" value="0"/>
      </propertyvector>
      <propertyvector name="RINDEX" energyunit="eV">
        <ve energy="2.93" value="2"/>
        <ve energy="3.11" value="2"/>
        <ve energy="3.31" value="2"/>
        <ve energy="3.55" value="2"/>
      </propertyvector>
    </propertytable>
  </surface>
</surfaces>
A.2. SUPPLEMENTARY SETTINGS

<propertiestable>
  <propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
    <ve energy="1.84" value="1"/>
    <ve energy="4.08" value="1"/>
  </propertyvector>
  <propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
    <ve energy="1.84" value="0"/>
    <ve energy="4.08" value="0"/>
  </propertyvector>
  <propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
    <ve energy="1.84" value="0"/>
    <ve energy="4.08" value="0"/>
  </propertyvector>
  <propertyvector name="RINDEX" energyunit="eV">
    <ve energy="1.84" value="1"/>
    <ve energy="4.08" value="1"/>
  </propertyvector>
  <propertyvector name="REFLECTIVITY" energyunit="eV">
    <ve energy="1.84" value="0.98"/>
    <ve energy="4.08" value="0.98"/>
  </propertyvector>
  <propertyvector name="EFFICIENCY" energyunit="eV">
    <ve energy="1.84" value="0"/>
    <ve energy="4.08" value="0"/>
  </propertyvector>
</propertiestable>

<surface name="rough_teflon_wrapped" type="dielectric_dielectric" sigmaalpha="6.0" finish="groundbackpainted">
  <propertiestable>
    <propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
      <ve energy="1.84" value="1"/>
      <ve energy="4.08" value="1"/>
    </propertyvector>
    <propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
      <ve energy="1.84" value="0"/>
      <ve energy="4.08" value="0"/>
    </propertyvector>
    <propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
      <ve energy="1.84" value="0"/>
      <ve energy="4.08" value="0"/>
    </propertyvector>
    <propertyvector name="RINDEX" energyunit="eV">
      <ve energy="1.84" value="1"/>
      <ve energy="4.08" value="1"/>
    </propertyvector>
    <propertyvector name="REFLECTIVITY" energyunit="eV">
      <ve energy="1.84" value="0.98"/>
      <ve energy="4.08" value="0.98"/>
    </propertyvector>
    <propertyvector name="EFFICIENCY" energyunit="eV">
      <ve energy="1.84" value="0"/>
      <ve energy="4.08" value="0"/>
    </propertyvector>
  </propertiestable>
</surface>

<surface name="perfect_apd" type="dielectric_metal" sigmaalpha="0.0" finish="polished">
  <propertiestable>
    <propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
      <ve energy="1.0" value="0.0"/>
      <ve energy="2.34" value="0.0"/>
      <ve energy="4.13" value="0.0"/>
    </propertyvector>
    <propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
      <ve energy="1.0" value="0.0"/>
    </propertyvector>
  </propertiestable>
</surface>
<ve energy="2.34" value="0.0"/>
<ve energy="4.13" value="0.0"/>
</propertyvector>

<propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
<ve energy="1.0" value="0.0"/>
<ve energy="2.34" value="0.0"/>
<ve energy="4.13" value="0.0"/>
</propertyvector>

<propertyvector name="REFLECTIVITY" energyunit="eV">
<ve energy="1.0" value="0.0"/>
<ve energy="2.34" value="0.0"/>
<ve energy="4.13" value="0.0"/>
</propertyvector>

<propertyvector name="EFFICIENCY" energyunit="eV">
<ve energy="2.48" value="0.33"/>
<ve energy="2.75" value="0.34"/>
<ve energy="3.10" value="0.33"/>
<ve energy="3.30" value="0.31"/>
<ve energy="3.54" value="0.29"/>
<ve energy="3.81" value="0.25"/>
<ve energy="4.13" value="0.2"/>
</propertyvector>

</propertiestable>
</surface>

<surface name="smooth" type="dielectric_dielectric" sigmaalpha="0.1" finish="ground">
<propertiestable>
<propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
<ve energy="1.84" value="1"/>
<ve energy="4.08" value="1"/>
</propertyvector>

<propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>

<propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>
</propertiestable>
</surface>

<surface name="rough" type="dielectric_dielectric" sigmaalpha="6.0" finish="groundbackpainted">
<propertiestable>
<propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
<ve energy="1.84" value="1"/>
<ve energy="4.08" value="1"/>
</propertyvector>

<propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>

<propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>
</propertiestable>
</surface>

<surface name="smooth" type="dielectric_dielectric" sigmaalpha="0.1" finish="ground">
<propertiestable>
<propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
<ve energy="1.84" value="1"/>
<ve energy="4.08" value="1"/>
</propertyvector>

<propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>

<propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>
</propertiestable>
</surface>
A.2. SUPPLEMENTARY SETTINGS

<propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>

<propertyvector name="RINDEX" energyunit="eV">
<ve energy="1.84" value="1"/>
<ve energy="4.08" value="1"/>
</propertyvector>

<propertyvector name="REFLECTIVITY" energyunit="eV">
<ve energy="1.84" value="1"/>
<ve energy="4.08" value="1"/>
</propertyvector>

<propertyvector name="EFFICIENCY" energyunit="eV">
<ve energy="1.84" value="0"/>
<ve energy="4.08" value="0"/>
</propertyvector>

</propertiestable>

</surface>

<surface name="black" type="dielectric_dielectric" sigmaalpha="6.0" finish="groundfrontpainted">
<propertiestable>
<propertyvector name="SPECULARLOBECONSTANT" energyunit="eV">
<ve energy="1.97" value="1"/>
<ve energy="2.34" value="1"/>
</propertyvector>

<propertyvector name="SPECULARSPIKECONSTANT" energyunit="eV">
<ve energy="1.97" value="0"/>
<ve energy="2.34" value="0"/>
</propertyvector>

<propertyvector name="BACKSCATTERCONSTANT" energyunit="eV">
<ve energy="1.97" value="0"/>
<ve energy="2.34" value="0"/>
</propertyvector>

<propertyvector name="REFLECTIVITY" energyunit="eV">
<ve energy="1.97" value="0.05"/>
<ve energy="2.34" value="0.05"/>
</propertyvector>

<propertyvector name="EFFICIENCY" energyunit="eV">
<ve energy="1.97" value="0"/>
<ve energy="2.34" value="0"/>
</propertyvector>
</propertiestable>
</surface>
</surfaces>
Appendix B

Postprocessing scripts (MATLAB)

B.1 Reading GATE output files

```matlab
% preprocessing: extract scintillation data from rawdata & extract
detector
clear
fname0 = 'filenameaa';
fname = 'allhitsHits.dat';
scintiphons = 5; % approximate number of scintillated photons
scintiT = zeros(scintiphons,1); %timestamp of interactions
scinti = zeros(scintiphons, 3); %coordinates of interactions
nP = 0; % total number of photoelectric effect
optT = zeros(scintiphons*1700, 1); % timestamps of these optical photons
optposition = zeros(scintiphons*1700, 3); % positions of optical photons
nopt = 0; % total number of optical photons

%% to read the datafile
for nfile=0:9, %how many files to be read
    fname = char(fname0+[0 0 0 0 0 0 0 0 nfile]); % name of the next
    fid = fopen(fname);
    T = textscan(fid, '%*s %*s %*s %*s %*s %*s %*s %f %*s %*s %f %f %f
    %*s %*s %*s %*s %*s %*s %s %*s %*s');
    fclose(fid);
    interaction=T(5);
    timestamp=T(1);
    xposition=T(2);
    yposition=T(3);
    zposition=T(4);
    clear T;

    scintind = strcmp('PhotoElectric', interaction)==true; % index of
    scintillation
detection = strcmp('Transportation', interaction)==true; % detected
    optical photons
```

75
clear interaction;

nPE0 = sum(scintind); % total number of scinti in this subset
nopt0 = sum(detection); % total number of optical photons in this subset

scinti(nPE+1:nPE+nPE0, :)=[xposition(scintind) yposition(scintind)
zposition(scintind)]; % where scintillation happens
optposition(nopt+1:nopt+nopt0, :)=[xposition(detection) yposition(detection)
zposition(detection)]; % where photons detected

clear xposition yposition zposition

scintiT(nPE+1:nPE+nPE0)=timestamp(scintind); % time of each scintillation
optT(nopt+1:nopt+nopt0)=timestamp(detection); % timestamp of all photon transportations

clear timestamp

nPE=nPE+nPE0;
nopt=nopt+nopt0; % renew the counts

end

scinti = scinti(1:nPE, :);
optposition = optposition(1:nopt, :);
scintiT = scintiT(1:nPE);
optT = optT(1:nopt); % clear the redundant preassigned space

nn = nopt;
thrphoton = 1000; % threshold of photon number; remove obvious Compton
scintiresults=cell(2, ceil(nn/thrphoton)); % a cell to storage all proved light distribution

nscinti=0; % total number of scintillations recorded

while nn > thrphoton,
    nowtime=optT(1); % timestamp of the current first record
    eventn=(optT>nowtime-50e-9)&(optT<nowtime+150e-9); % consider all photons in this scintillation are detected within [-50 150] ns
    optT=optT(~eventn); % remove this scintillation from timestamp
    nn=numel(optT); % number of events left
    optposition0=optposition(eventn,:); % extract this scintillation position
    optposition=optposition(~eventn,:); % remove this scintillation position

    if length(optposition0)<thrphoton % if number of photons detected in this event is smaller than a certain threshold, discard this event.
        continue;
    else
        nscinti=nscinti+1; % number of recorded scintillation
        scintiresults{1,nscinti}=optposition0; % record the optical photon coordinates in this event

        scintievent=(scintiT>nowtime-150e-9)&(scintiT<nowtime+150e-9); % speculate the scintillation of the current photon yieldings
        scintiresults{2,nscinti}=scinti(scintievent ,:); % extract the position of the scintillation
    end

end
scintiresults = scintiresults(:, 1:nscinti);
clear optposition optT scinti
scinti = scintiresults(2, :); % coordinates of scintillation position
optposition = scintiresults(1, :); % the light distribution is of most interest
clear scintiresults

% to sort the detector responses
projectionset=zeros(nscinti, 72); % a set of projections in a series of scintillation
planeset = cell(nscinti, 1); % a set of light distributions in a series of scintillation
for nn=1:nscinti,
    [projectionset(nn, :), planeset{nn}] = SORTPROJECTION(optposition{nn}); % obtain projections & plane images with noise sigma=1
end
save('planesetfilename.mat', 'planeset')
save('scintisetfilename.mat', 'scinti')

B.1.1 Converting photon coordinates to detector pixel counts (SORTPROJECTION.m)

function [projections, plane] = SORTPROJECTION(optposition, noiselevel)
%SORTPROJECTION [projections, plane] = SORTPROJECTION(
%optposition, noise level )
%To convert detected optical photon coordinates to detector pixel counts on z=0 and z=8 detectors. Also to add on noise
% optposition=[xposition yposition zposition]
% plane=[plane0; plane8] whole light distribution on detectors
% projections=[x0 y0 x8 y8] projections of light distribution on detectors
% noiselevel is the standard deviation of the gaussian noise
% xposition = optposition(:, 3);

zopt0 = zposition == 2; % z=0 & z=8 on the geometry with light guide will be z=-10 and z=2
zopt8 = zposition == -10;
N0 = sum(zopt0); % the number of photons on z0 plane
xyplane0=optposition(zopt0,1:2); % refresh position array: keep z=0mm
xyplane8=optposition(zopt8,1:2); % refresh position array: keep z=8mm
xyplane=[xyplane0;xyplane8]; %
N=length(xyplane); % number of optimal photon detection
plane0=uint16(zeros(18,18)); % original pixel values of the two detectors
plane8=uint16(zeros(18,18)); %
for nn=1:N,
    x=xyplane(nn,1)+22.95; % the location of optical photons in 0-51mm range
    y=xyplane(nn,2)+22.95;
Xmacro = round((x-2.55)/5.1);  % get the which macropixel position this photon belongs to
Ymacro = round((y-2.55)/5.1);
midXmacro = Xmacro*5.1+2.55;  % midline of the current macropixel (seperating micropixels)
midYmacro = Ymacro*5.1+2.55;
if x < midXmacro,
    Xmicro = 2*Xmacro + 1;
else
    Xmicro = 2*Xmacro + 2;
end
if y < midYmacro,
    Ymicro = 2*Ymacro + 1;
else
    Ymicro = 2*Ymacro + 2;
end
if Xmicro > 18, Xmicro = 18; end  % avoid extreme position
if Ymicro > 18, Ymicro = 18; end
if Xmicro < 1, Xmicro = 1; end
if Ymicro < 1, Ymicro = 1; end
if nn <= N0,
    plane0(Ymicro, Xmicro) = plane0(Ymicro, Xmicro) + 1;  % detector 0
else
    plane8(Ymicro, Xmicro) = plane8(Ymicro, Xmicro) + 1;  % detector 8
end

% NOISE
if nargin > 1,
    enoise0 = random('Poisson', noiselevel, [18, 18]);  % generate noise
    plane0 = double(plane0) + enoise0 - noiselevel;  % add noise
    plane0 = uint16((plane0 + abs(plane0))/2);  % set the negative value zero
    enoise8 = random('Poisson', noiselevel, [18, 18]);  % generate noise
    plane8 = double(plane8) + enoise8 - noiselevel;
    plane8 = uint16((plane8 + abs(plane8))/2);
end
xcounts0 = sum(plane0, 1);  % projection in x direction
ycounts0 = sum(plane0, 2);  % projection in y direction
xcounts8 = sum(plane8, 1);  % projection in x direction
ycounts8 = sum(plane8, 2);  % projection in y direction

% Output
if nargout <= 1,
    projections = [xcounts0 ycounts0' xcounts8 ycounts8'];
else
    projections = [xcounts0 ycounts0' xcounts8 ycounts8'];
    plane = [plane0 plane8];
end
end
B.2 Positioning algorithms

% Calculate DoI from a series of events (By similarity measurement of projections)
clear
scinti = importdata('scintisetsamples.mat'); % coordinates of scintillation position
referprojection = importdata('projectionslibrary.mat'); % library of photon distribution projection
planeset = importdata('planesetsamples.mat');
[projectionset, planesetnoisy] = projpoiss(planeset, 5, 5);

nsinti = length(scinti); % number of scintillations
readouttype = 'noSingle';

xpo = zeros(nsinti, 1); % computed scintillation position
ypo = zeros(nsinti, 1);
zpo = zeros(nsinti, 1);
xtrue = zeros(nsinti, 1); % true scintillation position
ytrue = zeros(nsinti, 1);
ztrue = zeros(nsinti, 1);

[libx, liby, libz] = size(referprojection); % size of the library
for nn=1:nsinti,

% rough estimation
thresrough = 3; orderrough = 2;
[xrough, yrough] = roughest(projectionset(nn, :), readouttype, thresrough, orderrough, libx); % xpeak, ypeak are in the library coordinate (libx * liby)

% obtaining similarity matrix
[neighbourefer, rangex, rangey] = neighbourdeter(xrough, yrough, referprojection, 'projection', libx); % neighbouring reference
 rangexy = [rangex 18+rangey 36+rangex 54+rangey]; % if subset, use this rangexy
 simatrix = simi(projectionset(nn, :), neighbourefer, rangexy, readouttype); % compute the similarities/likelihoods

% estimating xy-position
orderxy = 2; levelxy = 16; thresxy = 16;
[xpo(nn), ypo(nn)] = xyinterpo(simatrix, xrough, yrough, orderxy, levelxy, thresxy); % XY locate as sum of columns

% estimate DoI
orderz = 3; threz = 3; % whole projection: orderz = 3; threz = 3;
subset projection: orderz = 2; threz = 4
zpo(nn) = zinterpo(simatrix, orderz, threz);

% scintillation position(true)
xtrue(nn) = scinti{nn}(1);
ytrue(nn) = scinti{nn}(2);
ztrue(nn) = -scinti{nn}(3);
APPENDIX B. POSTPROCESSING SCRIPTS (MATLAB)

%% digitization and plotting
xdigi = round((xpo-1) / (libx-1) * 256 + 0.5);  % digitize 1--18 to 1--256
ydigi = round((ypo-1) / (libx-1) * 256 + 0.5);
zdigi = round((zpo-1) / 5 * 32 + 0.5);  % digitize 1--6 to 1--32
ztruedigi = round(ztrue * 32 / 8 + 0.5);  % digitize 0--8 to 1--32

% digitization the XY results to plot the result in a image
xyplane = hist3([xdigi ydigi], 'CTRS', {(1:256)' (1:256)'});
xyplaneimage = flipud(xyplane');
figure(1);showjet(xyplaneimage);

% digitization the DoI results to plot the result in a image
DoIlinear = hist3([zpo ztrue], 'CTRS', {(1+2.5/32):5/32:(6-2.5/32)'});
DoIlinearimage = flipud(DoIlinear);
figure(3);showjet(DoIlinearimage);

B.2.1 Adding Poisson noise (projpoiss.m)

function [projectionset, planesetnoisy] = projpoiss(planeset, noiselevel, thres)
%projpoiss To add Poisson noise to the planes in a plane set, get the
%projections and normalize it
%[projectionset, planesetnoisy] = projpoiss(planeset, noiselevel, thres)
% planeset: a set of noiseless plane distributions
% noiselevel: mean and variance of Poisson noise
% thres: a constant value to be subtracted
N = length(planeset);
projectionset = zeros(N,72);
planesetnoisy = cell(N,1);
for nn=1:N,
    plane0=planeset{nn}(:, 1:18);
    plane8=planeset{nn}(:, 19:36);
    enoise0 = random('Poisson', noiselevel, [18, 18]);  %generate noise
    plane0 = double(plane0) + enoise0 - thres;  %add noise
    plane0 = uint16((plane0 + abs(plane0))/2);  %set the negative value zero
    enoise8 = random('Poisson', noiselevel, [18, 18]);  %generate noise
    plane8 = double(plane8) + enoise8 - thres;
    plane8 = uint16((plane8 + abs(plane8))/2);
    xcounts0 = sum(plane0, 1);  %projection in x direction
    ycounts0 = sum(plane0, 2);  %projection in y direction
    xcounts8 = sum(plane8, 1);  %projection in x direction
    ycounts8 = sum(plane8, 2);  %projection in y direction

    % Normalization
    N0 = sum(xcounts0);  % total number of counts
    N8 = sum(ycounts8);
    plane0 = double(plane0);
B.2. POSITIONING ALGORITHMS

plane8 = double(plane8);
"projectionset(nn, :) = [xcounts0/N0 ycounts0’/N0 xcounts8/N8 ycounts8’/N8];
planesetnoisy{nn} = [plane0/N0 plane8/N8];
end

B.2.2 Rough estimation (roughest.m)

function [xrough, yrough] = roughest(projection, readouttype, thresrough, orderrough, libx)
%ROUGHEST A rough estimation of the scintillation XY coordinates to
%limit
%the range of searching
% INPUT:
% projection: projections of light distribution
% readouttype: Single or Double
% libx: the size of the library along x direction (and y)
% OUTPUT:
% xrough, yrough: rough estimated coordinates in library system
% with
% size libx*liby
x0 = projection(1:18);
y0 = projection(19:36);
x8 = projection(37:54);
y8 = projection(55:72);

% rough localization of scintillation
% 1st plane0
[xdesc0, xindex0] = sort(x0, ’descend’);
[ydesc0, yindex0] = sort(y0, ’descend’); %find the top 3 points
xpeak0 = (xdesc0(1:3).^2 * xindex0(1:3)’) / sum(xdesc0(1:3).^2); % a
weighted average of top three position
ypeak0 = (ydesc0(1:3).^2 * yindex0(1:3)’) / sum(ydesc0(1:3).^2);
% 2nd plane8
[xdesc8, xindex8] = sort(x8, ’descend’);
ydesc8, yindex8 = sort(y8, ’descend’); %find the top 3 points
xpeak8 = (xdesc8(1:thresrough).^orderrough * xindex8(1:thresrough)’) / sum(xdesc8(1:thresrough).^orderrough); % a weighted average of top
three position
ypeak8 = (ydesc8(1:thresrough).^orderrough * yindex8(1:thresrough)’) / sum(ydesc8(1:thresrough).^orderrough);

if strcmp(readouttype, ’Single’)
xrough=(xpeak8 - 1)/17*(libx-1) + 1;% rough X position to adjust
to 37x37x6
yrough=(ypeak8 - 1)/17*(libx-1) + 1;% rough Y position to adjust
to 37x37x6
else
 xrough=((xpeak0+xpeak8)/2 - 1)/17*(libx-1) + 1;% rough X position
to adjust to 37x37x6
 yrough=((ypeak0+ypeak8)/2 - 1)/17*(libx-1) + 1;% rough Y position
to adjust to 37x37x6
end
B.2.3 Similarity comparison algorithm (simi.m)

```matlab
function [ similarity ] = simi(sample, reference, rangexy, readouttype)
% simi Calculate the similarity between one response and the library
% INPUT:
% sample: a vector of the discription of an event (projection)
% reference: a local reference set; can be projection or projim
% rangexy: the range of xy projections (18x18 detector by default)
% used in estimation.
% readouttype: Single or Double(default).
% OUTPUT:
% similarity: the similarity between this sample and references.

if strcmp(readouttype, 'Single')
    responlength = length(rangexy);
    rangexy = rangexy((responlength/2 + 1):responlength); % for Single readout only half of the response is considered
end

sample = sample(rangexy);

xmax=6; ymax=6; zmax=6; % size of the reference (usually 6x6x6)
similarity = zeros(xmax, ymax, zmax);

for xx=1:xmax,
    for yy = 1:ymax,
        for zz = 1:zmax,
            onerefer = reference{xx,yy}{zz}; % one reference response (probability function)
                onerefer = onerefer(rangexy); % reduce the reference response according to the rangexy
            similarity(xx,yy,zz) = 1/sum((sample-onerefer).^2); % similarity is defined as the reciprocal of Euclidean distance
        end
    end
end
```

B.2.4 Maximum likelihood algorithm (hbml.m)

```matlab
function [ likelihood ] = hbml(sample, reference, rangexy, readouttype)
% HBML Maximum Likelihood scintilation positioning algorithm by Hunter & Barret. Modified by applying gaussian function. Can be also used as estimation with only a subset of the readout projections and Single readout.
% INPUT:
% sample: a vector of the discription of an event (projection)
% reference: a local reference set; can be projection or projim

sample = sample(rangexy);

xmax=6; ymax=6; zmax=6; % size of the reference (usually 6x6x6)
similarity = zeros(xmax, ymax, zmax);

for xx=1:xmax,
    for yy = 1:ymax,
        for zz = 1:zmax,
            onerefer = reference{xx,yy}{zz}; % one reference response (probability function)
                onerefer = onerefer(rangexy); % reduce the reference response according to the rangexy
            similarity(xx,yy,zz) = 1/sum((sample-onerefer).^2); % similarity is defined as the reciprocal of Euclidean distance
        end
    end
end
```
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10 B.2.5  XY-plane position interpolation (xyinterpo.m)

8 % rangexy: the range of xy projections (18x18 detector by default)
9 % used in estimation. The default
10 % readouttype: Single or Double(default).
11 % OUTPUT:
12 % likelihood: the likelihood of this sample
13 if strcmp(readouttype, ’Single’)
14 responlength = length(rangexy);
15 rangexy = rangexy((responlength/2 + 1):responlength); % for Single
16 readout only half of the response is considered
17 end
18 pseudolightyield = 8000; % pseudo# of scintilated photons
19 sample = sample(rangexy)*pseudolightyield; % distri. of these photons;
20 with subset
21 xmax=6; ymax=6; zmax=6; % size of the reference(usually 6*6*6)
22 likelihood = zeros(xmax, ymax, zmax);
23 for xx=1:xmax,
24 for yy = 1:ymax,
25 for zz = 1:zmax,
26 onerefer = reference{xx,yy}{zz}; % one reference response(probability function)
27 if sum(onerefer) == inf, % avoid extreme value at the edge.
28 likelihood(xx,yy,zz) = -inf;
29 continue
30 end
31 onerefer = onerefer(rangexy)*pseudolightyield; % round them to
32 likelihoodone = -(sample-onerefer).^2 ./ (2*onerefer) - 0.5*
33 log(2*pi*onerefer); % gaussian
34 likelihood(xx,yy,zz) = sum(likelihoodone(:));
35 end
36 end
37 end

1 function [xpo, ypo] = xyinterpo(simatrix, xpeak, ypeak, orderxy, level, threshold)
2 %xyinterpo XY-position estimation based on similarity/likelihood
3 % [xpo, ypo] = xyinterpo(simatrix, xpeak, ypeak, orderxy)
4 % OUTPUT
5 % xpo, ypo: estimated xy position
6 % INPUT
7 % simatrix: simalarity/likelihood matrix of neighbourhood
8 % xpeak, ypeak: the rough estimated peak position, orderxy is interpolation
9 % orderxy: order of weights (1~2 are good)
10 % threshold: how many reference positions are considered in the
11 % interpolation. the rest is to be substracted (16 better at centre; 4
12 % better at edge)
13 levelmax = sum(simatrix,3); %sum of all similarities along each z
14 [maxsimaxa, maxind] = sort(levelmax(:), ’descend’);
APPENDIX B. POSTPROCESSING SCRIPTS (MATLAB)

B.2.6 DoI interpolation (zinterpo.m)

function [zpo] = zinterpo(simatrix, orderz, threshold)
% zinterpo DoI interpolation based on similarity/likelihood

% OUTPUT
% zpo: estimated z position

% INPUT
% simatrix: similarity/likelihood matrix of neighbourhood
% orderz: order of weights (3?)
% threshold: how many reference positions (with biggest similarity)
% are considered in the interpolation. the rest is to be subtracted (3?)
levelmax = sum(simatrix,3); % sum of all similarities along each z
[maxsima, maxind] = sort(levelmax(:), 'descend');
[xnow, ynow] = ind2sub(size(levelmax), maxind); % record the xy position of this column

simatrix1 = squeeze(simatrix(xnow(1), ynow(1), :)); % the four most similar columns
simatrix2 = squeeze(simatrix(xnow(2), ynow(2), :));
simatrix3 = squeeze(simatrix(xnow(3), ynow(3), :));
simatrix4 = squeeze(simatrix(xnow(4), ynow(4), :));

[simatrix1z, ind1z] = sort(simatrix1, 'descend'); % sort the four columns along z
[simatrix2z, ind2z] = sort(simatrix2, 'descend');
[simatrix3z, ind3z] = sort(simatrix3, 'descend');
[simatrix4z, ind4z] = sort(simatrix4, 'descend');

simatrix1z = simatrix1z(1:threshold) - simatrix1z(threshold + 1); % maximum threshold number of similarities are substracted by the rest
simatrix2z = simatrix2z(1:threshold) - simatrix2z(threshold + 1);
simatrix3z = simatrix3z(1:threshold) - simatrix3z(threshold + 1);
simatrix4z = simatrix4z(1:threshold) - simatrix4z(threshold + 1);

ind1lz = ind1z(1:threshold); % index of maximum threshold refers
ind2lz = ind2z(1:threshold);
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\begin{verbatim}
31 ind3z = ind3z(1:threshold);
32 ind4z = ind4z(1:threshold);
33 simatrixzall = [simatrix1z' simatrix2z' simatrix3z' simatrix4z'];
34 indzall = [ind1z; ind2z; ind3z; ind4z];
35 zpo = simatrixzall.^orderz * indzall / sum(simatrixzall.^orderz);% interpolation
36 end
\end{verbatim}