Assessing the Benefits of a Virtual Transshipment Hub in the Swedish Forestry Industry

Gustav Danell

October 19, 2015
## Contents

1 Acknowledgements

2 Introduction
   2.1 Summary
   2.2 Sammanfattning
   2.3 Aim
   2.4 Background
   2.5 Classification of lumber
   2.6 The forest supply chain
   2.7 A virtual transshipment hub

3 Problem description
   3.1 Methodology
   3.2 Mapping the current system
   3.3 Scenario 1
   3.4 Scenario 2
   3.5 Scenario 3

4 Mathematical theory
   4.1 Mathematical optimization
   4.2 Linear programming

5 The simplex algorithm
   5.1 The standard form
   5.2 Convexity

6 Integer programming
   6.1 Solving the integer program
   6.2 Branch and cut
   6.3 Relaxation
   6.4 Cutting plane algorithms
   6.5 Branch and bound

7 Previous work
   7.1 Cost allocation models
   7.2 The Shapley value
   7.3 The nucleolus
   7.4 Allocations based on separable and non-separable costs
   7.5 Equal profit method

8 The chosen model
   8.1 Interpreting the model
   8.2 Supply and assortments
   8.3 Demand and assortment groups
   8.4 Trucks and drivers
   8.5 Distances and geographical nodes
   8.6 Objective and costs

9 Results
   9.1 Mapping the present
   9.2 The proposed model
   9.3 Testing the model
      9.3.1 Scenario 1
      9.3.2 Scenario 2

10 Discussion

11 References
For my lovely children
Hopefully my achievement can inspire you
To reach your dreams
And for my wonderful fiancé
For all your tender support and patience
For your wit and understanding
It is my love for you, my family
Which made me pursue my diploma
1 Acknowledgements

I wish to thank my supervisor at Umeå university, Leif Persson, for all his time invested into this paper. Thank you also Leif Nilsson who made a great effort in creating an interesting educational program. I would also like to thank my external supervisor, Anders H. Jonsson at Länsstyrelsen in Västerbotten as well as Robin Norrman at Process IT and Innovations for the opportunity to conduct this paper and to work with such an interesting problem. A big thank you goes to Dimitris Athanassiadis at SLU who helped me with questions and data. I would also like to thank Anders Ringsell at Martinsons as well as Anna Hansson and Peter Lundström at NK Lundströms for your advice and help to understand the industry. A thank goes out for all those people I had contact with over the telephone to assist me in my work. In particular Gert Andersson at Skogforsk who I had long discussions with to try to solve the impossible task of adapting the nonreal data to become plausible. I would also like to take this opportunity to thank all the fantastic people and teachers which I have had contact with during these five years. Your presence has embellished these years and all the laughter we’ve shared will be remembered forever. I’d also like to thank my parents for allowing me to stay at their home during some of the period, and once again my fiancé who has helped tremendously, making the household function and the children happy when I was away working with my paper.
2 Introduction

2.1 Summary

The purpose of this master thesis is to reduce the transportation costs and environmental distress by improving the transportation routes needed to supply the sawmills in the county of Västerbotten, Sweden, their raw material. It is of particular interest to explore the possible benefits of implementing a so called virtual transshipment hub for this purpose. The outline of the hub is to allow the companies within the system to deduce raw materials from other companies’ contracted harvesting areas. The hub would thus create a pool of the total raw material appointed by each specific company. These companies may deduce the raw material needed but can have it transported from a closer site than their own contracted lumbering area. Sawmill companies rely on the use of harvesting areas to provide the needed raw material and it is of common practice to contract different harvest areas. The thesis is that the permission to use other companies’ harvest areas would create new possible routes resulting in better planning and as an extension more efficient routes. This master thesis will investigate the current situation in the industry and adapt a model suitable for the purpose from the information gathered. This thesis work will also provide a number of cost allocation models which are used in cooperations between different companies in order to determine how to allocate the savings / costs between the companies.

It was revealed in an interview that it was desired to include an exploration of the ETT trucks in this paper. The ETT trucks is a new type of truck which is not currently permitted in Sweden. It has a higher loading capacity than the conventional trucks used today.

Gathering data proved much more difficult than initially anticipated. As a result, this paper will not provide any actual data testing, but the Results section will show that the model is working as intended by using with trivial data. More on the difficulties associated with the data in the section Discussion.

Due to the trivial data it is impossible to express an actual cost saving in using a numerical value or percentage. The results from this survey did however show that there were a signification reduction of the cost associated with the transportation of raw material when the two companies tested cooperated in comparison to when they worked separately.

The use of the ETT trucks would reduce the transportation costs and CO$_2$ emissions by 20 % respectively, as presented in a survey conducted by Lofroth and Svenson (2012). The conclusion is that regardless if the government allows the use of ETT trucks, it should lie in the best interest to further explore the implementation of a virtual transshipment hub using real data and a thorough investigation of eligible participants through the cost allocation models and a subsequent maintenance of the system using supply chain management.

2.2 Sammanfattning


Hypotesen är att användningen av hubben kommer att skapa nya, förbättrade rutiner och en mer överskådlig och hanterlig logistik för de berörda företagen, vilket i sin tur kommer att skapa en mer effektiv logistik i allmänhet. Denna rapport kommer att undersöka den rådande situationen och anpassa en matematisk optimeringsmodell efter denna. Rapporten kommer även diskutera några kostnadsallokeringsmodeller för att ge ett ramverk för hur företagen kan allokera sina kostnader alternativat vinster.

Under en intervju blev jag även varse om att det fanns ett intresse från industrin att undersöka de effekter ETT lastbilar har på utsläpp och transportkostnader. ETT lastbilar är för närvarande inte tillåtna i Sverige. Dessa lastbilar har en högre lastningskapacitet än de lastbilar som för närvarande används.

Det visade sig att relevant data var en omöjlighet att finna, varför rapporten inte innehåller några numeriska resultat. Resultatdelens syfte är därför att visa att modellen fungerar som avsett med hjälp av trivialt data. Rapporten visar emellertid att det kan föreligga...
en möjlighet att reducera sina transportkostnader signifikant då företagen samarbetar i jämförelse med när företagen arbetar individuellt. Då testdatat som modellen testades på var fabricerat och kraftigt förenklat är det dock inte möjligt att presentera en numerisk eller percentuell besparing. En undersökning av ETT lastbilarna som presenterade av Löfroth och Svenson (2002) visar att dessa lastbilar, i jämförelse med de konventionella lastbilarna, ger en besparing på 20 % gällande både reducerade transportkostnader och CO2 utsläpp. Min rekommendation är således att fortsätta undersöka den virtuella hubben och att ifall det visar sig att en hubb är implementerbar skapa en virtuell hubb genom noggrann övervågning av medlemmar med hjälp av kostnadsallokeringsmodellerna och att sedermera underhålla systemet med hjälp av supply chain management.

2.3 Aim

The aim of this paper is to survey the present in the forestry industry logistic systems. This mapping will include an evaluation of how the information is shared and through which forums. This is of importance as theories in Supply chain management has credited information sharing as a very valid and important factor for implementing interdependent systems. It is also of interest to survey how optimized the routes presently are and to assess the beneficial (by thesis) impact the transshipment hub has on the companies within the system. Lastly an assessment of the virtual transshipment hub will be performed in combination of the imposition of a new kind of trucks – the ETT trucks which has a higher loading capacity than the traditional trucks used today.

This project is included in Kompass 2020; a Swedish cluster of projects whose focus is to develop the process integration in the refinement industry in Northern Sweden. It is compromised by four universities, governmental authorities, individual companies and industry groups. The projects’ structures rely on different ”rooms”, or sections of interest, such as new packaging solutions, new materials stemming from the forest industry, industrial coordination on a global scale and industrial building to name some.

2.4 Background

The master thesis is proposed by Västerbotten Länsstyrelse and to some extent by ProcessIT Innovation and SLU – Sveriges Lantbruks Universitet, The Swedish Rural University. This master thesis will act as a framework to see if there exists any incentives for Kompass 2020 to further analyze and possibly implement similar systems to a bigger extent. Why for instance a municipal institution shows interest in these sorts of questions will be presented in this section. Concisely, the county governs the forestry industry due to its positive impact on the county in terms of job opportunities and monetary income. The sector is however operating under low profit margins. The industry is also subjected to uncertainty which is not present in many other industries. The logistics in this aspect is of great importance as it saves costs which has direct impact on profit margins. It has been mentioned that the industry is currently exposed to the worst crisis in over 40 years (Skogsindustrierna, 2012).

The forest industry employs vast volumes of forest every year and is therefore very transport intense. The logistics connected to the industry accounts for approximately 25 % of the total land based transportation in Sweden (Forsberg et al., 2005). Of these, approximately half are attributed to the transportation of raw material and the other half from finished wood-based products.

In Västerbotten alone approximately 7 million $m^3$ lumber is harvested each year. The regional sawmills have a total production of 1.5 million $m^3$, the cellulose companies refine 3 million $m^3$ and the energy sector in Västerbotten consumes 1.5 million $m^3$ annually.

The forest industry is exposed to a major source of complexity not present in many other industries. This complexity stems from the uncertainty regarding the input to the industries. Trees are living beings and as such vary substantially in appearance and quality. The forests’ heterogeneous characteristics and the fact that a log must be processed in the sawmills until the quality can be deterred as it requires internal investigations causes difficulties in assessing what kind of material one may expect from a specific region.

The uncertainty regarding the input naturally causes problems. Each type of combination of different dimensions and quality is typically only suited for a limited set of end products. This causes problems with, for instance, process adaptations. The uncertainty also causes the industry to operate from a PUSH production rather than a PULL.

The difficulty in the assessment of lumber comprises two additional causes of distress. The first governs the complications that arise from not being able to comply a customer need. The other is the tension in work flow created if too much raw material is transported to a certain sawmill. Too much timber would produce a need for external stocking, due to stocking possibilities at the
sawmills being prone to be very limited, with additional driver costs and transshipment costs as consequences. The addition of new operations may also cause delays with corresponding costs.

The forestry industry is operating its lumbering in relatively narrow time frames while trying to fulfill the demands on a daily basis. During the summer it is not possible to have lumber in the forests as it becomes ruined as a result of the damp climate, mold and insects. The ground in the forest must also be able to carry the heavy machinery used to lumber which is not always possible. For the ground to be able to keep up the heavy machines it must be solid - preferably by frost. As a consequence it is desirable to conduct as much lumbering as possible during the fourth quarter when the weather permits lumbering. Due to this extensive lumbering the industry receives large volumes of logs during February, March and April which is then used as a buffer during the summer when lumbering is not possible. This strains the companies’ planning. Due to the uncertainty of the weather the practice is to plan more harvesting than necessary and subsequently “jump between” the lumbering areas where the area permits gathering of logs.

Due to the current structure in the forest industry companies are heavily dependent on entering contracts regarding lumbering areas. As a result several different companies may be negotiating for the same lumbering area. This competition may cause a focal company’s lumbering area to become spread out over great geographical distances. Another common practice is to create big safety stocks to reduce the damage caused by not being able to comply to customer needs in time.

Both these practices have shortcomings in both economic and environmental terms; the economy suffers for the focal companies due to the capitalization in safety stocks or lumber areas which essentially acts as safety stocks, and the environment suffers when sawmills must deduce their lumber from harvest areas far away when a harvest area in its vicinity has the same supply, but is not contracted.

An interview with a smaller sawmill in the county also provided insight into the concerns of the future in these subjects. The interviews revealed in particular three concerns regarding changes in the future. These concerns governed the implementation of the sulfur directive and the government’s reluctance to allow a higher loading capacity for lumber trucks, i.e. to allow the ETT trucks.

The imposition of the sulfur directive, which regulates how much sulfur marine transports may emit, might become a big concern for the industry according to the company representative. This company worries that the export, which in Sweden utilizes maritime export to a high degree (approximately 90%) must find alternatives for their transport which is not as efficient and economic. The representatives at the sawmill worried that the end-users of the products must pay an additional cost accounted to the additional transportation costs, which would then become a competitive disadvantage. Especially considering that the sulfur regulation forces a lower tolerance of sulfur in Sweden compared to other parts of Europe, such as Russia which also exports lumber. The sawmill feared that companies in countries which is not affected by the directive may keep their prices unchanged and thus create a competitive advantage.

2.5 Classification of lumber

Throughout this paper the terms classification or assortments will often be mentioned. This section describes this phenomenon. The need for classification is a major factor in the industry and permeates the industry as a whole.

The sawmills production parameters are often adapted to suit a specific classification of timber. These parameters includes how the lumber will be cut. The most common practice according to Grönlund (1992) is illustrated in Figure 1. The figure shows that two cuts are initially performed before the lumber is rotated 90 degrees and subsequently cut again. The block extracted from the middle is called the center yield and is the most valuable part of the log. In practice it is a calibration to gain as much center yield possible while still getting as much exchange from a log as possible. It may sometimes be necessary to adapt the cuts in order to quickly gain material to a specific product to meet customer needs.

The different cut patterns yield logs suitable for different products. The total amount of possible cut patterns are very high but given a certain diameter of a log only a few is deemed adequate.

Some timber is due to its small diameter not suitable to be processed at a sawmill. This timber is referred to as pulp wood and is used in manufacturing paper and carton.

Due to the different dimensions’ high correlation with the end-products the term adaption is widely used in the industry. When the harvesters in the forests timber they conduct a mathematical optimization as to how they should cut the tree. The parameters for these optimizations are provided by the refinement stations to ensure that the best suited dimensions ends up at their facilities. These adaptions are unique to that refinement station to be the most suitable for the internal processes.
2.6 The forest supply chain

Historically it has been a common practice to conduct improvements explicitly inside focal companies and within concerned departments.

However it has been seen that higher efficiency can be achieved by adapting so-called supply chain management. A supply chain begins with supply of raw material, passes through one or multiple steps for manufacturing, storing and manufacturing and subsequently the arrival at a customer. In green supply chains it is also common to consider the disposal of said products.

Figure 2 illustrates the supply chain in a forestry context. It can be seen from the figure that the companies within have several options regarding how to transport their products.

Figure 2: An illustration of the forest supply chain. Source: Gunnarsson (2007)

There are some necessary activities that must be undertaken in order to ensure future harvesting possibilities. These activities are planting, cleaning, thinning and harvesting (Gunnarsson, 2007). Note that Figure 2 does not represent every tier in the supply chain management. A sawmill may for instance sell the finished planks into a furniture manufacturer which has a supply chain of logistic partners, intermediate stock terminals, wholesalers, retailers and end customers.

An initial assortment is undertaken of the raw material depending on their use. The three major categories for these assortments are saw logs, pulp wood and forest residues. These can then be assorted further into different subgroups depending on dimension and quality (Gunnarsson, 2007).

Figure 2 depicts the post harvesting transportations for the different classes. Timber is either transporter to terminals for storage or sawmills. Similarly pulp wood is either transported to terminals or pulp mills. After harvesting residues in the form of tops and branches are left behind, typically for a year until they are chipped (Gunnarsson, 2007). These residues are then transported either directly or through intermediary storage to their refinement stations - heating plants.

These processes creates byproducts. Specifically the byproducts from the sawmills, for instance chips, are transported to pulp mills and heating plants to be processed further. Bark is a byproduct of pulp making which are used as fuel at the pulp mill or heating plants (Gunnarsson, 2007). Finally, the pulp products can be transported to the paper mills where they will be made into paper products.

2.7 A virtual transshipment hub

This paper has presented some problematic and complex factors which must be taken into account for the companies. The thesis is that a virtual transshipment hub would provide benefits for the companies within the system. In particular the transshipment hub is deemed to provide lesser transportation costs. The system is to be implemented in a manner closely related with how energy is transferred today. In the energy industry it is impossible to tell which company that actually produced a specific energy current. Instead all currents from all companies are tied up to an energy pool which the end user extracts some current from and pays the company he or she is contracted to.

The same thought is initialized in this instance. Instead of forcing all the focal companies to create big safety stocks, a pool of safety stocks collaboratively constructed by all the coordinating companies is emerged. The focal companies employ the
same strategies and procedures as they presently do regarding their purchases. However, the idea behind the virtual hub stipulates that every company within the hub is entitled to transport what they have ordered regardless of from which lumbering area the tree is gathered. Figure 3 illustrates an conceptualization of this system. Figure 3 shows the thesis that the companies may experience lesser transportation costs when other companies’ contracted lumbering areas can be used to extract the needed lumber from. Without the hub, the companies would have to visit the sparse circles to gather their needs. Note that the illustration shows a “perfect” balance in the sense that the demand for all assortments can be gathered within the new hub. In reality the company may need to visit some of their own lumbering areas further away, as a consequence of lacking supply of some assortment(s) or to withhold the equilibrium constraint.

![Figure 3: An illustration of an virtual transshipment hub. The triangle represents a sawmill and the lumbering areas is represented as circles. The different companies is represented as colors and the colored rectangles represents the area the companies may extract their lumber from.](image)

This would impose that a certain company may have the possibility to fulfill a need using lumber a competitor has lumbered, provided that the company that extracts material from a competitor provides the system with at least the same quantity of the same assortment(s) extracted. The latter is a necessity to ensure that the system does not get drained of its capacity but instead always strives to be in equilibrium. This will be regulated by the proposed mathematical model presented in Results in more detail.

With this practice it may be possible for the focal companies to take advantage of, for instance, the vicinity of competitors’ lumbering areas compared to their own lumbering sites. One may for example be able to provide more necessary lumber from a confined area with the help of the competitors rather than from a bigger area when only the own contracted lumbering areas is considered. The possibility to provide the focal companies with raw material from more limited areas is deemed to create better utilization possibilities – for both the transportation companies and the harvest companies.

Allowing the gathering of raw material of a whole industry to more confined areas may also provide indirect effects such as higher efficiency in the lumbering areas themselves when setup times and processes for different types of products can take economies of scale into account.

The narrowing of the gathering areas can also be used to an advantage by making clusters of lumbering areas in the vicinity of each other to provide raw materials for the industry as a whole and subsequently shift these clusters when they have been drained. This would allow a better overview of the transportation and hence a greater possibility for coordination.

The concept of constructing a virtual transshipment station relies heavily on the need to perform massive structural changes in the forestry industry. For instance, today the competitors have little to no coordination with each other forcing such an implementation to require cultural changes in the industry. Interviews has revealed that the information sharing mainly relied on e-mail correspondence and the use of cellular phones.

The lack of company coordination also causes concerned IT systems and coordinating system to be non present which implies that an implementation of the virtual hub requires financial obligations.

3 Problem description

This paper will assess the benefits of implementing a virtual transshipment hub by comparing the present situation with two additional scenarios. These scenarios correspond to optimizing the current routes in order to salvage information regarding the present efficiency. Subsequently an optimization will be performed to investigate the benefits of a virtual transshipment hub. The scenarios will be investigated by planning the routes of the logging trucks, using mathematical optimization to construct driving schedules to match the supply and demand within some time constraints.

The supply is described as the physical log piles in the vicinity of the logging areas. These are located adjacent to the forest roads. The demand are presented in the descriptions of the industrial orders and can as such extracted from databases.

The participants within the transshipment hub wish to invoke some kind of restriction of allowing competitive companies to withdraw more information in terms of supplies and demand than necessary. For instance, companies has expressed a reluctance to use flow maps as the localization of harvest areas may reveal company secrets such as volumes, quality thinking etc. The participants do
not wish that the hub would be too liberal either. The focal companies wish to find possible better routes for their own operations, but not a full-out cooperation. The focal companies do not want competitive companies to extract too much material from their contracted lumbering areas, rendering the focal company unable to comply to their own customer needs. Hence such restrictions must be present.

3.1 Methodology

As a prelude to this chapter, I encountered a situation where actual data were not possible to use, which forced me to use artificial, trivial, data for evaluating my proposed model with the purpose of merely showing that the model worked as intended.

The necessary steps conveyed throughout this project was to first have a discussion with the different companies. This was mainly done through telephone contact with the purpose of collecting data, and gather information regarding the industry. These telephone discussions can be referred to as informal discussions. The project has also involved one formal discussion, i.e. a physical meetings to discuss different topics.

These discussions were then used as a foundation to analyze the present, and to construct an optimization model with the purpose of finding improvements.

The actual optimization were conducted using AMPL along with the solver Gurobi. A literature review concerning different optimization techniques and models preceded the optimization. The focus of this review were mainly to find a suitable "start" model, which could be adapted to suit this particular instance.

Cost allocation models were also investigated in order to find a suitable model for this particular instance where the companies want to convey coordination with each other, but to a limited degree. The literature search also included finding information regarding the efficiency of the ETT trucks and which impact these had on the transportation costs and the environment.

3.2 Mapping the current system

In order to come up with suggestions for improvements it was deemed of paramount importance to identify how the present situation was. As such the first assignment of this project was to identify how the system were spanning. How were the driving routes conducted; was it by calling outsourced logistic operators (3PL) or was it done automatically, to full or to some degree with implemented information systems? Were there any strategies involved regarding how the material were driven; was it for instance more profitable to fulfill certain demands for one assortment first before starting to drive the rest?

An implementation relies on the tenet that information systems will be used. As such the initial examination also involved an overview of how information was shared and when.

This will constitute the ground for analyses regarding some subsequent scenarios.

How is the system currently spanning; how long is the drivers routes presently? How are the transportations ordered? Is there any strategies involved in the procurement? How does the information get shared?

3.3 Scenario 1

Given how the system is presently outlined, portrayed by examining the present, is the system operating under optimized conditions? In this first scenario a mathematical optimization was undertaken with emphasis to see if a different route than presently conducted would provide a better result. This optimization would be under the perimeters that some logging areas were contracted to different companies and as such were obliged to only serve those companies’ needs. Initially the optimal solution of the mathematical model conducted for Scenario 1 would be compared to historical transportation routes taken by the companies using the same data. This was not possible however due to the impossibility of finding data as mentioned earlier. The scope of Scenario 1 thus becomes to find a mathematical solution while not allowing company cooperation.

Is the system optimized taking the findings in identifying the present into account? How can it be improved? How are the routes conducted if there is cooperation, using the proposed mathematical model?

3.4 Scenario 2

Scenario 2 will consist of a mathematical optimization which will be conducted to provide an insight in whether the virtual transshipment station would yield any economic enhancements compared to Scenario 1 where cooperation is inhibited. This scenario will be made under the assumptions that the adaptions are no longer unique. This is due to the high requirement on information technology in order for this to work, and the fact that the optimizations carried out in the adoption
process by the harvester can be done in matter of seconds. The assumption is therefore that the companies interact in a higher extent in order to receive more benefits, using real-time information sharing coupled with immediate optimization of the saw patterns by the harvesters, allowing a sawmill who relies on unique adoptions patterns to be able to use lumber processed by other companies’ harvesters. This scenario will however have constraints to prevent other companies to for instance drain the focal companies’ supply of its assortments.

By lifting the restrictions of how the lumbering areas is contracted to different companies, is there any room for improvements in the system as a whole by converging the system into the virtual transshipment station with some constraints restricting too liberal coordination?

3.5 Scenario 3

The initial thought for scenario 3 was to have an additional optimization performed to investigate the differences in terms of transportation costs when allowing the so-called ETT trucks with a much higher capacity than the currently legislated trucks, as this was requested by one of the companies to elucidate the (presumed) positive effects of these types of vehicles. Due to the difficulties with the data, such an optimization would not produce anything of value using the trivial data this model will use. The focus of this scenario thus shifted to a literature study, rather than an optimization with numerical results in order to investigate the impact the ETT trucks has.

If it became allowed to operate the ETT rigs, what would the implications be in terms of transportation costs when combined with the virtual transshipment hub?

4 Mathematical theory

4.1 Mathematical optimization

The concept of mathematical optimization is to find the best possible solution to a given problem. The idea of finding the best possible solution is of course governed in many practical applications and mathematical optimization is therefore widely used in several different areas be it theoretical or practical.

Because of the globalization of companies, the creation of multicompanies and more and more complex processes surrounding companies mathematical optimization is used in very big and complex systems. As such, research often focuses on developing algorithms fast enough and accurate enough to keep in pace with the highly dynamic and network based reality.

Mathematical optimization as a concept is rather new. Subsequent to World War II mathematicians such as Dantzig began to investigate the possibilities of optimization. In 1947 Dantzig introduced the simplex algorithm which is an algorithm for finding optimal solutions for linear programs. This algorithm is widely used even today and is often the core of optimization in modern software.

Similar for all optimization models is the syntax. All optimization models govern to maximize or minimize a function, the so-called objective function with respect to some constraints. A general model thus has the appearance

\[
\begin{align*}
\text{minimize } f(x) & \quad (2.1) \\
\text{subject to } g_i(x) & \leq b_i, \quad i = 1, \ldots, m
\end{align*}
\]

where \( f(x) \) is the objective function which is to be minimized over the vector \( x \). In this representation \( \{g_i\}_{i=1}^m \) denotes \( m \) functions corresponding to the \( m \) upper limits, \( \{b_i\}_{i=1}^m \) which are considered to be constants. For those unfamiliar with optimization it may be noteworthy that the inclusion of \( i = 1, \ldots, m \) or by analogy \( \forall i \in S \) where \( S \) is some set containing all elements \( i \in 1, \ldots, m \) means that we have more than one constraint. In this example we would have the constraints \( g_i(x) \leq b_i \) up to \( g_m(x) \leq b_m \). In total there would be \( |S| \) constraints, where \( |S| \) is the amount of elements in the set \( S \).

The values of the variables \( x \) differ depending on the outline of the problem. Three often conceptualized models include the linear program, the integer program and the non-linear program where the latter has a non-linear function corresponding to \( \{g_j\}_{j=1}^m \). Linear programs and integer programs will have a more in-depth discussion in the next section in order to elucidate the differences in approaches to solve those particular problems. Integer programs in themselves can be divided into different categories. Those of interest in this work will be BIP and MIP. BIP refers to Binary Integer Programs where the variables, say \( x \) can only take values of 0 or 1, or \( x \in \{0,1\}^n \) where the power of \( n \) refers to \( x \) being a vector of \( n \) elements, i.e. we would have the variables \( x_1, \ldots, x_n \). MIP or Mixed Integer Programs allows for instance two sets of variables, \( x \) with \( n \) variables to take on integer values and \( y \) with \( m \) variables to have real values, or \( x \in \mathbb{N}^n \) and \( y \in \mathbb{R}^m \).
4.2 Linear programming

Following the general outline of a mathematical optimization problem as in (2.1) we declare that in order for a particular problem to be a linear program problem the vector $x$ to be maximized must be real i.e. $x \in \mathbb{R}^n$. Recall that the $m$ functions \( \{g_j\}_{j=1}^m \) must be linear.

The general approach for solving linear programs is rather straightforward using the so-called simplex algorithm briefly mentioned earlier.

5 The simplex algorithm

5.1 The standard form

The simplex algorithm was first introduced by Dantzig in 1947. It requires the problem to be expressed in the standard form

\[
\begin{align*}
\text{minimize } & f(x) \\
\text{subject to } & g_i(x) = b_i, \quad i = 1, \ldots, m
\end{align*}
\]

Where (2.2) differs from (2.1) by applying a constraint that $x \in \mathbb{R}^n_+$, i.e. that all $n$ $x$-variables are real and greater than or equal to zero. Recall that since $\mathbb{N} \subset \mathbb{R}$ the $x$ variables may have integer values. It is of utmost importance that the model which the simplex algorithm is applied on must consist of $m$ equations rather than inequalities as in (2.1). By adding so-called slack variables one can transform inequalities into equations. These slack variables will then indicate how good a given solution is. If the slack variables are minimized (and the objective function is to be maximized) the solution has been allowed to use as much of the variables as possible and hence performed the best solution. For instance by applying a slack variable, $s_1$, at the right-hand side of the inequality $x_1 + x_2 \leq 3$ one derives at the equation $x_1 + x_2 = 3 + s_1$. By analog if one considers the inequality $x_1 + x_2 \geq 3$ it can be transformed into the equation $x_1 + x_2 + s_2 = 3$ by adding the slack variable $s_2$

5.2 Convexity

The accuracy and swiftness of the Simplex algorithm is credited to the properties of the linear program. The fact that the feasible region of a linear program is convex is one very well suited property.

Definition 5.1 (Convexity). A set $S \subset \mathbb{R}^n$ is called a convex set if for all points $u, v \in S$ and all $\lambda \in (0,1)$ we have that $\lambda u + (1-\lambda)v \in S$.

Or, when expressed in words, Definition 5.1 claims that if a straight line can be drawn between any two arbitrary chosen elements $u$ and $v$ in a set $S$ and if every element this line passes also is an element of the set $S$, then the set $S$ is a convex set. Figure 4 illustrates Definition 5.1.

Theorem 5.2 (The convex feasible region). The feasible set $S$ corresponding to a linear program $S = \{ x \in \mathbb{R}^n : Ax = b, x \geq 0 \}$ is a convex set where $S$ is the standard form expressed in matrix form.

Definition 5.3. A point $\Psi$ in a convex set $S$ is called an extreme point of $S$ if it does not exist any two points $u, v \in S$ and some $\lambda \in (0,1)$ such that $\Psi = \lambda u + (1-\lambda)v \in S, u \neq v$

Theorem 5.2 and Definition 5.3 are quite remarkable and certainly very convenient when trying to find optimal solutions. Every optimal solution on a convex set is located in corners of that set, i.e. not interior of any line segment between some points $u$ and $v$. This is certainly a very convenient property when one wishes to find optimal solutions. It implicates that instead of checking every possible element in the feasible region it is sufficient to investigate only the corners. It also implicates that since the feasible set is convex one can traverse these extreme points following a path of ascending (or descending depending on objective function) corner points until it is not possible to find a better solution; the optimum is found.

This is the general idea behind the simplex algorithm; to utilize the convexity of the feasible region to traverse these points and ultimately find an optimal solution. This concept is illustrated in Figure 5.

6 Integer programming

The integer program has the same general outline as the linear program with the exception that in integer programs the vector $x$ to be maximized is a vector of $n$ integer variables, $x \in \mathbb{Z}^n$. However
slight the difference between forcing the vector to only assume integer values and allowing real solutions may seem, the implications in terms of solving these models vary substantially. In fact this small difference creates an entirely different feasible region for the integer program in comparison to the linear program. The feasible region of an integer program is more accurately described as a feasible set consisting of some integer points rather than a coherent region as in the linear program. This appearance of the feasible region for an integer program causes it to lack the convexity property. See Figure 6 where the blue region is the feasible region of the linear program and the red circles is the feasible set for the integer program.

From Figure 6 it becomes evident that

1) the feasible set of an integer program is not convex and
2) one cannot apply the simplex algorithm to find optimal integer solutions.

In other words, one must rely on different approaches in order to actually solve the integer program.

6.1 Solving the integer program

Solving the integer program requires some different techniques in comparison with solving the linear program. An interested reader is referred to Wolsey (1998) who explains the topics presented in this section in more detail.

6.2 Branch and cut

Branch and cut is widely adopted when trying to find optimal integer solutions. It is not a procedure in itself but rather a collaboration or a merge of different techniques.

The techniques included in branch and cut are relaxation, cutting plane algorithms, branch and bound and preprocessing. These techniques, except preprocessing, is presented below. Preprocessing is not presented as it in practice is the same as a cutting plane algorithm.

6.3 Relaxation

The concept of relaxation is to perform a modification of the feasible set into a feasible region, as in the linear program. Two widely used relaxations are linear relaxation where one allows $x$ to assume real values and thus enlarge the feasible set into a feasible region (as seen in Figure 6 where both the blue and red region contains the red dots corresponding to the feasible set of the integer program) and Lagrangian relaxation where one performs a linear relaxation and then drops some constraints and subsequently add these into the objective function.

A sought-after property of these relaxations is that different techniques, such as cutting plane algorithms, can be used to alter the feasible region of the relaxed problem to find integer corner points and thus optimal integer solutions.

6.4 Cutting plane algorithms

One way of solving an integer program is to conduct a linear relaxation, run the simplex algorithm and subsequently add new inequalities in order to remove redundant regions in the corner points so that an integer point becomes an optimal point. Different algorithms deals with this under the shared name cutting plane algorithms. It can be seen in Figure 6 that the blue region corresponding to a relaxation of the feasible set (the red dots) has been cut by the green line, creating a smaller feasible region.
6.5 Branch and bound

Branch and bound deals with breaking a certain problem into smaller problems and solving these.

Definition 6.1. Let \( S = S_1 \cup \ldots \cup S_K \) be a decomposition of \( S \) into \( K \) smaller sets. Also let \( z^k \) be the optimal solution for decomposition \( k \) such that \( z^k = \max \{ cx : x \in S_k \} , k = 1 \ldots K \). Then the optimal solution for the overall problem is \( z = \max_k z^k \).

Where Definition 6.1 entails that one may assign a decomposition algorithm to create \( k \) decompositions by for instance forcing some variables to assume some values, and then find the optimal solutions for these. Then, the optimal solution for all the \( k \) decompositions is the optimal solution for the whole program.

7 Previous work

Improvements in transportation planning has become more and more important in the forestry industry, due to large volumes and long transportation distances coupled with increasing fuel prices and environmental awareness. An interested reader may want to read Epstein et al. (1999) and Weintraub et al. (1969). The forest industry generally employs numerous applications of optimization techniques in their operations. Rönqvist (2003) highlights the need for optimization in the forest industry showing eight different areas where it is applied, ranging from the board cutting in the forests to annual harvesting, scheduling, road investments and forest evaluation. Rönqvist (2003) also categorizes the different applications in terms of level of implementation; be it on a strategic level or a planning level.

The concept of modelling a system with the particular characteristics of this master thesis with a mathematical approach has been evolved from the classical travelling salesman problem (TSP) in which one agent, or vehicle, visits a certain set of nodes in such a way that the overall travel route can be minimized. This concept was later enhanced further by Dantzig and Ramzor (1959) to a model often referred to as the Vehicle Routing Problem (VRP) in which case these nodes includes certain demands and the vehicle has certain capacities. Because of the complexity involving how to plan and schedule these routes along with the possible beneficial outcomes of implementing these kinds of problems, many extensions of the classic VRP has been developed. One of the extensions of the VRP is to include time windows which are time intervals when the vehicles can visit certain nodes. This can be used for instance in Just In Time, JIT, planning. This model is called VRPTW, or Vehicle Routing Problem with Time Windows.

Another interesting extension of the VRP which could be used to solve the problem of this master thesis is the VRPTWTS where the VRPTW has been extended to include transshipment nodes, i.e. intermediate stock locations to be used as hubs. PDPTW – the Pick Up and Delivery Problem with Time Windows is another extensions where the vehicles is given the possibility to deduct some subsets of the demands at different supply nodes to be transported to a demand node. Cordeau et al. (2002) portrays a general survey of different VRP models. Bräysy & Gendreau (2005) provide a survey of methods concerning VRPTW.

There exist many evocations in terms of optimization within logistics and supply chain management in the forest industry. The term DSS is often used to describe these types of models. A DSS, or Decision Support System is a system which tries to aid in the planning of the routes – eliminating the need of manually designing them. ASICAM (Epstein et al., 1996) is an early DSS created to ease the planning of logging trucks. It is used by many forest companies in various South American countries. It assigns the drivers a schedule on a daily basis which is created by a simulation based heuristic. The methodology employed does not take the entire day's operations into account and instead optimizes sequentially. It can only consider one day scheduling at a time. Åkarweb (Eriksson and Rönqvist, 2003) is a system which solves potential transport orders on a daily basis which then becomes routes when the transport managers select the actual transport orders. It is based on a linear program backhauling problem. Åkarweb does not produce any schedule but rather determines the destinations of the logs. Gingras et al. (2006) describes a system called MaxTour which is developed for the forest industry in Quebec, Canada. The system establishes routes with predefined loads with already initialized origin-destination pairs. The destinations of the logs are therefore already determined and MaxTour establishes single backhauling routes. MaxTour does not produce any schedule. RuttOpt (Andersson et al., 2008) is another system developed with the aim of reducing transportation costs within the industry. It is carried out in two stages. Initially a pick up and delivery model is used to identify the best way to drive fully loaded trucks. A unified tabu search, adapted from Cordeau et al. (2001) modified to better suit the needs for the article, is then employed to model how to drive the remaining piles in the best manner.
RuttOpt describes a system with much detail. The mathematics behind RuttOpt is described in detail in Flisberg et al. (2009). Forsberg et al. (2005) describes FlowOpt and evaluates the system using some case studies. This is done using an optimization which involves backhauling. The technique to solve the system is to use so-called column generation.

Frisk et al. (2010) performed a survey in the subject of cost allocation models within company cooperation as well as a survey of the benefits of backhauling in the forestry industry. The survey consisted of case studies of different cost allocation methods. The authors also proposed a new method for cost allocation models in the premises of the forestry industry. This new method presented is called EPM. In particular the concepts of Shapley’s value, Nucleous, CGM and EPM had properties suitable for this project. The survey conducted by Löfroth and Svenson (2012) of the use of the ETT trucks concluded that the transportation costs and CO2 emission were reduced by 20% and the need for transportation by 35% when comparing to solely rely on the conventional trucks.

In supply chain management theory it is often considered a prerequisite to include cooperation throughout the whole supply chain if one wishes to employ a pull system, which means that products are produced when it is established that there is a demand for them. Because the pull system rely heavily on the pillar of cooperation it is often considered to be a necessity to provide the companies within the supply chain to have valid information sharing systems. These are often employed through BI (Business intelligence) or IT (Information technology). Some scholars elucidates the importance of these systems being electronic and automatic (Tseung et al., 2012).

7.1 Cost allocation models

This section will describe the concepts and the mathematical models of some cost allocation concepts. The concept of a cost allocation model is to find how big cost or saving each participating company within a cooperation is responsible for or entitled to (depending on if you use the model to calculate the costs or savings respectively). Cost allocations can be calculated in many ways, and it therefore exists some desired properties which can be used to evaluate a specific model. There are several different criterias for a cost allocation model. So many in fact that Frisk et al. (2010) claims that no model actually fulfill them all. There are however some which are more commonly used which will be presented here.

We define a subset of participants from the grand coalition, $N$ to be a coalition $S$ of participants i.e. the grand coalition is the set of all eligible companies to join and participate in a cooperation, and a coalition $S$ is a subset of these. We introduce the coalition because it is not always preferred to allow all eligible participants to join a cooperation. The total cost may for instance become greater if a company joins the participation, while that participant does not contribute any positive net value. Every participant within $N$ is assumed to have the opportunity to form a coalition. When there is a coalition, $S$, can the total cost for that specific coalition be described as $c(S)$. A cost allocation model that splits the total cost $c(N)$ is said to be efficient if each company’s individual cost sums up to the total cost, i.e. if $\sum_{j=N} y_j = c(N)$ where $j \in N$ is a participant of the coalition and $y_j$ is the total cost occuring for participant $j$. The model is said to be individually rational if, for every participant, the inclusion in the coalition implies lower costs than if the participant were independent of the coalition, i.e. if $y_j \leq c_j$. The core is defined as those cost allocation models, $y$ which satisfies the efficiency condition as well as rationality for all the participants within the core, i.e. $\sum_{j \in S} y_j \leq c(S), S \subseteq N$. If one cost allocation model is in the core, we say that the cost allocation is stable.

7.2 The Shapley value

The Shapley value is computed as if every participant in the coalition were to enter it one-by-one. When the participant enters the coalition, the player is assigned a marginal cost, which implies that the total cost of the coalition increases as more participants joins. If the participants where to enter the coalition one at a time randomly, we could calculate the cost allocation as:

$$y_j = \sum_{S \subseteq N \setminus j \in S} \frac{(|S| - 1)!(|N| - |S|)!}{|N|!} [c(S) - c(S - \{j\})]$$

In this equation, $|S|$ denotes the number of participants in the considered coalitions, i.e the amount of elements in the set $S$. The summation is carried out over all coalitions, $S$, which contains participant $j$. The expression $c(S) - c(S - \{j\})$ describes the additional cost which occurs if participant $j$ enters the coalition. The Shapley value is not guaranteed to grant the individual rationality property and as such there is no guarantee that the model meets the criterion of a stable value. It has however been proved to be the only value that satisfies four axioms formulated by Shapley (1953), which will not be presented here.
7.3 The nucleolus

Schmeidler et al. (1969) presented a cost allocation model which is surveyed in the works of Frisk et al. (2010). In describing this model we proceed with the terminology for $N = \{1, 2, ..., n\}$ representing the grand coalition with $n$ players and $S$ being a coalition, where $S \subset N$. We also define $v(S)$, called the value of coalition $S$. By assumption $v(S) = 0$ if $n \leq 2$. The outcome of a game will be a $n$-tuple $x = (x^1, x^2, ..., x^n)$ where each $x_i$ represents the outcome of player $i$. Schmeidler et al. (1996) defines the payoff vector to be a vector where $x_i \geq 0$, $i = 1, 2, ..., n$, and where $x(N) = v(N)$ where $x(S) = \sum_{i \in S} x_i$. We denote $X$ to represent the set of all payoff vectors.

The idea behind the nucleous is to find the payoff vector that is accepted as a compromise between the players within the cooperation. Suppose we have two payoff vectors, $x$ and $y$. Then the procedure of finding the accepted payoff vector is done by computing the so-called excess, which is defined as $v(S) - x(S), S \subset N$ with respect to $x$. This number reflects the "attitude" of coalition $S$ in regards of the proposed payoff vector $x$. If one coalition has a great excess it is interpreted as that coalition being the most reluctant to a suggested payoff vector. If $\max \{v(S) - x(S) \mid S \subset N\} > \max \{v(S) - y(S) \mid S \subset N\}$ we can conclude that the payoff vector represented as $y$ is more accepted than that proposed by $x$.

7.4 Allocations based on separable and non-separable costs

Tijs and Driessen (1986) describe different allocation models based on procedures which differates the separable and non-separable costs for the associated companies in the system. The common methodology for these models is to allocate to each company its separable cost, $m_j$ and subsequently its non-separable cost, $w_j$. The non-separable costs are distributed according to different weights. The definition of these weights differs between different models based on separable and non-separable costs.

The separable costs equals $m_j = c(N) - c(N - \{j\})$, i.e. the marginal cost of participant $j$ upon joining the grand coalition, $N$. Of the models presented by Tijs and Driessen, only the Cost Gap Method, CGM were found stable in the survey conducted by Frisk et al. (2010). In this model the weights, $w_j$ are computed as $w_j = \min_{S, j \in S} g(S)$ where $g(S) = c(S) - \sum_{j \in S} m_j$. The separable cost $m_j$ constitutes a lower bound for the associated cost of participant $j$ when joining the grand coalition.

The amount $m_j + w_j$ can be seen as an upper bound associated to participant $j$’s inclusion into the grand coalition. It is what participant $j$ would pay if all other participants pay their marginal cost in the best coalition $S$. This model assumes that $g(S) \geq 0$, $\forall S$ and $\sum_{j \in N} w_j \geq g(N)$. Hence, models based on separable and non-separable costs distributes the costs according to

$$y_j = m_j + \frac{w_j}{\sum_{i \in N} w_i} g(N)$$

7.5 Equal profit method

Frisk et al. (2010) found some reluctance with these models when presenting these to the companies within the studies. According to Frisk et al. (2010) it would be beneficial in negotiation situations to be able to present a model where the relative cost savings is as similar as possible for the different participants. This led to the development of the equal profit method, EPM. It is defined as:

$$\min \frac{y_i}{c(i)} \quad \text{s.t.} \quad f \geq \frac{y_j}{c(i)} - \frac{y_j}{c(j)} \quad \forall (i, j)$$

$$\sum_{j \in S} y_j \leq c(S), S \subset N$$

$$\sum_{j \in N} y_j = c(N)$$

$$y_i \geq 0, \forall i$$

Where the relative saving of participant $i$ can be expressed as $(c(i) - y_i)/c(i) = 1 - y_i/c_i$. This quantity, while under the assumption that the value is stable, has the property that $c(i) \geq y_i$. Thus we get the difference in relative savings which is shown in the first constraint. The two remaining constraints defines all stable allocations, rendering this model to calculate the allocation which minimizes the difference in relative savings while ensuring that the allocation is stable.

8 The chosen model

This section will describe the model which my own mathematical model will be based upon. This section will describe how to interpret the model in general and how to visualize it. My own model will however be described mathematically in the section Results. The model I will propose will be modified, but the general description in this section will still apply. The model I will present in Results will be based on phase one of Flisberg et al. (2009) – as such phase two will be omitted in this paper. More
on this in Discussion.

The model used as a fundament in this master thesis is the model used in RuttOpt (Andersson et al., 2008), which is a program employed in many case studies in the forestry industry. It is provided and developed by Skogforsk. The model itself is developed by Flisberg et al. (2009). The scope of the model is to minimize the transportation route required in order to provide each company within the system their demand (if it can be met) within some time constraints. It is a two-phased model. The first phase accounts the flow between the supply and demand points for individual trucks during individual days. The model's constraints focus on supply, demand and time availability of the trucks. According to Flisberg et al. (2009) it is a model is more detailed than the traditional models as it involves decisions about individual trucks. The aim of phase one is to form transport nodes which are full truck loads loaded at one or several supply points and subsequently transported to one demand point. It is the solution of phase one which provides the basis from where these nodes can be generated. These will then be used as a basis in phase two, where a route will be created, being connected from the departure from a demand node to the arrival to a demand point. The reasoning behind this is to initially proclaim an initial optimization leaving the "remaining" optimization to become a VRPTW which can be solved using the unified tabu search algorithm (UTSA) developed by Cordaeau et al. (2001). This algorithm is however extended by Flisberg et al. (2009) to enable differences in supply and demand and multiple home bases. Phase two focuses on how to drive the remaining piles of lumber which is left after only the full truckloads from phase one has been undertaken.

8.1 Interpreting the model

A description of the actual routing of a lumber truck is presented in Figure 7. The truck is loaded with the supply to be delivered at the different refinement stations D1-D4 from the supply points S1 through S6. The square H denotes the home location where the vehicle starts and ends the work day. This can either be a company’s location or the personal address of the driver. It can be noted from Figure 7 that the problem can be presented as a pick up and delivery problem as several supply points may be needed to visit in order to fulfill one customer’s demand. In the example portrayed by Figure 7 C1 represents a location where a change of driver is done.

8.2 Supply and assortments

Each harvest area is defined by a geographical node and supplies logs divided into different categories, or assortments presented earlier. Logs of equal or similar assortments are divided into piles in the vicinity of the roads. For the purpose of modelling we define a supply point to be a geographical node which can contain different assortments. The information of a supply point also includes volume of the assortments. The volume parameter will keep track of the volume of the specific assortment, which can be increased due to production at the harvest area, and a reduction caused by transportations to refinement stations. A harvest area is subordinated to two time windows – the first corresponding to the general availability (which is the same for all trucks) and the loader availability. The latter time window is invoked because trucks without a crane depend on a loader to facilitate the loading, which is available during working hours. The general availability of a harvest area is twenty four hours a day.

8.3 Demand and assortment groups

The definition of a demand point is a customer order carried out by a refinement station - for instance a sawmill. The customer order includes an assortment group and a volume which can be divided into different demand points if there are limits on the proportions of different assortments in an order. The demand points are open during specific hours which typically varies from industry to industry. Paper mills, for instance, are typically open 24 hours a day whereas small sawmills may only be open during regular office hours. Figure 8 illustrates how weekly demands can be decomposed into daily minimum and maximum demand.
8.4 Trucks and drivers

The trucks may differ in terms of having a crane or not. The law regulates the total weight of the trucks, i.e. if a crane is added, the loading capacity is lowered. In Sweden the loading capacity of traditional trucks without a crane is 38 tonnes. The loading capacity when a crane is used is reduced by the weight accounted by the crane. According to interviews using a crane lowers the loading capacity with 2.5 tonnes, i.e. a maximum loading capacity of 35.5 tonnes if the use of a crane is adopted. If the truck uses a crane no loader is required at the supply points. As presented earlier refinement companies tend to employ several logistics companies, or hauliers. These may have a single truck at their disposal or a vehicle fleet. The availability of a truck depends on how many drivers that uses that specific truck. For instance a single driver is restricted by law to work a maximum of 10 hours during a day. If the truck is used in shifts of for instance three people it can be used during every hour of the day. In the case of multiple drivers, they change at specified change-over nodes. The vehicles start and end the day’s total driving routes at specific home bases. As a consequence of the points presented in this section, we can conclude that the time required (and costs) differs for loading at the supply points which stipulates the need of separated costs and working hours for the loading / unloading procedure. Unloading is most commonly done through ”industrial unloading”, i.e. that the company that receives the delivery has cranes disposable to conduct the unloading themselves. The working hours for each vehicle is gathered from information from the companies involved in the study.

8.5 Distances and geographical nodes

There exist four types of nodes; the supply points, the demand points, the change-over points and home bases. Due to the mathematical nature of this problem it is of importance to compute both the distances (which will be minimized in the model) and the driving time between all pairs of nodes (which must comply to the working hours of the trucks). For this purpose the Swedish national road database, NVDB, may be used. This database has detailed information of all roads in Sweden, including private roads which are often used to access remote forest areas.

8.6 Objective and costs

The objective is to balance the most efficient route which minimizes the transportation costs while still fulfilling the demand put onto the system. This minimization will be performed to minimize the costs for the entire vehicle fleet. In alignment with Flisberg et al. (2009) the plain transportation costs has been extended to include a set of different costs. This is in order to mimic the practical aspects of the mathematical model to a higher extent. The plain transportation cost is defined as a unit distance cost which differs depending on if the trucks are fully loaded or not. Included in the model is also a cost associated to the working hours, i.e. a cost connected to the time the truck operates. A penalty cost is also associated with the model which will be activated if the demand for some reason could not be met. There are also two bonuses associated with the model; the first is a demand based bonus and the other a supply based bonus. A demand point can specify a bonus value for each ton of logs supplied if it is desired to receive more supplies, if it is possible, than indicated by the lower daily demand. The forest industry stipulates a fast removal of all piles connected to a lumbering area to prevent the material to become ruined. A bonus is implemented for certain supply points. This bonus is however only applied to supply points specified by the planner.
9 Results

As a prelude to this chapter; this section shows an example of an instance with corresponding values of parameters and its optimal solution. The intent is to show that the model is working as intended, and this section is therefore not connected to any reality-based data. The theories of different cost allocation models has already been presented in a separate section and will not considered here to prevent a result chapter stretching several pages of theory.

My proposed model will also be described here, and it will be used in the results of Scenario 1 and Scenario 2.

9.1 Mapping the present

An interview with the CEO of one of the major hauliers to one refinement company revealed additional complexity and information regarding order handling and how the transports are conducted. The additional complexity stems from the roads which are used to gather the harvested lumber. According to the interviewee private roads are used almost exclusively to transport the lumber from lumbering areas to the refinement stations. The availability of these types of roads are restricted and it is up to the committee of the specific road to grant access or to decline transports over the road. The incitement to decline access is mainly due to the roads being too weak, especially during the spring. Problems also arise in the planning of the routes due to lack of information about maintenance. For instance, ploughing the roads during the winter is a necessity for availability.

The haulier provides outsourced logistic solutions to five companies in the county. Typically an order is presented by the refinement company and the outsourced logistic companies then decide how to complete this order. For this purpose a mathematical optimization model is used to plan the routing. The driving route is then e-mailed to the specific vehicle(s). The recipient has access to a computer in the truck. It is also common that the recipient uses personal cell-phones or home computers to access the driving schedule. Cell phones are widely used to aid in the information sharing between the logistic company and the vehicles.

The interviewee also mentioned that they practice lumber exchanges with other logistic companies. These are carried out in an informal matter; it is based on knowledge about how and where other companies operates. It is based on personal relationships. The information forum used is cellular phone contacts. According to the interviewee, using cellular phones and e-mail correspondence is sufficient.

The interviewee stressed that the logistic companies are subjected to a highly dynamic industry. For them, the most beneficial information system would be a system which rapidly updates road conditions and if specific roads has been ploughed etc. The interview revealed how sensitive logistics companies are to external conditions, and how these causes distress in the routes. Delays coupled with daily time restrictions on the drivers enforced by the law, can cause huge problems. It can for instance cause high complications when two people shares the same vehicle and works in shifts.

9.2 The proposed model

The variables used in the model are:

\[ x_{ijvt} \] Flow of assortment \( a \) from supply point \( i \) to demand point \( j \) using vehicle \( v \) at the end of period \( t \)

\[ l_{ita} \] Storage of assortment \( a \) at supply point \( i \) at the end of period \( t \)

\[ h^+ \] Total time to perform all transportations available

\[ s_{jta} \] Volume of not fulfilled demand of assortment \( a \) at demand point \( j \) at the end of period \( t \), i.e. demand not met at \( t \).

The sets and parameters of the mathematical model are defined as:

\[ V \] The set of vehicles
$T$ The set of time periods

$I$ The set of supply points

$J$ The set of demand points

$A$ The set of assortments

$L$ A set of ordered pairs $\langle i, j \rangle, i \in I, j \in J$ where each ordered pair describe the relationship between a supply point $i$ and the demand point $j$ which has the supply point contracted

$W^+$ The set of all combinations of ordered pairs, $W^+ = I \times J$, where $\times$ is the Cartesian product

$R$ The set of all ordered pairs which are present in $W^+$, but not in $L$, i.e. $R = W^+ \setminus L$, where $\setminus$ is the set difference

$s_{ita}^+$ Additional supply of assortment $a$ at supply point $i$ provided during time period $t$

$d_{jta}^–$ Accumulated lower demand of assortment $a$ at demand point $j$ in period $t$

$d_{jta}^+$ Accumulated upper demand of assortment $a$ at demand point $j$ in period $t$

$c_{ijv}$ Unit transportation cost between point $i$ and $j$ using vehicle $v$

$u_i$ Bonus for loading 1 ton at supply point $i$

$v_j$ Bonus for delivering 1 ton at demand point $j$

$p_{ij}$ Unit transportation (and loading/unloading) time between supply point $i$ and demand point $j$

$h_{vt}$ % of total transportation time ($h^+$) vehicle $v$ is allowed to utilize in period $t$

$M_{j,t}$ Penalty for each ton of unfulfilled demand at demand point $j$ in period $t$

$\epsilon$ A constant value which determines how much a company may deduce from other companies in relation to how much the company supplies.

$cap_v$ The maximum loading capacity of vehicle $v$
The mathematical model created is:

\[
\begin{align*}
\text{min} & \quad z = \sum_{t \in T} \sum_{v \in V} \sum_{a \in A} \sum_{(i,j) \in A^+} \left( c_{ijv} - u_i - v_j \right) x_{ijvta} + \sum_{j \in J} \sum_{t \in T} M_{jt} s_{jt} \\
\text{s.t.} & \quad l_{i,t-1,a} + s_{it}^+ - \sum_{v \in V} \sum_{a \in A} x_{ijvta} = l_{ita}, \quad \forall i \in I, t \in T, a \in A \\
& \quad \sum_{v \in V} \sum_{p \in T} \sum_{p \leq t} \sum_{a \in A} \sum_{(i,j) \in W^+} x_{ijvpa} + s_{jt} = d_{jt}^+, \quad \forall j \in J, t \in T \\
& \quad \sum_{v \in V} \sum_{p \in T} \sum_{p \leq t} \sum_{a \in A} \sum_{(i,j) \in W^+} x_{ijvpa} \leq d_{jt}^+, \quad \forall j \in J, t \in T \\
& \quad \sum_{t \in T} \sum_{v \in V} \sum_{a \in A} \sum_{(i,j) \in W^+} p_{ij} x_{ijvta} \leq h^+, \quad \forall v \in V, t \in T \\
& \quad \sum_{(i,j) \in W^+} \sum_{a \in A} x_{ijvta} \leq cap_v, \quad \forall v \in V, t \in T \\
& \quad l_{ita} \geq \sum_{v \in V (k,l) \in R} \sum_{g \in T} \sum_{g \leq t} x_{kjvga} - \sum_{v \in V (i,j) \in L} x_{ijvta} + \epsilon, \forall \forall l \in T, j \in J, a \in A \\
& \quad l_{it} \geq 0, \forall i \in I, t \in T \\
& \quad x_{ijvta} \geq 0, \forall (i,j) \in A^+, v \in V, t \in T, a \in A \\
& \quad s_{jt} \geq 0, \forall j \in J, t \in T \\
\end{align*}
\]  

Constraint (2) models that the stock of an assortment \( a \) at a supply point \( i \) at time period \( t \) is the stock level for that specific assortment at the end of the previous time period in addition with the "net flow" in and out from the supply point at time \( t \).

Constraint (3) describes that the total volume transported of assortment \( a \) must equal time step \( t \)'s lower demand limit. If it does not, the difference is tracked as \( s_{jt} \) which will activate a penalty in the objective function \( z \).

By analogue constraint (4) emphasises that it is not allowed to deliver more lumber to demand point \( j \) than the upper demand limit permits.

Constraint (5) and (6) provides the time capacity for each truck and time period. These constraints forces all vehicles to perform similar work loads in relation to the total time available for the tasks. In particular (5) tells that the summation of all time used individually for all vehicles must sum to the total time available. Constraint (6) is closely related to (5) but on a more individual level. It claims that every vehicle’s work load at every time period must not exceed the % of the total workload assigned to vehicle \( v \) at time \( t \).

The constraints added to this model in comparison to the one provided by Flisberg et al. (2009) are constraint (7) and constraint (8). Constraint (7) describes that for every timestep \( t \) each vehicle \( v \) must not exceed the vehicle’s capacity limit, \( cap_v \). Constraint (8) describes that a company must be able to provide the system with at least the total amount it has deducted from the other companies for a specific assortment \( a \) up to and including time period \( t \). The addition of \( \epsilon \) in the added constraint is to prevent a company from deducting lumber from other companies’ harvesting areas, whilst only providing the exact same amount from its own harvesting areas. The remaining constraints (9), (10) and (11) are the non negativity constraints on the system. The model presented aims to minimize the total transportation costs for the total route during the entire time period and for all vehicles. This is presented in (1) which also "benefits" the objective function by subtracting a bonus for loading and unloading at specific supply and demand points respectively.
9.3 Testing the model

This section will test the model with the sets:

\[ T = \{ 1, 2 \} \]
\[ V = \{ 1, 2 \} \]
\[ V = \{ 1, 2 \} \]
\[ I = \{ 3, 4 \ldots 7 \} \]
\[ J = \{ 1, 2 \} \]
\[ A = \{ 1 \} \]
\[ L = \{ (2, 1), (3, 1), (4, 1), (5, 2), (6, 2), (7, 2) \} \]
\[ A^+ = I \times J \]
\[ R = \{ (2, 2)(3, 2)(4, 2)(5, 1)(6, 1)(7, 1) \} \]

and the parameters:

\[
s^T_{it} = 0, \forall i \in I, t \in T
\]
\[
d^+_{jt} = 1, \forall j \in J, t \in T
\]
\[
d^+_t = 3, \forall j \in J, t \in T
\]
\[
h_{vt} = 0, 3, \forall v \in V, t \in T
\]
\[
l_{0a} = 5, \forall i \in I, a \in A
\]
\[
M = 5000
\]
\[
cap_v = 5, \forall v \in V
\]
\[
\epsilon = 1
\]

In this trivial example there are only two demand nodes which (for simplicity) is a separate company. Therefore \( j \) will be sometimes be described as a company rather than a demand point in the following sections, in order to portray the model in a more practical manner.

9.3.1 Scenario 1

Using the set \( L = A^+ \), in order to avoid a situation where a company uses other companies’ contracted lumbering areas, the optimal value becomes \( z = 330 \). The time for the drivers to complete this route is \( h^+ = 3 \). The optimal route is shown in Figure 9.

In accordance with Scenario 1’s problem description it can be seen from Figure 9 that this optimal solution does not use other companies’ contracted lumbering areas – each node \( ij \) (in the figure) delivers to its corresponding company \( j \). In the solution it can be seen that both company 1 and 2 utilizes a full truckload from its closest valid harvest area of 3 units, and subsequently a load of 2 units from the same harvest area. Due to each supply point \( ij \) having an initial supply of 5 units, the program then chooses to utilize the remaining one unit in order to fulfill the company demand from its closest valid harvesting area.
9.3.2 Scenario 2

Using the model with the test data using the actual $A$ and $W^+$, the corresponding optimal value is $z = 221$. The time required by the truck drivers is $h^+ = 2.4$ time units to complete the route.

From Figure 10 it can be seen that if we allow deduction of lumber from harvest areas contracted by other companies, it is possible to achieve a better solution. From Figure 10 it can be seen that both companies utilizes the focal company’s cheapest node to the fullest – company 2 takes 4 units in total from its closest node 3 and company 1 takes a total of 5 units from its closest node 7 as these nodes are valid in this scenario. The reason why company 2 utilizes node 6 instead of emptying node 3 is because of the objective function, which is written in such a way that the optimization program optimizes the system. Therefore, when node 7 is empty it is better for the system to allow company 1 to use its own node 3 instead of the second best alternative node 4. Hence, company 2 must deduce two units from 6 to meet its own demand, and to not empty 3 in order for company 1 to meet their demand.

In order to test whether constraint (8) worked as intended, the initial values of $l_{i6a}$ were adjusted so that $l_{i6a} = 0$ for $i = \{6, 7\}, a = \{1\}$ and $l_{50a} = 3$ for $a = \{1\}$ ceteris paribus. The corresponding optimal value is $z = 5750$ and the required time for the truck drivers to perform the routes is $h^+ = 2.2$.

From Figure 11 it can be concluded that the total amount of flow to company 2 were 5 units in $t = 1$ and 3 in $t = 2$. This caused the variable $s_{21}$ to be activated, showing that the demand in $t = 1$ were unfulfilled by one unit for company 2, which in turn caused a penalty of $M = 5000$ in the objective function. From Figure 11 it can also be seen that company $j = 2$ utilized its closest valid node $i = 3$ to the fullest by deducing 2 units in $t = 1$. The reason why company 2 could not deduce more than 2 units from node 3 is due to constraint (8), as the total supply of company $j = 2$ were 3 units, and constraint (8) requires that a company has at least $\epsilon = 1$ more unit in its supplies than the total amount deduced from other companies. Company 1 utilized its closest node 3, while still allowing company 2 to deduce its maximum amount of units. When node 3 were depleted, company 1 utilized its next-closest valid node 4 in order to fulfill the demand in $t = 2$.

9.3.3 Scenario 3

As pointed out by Löfroth and Svenson (2012) the transportation costs were reduced by 20 %, the $CO_2$ emissions by 20 % and the need for transportation by 35 % (as opposed to if the same volume were to be transported solely by conventional trucks). One can model the inclusion of the ETT trucks using the proposed model by simply redefining the preset parameter $cap_v$ corresponding to the capacity of the trucks.
Figure 10: The optimal transportation nodes of Scenario 2 shows the optimal route from the nodes \( ij \) to company \( j \). Red arrows represent vehicle 1 and blue vehicle 2. The supply points are represented as \( ij \) for easier view.

Figure 11: The optimal transportation nodes of Scenario 2 where the initial supply has been adjusted in order to obtain a solution whose optimal solution does not fulfill constraint (8). Red arrows represent vehicle 1 and blue vehicle 2. The supply points are represented as \( ij \) for easier view.

10 Discussion

Much emphasis throughout this project has been on receiving the required data. Initially the data collection was vastly delayed rendering actual data to be impossible to use due to time limits. Instead the intention was to use data from a different optimization conducted by SLU. This data were not actual data but rather estimations over some small lumber areas’ harvest capacities. These capacities were then assumed to be homogeneous for a bigger area.

The data itself were not in alignment with this particular project. It showed total capacities, without consideration for different dimensions in the different lumbering areas. The data also showed lumbering areas and refinement stations over a much bigger area than this project intended along with data spanning a year. For this, I had to narrow the data down to only represent lumbering areas and refinement stations within a 12.5 metric miles radius. Estimations over the dimensions were collected from the different companies (i.e. how many percent of the total production is typically credited to a certain dimension). These parts were done using Python. Python were also used to assign each company a set of lumbering areas. This was done by shuffling a list of indices in a random order and subsequently assign a subset of the indices to the different companies. How many lumbering areas a company were given
The answer is no, a more formal cooperation ought to provide better results.

Is the system optimized taking the findings in identifying the proposed model, answers Scenario 1; Is the system optimized taking the findings in identifying the present into account?

What proved to be impossible was to find a plausible distribution of the different dimensions at the lumbering areas. This was due to the companies not having any general perceptions regarding approximately how many percentages that represented a particular dimension in some arbitrary lumbering area, and the lack of data which did not show lumber already adapted. The latter implicated that it did not exist any data suitable for my need to be found in any database. I did not want a situation were I simply used the weights given for the dimensions and claim that every lumbering area would carry all dimensions in some even distribution. It is commonly known that certain dimensions are almost nonexistent in some lumbering areas, even though they may be quite common in other areas. I tried to simulate this phenomenon under the assumption that the distribution of trees would converge in probability to a normal distribution, under the Law of big numbers. This was not however possible, due to no data to adjust the parameters to (to find a suitable estimated mean value, \( \mu \) or a estimated variance, \( \sigma^2 \), or to validate the assumption of normal distribution). I also tried to create a vector for this purpose using integer partitioning, i.e. a program which finds all combinations (terms) to receive a specific sum. This however proved to be very complex. When I tested with much smaller instances of integer partitioning, I still got a program which continued to calculate for several hours without any result. It would also cause disambiguity due to that I essentially had to choose which one of hundred different combinations would be most suited for the instance. I also tried to create two integer programs whose purpose were to minimize and maximize the difference between the harvest area with the highest amount of supply and the lowest. The intentions were then to interpolate these results. The implementation however only assumed trivial optimal solutions without a constraint which could provide a plausible distribution of the different dimensions, essentially rendering me to be back at square one.

My findings suggests that the industry must be able to track lumber from different areas using pre-adapted data. In order to capture the most benefit from a virtual transshipment hub this must be considered a necessity. The reason is that adapted lumber cannot, generally, be matched with the demand of other companies. Using pre-adapted could also induce more possibilities to keep company secrets (such as adaption schemes). The adaption itself is optimized and carried out in matter of seconds to comply to the company needs, of whomever uses the harvest area be it the contracted company or some other. From this, emphasis must be placed upon a solid supply chain management to maintain a good system as well as information sharing system to manage the harvesters in order to create the most beneficial adaptions.

Interviews with managers and other responsible employees at the different refinement stations has throughout the project given a picture of the industry where the companies has a limited cooperation with each other. The companies were reluctant to share too much information between each other. Information which could reveal any strategic planning were especially sensitive. For instance, information concerning the planning regarding where to have harvest areas and how well these produced and how the companies adapted their lumber as well as the distribution of this adaption scheme were not shared. The interviews also gave an indication of the companies in the industry being segregated. Cooperations are practiced in the industry, but it seems that it is not something integrated at a strategic level. Instead, it comprises of actions undertaken due to personal knowledge of other companies’ routes. Due to the cooperations being on personal levels, it is not practiced between all companies. Instead there are clusters of companies “eligible” to induce bartering of lumber between eachother. This reveals an industry which is not operating under optimized conditions, as the companies “eligible” to cooperate with eachother depends on mutual contacts and past experiances, rather than mathematical calculations (such as for instance the Shapley value to calculate which companies would benefit from a cooperation). This, along with the result of Scenario 1 showing that all else equal the model still yields a better solution using the proposed model, answers Scenario 1; Is the system optimized taking the findings in identifying the present into account? The answer is no, a more formal cooperation ought to provide better results.
The proposed model ought to yield a cost saving, but not as high as the saving presented by Flisberg et al. (2009). The reasoning behind that savings should occur is due to a more formal bartering system. The reason why it is assumed that the savings would not be as good as in the model provided by Flisberg et al. (2009) using the same data sets is due to the introduction of constraint (8) and the very nature of the integer program. The integer program itself will be relaxed in order to be solved, causing the feasible set to become a feasible region which has been stated to always hold the convexity property, according to Theorem 5.2. If the optimal solution provided by the model of Flisberg et al. (2009) would produce an integer solution which would be infeasible in the extended model proposed by me, we can state that at least one of the initial set of constraints has become redundant in the new description. This would mean a lesser numerical optimal solution in the new model in comparison to the original. If the optimal solutions coincides, the new constraints does not dominate the original constraints rendering the solution to take the same optimal solution. As a conseuence, adding constraint (8) will either cause the model to receive a lesser optimal solution or the same. It has the possibility of creating a smaller feasible region and thus not allow as many possible routes as without these constraint (8). Keep in mind though that Flisberg et al. (2009) did not show any numerical tests for their phase 1 model exclusively, so there are no quantitative results to compare. Scenario 2: *By lifting the restrictions of how the lumbering areas is contracted to different companies, is there any room for improvements in the system?* can therefore be concluded that, yes, there are room for improvements using an virtual transhipment hub, looking at the results for Scenario 2 but the optimal value can never be better than those presented by Flisberg et al. (2009).

The inclusion of the ETT trucks seems to be very efficient, and from the empirical study referenced in this paper, it seems that the additional capacity of the vehicles exceeds the cost savings achieved by a higher extent of cooperation. An actual attempt to use my proposed model with the ETT rigs is merely to define the parameter for cap accordingly. To conclude Scenario 3; I deem the implementation of ETT rigs to provide much better results in terms of reduced costs as well as in terms of environmental distress.

The choice of cost allocation model boils down to politics and preferences between the different companies. As pointed out by Frisk et al. (2010), the most "obvious" choice of model would be EPM as it provides the companies the same incitament to perform well, as well as easier negotiations. However, due to the reluctance to have too much cooperation, different companies may feel entitled to higher relative savings, due to perhaps company size, monetary or knowledgebased investments into the creation of the system. This could stipulate the use of different cost allocation models. The nucelous' strength is that it can be "customized" in order to capture discrepancies from models where savings are distributed evenly. This is because it is based on the best suited comprimize. Problems naturally occurs, however, if the associated companies has high gaps in terms of entitlement to the cost savings, rendering no compromise to be in alignment with many companies.

Due to the current nature in the industry, it may be desirable to create more than one virtual transhipment hub. The interviews revealed that certain companies does not cooperate with others. One reason behind this may be that the companies feel that they do not receive any benefits from a cooperation with eachother. For this purpose, an initial survey can be conducted using the Shapley value to deem which companies would be eligible to be included in a system and subsequently some other model to determine the actual cost allocation for those companies.

Using the Shapley value by itself can also be done, but as stated in the theory it does not guarantee a stable model which in practice means that the system could yield a higher saving by omitting some companies, or that some companies could yield a higher saving operating on its own.
11 References


