Fluid Structure Interaction
Modeling of the Dynamic of a Semi Submerged Buoy

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Abstract:
This thesis presents a study of buoy systems for wave’s energy by focusing on the development of a model in which modeling of a wave energy conversation is in operation. Throughout the thesis, the buoyancy and motion of the submerged body has been used to describe the wave-buoy interaction. The mathematical model for an investigating buoyancy and the dynamic heave response of this buoy under the two different load cases be considered as a single degree of freedom, which have natural characteristic, mass (M), stiffness (K) and damper (C) of the oscillating buoy which were estimated from the numerical method, because it is very important in the ocean wave stage and essential for the design of offshore installation. By using the Finite Element analysis in the software COMSOL Multiphysics 4.2.a, the fluid structure interactions established on two dimensional potential flow theories and linear are simulated and modeled. Then compared the numerical results which are obtained by transient harmonic oscillations with these of different geometric.

Comparison the analytical results and experimental system lent validity to the numerical models.

Keywords
COMSOL Multiphysics, wave energy converter, Dynamic characteristic, Submerged body, Finite Element method (FE), SDOF, ALE,
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Notations

\( c_a, \) Added Damping [Ns/m]

\( c \) Damping of a System [Ns/m]

\( E \) Modulus of Elasticity (Young’s Modulus) [Pa]

\( f_{ext} \) User defined Free Energy [J/m^3]

\( g \) Gravitational Force [9.81 N/m^2]

\( j \) Imaginary Unit \([j=\sqrt{-1}]\)

\( k_b \) Buoyancy Stiffness [N/m]

\( k \) Stiffness of the System [N/m]

\( m_a \) Added Mass [kg]

\( m_b \) Mass of the Floating Buoy [kg]

\( m \) Mass of the System [kg]

\( P \) Pressure [N/m^2]

\( p_o \) Atmospheric Pressure [N/m^2]

\( s \) Complex Variable

\( v \) Poisson’s ratio

\( X \) Mobility Tuning Parameter

\( \gamma \) Mobility \([m^2.s/kg]\)

\( \zeta \) Damping Ratio \([\zeta = \frac{c}{2} \sqrt{mk}]\)
\( \eta \) Dynamic Viscosity [Ns/m²]
\( \rho \) Density of Fluid [kg/m³]
\( \sigma \) Surface Tension Coefficient in Fluid Dynamic
\( \omega \) Angular Frequency [rad/s]
\( \theta_{\omega} \) Contact Angle [rad]
\( \varnothing \) Dependent Variable, the Phase field Variable
\( \Psi \) Dependent Variable, the Phase field Help Variable
\( \lambda_{mf} \) Mean Free Path [m]
\( \varepsilon \) Parameter Controlling Interface Thickness, Stress Tensor in Structural Mechanics
\( \varepsilon_r \) Relative Permittivity

**Abbreviations**

FSI Fluid Structure Interaction
FEM Finite Element Method
ALE Arbitrary Lagrangian – Eulerian
SDOF Single Degree of Freedom System
CFD Computational Fluid Dynamics
CSD Computational Structural Dynamics
1. **Introduction**

The present moment, there is an intense interest to be working in the field of wave energy research. Due to increasing energy demand in the world at an alarming rate and production of electricity from renewable energy sources which is becoming an important in the idea of the development of this technology to become a major player in the energy industry. Many countries are setting ambitious targets in this topic. This growing enthusiasm can be seen in several forms, such as attendance of wave energy conferences, academic research funding and political support. With those support of mechanisms, the wave energy technologies has development. Therefore it is very important to find ways to harness energy from sustainable and clean sources.

Wave energy has been under investigation by researchers for decades and continues to be challenging problem. There are many forms of renewable energy system around the world, such as wind mills, ocean energy, tidal and offshore wind, nuclear power and solar power.

Robertson [In the late 1860’s] discovered the first wave power boat by harnessing the power of the wave, also inventors and engineers have focused on wave powered technologies over the last century. Their studies are important to understand this resource to extract its energy to produce electricity.

1.1 **Background**

This thesis “Fluid Structure Interaction Modeling of the Dynamic of a Semi Submerged Buoy” is an extended of the previous work, in which the floating buoy has a stable mass [9]. The mechanical of single degree of freedom model was present the hydrodynamic parameters which was estimated from the numerical model. In this work, COMSOL Multiphysics software was used to simplify the Finite Element (FE) and numerical single degree of freedom (SDOF) basic on the knowledge of the dynamic of the submerged buoy. The basic for the linearized forces which lets superposition of exciting waves, added damping, added mass, and restoring force was established by the linear theory [9, 10, 11].

[8]
To formulate the mathematical model of the buoy, assumptions and approximation were involved; therefore the model was establish and assessed by many ways of validation against its analytical model. A complex phenomenon which happened in Fluid Structure Interaction (FSI) analysis was the coupling between the structural dynamics and the fluid dynamics. Both of the computational structural dynamics (CSD) and computational fluid dynamics (CFD) must couple through the governing equations of interaction between the floating buoy and the water. First must be created the governing equations for the water in the absence of the floating. Stress-Strain application and strain-displacement from structural mechanics were used to define the floating buoy movement. The model equations of the fluid were described by generalized Navier Stokes equation which solved both of velocity and pressure. The traction and the displacement compatibility should be specified to resolve the coupling of the buoy-fluid interface in order to ensure the equilibrium.

1.2 **Aim and Objective**

The aim of this work is to model the dynamic characteristic parameter of the submerged buoy with the variables m (Mass), c (Damping) and k (Stiffness). On the basic of this linear theory, single degree of freedom (SDOF) mathematical models of the floating buoy are formed to demonstrate the motion of oscillation. The main section of its objective to achieve a mathematical representation of the semi submerged cubic buoy which was formed upon single degree of freedom with it is characteristic, mass (M), stiffness (K) and damper (C) of the oscillating buoy which were estimated from the numerical method solved by carrying out the Finite Element analysis in the software COMSOL Multiphysics 4.2.a. Comparison with numerical models lent validity to the experimental and analytical results which are investigated from the work.
1.3 Scope

The scope of this thesis is to present the model of the dynamic analysis of the buoy oscillating in finite water depth to a single mode of the oscillation, and dealing with the coupling effect of the submerged floating buoy. The numerical models and the experimental investigation are present here with two geometries with the presumption of having different masses. The added mass and added damping coefficients are obtain from determining the dynamic pressure and energy relations, but the buoyancy stiffness was separately assessed in relation to hydrostatic pressure. On the fundamental of linear theory, the single degree of freedom model of the planar oscillating buoy is design to demonstrate the vertical oscillation in the water of constant depth. From the motion of a submerged body and the modeling of the buoyancy, SDOF model is validated against the experimental results and analytical model [9].
2 Theoretical and Basic Notations

2.1 Scope Conceptual

This thesis is an extension to find a method to predict the floating buoy motion and find a simulation model. It is important to know the wave of the force coefficients, the added damping coefficients and the added mass coefficients to predict that motion of the floating buoy which are subjected ocean waves. By the SDOF system, the heave motion of the floating cube buoy in fluid domain with the constant depth water can be modeled. The computational fluid-structure interaction (FSI) analysis was detailed in relation with software COMSOL Multiphysics. With the accommodation of the Arbitrary Lagrangian-Eulerian (ALE), the problems due to exact delineation and deformations form the general moving mesh method are find out. The FSI predefined Multiphysics included a continuous application mode Stress-Strain, Laminar, Two-Phase Flow, Phase Field (tpf), and Moving Mesh (ale). The problems which are dealing with (FSI) requires coupling of Computational Structural (CSD) and Computational Fluid Dynamics (CFD). In several industrial applications, (FSI) plays a very significant role such as biomedical, nuclear engineering, material processing and automotive etc. The coupling between the structure and the fluid requires sharing of the boundary condition between them. The coupled fluid-structure governing equations can be resolved by using COMSOL Multiphysics solver [12].

2.2 Buoy Dynamics

The figure (2.1) below shows the description of the motion of a submerged buoy when been excited by surface wave; due to that excitation the buoy experiences six degrees of motion; two for each direction rotation and translation, as:

X direction: roll as rotation and surge as translation
Y direction: Pitch as rotation and sway as translation
Z direction: Yaw as rotation and heave as translation
Due to the heave motion is necessary with regard to the oscillations of the buoy; the buoy motion will investigate only in the z-direction.

### 2.2.1 Hydrodynamic Issues

The assumptions taken for the governing equation are:

1. The oscillation is only in heave direction.
2. The friction losses and viscosity been ignored.

Hydrodynamic loads are the classifier of the hydrodynamic problems [13]:

I. The incident regular waves restrained the structure from oscillation while the excitation load waves cause the oscillation.
II. The hydrodynamics loads are defined as added damping, added mass and restoration terms.

The added damping and mass were due to the FSI states, where the added mass was described by a matrix representing the modeling of interaction between the structure and the surround fluids, which that surround fluid increase the effective mass of the structure where the added mass is equal to that fluid displaced mass, taking in consideration that the added mass term assume for an ideal fluid independent on the viscosity. [5]

2.2.2 Boundary Conditions

Pressure fields in the fluid are caused due to the oscillation of the structure which would form outgoing waves, under the absence of incident waves where the oscillation assumed to be formed due to that forced harmonic heave motion of the floating buoy.
Where:

\( \Gamma_0 \): The boundary condition of the linearized free surface.

\( \Gamma_{\text{Bouy}} \): The buoy surface.

The boundary conditions \((\Gamma)\) were specified as \([14,17,19,20]\):

1. The boundary condition \( \Gamma_0 \) free-surface are consider to be linear

\[
\frac{\partial \phi}{\partial z} = k \phi
\]  

(2.1)

Where \([k]\) represent the number of wave, which has the equation of \( k = \frac{\omega^2}{g} \) [20]. The water pressure on the free surface was assumed to be equal to the atmospheric pressure \( p_0 \) to satisfy the dynamic free-surface condition.

\[
p = p_0
\]  

(2.2)

Velocity potential which is satisfying the Laplace equation for the fluid domain is specified by determining the initial conditions and boundary conditions.

2. At the surface, the normal vector for the forced heave velocity and the fluid velocity are required to be equal, figure (2.2).

\[
\frac{\partial \vec{\phi}}{\partial n} = \sum_{k=1}^{6} \hat{u}_k n_k
\]  

(2.3)

\[
\frac{\partial \vec{\phi}}{\partial n} = 0 \text{ and } \frac{\partial \vec{\phi}}{\partial n} = u_n
\]  

(2.4)

Where:

\( u \): The velocity of the buoy.

\( u_n \): The normal vector component of velocity of this buoy.

\( n \): The normal vector for the wetted surface directed in the \( z \)-direction.
3. On the bottom at depth \( h \), the normal vector of the fluid velocity is equal to zero considering the fluid as an ideal.

\[
\left[ \frac{\partial \phi}{\partial z} \right]_{z=-h} = 0 \tag{2.5}
\]

4. At infinite distance, the radiations of the leaving wave were considered to be as:

\[
\frac{\partial \phi}{\partial z} \pm jk\phi \rightarrow 0 \text{ as } r \rightarrow \pm \infty \tag{2.6}
\]

### 2.2.3 Added Mass and Damping

By determining the dynamic pressure after solving the potential velocity, the added mass can be obtained. Ignoring the hydrostatics pressure and integrating the remaining pressure covered the buoy; the vertical force should be as following [14]:

\[
F_z = -m_a \frac{d^2z}{dt^2} - c_a \frac{dz}{dt} \tag{2.7}
\]

Where:

- \( F_z \): The force which is obtained from integration of the linear pressure.
- \( m_a \): The added mass in the heave mode.
- \( Z \): The displacement in the heave direction.
- \( C_a \): The coefficient of the heave damping.

Damping coefficient can be obtained from the energy relations. Since the damping is related to the energy, so the total energy written as summation of potential and kinetic energy [14].

\[
E(t) = \rho \iiint_{\Omega} \left( \frac{1}{2}V^2 + gz \right) dt \tag{2.8}
\]

The derivative of the total energy over the time
\[ \frac{dE(T)}{dt} = -\rho \int_{\text{Sur}} \left( \frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial n} - \frac{p - p_0}{\rho} + \frac{\partial \phi}{\partial t} \right) U_n ds \]  

(2.9)

Where:

**E**: The total energy for the fluid volume \([\Omega]\).

\(\Omega\): The volume integration.

**Sur**: The boundary surface of the fluid volume.

\(\frac{\partial}{\partial n}\): The derivation on along the unit normal vector with the surface boundary.

\(P_0\): The atmospheric pressure.

**Un**: The normal component of the mean velocity for the surface boundary.

With respect to the above boundary conditions for this problem, which consisted of the wetted buoy surface \(\text{Sur}_b\), the \(\text{Sur}\) is bounded from \(-\infty\) to \(+\infty\), the derivative of the total energy should be as following:

\[ \frac{dE}{dt} = -\int_{\text{Sur}_b} (p - p_0) U_n ds - \rho \int_{\text{Sur}} \frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial n} ds \]  

(2.10)

\[ \int_{\text{Sur}_b} (p - p_0) U_n ds = \frac{dz}{dt} \int_{\text{Sur}_b} (p - p_0) z ds \]  

(2.11)

\[ \int_{\text{Sur}_b} (p - p_0) U_n ds = \frac{dz}{dt} \left( m_a \frac{d^2 z}{dt^2} + c_a \frac{dz}{dt} + k_b z - \rho g V \right) \]  

(2.12)

The two terms \([k_b z]\) and \([\rho g V]\) originated due to the hydrostatic pressure. Where the \([k_b]\) represented the change in the buoyancy due to the vertical movement. The fluid volume displacement at \(z=0\) was specified for \([V]\).

Integrated the total energy over the oscillation period \([T]\), the average energy can be obtained as:

\[ \int_{0}^{\tau} \frac{dE}{dt} dt = 0 \]  

(2.13)
\[
\int_0^T dt \left[ \int_{\text{Surf}} (p - p_0)U_n ds \right] = \int_0^T dz \left( m_a \frac{d^2z}{dt^2} + c_a \frac{dz}{dt} + k_b z - \rho g V \right)
\]  
(2.14)

2.3 FSI Consideration

Fluid-structure interactions (FSI) could be stable or oscillatory. The big idea in the FSI analysis was determine damping terms and added mass in the mechanical system oscillating in the liquid. The hydrodynamic added damping and damping mass is the harmonic oscillation of the floating buoy generated waves which make dynamic pressure around the buoy to volatile resulting in moments and forces exerting on that submerged part of the floating buoy. In this thesis, the hydrodynamic added damping and added mass in heave mode of the oscillation basis on the numerical method. Through the Fluid-structure interaction (FSI) application mode in this MEMS module, the simulation and modeling are carried out by using COMSOL Multiphysics. The (FSI) application included a continuum application mode Stress-Strain, Two- Laminar, Tow-Phase Flow, Phase Field (tpf), and Arbitrary Lagrangain-Eulerain (ALE) Physics Application.

2.3.1 Laminar, Two-Phase Flow, Phase Field (tpf) Physics Application

Using the Navier Stokes equation, equation of the continuity and moment transport equation, the model equations of fluid are obtained to solve the velocity and pressure [18]:

\[
\rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla) u = \nabla [-p + \eta \{\nabla u + (\nabla u)^T\}] + F_g + F_{st} + F_{ext} + F
\]  
(2.15)

This means that:

\[
\nabla \cdot u = 0
\]  
(2.16)

Where:

\(\rho\): Fluid Density in [kg/m\(^3\)]

\(u\): Fluid Velocity in [m/s]

[17]
\( \eta \): Dynamic Viscosity in \([N.s/m^2]\)

\( F \): Body Force in \([N]\)

\( F_g \): Gravity Force in \([N]\)

\( F_{st} \): Surface Tension Force in \([N]\)

\( F_{ext} \): Force due to External contribution.

The gravity force defined as:

\[
F_g = \rho \cdot g
\]  
(2.17)

Surface tension force \( F_{st} \) is suitably considered as a body force which is defined as:

\[
F_{st} = G \cdot \nabla \cdot \phi
\]  
(2.18)

But the force due to an external contribution \( F_{ext} \) to the free energy:

\[
F_{ext} = \left( \frac{\partial f_{ext}}{\partial \phi} \right) \nabla \phi
\]  
(2.19)

Basis on the density of the fluid 1 \((\rho_1)\) and the fluid 2\((\rho_2)\), the density of the fluid \((\rho)\) is calculated as:

\[
\rho = \rho_1 + (\rho_2 - \rho_1) \cdot V_f
\]  
(2.20)

Also, the dynamic viscosity \((\eta_1)\) of the fluid 1 \((\eta_1)\) and the fluid 2\((\eta_2)\), the dynamic viscosity of the fluid \((\eta)\) is calculated as:

\[
\eta = \eta_1 + (\eta_1 + \eta_2) \cdot V_f
\]  
(2.21)

Where:

\( V_f \): The volume fraction of fluid 2.

The volume fraction of fluid 2 can calculate as:

\[
V_f = \min \left\{ \max \left\{ \left( \frac{1 + \phi}{2} \right), 0 \right\}, 1 \right\} \quad \text{where} \quad 0 < V_f < 1
\]  
(2.22)
According to [11], the $F_{\text{ext}} = 0$, so the Navier-Stokes equation for Laminar, Tow-Phase Flow, and Phase Field obtained as:

$$
\rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla) u = \nabla [-p l + \eta \{\nabla u + (\nabla u)^T\}] + \rho g + \frac{\partial \nabla \phi}{\partial t}
$$

(2.23)

### 2.3.2 Stress-Strain Physics Application

In the Stress-Strain physics application assumed at the components of both of stresses $(\tau_{x\phi}, \tau_{y\phi})$ and strains $(\gamma_{x\phi}, \gamma_{y\phi})$ are zero. The global displacement $(u, w)$ in $z$ direction can be solved. By assuming that the allowed loads are in $x$ & $z$ direction, the equilibrium Stress-Strain equation can be [18]:

$$
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\sigma_x - \sigma_\theta}{x} + K_x = 0
$$

(2.24)

$$
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\sigma_x - \sigma_\theta}{x} + K_z = 0
$$

(2.25)

In order to the deformation components $(u, v, w)$ which are associated with the strain-displacement, the assumed of the small displacement in are:

$$
\varepsilon_x = \frac{\partial u}{\partial x} \quad \varepsilon_z = \frac{\partial w}{\partial z} \quad \varepsilon_\phi = \frac{\partial u}{\partial x}
$$

(2.26)

$$
\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}
$$

(2.27)

By Newton’s second law, the time-dependent equation to run transient analysis of the buoy can obtain [18]:

$$
\rho \frac{\partial^2 u}{\partial t^2} - \nabla . c \nabla u = F
$$

(2.28)
2.3.3 **Arbitrary Lagrangain-Eulerain (ALE) Physics Application**

Due to structural mechanics consideration and point of concentrating the studies, following up the motion of the particle for a structure when dealing with meshing for the numerical analysis, the mesh must follow the motion of the structure in the domain of study.

A material coordinate system which know as, the Lagrangian method, which is a technique used for a mesh movement such that each material particle coincided with the node of the mesh (see Figure).

From fluid dynamics consideration, Lagrangian method was not applicable, since the properties of the flow have apart relation of interest than particles, and where the deformation was large too.

The Eulerian method was used since the mesh was fixed in the space (see Figure2.3), in this method, the fluid motion and the flow properties changes of the shell be noticeable.

ALE method is used for handling the moving boundaries with a moving grid, and the deforming geometry, where ALE algorithm allows a means of exploiting the duel features (see Figure2.3).

The calculation was the dependent source for the movement of the mesh of the structure which solved by Lagrangian algorithm.

Calculations for the fluid domain were solved by ALE algorithm; the mesh displacement adjusted for a free displacement setting which was solved by Winslow smoothing method [1], with constraints boundaries for solving the specified PDE. This significant that the mesh has little deformation, where there was no presence of ill-shaped mesh elements.
2.4 Finite Element Analysis

Finite element analysis is a numerical solving way that resulting in an approximate solution and percentage error. Finite element method use to solve the PDE (Partial Differential Equation) either by using several numerical integration techniques like Range-Kutta, Midpoint theorem, Euler’s method, etc. Today, engineering conditions find that it is important to get approximate numerical solutions to the problems instead of the exact closed-form solution. Therefore using of the finite element method (FEM) is very developing in the engineering analysis and they can expect this use to greatly in coming year.

Figure 2.3 Lagrangian, Eulerian, and ALE description moving meshes [3]
2.4.1 Finite Element Analysis Description

The main issue in the Finite element method while solving is first is to finding the suitable weak forum equation. The second issue to specify the type of element that will be used in the meshing if it is square or rectangular, etc... The result of the nodes in the specified mesh gives the distribution of the desired solution [16].

2.4.2 Engineer Analysis Kinds

There is two considerations should be taken when analyzing any structure by modeling. First, if the structure considered to behave in linearity that’s mean the material is not enter the plastic region by having the ordinary parameters, where the other to behave is in a non-linearity that’s mean the material deformed due to the capacity of stresses after it overtaking the elastic region. Other type of analysis for a structure is to study its resonance frequencies due to the vibrations occur of the body due to an impact, or dynamic load, rotations, etc... It is important to know the resonance frequencies to avoid failure in the system.

2.4.3 Practical Application

Due to unknown failure in the structure due to unspecified loads or stresses, the FEA give the ability to predict that failure. Which shell reduces the cost of the manufacturing and reduce the time of designing when dealing with well created modeling and well estimating testing. [16]

2.5 Theoretical Model

A theoretical model is a general theory designed to explain the mechanical structures characteristic and parameter effect to correct understanding of structure. In this part the concept of the single degree of freedom [SDOF] system was present as the bases of the theoretical model to model the dynamic structure of the body. The theoretical model of
the floating buoy was considered as a [SDOF] system corresponded to the added damping, the buoyancy stiffness and added mass coefficient.

2.5.1 Single Degree of Freedom Model

The concept of a single degree of freedom (SDOF) is very important to develop the dynamic structural analyses. Since it called Single degree due to the variation resulted on one dimension. A single degree of freedom model is consisting of a mass, damper, and spring as represent in figure 2.4.

A single degree of freedom system can be analyzed by the Newton’s second low equation (2.29).

\[ f(t) = m \cdot a \]  

(2.29)

By the help of the free body diagram, figure 3.5, and the equation of motion (2.31) express as:

Figure2.5 Free Body Diagram of SDOF System
\[-k x(t) - cx(t) + f(t) = m\ddot{x}(t)\]  \hspace{1cm} (2.30)

\[m\ddot{x}(t) + cx(t) + kx(t) = f(t)\]  \hspace{1cm} (2.31)

Where:

- \(m\): Mass of the body in (kg).
- \(k\): Spring stiffness, \([ k = \text{Force} \over \text{spring} \over \text{displacement}(x) ]\) in (N/m).
- \(c\): Coefficient of viscous damping, \([ c = \text{Force} \over \text{damper} \over \text{velocity}(v) ]\) in (N.s/m).
- \(x\): The displacement of the mass from its equilibrium in (m).
- \(f\): The input force applied to the system in (N).

By taking the Laplace Transform of the equation 2.2, the system of equations has the general solution of:

\[
\{x\} = \{X\}. e^{st}
\]

\[
\{\dot{x}\} = s\{X\}. e^{st} = s.x
\]

\[
\{\ddot{x}\} = s^2 \{X\}. e^{st} = s^2 . x
\]

Where \(s\): complex valued frequency

The system equation represented as:

\[(m s^2 + cs + k).X(s) = f(s)\]  \hspace{1cm} (2.32)

By relating the Laplace transform of system to the Laplace transform system response, the transform equation can be defined as:

\[H(s) = \frac{X(s)}{f(s)} = \frac{1}{(m s^2 + cs + k)} = \frac{1}{s^2 + \frac{c}{m}s + \frac{k}{m}}\]  \hspace{1cm} (2.33)

Where \(H(s)\) is known as a system transfer function of the SDOF system and that can be written as the dynamic transfer function:
\[ H(s) = \frac{1}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]  

(2.34)

Where:

\( \zeta \): The relative damping \([\zeta = \frac{c}{2\sqrt{mk}}]\) in (rad/sec).

\( \omega_n \): The un-damping angular response frequency \([\omega_n = \sqrt{\frac{k}{m}}]\) in (rad/sec).

A simple way to obtain the Frequency Response Function of the SDOF system from equation (2.33) which is represented in frequency domain as:

\[ H(s) = \frac{R}{s-\lambda} + \frac{R^*}{s-\lambda^*} \]  

(2.35)

Where:

R: Residues for mode shape of the system.

\( \lambda \): Poles of the system damping and natural frequency.

*: Complex conjugate.
2.5.2 Mathematical Model

A mathematical model describes the significant characteristics by applying the physical laws of the system to describe the dynamics for that system. The mathematical model of the floating buoy was based on SDOF mass-damper-spring system. The hydrodynamic influence force which applied on the buoy can be founded from the integrating hydrodynamic pressure. By solving velocity potential, the hydrodynamic pressure can calculated over its wet surface. From the restoring force as state, the hydrostatic force can obtained. By assumption about zero-forward-speed, the dynamic equation of motion (2.36) can be shown as [13]:

\[
[M_{jk} + m_{a,jk}] \ddot{s}_k + c_{a,jk} \dot{s}_k + k_{b,jk} s_k = f_j
\]  

(2.36)

Where:

- \(J, k\): Signified of the mode no. \([J, k= 1, 2, 3, 4, 5, 6]\) for sway, surge, roll, heave, pitch, and yaw respectively.
- \(M_{jk}\): Expression for body inertia matrix.
- \(m_{a,jk}\): Frequency-dependent added mass coefficient.
- \(c_{a,jk}\): Damping coefficient.
- \(k_{b,jk}\): Hydrostatic restoring force coefficients.
- \(s_k\): Designated translator and angular displacements of the body.
- \(f_j\): Exciting force for \(j= 1, 2, 3\) and moment for \(j = 4, 5, 6\) arose from the presence of the incident wave elevation.

To present the heave mode of oscillation of the floating buoy, the equation (2.36) can be rewritten as following equation according to the Figure 3.6:

\[
[m_b + m_a(\omega)] \ddot{x} + c(\omega) \dot{x} + k_b x = f_e(t)
\]  

(2.37)
Where:

\( f_e(t) \): External exciting force considering to the hydrodynamic force in terms of restoring force coefficient, damping coefficient and added mass.

Figure 2.6 A SDOF model considering heaving mode of oscillation of the floating buoy
3 Study Methodology

In this Study, the COMSOL Multiphysics software was used for analyzing the floating buoys modeling numerically and solved the Finite Element Analysis (FEA) techniques.

3.1 Modeling

We will have two different domains when we used fluid structure interaction module which called solid and fluid domain. In our model, we have two fluid domains air and water and two solid domain for a cubic buoy which was considered for the modeling of the floating buoy, and rectangular cubic buoy, this buoy would be excited by a harmonic perturbation force of 700 [N] to bounce vertically up and down as a way to generate ocean wave. The following below specified the model in detail.

3.1.1 Model Specification

The two solid floating buoys considered to be surrounding by two fluids the air, and part of it is immersed in the water. The modeling was considered to be drawn in the COMSOL Multiphysics by study the case in 2D space dimension. The air domain were considered as a rectangle of width 6 [m] and height 1 [m] and the water domain is considered as rectangle of width 6 [m] and height 2 [m] where the buoy domain is considered as cubic with the 0.4 [m] side and wave generator domain is considers as rectangular cubic with width 0.6[m] and height 0.4[m].

Figure 3.1 Dimension of the Model
3.1.2 Boundary conditions

For Arbitrary Lagrangian – Eulerian (ALE) branch, the moving mesh, solid, and water domains boundary condition are discussed as well.

3.1.2.1 Moving Mesh Boundary Condition

In the modeling, the buoys that were represented as a solid domain were restricted to move in the direction of the heave z-direction. Where the water and the air that represented in the modeling as a two fluid domains where allowed to displace in all direction after applying their boundary condition where the Moving Mesh ALE were solved by the prescribed Winslow smoothing function.[1]

The fluid domain were formed by the Navier-Stokes equations, where the solution appears in a on a freely moving deformed mesh, where the deformation were computed by Winslow smoothing equation with respect to the initial shape. [1]

3.1.2.2 Solid Domain Boundary Condition

The two solid floating buoys were considered as a solid domain in our modeling, where the Archimedes principle were used to figure out the density if the solid domain. The principle tends that: any object immersed in a fluid partially or totally, is effected by a force buoyed up, equal to the weight of the fluid displaced by the object [2]. The fluid volumes $V_w$ that substituted from the submerged body were calculated as shown:

- For the cubic buoy:

The mass of the buoy was denoted by $m_b$ and considered to be 20 kg and the density of the water was denoted by $\rho_w$.

\[ \rho_w \times V_w = m_b, \]

\[ V_w = \frac{20}{1030} = 0.01942 \text{ m}^3 \]  \hspace{1cm} (3.2)
By apply Archimedes Principle to get the density of the buoy, where the density and the volume of the buoy are represented by $\rho_b$ and $V_b$ respectively.

$$V_b = 0.4^3 [m^3], \quad \rho_b = \frac{(0.01942 \times 1030)}{0.4^3} = 312.5 \left[\frac{kg}{m^3}\right]$$ (3.3)

- For the rectangular cubic buoy (Wave generator):

The mass of the wave generator was denoted by $m_{WG}$ and considered to be 40 kg and the density of the water was denoted by $\rho_w$.

$$\rho_w \times V_w = m_{WG},$$

$$V_w = \frac{40}{1030} = 0.0388 m^3$$ (3.4)

By apply Archimedes Principle to get the density of the wave generator, where the density and the volume of the wave generator are represented by $\rho_{WG}$ and $V_{WG}$ respectively.

$$V_{WG} = 0.4^2 \times 0.6 = 0.096[m^3],$$

$$\rho_{WG} = \frac{(0.0388 \times 1030)}{0.096} = 416.67 \left[\frac{kg}{m^3}\right]$$ (3.5)
### Table 3.1: Solid domain parameter setting

<table>
<thead>
<tr>
<th>Description</th>
<th>Cube</th>
<th>Rectangular cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Height</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Weight</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>Density</td>
<td>312.5</td>
<td>416.67</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>205e9</td>
<td>2750e6</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The Boundary load in the solid domain is applied due to fluids surround it by the force written as:

\[
F_1 = -10e4 \times \sin(2 \times \pi \times t \times v) \text{ [N]} \tag{3.6}
\]

\[
F_2 = -10e4 \times \sin(2 \times \pi \times t \times v2) \text{ [N]} \tag{3.7}
\]

*Where:*

\( v \): *The displacement of the buoy in the y-direction*

\( v2 \): *The displacement of the rectangular cubic buoy in the y-direction*

*\( F_1 \): The Boundary load for the buoy*

*\( F_2 \): The Boundary load for the rectangular cubic buoy*
3.1.2.3 Water and Air Domain Boundary Condition

In the fluid domain with respect to the water, it set to be incompressible constrained by Navier-Stokes equation.

There were three walls, constrained as a no slip wall, the two edges boundaries of air and water, and the bottom of the water, where No slip is the default boundary condition for the stationary solid wall. This condition prescribes the velocity equal to zero \( u=0 \).

The walls of the two buoys considered as the Moving wetted wall for the solid domain constrained with their velocity field, \( y \) component.

An outlet boundary was selected at the top of the modeling domain with a boundary condition of Pressure \( P_0=0 \) pa, with no viscous stress.

![Figure 3.2.Fluid 1 and 2 for the floating buoy and wave generator](image)

3.2 Model Meshing

In solving the model, the mesh shown below Fig 3.3 have been taken in consideration; for the Time-dependence solver of a triangular type element where the mesh element was 831 elements as given in the table 3.2 below:
Figure 3.3 Mesh 1

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum element quality</td>
<td>0.5083</td>
</tr>
<tr>
<td>Average element quality</td>
<td>0.8957</td>
</tr>
<tr>
<td>Triangular elements</td>
<td>831</td>
</tr>
<tr>
<td>Edge elements</td>
<td>115</td>
</tr>
<tr>
<td>Vertex elements</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 3.2 Mesh statistics
3.3 Model Solver

For solving the modeling a solvers were chosen for the solution as explained below:
The Time Dependent were have the range from 0 to 10 sec with time step 0.001 sec, where the time stepping were depend on the Generalize alpha as the method for time stepping where the steps taken were free unconstrained, where the Predictor was considered as linear.

The solver with respect to the Direct where controlled in it the PARDISO method, where a Factor in error estimate were 40 [18].
4 Result and Discussion

4.1 Results

While solving the model in the Time-dependence solver, the mesh have been taken for a triangular type meshing having a big size elements in most of the area while around the buoy it become finer at the area of interest of the study.

![Figure 4.1 Mesh for Time-Dependence](image)

The following figures shows the postprocessing of model in surface von Mises stress plots and contour and transient heave oscillation plots for both of buoy and wave generator:

![Figure 4.2 von Mises stress (N/m²) Surface (Wave Generator)](image)  ![Figure 4.3 von Mises stress (N/m²) Surface (Buoy)](image)
A unique excitation by point load [-700 N] where the minus sign tend to the vertical downward direction of the load, as shown in figure 4.2 had been applied. The movement of wave generator buoy after this unique excitation will be generates the wave in the domain of this study. And the two buoys will be bouncing with damping.

The buoy start damped bouncing due to the wave generated, as figure 4.3 shown. By this bouncing, the displacement of the buoy was plotted in figure 4.7 to trace the frequencies of the buoy.

The volume fraction of the fluid was been calculated by this equation:

\[1 - \min(\max(0.5*(1+\phi_{pf}), 0), 1)\]

where \(\phi_{pf}\) is the phase field variable which is unitless varies by means of time in the sinusoidal damped shape as shown in figure 4.5 where the equation show the maximum and minimum of the phase field to plot the volume fraction of unit 1 as figure 4.4 shown.
The velocity magnitude of the fluid was been calculated by $\sqrt{(u_3^2 + v_3^2)}$, where $u_3$ and $v_3$ of the velocity field component of $x$ and $y$ directions, as figure 4.6 shown.

To ensure that the modeling of the tank behave as the real ocean where the wave in the ocean don’t reflect an equation of the viscosity have been implemented in the COMSOL to prevent the reflection of the wave as shown in figure 4.7 and figure 4.8. This equation will raise the
viscosity of the water at the edges and keep it normally elsewhere to guarantee no reflection of the wave and the only wave that will excite the buoy is the wave generated by the wave generator as shown below.

Since in the real life the buoy is bouncing in an infinite surround that’s mean the wave will generated will not stuck and reflected back, they will keep moving until the damped to zero when reach edges. In order to perform that the viscosity at the edges should be high to damp the waves and won’t let them reflected back. The equation of the viscosity was written as:

$$Vd \times exp1 \times exp2$$

where \(exp1 = 2.7183^{(1e-7/x)}\)

\(exp2 = 2.7183^{(1e-7/abs(6-x))}\)

\(Vd = 8.9e-4 [\text{Pa}\times\text{s}]\)

At the two side edges we consider it as a no slip wall and the upper edges as an outlet since the tank its open to the air in real life.

![Figure 4.9 Displacement of the Wave Generator](image)

This figure 4.9 shows the displacement of the wave generator after excitation by the load, it’s moving in a damped sinusoidal wave shape bouncing up and down.
This figure 4.10 shows the displacement of the buoy after excitation by the wave, it’s moving in a damped sinusoidal wave shape bouncing up and down.
Since this study is held in one dimension in vertical direction, the displacement of the buoy up and down was taken in consideration a single degree of equation was control the study to get the frequency of the system.

\[25\ddot{z} + 25\dot{z} + 490.5z = 0\]

Where:

1- The Mass of buoy= 20 kg and the Added mass=5kg, the total mass= 25 kg

2- Damping: \( C = \alpha m + \beta k = 1 \text{ (1/s)} \times 25 \text{ (Kg)} = 25 \text{ N.s/m} \)

where

Beta : Rayleigh damping
Alfa : Mass damping parameter

3- Stiffness damping parameter:

\[ K = \frac{F_{\text{Spring}}}{\text{Displacement} (z)} = 490.5 \text{ N/m} \]

By the help of the free body diagram, the equation of motion express as:

Where:

m: Mass of the body (kg).

k: Spring stiffness (N/m) where,

c: Coefficient of viscous damping( N.s/m) where

z: The displacement of the mass from its equilibrium in (m).
The Frequency of the buoy is calculated by the reciprocal of the time taken to finish the first cycle from the periodic displacement graph shown above.

\[ Freq = \frac{1}{1.234 \text{ sec}} \cong 0.81 \text{ Hz} \]

The main issue in this physics is to specify the viscosity of the fluid, specify the walls for the boundary condition, and the volume force.

**Results:**

The frequency where equal to 0.81 Hz

**4.2 Parameters Estimation**

The parameters estimation was handled by means of the spectrum analysis, where the heave oscillation was evaluate by the discrete Fourier transform (DFT) by means of computational algorithms fast Fourier transform (FFT). The spectrum of the heave response was seized by the weight of the buoy, the spectrum of the input force, as a result of the frequency response receptance. The added mass was evaluated by log-log plot of the receptance of the slope for high frequency. The added damping is then calculated through the damping ratio definition.
5. Conclusion

This model was handled under SDOF, simplified to mass, spring and damper, the model was describe the dynamics motion for a semi-submerged floating buoy, where an additional terms were added like added mass and added damper.

Where these variables were depend on the frequency of oscillating and their modes, the geometry of the floating buoy, and water-depth.

The Model was considered as the buoy was affected by a load on the buoy and affected by the wave created in the fluid domain to oscillate the buoy and the spongy wall of the tank to prevent the reflection of the wave after inserting the equation of the viscosity of the water.

The heave displacement of the model under the time-dependent was obtained by means of FEM, solving the coupled partial differential equations of ALE, stress-strain equations, and Navier-stoke equations.

By the result of the displacement of the buoy the frequency was extracted from the graph of the displacement.

The numerical model SDOF was compared by the results from the previous work [9, 10]. The results were within the acceptable range.

Future work

A suggestion for further work is extending of this work can be by researching about considered the heavy wind and the strong waves and their effect on the oscillating floating buoy. Researchers could confirm the results with any other numerical software. Also may perform two or multi degree of freedom for this study. Held this study in salty water instead of fresh water and observes the bouncing and the frequency. Perform other way of generating wave like moving wall.
6. Reference


16. *Finite element analysis, site:*


18. COMSOL Multiphysics 4.0a, (June 2010), COMSOL *Multiphysics Reference Guide* 4.0a, Part number: CM020005

Appendix 1

1. **Selected Space Dimension**
   > 2D

2. **Add Physics**
   > Moving Mesh (ale) > Solid Mechanics (solid) > Laminar Two-Phase Flow, Moving Mesh (tpfmm)

3. **Selected Study Type**
   > Time Dependent > Finish

4. **Model 1 (mod1)**

A. **Definitions**

i. **Geometry**

<table>
<thead>
<tr>
<th>Geometry statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Space dimension</td>
<td>2</td>
</tr>
<tr>
<td>Number of domains</td>
<td>6</td>
</tr>
<tr>
<td>Number of boundaries</td>
<td>23</td>
</tr>
</tbody>
</table>

ii. **Water (r1)**

<table>
<thead>
<tr>
<th>Settings</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Position</td>
<td>{0, 0}</td>
</tr>
<tr>
<td>Width</td>
<td>6</td>
</tr>
<tr>
<td>Height</td>
<td>2</td>
</tr>
<tr>
<td>Size</td>
<td>{6, 2}</td>
</tr>
</tbody>
</table>
iv. **Air (r2)**

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Position</td>
<td>{0, 2}</td>
</tr>
<tr>
<td>y</td>
<td>2</td>
</tr>
<tr>
<td>Width</td>
<td>6</td>
</tr>
<tr>
<td>Height</td>
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</tr>
<tr>
<td>Size</td>
<td>{6, 1}</td>
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</tbody>
</table>

v. **Buoy (r3)**

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</thead>
<tbody>
<tr>
<td>Position</td>
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</tr>
<tr>
<td>x</td>
<td>4.5</td>
</tr>
<tr>
<td>y</td>
<td>2</td>
</tr>
<tr>
<td>Base</td>
<td>Center</td>
</tr>
<tr>
<td>Side length</td>
<td>0.4</td>
</tr>
<tr>
<td>Side length</td>
<td>0.4</td>
</tr>
<tr>
<td>Size</td>
<td>{0.4,0.4}</td>
</tr>
</tbody>
</table>

vi. **Wave Generator (r4)**

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<td>0.6</td>
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<td>y</td>
<td>2</td>
</tr>
<tr>
<td>Base</td>
<td>Center</td>
</tr>
<tr>
<td>Side length</td>
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</tr>
<tr>
<td>Side length</td>
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</tr>
<tr>
<td>Size</td>
<td>{0.6,0.4}</td>
</tr>
</tbody>
</table>
Moving Mesh (ale)

Moving Mesh selection

<table>
<thead>
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<tbody>
<tr>
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</tr>
<tr>
<td>Mesh smoothing type</td>
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</table>
vii. **Fixed Mesh 1**

1. Selection

![Fixed Mesh 1](image)

viii. **Prescribed Mesh Displacement 1**

1. Settings

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Prescribed # displacement</td>
<td>{1, 0}</td>
</tr>
</tbody>
</table>

2. Selection

![Prescribed Mesh Displacement 1](image)
ix. *Prescribed Deformation 1*

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prescribed mesh displacement</td>
<td>{0, v}</td>
</tr>
</tbody>
</table>

2. Selection

---

*x. Free Deformation 1*

1. Selection

---
xi. **Prescribed Mesh Displacement 4**

1. Selection

   ![Prescribed Mesh Displacement 4](image)

xii. **Prescribed Mesh Displacement 2**

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prescribed mesh displacement</td>
<td>{0, v}</td>
</tr>
</tbody>
</table>

2. Selection

   ![Prescribed Mesh Displacement 2](image)
B. Solid Mechanics (solid)

C. Solid Mechanics

<table>
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<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
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</tbody>
</table>

i. Linear Elastic Material Model 1

1. Selection
Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Young's modulus</td>
<td>User defined</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>205e9</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>User defined</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.4</td>
</tr>
<tr>
<td>Density</td>
<td>User defined</td>
</tr>
<tr>
<td>Density</td>
<td>DensityBouy</td>
</tr>
<tr>
<td>Include geometric nonlinearity</td>
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</table>

**ii. Free 1**

**1. Selection**
iii. *Initial Values 1*

1. Selection

![Initial Values 1]

iv. *Linear Elastic Material Model 2*

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson's ratio</td>
<td>User defined</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.4</td>
</tr>
<tr>
<td>Density</td>
<td>User defined</td>
</tr>
<tr>
<td>Density</td>
<td>DensityBouy</td>
</tr>
</tbody>
</table>
2. Selection

![Diagram 1](image1)

*Linear Elastic Material Model 2*

v. **Body Load 1**

1. Settings

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<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Load</td>
<td>{0, -BodyLoadB, 0}</td>
</tr>
</tbody>
</table>

2. Selection

![Diagram 2](image2)

*Body Load 1*

[55]
vi. **Boundary Load 1**

1. **Settings**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Force</td>
<td>({0, -0.1 \times 10^4 \times \sin(2\pi/21 \times t \times v), 0})</td>
</tr>
</tbody>
</table>

2. **Selection**

---

**Boundary Load 1**

**Roller 1**

1. **Settings**

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<thead>
<tr>
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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Constraint type</td>
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</tr>
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<td>Use weak constraints</td>
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</table>
2. Selection

2.1.1. Added Mass 1

1. Settings

<table>
<thead>
<tr>
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<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass</td>
<td>{0, 5, 0}</td>
<td></td>
</tr>
</tbody>
</table>

2. Selection

Added Mass 1
D. Laminar Two-Phase Flow, Phase Field (tpf)

![Diagram of Laminar Two-Phase Flow, Phase Field]

### Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Two-phase flow, phase field</td>
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<tr>
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<td>std1/time</td>
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</table>

i. Fluid Properties 1

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>User defined</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>Vs</td>
</tr>
<tr>
<td>Surface tension coefficient</td>
<td>User defined</td>
</tr>
</tbody>
</table>
Parameter controlling interface thickness | 0.07

1. Selection

![Fluid Properties 1](image)

**i. Fluid Properties 3**

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>User defined</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>Vs</td>
</tr>
<tr>
<td>Surface tension coefficient</td>
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<td>Parameter controlling interface thickness</td>
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</table>

2. Selection

![Fluid Properties 3](image)

[59]
ii. **Volume Force 1**

1. **Settings**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

2. **Selection**

![Volume Force 1](image)

iii. **Volume Force 2**

1. **Settings**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume force</td>
<td>{0, -9.81*if(\phi pf&lt;0, 1.25, 1030), 0}</td>
</tr>
</tbody>
</table>
2. Selection

**Volume Force 2**

**iv. Initial Values 2**

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid initially in domain</td>
<td>Fluid 2</td>
</tr>
</tbody>
</table>

2. Selection

**Initial Values 2**

**v. Wall 2**

1. Settings
2. Selection

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary condition</td>
<td>Moving wetted wall</td>
</tr>
<tr>
<td>Velocity of moving wall</td>
<td>{0, vt, 0}</td>
</tr>
</tbody>
</table>

Wall 2
vi. Wall 3

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary condition</td>
<td>Moving wetted wall</td>
</tr>
<tr>
<td>Velocity of moving wall</td>
<td>({0, v2t, 0})</td>
</tr>
</tbody>
</table>

2. Selection

![Wall 3](image)

vii. Wall 4

1. Selection

![Wall 4](image)
viii. Wall 5

1. Selection

![Wall 5 Diagram]

ix. Outlet 1

1. Settings

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary condition</td>
<td>Pressure</td>
</tr>
</tbody>
</table>

2. Selection

![Outlet 1 Diagram]
b. **Meshes**

i. Mesh 1

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum element quality</td>
<td>0.5083</td>
</tr>
<tr>
<td>Average element quality</td>
<td>0.8957</td>
</tr>
<tr>
<td>Triangular elements</td>
<td>831</td>
</tr>
<tr>
<td>Edge elements</td>
<td>115</td>
</tr>
<tr>
<td>Vertex elements</td>
<td>18</td>
</tr>
</tbody>
</table>

ii. Selection

*Outlet 1*
1. **Size (size)**

<table>
<thead>
<tr>
<th>Settings</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Calibrate for</td>
<td>Fluid dynamics</td>
</tr>
<tr>
<td>Maximum element size</td>
<td>0.6</td>
</tr>
<tr>
<td>Minimum element size</td>
<td>0.012</td>
</tr>
<tr>
<td>Resolution of curvature</td>
<td>0.4</td>
</tr>
<tr>
<td>Maximum element growth rate</td>
<td>1.4</td>
</tr>
<tr>
<td>Predefined size</td>
<td>Coarse</td>
</tr>
</tbody>
</table>

**c. Study 1**

**i. Time Dependent**

Times: range(0,0.001,10)

**Mesh selection**

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry 1 (geom1)</td>
<td>Mesh1</td>
</tr>
</tbody>
</table>

**Physics selection**

<table>
<thead>
<tr>
<th>Physics interface</th>
<th>Discretization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Mechanics (solid)</td>
<td>physics</td>
</tr>
<tr>
<td>Solid Mechanics 2 (solid2)</td>
<td>physics</td>
</tr>
<tr>
<td>Moving Mesh (ale)</td>
<td>physics</td>
</tr>
<tr>
<td>Laminar Two-Phase Flow, Phase Field (tpf)</td>
<td>physics</td>
</tr>
</tbody>
</table>