Simulation of Solutions to Excessive Vibration Problems of Pedestrian Footbridges

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Abstract

Footbridges are often prone to vibrations induced by pedestrian loading. They are long and slender and their weight is moderate. One of the classical and famous examples is the transverse vibrations encountered on the millennium bridge in London. Horizontal vibration problems have received considerable attention due to millennium bridge vibration problem, but potential problems relating to vertical vibrations are more common.

Vibrations from pedestrian loads are mostly limited to vibration frequencies below 5 Hz. Most pedestrian bridges have several vibration modes below the above mentioned frequency. Increasing the stiffness could be a remedy to this vibration problem. And for a slender structure, increasing the stiffness often implies a corresponding increase in mass of the structure, whereby the desired effect is not achieved. Instead damping is often used to reduce the excessive vibration problem of footbridges by attaching additional dampers to the structure.

London Millennium bridge was modelled in this thesis in order to show the effect of tuned mass damper. There was a substantial reduction of the displacement of the bridge when tuned mass dampers were added to the bridge model.

Keywords:

Damping, Modelling, Footbridges, Resonance, ABAQUS, MATLAB
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1 Notation

\( c \)  Damping Ratio

\( c_c \)  Critical damping ratio

\( C \)  Modal Damping

\( E \)  Young’s Modulus

\( F \)  Force

\( F_d \)  Damping Force

\( FE \)  Finite Element

\( F_i \)  Inertia Force

\( F_s \)  Stiffness Force

\( F(t) \)  Externally Applied Force

\( f \)  Frequency

\( f_n \)  Fundamental Natural Frequency

\( H(f) \)  Frequency Response

\( H(s) \)  Dynamic Transfer Function

\( I \)  Inertia

\( K \)  Modal Stiffness

\( k \)  Stiffness

\( k_l \)  Lateral Working Force

\( L \)  Length

\( l_0 \)  Equilibrium Position

\( M \)  Modal Mass

\( M_{Pg} \)  Modal Self weight of Pedestrians
\( m \) Mass
\( n \) No of pedestrian uniformly distributed
\( Q \) Pedestrian load
\( t \) Time
\( U \) Deflection
\( V \) Relative Velocity across the Damper
\( \omega \) Natural Frequency
\( z \) Distance
\( z(t) \) Deflection of the beam at a selected location
\( \ddot{z} \) Acceleration
\( \dot{z} \) Initial Velocity
\( \zeta \) Estimated Damping Ratio
\( \lambda \) Eigenvalues
\( \sim \) Generalized System
\( \phi \dot{z} \) Modal Velocity
\( \phi \) Modeshape
\( \psi(x) \) Shape Function
\( \Delta t_i \) Time step
\( \omega_0 \) Undamped Natural (angular) Frequency
Abbreviations

DOF     Degree of Freedom
FEM     Finite Element Method
MDOF    Multiple Degree of Freedom
SDOF    Single Degree of Freedom
TMDs    Tuned Mass Dampers
2 Introduction

2.1 Background

With the emergence of new materials and advanced engineering technology, modern footbridges can be designed and constructed to have longer spans and greater slenderness than ever to satisfy the transportation needs and the aesthetical requirements of society. Such slender footbridges have often low stiffness, low mass, low damping and are prone to vibration induced by human activities. The decreasing of mass and stiffness in footbridge structures leads to low natural frequencies for footbridges with greater danger of resonance [1] and larger lateral dynamic response [2]. Severe vibration serviceability problems can arise, particularly in the lateral direction as pedestrians are more sensitive to the low frequency lateral vibration than the vertical one [3]. This phenomenon has been evidenced by the excessive lateral vibrations of many footbridges worldwide such as Millennium Bridge in London and the T-Bridge in Japan, etc. It is generally accepted nowadays that modern slender footbridges have greater danger of suffering vibration serviceability problem rather than safety or strength problems [4].

In general, lateral deflection is considered as a response to lateral loads. However, vertical loads can also induce lateral deflections, particularly in slender structures such as frames and long bridges.

Footbridges have greater danger of suffering vibration problems rather than safety or strength problems [4]. Several cases of footbridges experiencing great vibration have been reported. Examples are as follows:

1. Auckland Harbour Bridge.
2. The Toda Park Bridge.
3. The London Millennium Bridge.

If footbridges are designed for static loads only, they may be susceptible to vertical and horizontal vibrations. Recent experiences, for example with the London Millennium Bridge, have shown just how an important subject the dynamics of footbridges is.
Dynamic analysis of the London Millennium Bridge was carried out at Lund University as a thesis. But they did not focus on the solutions to the excessive vibration problem. This thesis will focus on how excessive vibration problems of the pedestrian bridge can be attenuated using tuned mass dampers.

2.2 Aim of the Work

The aim of this master thesis is to show the effect of TMDs on the dynamic response due to pedestrian induced vibrations of the London millennium bridge.

2.3 Methodology

This thesis comprises of three main parts and they are described as follows:

- Theoretical study of structural dynamics and dynamic loads induced by pedestrians.

- Structural description of London millennium footbridge and dynamic analysis of the bridge using finite element modelling and SDOF-model.

- Effects of suggested solution techniques.

For the simulations in this thesis, ABAQUS has been used to model London Millennium Bridge from which the FRFs needed to show the effect of TMDs have been extracted after applying a unit horizontal force to the bridge model. MATLAB were used to evaluate results.
3 Structural Dynamics and Dynamic Loads Induced by Pedestrians

3.1 Structural Dynamics

In the above context, dynamic means time-varying. Structural dynamics describe the behaviour of a structure due to dynamic loads. A dynamic load is one whose magnitude, direction, or point of application varies with time; while the resulting time varying deflections and stresses constitute the dynamic response. Perhaps the most demanding step in any dynamical analysis is the creation of a mathematical model of the structure. The structure is first analysed analytically. The analytical model consists of:

1. A set of the simplifying assumptions made in reducing the real system.
2. A set of drawings depicting the analytical model of the real system.
3. A list of the design parameters (sizes, materials, etc).

The number of displacement quantities which must be considered in order to represent the effects of all significant inertia forces is called the number of degrees of freedom (DOF) of the system [5].

In order to create a useful analytical model you must have clearly in mind the intended use of the analytical model, that is, the types of behaviour of the real system that the model is supposed to realistically represent. Simplicity of the analytical model is most desirable, so long as the model is adequate to represent the necessary behaviour. Once you have created an analytical model of the structure you wish to study, you can then apply physical laws (e.g., Newton’s laws, stress-strain relationship, etc.) to obtain the differential equation(s) of motion which describe in mathematical language, the analytical model.

Once a mathematical model has been formulated the next step in a dynamical analysis is to solve the differential equation(s) to obtain the dynamical response. The two parts of dynamical behaviour which are of primary importance in structural applications are free vibration and forced response, the former being the motion resulting from specified initial
conditions, and the latter being motion resulting from specified inputs to the system from external source. Thus, you solve the differential equations of motion subject to specified initial conditions and obtain the resulting time history of the motion of the structure. This constitutes the predicted behaviour of the (real) structure, or the response.

In this thesis, forced response will be treated. Almost any type of structural system may be subjected to one form or another of dynamic loading during its lifetime. The solution of a typical structural dynamics problem is considerably more complicated than its static counterpart due to the addition of inertia and damping forces to the restoring forces and due to the time dependency of all force quantities.

In order to obtain the responses of a structure, a dynamic analysis is performed with the objective to solve the equation between the inertia force, damping force, and stiffness force together with the externally applied force [5].

\[ F_i + F_d + F_s = F(t) \]  \hspace{1cm} (3.1)

Where

\( F_i \) is the inertial force of the mass and is related to the acceleration of the structure by

\[ F_i = m\ddot{z}, \] \hspace{1cm} (3.2)

\( F_d \) is the damping force and is related to the velocity of the structure by

\[ F_d = m\dot{z}, \] \hspace{1cm} (3.3)

\( F_s \) is the elastic force exerted on the mass and is related to the displacement of the structure by

\[ F_s = kz \] \hspace{1cm} (3.4)
Where \( k \) is the stiffness, \( c \) is the damping ratio and \( m \) is the mass of the dynamic system. Further, \( F(t) \) is the externally applied force [5]. Substituting these expressions into Eq. 3.1 gives the equation of motion.

\[
m\ddot{z} + c\dot{z} + kz = F(t)
\]  

(3.5)

Dynamic response can be found by solving this equation of motion. Two different dynamic models are presented in the following sections. First the structure is modelled as a system with one degree of freedom (an SDOF-model) and a solution technique for the equations of the system is presented. Then the structure is modelled as a multi-degree-of-freedom system (an MDOF-model). Modal analysis is then presented as a technique to determine the basic dynamic characteristics of the MDOF-system and provide the required solution to excessive vibrations.

### 3.1.1 SDOF Model

In this section, equation of motion for a generalized SDOF system with distributed mass and stiffness is formulated. Then a numerical time-stepping method for solving this equation is presented. It is noted, that the analysis provides only approximate results for systems with distributed mass and stiffness [6].

### 3.1.2 Equation of Motion

A system consisting of a simple beam with distributed mass and stiffness can deflect in an infinite variety of shapes. By restricting the deflections of the beam to a single shape function \( \psi(x) \) that approximates the fundamental vibration mode, it is possible to obtain approximate results for the lowest natural frequency of the system. The deflections of the beam are then given by

\[
u(x,t) = \psi(x)z(t)
\]  

(3.6)

Where the generalized coordinate \( z(t) \) is the deflection of the beam at a selected location [7].
The equation of motion for a generalized SDOF-system is of the form

\[ \ddot{\tilde{m}} \ddot{z} + \tilde{c} \dot{z} + \tilde{k} z = \tilde{f}(t) \]  \hspace{1cm} (3.7)

Where,
\[ \tilde{m} \] = Generalized mass of the system
\[ \tilde{c} \] = Generalized damping of the system
\[ \tilde{k} \] = Generalized stiffness of the system
\[ \tilde{f}(t) \] = Generalized force of the system

The generalized mass and stiffness can be calculated using the following expressions

\[ \tilde{m} = \int_{0}^{L} m(x)[\psi(x)]^2 \, dx \]  \hspace{1cm} (3.8)

\[ \tilde{k} = \int_{0}^{L} EI(x)[\psi''(x)]^2 \, dx \]  \hspace{1cm} (3.9)

Where,
\[ m(x) \] is mass of the structure per unit length
\[ EI(x) \] is the stiffness of the structure per unit length \( L \) is the length of the structure.

Damping is usually expressed by a damping ratio, \( \zeta \), estimated from experimental data or taken from standards. The generalized damping can then be calculated from the expression

\[ \tilde{c} = \zeta (2\tilde{m}\omega) \]  \hspace{1cm} (3.10)

Where \( \omega \) is the natural frequency of the structure. Once the generalized properties \( \tilde{c} \), \( \tilde{m} \), \( \tilde{k} \) and \( \tilde{f}(t) \) are determined, the equation of motion (Eq. 3.7) can be solved for \( z(t) \) using a numerical integration method.
Finally, by assuming a shape function $\psi(x)$, the displacements at all times and at all locations of the system are determined from equation (3.6).

The general approach for the solution of the dynamic response of structural systems is to use numerical time-stepping methods for integration of the equation of motion. This involves, after the solution is defined at time zero, an attempt to satisfy dynamic equilibrium at discrete points in time. One method commonly used for numerical integration is the central difference method, which is an explicit method. Explicit methods do not involve the solution of a set of linear equations at each step. Instead, these methods use the differential equation at time $t_i$ to predict a solution at time $t_{i+1}$. The central difference method is based on a finite difference approximation of the velocity and the acceleration. Refer to section 4.3 for an example.

3.1.3 MDOF

Although SDOF models may adequately describe the dynamical behaviour of some systems, in most cases it is necessary to employ more “sophisticated models”, for example, MDOF models. It is possible to approximate all structures as an assemblage of finite number of mass-less members and a finite number of node displacements. The mass of the structure is lumped at the nodes and for linear elastic structures the stiffness properties of the members can be approximated accurately. Such a model is called a multi-degree-of-freedom (MDOF) system.

3.1.4 Modal Analysis

Modal analysis includes the formulation of the eigenvalue problem and a solution method for solving the eigenvalue problem. The goal of modal analysis in structural mechanics is to determine the natural mode shapes and frequencies of an object or structure during free vibration. Different types of frequency response function are measured experimentally in vibration analysis in order to identify the mechanical properties of the system. In some cases, frequency response functions between signals that are naturally occurring. In other case (In this thesis), frequency response function between an applied force and the resulting response are measured
which is used to identify the mechanical properties of a structure. Residues and poles are estimated from the extracted frequency response function and used to describe a linear system. It is common to use the finite element method (FEM) to perform this analysis because, like other calculations using the FEM, the object being analyzed can have arbitrary shape and the results of the calculations are acceptable. The types of equations which arise from modal analysis are those seen in eigensystems. The physical interpretation of the eigenvalues and eigenvectors which come from solving the system are that they represent the frequencies and corresponding mode shapes.

The displacements of the nodes are the degrees of freedom. By discretizing the structure in this way, a stiffness matrix $K$, a damping matrix $C$ and a mass matrix $M$ the structure can be determined, see for example [8]. The generalized equation of motion is for MDOF system given as:

$$M\dddot{U} + C\dot{U} + KU = F$$

(3.11)

Where,
- $M$ is the mass matrix,
- $\dot{U}$ is the 2nd time derivative of the displacement $U$ (i.e. the acceleration)
- $\dddot{U}$ is the velocity $U$ is a damping matrix
- $K$ is the stiffness matrix and
- $F$ is the force vector

In deriving the solutions to the multiple degree of freedom system, one often begins in structural dynamics with undamped systems, that is systems where $C = 0$. To find the free solutions, the solutions where the external forces are zero, an eigenvalue problem is defined by rewriting (Eq.3.11) as

$$\begin{bmatrix} M^{-1}K + S^2 I \end{bmatrix}\{U\} = \{0\}$$

(3.12)

The non-trivial solutions to equation (3.8) are obtained by setting the determinant equal to 0,

$$\det\left[ M^{-1}K + S^2 I \right] = 0$$

(3.13)
Which leads to a polynomial in $S^2$. The roots of this polynomial are known from linear algebra as, the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_N$ where N is the number of dimensions in the matrix equation, that is, the number of masses in the system. The vectors $\{x\} = \{\psi_k\}$ which satisfy equation (3.11) with $S^2 = \lambda_k$, are the eigenvectors of the system. Within structural dynamics, these are often called mode shapes or modal vectors, and are unique for each given structure and boundary conditions. Because eigenvectors can be arbitrarily scaled, the mode shapes are only determined in shape, that is, only the relative motion of the points is unique [9].

### 3.2 Dynamic Loads

Walking pedestrians induces considerable vertical and horizontal dynamic loads. Walking and running are harmonic loads that are prevailed by the pacing rate. These forces have components in all three directions, vertical, lateral and longitudinal and they depend on parameters such as pacing frequency, walking speed and step length [10].

Synchronous excitation is caused by the combination of high density of pedestrians and low natural frequencies of bridges within the frequency range of pacing rate. During synchronization, footbridges resonate close to or at the natural frequency within the frequency range of pacing rate, and part of pedestrians will change their footfalls to match the vibration. The force functions are frequency dependent. The typical pacing frequency for walking is around 2 steps per second, which gives a vertical forcing frequency of 2 Hz. Slow walking is in the region of 1.4 – 1.7 Hz and fast walking in the range of 2.2 – 2.4 Hz. This means that the total range of vertical forcing frequency is 1.4 – 2.4 Hz with a rough mean of 2 Hz [11]. Since the lateral component of the force is applied at half the footfall frequency, the lateral forcing frequencies are in the region of 0.7 - 1.2 Hz [6].

Lots of footbridges have their natural vertical and lateral frequencies within the limits mentioned above which makes them susceptible to excessive vibration. The synchronization phenomenon of people walking in groups and crowds will be handled in the next section.
3.2.1 Phenomenon of Synchronous  Lateral and Vertical  Vibration

Pedestrians are more resistant to vertical vibration than horizontal when walking over a bridge. They are sensitive to low frequency lateral motion on the surface on which they walk. Random horizontal pedestrian walking forces, combined with the synchronisation that occurs naturally within a crowd, cause small horizontal motion of the bridge and perhaps, walking of some pedestrians becomes synchronized to the bridge motion. If this small motion is perceptible, it becomes more comfortable for the pedestrians to walk in synchronisation with the horizontal motion of the bridge [12,13].

Because lateral motion affects balance, pedestrians tend to walk with feet further apart and attempt to synchronise their footsteps with the motion of the surface. The pedestrians find this helps them maintain their lateral balance [6]. The walking of more pedestrians is synchronized, increasing the lateral bridge motion further. When the amplitude of the motion increases, the lateral dynamic force increases, there is also an increase in the degree of synchronisation between pedestrians. In this sense, the vibration has a self-excited nature and it takes some time before the vibration is fully developed. However, because of the human behaviour of pedestrians, they reduce walking speed or stop walking when the vibration becomes uncomfortable. Therefore, the vibration amplitude does not become infinitely large [13, 14].

The movement at the Millennium Bridge was clearly caused by a substantial lateral loading effect which had not been anticipated during the design. The loading effect has been found to be due to the synchronisation of lateral footfall forces within a large crowd of pedestrians on the bridge [15]. This arises because it is more comfortable for pedestrians to walk in synchronisation with the natural swaying of the bridge, even if the degree of swaying is initially very small.

The pedestrians find this makes their interaction with the movement of the bridge more predictable and helps them maintain their lateral balance. This unthinking reaction of the pedestrian ensures that footfall forces are applied at the resonant frequency of the bridge, and with a phase such as to increase the motion of the bridge. As the amplitude of the motion increases, the lateral force imparted by individuals increases, as does the degree of correlation between individuals. It was subsequently determined, as
described below, that for potentially susceptible spans there is a critical number of pedestrians that will cause the vibrations to increase to unacceptable levels [15].

3.3 Load Model

A mathematical model of the pedestrian dynamic forces is needed in order to perform a dynamic analysis. Two different load models will be used in this thesis.

- Dallard’s load model
- Nakamura’s load model

These models are both results from observations and measurements of the phenomenon of horizontal human-structure synchronisation. These load models will be discussed in details in the next section.
2 Structural Description and Dynamic Analysis of London Millennium Bridge

London Millennium footbridge is sited on the River Thames in London, United Kingdom, between St. Peter’s Hill and St. Paul’s Cathedral on the north bank of the river, and the Borough of Southwark with the nearby Globe Theatre and Tate Modern Art Museum on the South. The Millennium bridge is the first new bridge across the Thames in London in more than a century. It was opened on the 10th June 2000 and during the opening day it suffered excessive lateral vibrations induced by pedestrians.

In this section, dynamic analysis of the London Millennium Bridge will be performed. The chapter is divided into three sections. In the first section, there is a general description of the bridge structure. The second section describes the finite element modelling of the bridge and the dynamic analysis performed using this model. The last section describes a dynamic analysis performed using a SDOF model of the bridge.

4.1 Structural Description

The bridge structural diagram is that of a shallow suspension bridge, where the cables are as much as possible below the level of the bridge deck to free the views from the deck. Two groups of four 120mm diameter locked coil cables span from bank to bank over two river piers. The lengths of the three spans are 81m for the north span, 144m for the main span between the piers and 108m for the south span[15]. The sag of the cable profile is 2.3m in the main span, around six times shallower than a more conventional suspension bridge structure. Fabricated steel box sections, known as the transverse arms, span between the two cable groups every 8m. The 4m wide deck structure comprises two steel edge tubes which span onto the transverse arms. The deck itself is made up of extruded aluminium box sections which span between the edge tubes on each side. Other finishes such as the lighting and handrail are also fixed onto the edge tubes. The groups of cables are anchored at each bank. Each abutment is founded on a 3m reinforced concrete pile cap anchored by a group of 2.1m diameter reinforced concrete piles. There are 12 piles on the north bank and 16 on
the south bank, where the site is constrained and the pile cap shorter in consequence. See figure 4.1.

The cable profile is very shallow and such a shallow cable profile results in large cable tensions. According to the designers [15], the dead load of the bridge is 2000 kg/m along the bridge axis and the resulting total dead load cable tension is 22,5 MN. The bridge deck is articulated at 16 meter long intervals and therefore it provides no lateral stiffness. Finally, it is concluded, that the structural system both in vertical and horizontal directions derives its stiffness from the cables [16].

*Figure 4.1. The London millennium footbridge [16]*
4.2 Finite Element model

A dynamic analysis of the London Millennium Bridge was performed using the Finite Element Method. ABAQUS (Student edition 6.7-2) software was used in the FE modelling of the London Millennium Bridge and dynamic analysis (as in 6). In order to analyse the structure dynamically, a 3-dimensional finite element (FE) model of the centre span of the Millennium Bridge was established. It was decided only to look at the centre span as this was the part of the bridge that experienced the largest vibrations on the opening day. Modal damping of 0.005 was used in this model. The geometrical structure of the FE model is shown below in figure 4.2. This is made up of two cables, two edge beams and secondary beams that spam between the cables every 8 meters.

![FE model of the centre Span of the millennium bridge](image)

*Figure 4.2. FE model of the centre Span of the millennium bridge*

The material and section properties used in the FE Model are shown below
Table 4.1. Material properties of steel

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>81 Gpa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>210 Gpa</td>
</tr>
<tr>
<td>Thermal Expansion</td>
<td>12. E-06</td>
</tr>
</tbody>
</table>

Table 4.2. Section properties

<table>
<thead>
<tr>
<th>Section</th>
<th>Secondary beams</th>
<th>Primary Beams</th>
<th>Cables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia</td>
<td>1016.16 E-06 M^4</td>
<td>19.6.E-06 M^4</td>
<td>0.01.E-06 M^4</td>
</tr>
<tr>
<td>Cross sectional inertia</td>
<td>32.8.E-03 M^2</td>
<td>7.31.E-03 M^2</td>
<td>45.2.E-03M^2</td>
</tr>
</tbody>
</table>

The dimensions and coordinates of the beams are taken from Arup’s structural drawings [17]. Boundary Conditions were chosen in such a way that the cables are free to rotate but are fixed against translation in any direction at the piers. There are no restrictions of the movements of the deck.

Frequency extraction procedure in ABAQUS was used to determine natural frequencies. The calculated natural frequencies are close to those measured on the real structure. The results are shown in table 4.3.

Table 4.3. Frequency extraction results

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Horizontal Mode</td>
<td>0.43Hz</td>
</tr>
<tr>
<td>First Vertical Mode</td>
<td>0.37Hz</td>
</tr>
<tr>
<td>Second Vertical Mode</td>
<td>0.71Hz</td>
</tr>
<tr>
<td>Second Horizontal Mode</td>
<td>1.08Hz</td>
</tr>
</tbody>
</table>
The first and second horizontal natural frequencies measured on the real bridge are 0.48Hz and 0.95Hz respectively. A unit concentrated force was applied in the horizontal direction and at the centre of the modelled bridge. The frequency response functions were extracted from the horizontal direction from node 27 and 19 for attenuation of first and second horizontal mode respectively. The extracted functions were used in the next chapter to show the effects of tuned mass damper.

### 4.3 SDOF Model of the Millennium Bridge

The Millennium Bridge is, in this section modelled as a generalised single degree of freedom system. The aim was to investigate the response of the bridge due to dynamic loading. The largest movement of the bridge took place on the centre span at a frequency of 0.95 Hz [18]. This frequency correspond with the second horizontal mode. Therefore, the SDOF model will be designed to describe the behaviour of that mode. The centre span was approximated as a single beam with distributed mass and stiffness. For the fact that the first mode shape of the bridge are close to sinusoidal, the shape function below was used in the modelling;

\[
\phi(x) = \sin\left(\frac{2\pi x}{L}\right)
\]  

(4.1)

Generalised mass was calculated from equation 3.8.

\[
\tilde{m} = \frac{mL}{2} = \frac{2234 \cdot 144}{2} = 160848 \text{ Kg}
\]  

(4.2)

\[f_2 = 1.08Hz \text{ From ABAQUS}
\]

\[\omega_2 = 2\pi f_2 = \sqrt{\frac{\tilde{k}}{\tilde{m}}} = 6.786\]

(4.3)

\[\tilde{k} = 7407017.6 \text{ Kg/ s}^2\]
The estimated damping ratio for each mode was measured around 0.006 to 0.008 after the opening of the bridge. Therefore an estimated damping ratio of 0.006 is used in the calculation below

\[
\tilde{c} = \zeta (2\tilde{m}\omega_2) = 13098.2 \text{ Kg/s} \quad (4.4)
\]

The generalised SDOF model was now used to calculate the dynamic response of the millennium bridge when subjected to dynamic load models proposed by Dallard and Nakamura. The equation of motion is solved for \( z(t) \) using finite difference method.

### 4.4 Dallard’s Load Model

Dallard et al conducted a test on the millennium bridge. The objective of the test was to provide the data needed to solve the vibration problem on the Millennium Bridge. The test showed that the dynamic force induced by pedestrians was approximately proportional to the lateral velocity of the bridge [18].

According to Dallard et al, the dynamic force per pedestrian \( \tilde{f}(t) \), can be related to the local velocity of the bridge, \( \dot{z}_{local} \) by:

\[
\tilde{f}(t) = k\dot{z}_{local} \quad (4.5)
\]

Where

\( k \) is a constant dependent on the bridge characteristics.

The pedestrians’ contribution to the modal force is \( \phi f(t) \)

\( \phi \) is the mode shape

The local velocity is related to the modal velocity by:

\[
\dot{z}_{local} = \phi \ddot{z} \quad (4.6)
\]

Single pedestrians’ contribution to the modal force is therefore:

\[
\phi \tilde{f}(t) = \phi k\dot{z}_{local} = \phi \ddot{z} \quad (4.7)
\]
the modal excitation force generated by \( n \) people uniformly distributed on the span is:

\[
\tilde{f}(t) = \frac{L}{2} \int_{0}^{L} \phi \, nk \tilde{z} \, dx 
\]  

(4.8)

Based on the field tests conducted on the London Millennium Bridge, \( k \) was found to be 300 Ns/m in the lateral frequency range 0.5 – 1.0 Hz.

Using \( \phi = \sin \left( \frac{2\pi x}{L} \right) \)  

(4.9)

The lateral pedestrian force becomes

\[
\tilde{f}(t) = \frac{L}{2} \int_{0}^{L} \phi \, nk \tilde{z} \, dx = 0.5nk \tilde{z}(t) 
\]  

(4.10)

The lateral pedestrian force is proportional to the bridge velocity.

### 4.4.1 Simulation of Dallard’s Load Model.

The aim of this simulation is to show how the lateral pedestrian forces proposed by Dallard’s increases linearly with velocity of the millennium Bridge. Assuming the pedestrians are distributed uniformly over the spans and that the mode shape (\( \phi \)) varies sinusoidally. The following values were used in the simulation:

Pedestrian density is 1.0 pers/m²

\[
\tilde{f}(t) = 0.5nk \tilde{z}(t) = 0.5 \times 576 \times 300 \tilde{z}(t) = 86,400 \tilde{z}(t)
\]
The displacement, velocity and acceleration of the millennium bridge were simulated using the dynamic load model proposed by Dallard et al. Lateral force of \(86.400\dot{z}(t)\) is used as input force. A small sinusoidal excitation force is used to get the system going. The M, C, K values of the Millennium bridge were used. Also shown below is the lateral force versus velocity graph. The equation of motion is solved using central difference method and was implemented in MATLAB.

Time step = \(\Delta t_i\).

The central difference expressions for velocity and acceleration are substituted into the general equation of motion, we have that

\[
m\left(\frac{z_{i+1} - 2z_i + z_{i-1}}{(\Delta t_i)^2}\right) + c\left(\frac{z_{i+1} - z_{i-1}}{2\Delta t_i}\right) + k(z_i) = f_i
\]  

(4.11)

Note that \(z_i\) and \(z_{i-1}\) are obtained from the preceding time step

The new displacement \(z_{i+1}\) is now calculated below

\[
z_{i-1} = \frac{\hat{f}}{\hat{k}}
\]  

(4.12)

\[
\hat{f} = f_i - \left[ \frac{m}{(\Delta t_i)^2} - \frac{c}{2\Delta t} \right] z_{i-1} - \left[ k - \frac{2m}{(\Delta t_i)^2} \right] z_i
\]  

(4.13)

\[
\hat{k} = \frac{m}{(\Delta t_i)^2} + \frac{c}{2\Delta t}
\]  

(4.14)

The simulated displacement time graph of Dallard’s load model on the Millennium bridge is shown below.
Figure 4.3. Displacement-time graph of the SDOF model
As can be seen in fig.4.4, the lateral force generated by pedestrians is approximately proportional to the velocity response of the bridge. A small disturbance will after some time give rise to a very large displacement, the bridge response will increase very rapidly towards infinity if the structure is not adequately damped. It is possible to reach a steady state solution using Dallard’s load model if the damping of the millennium bridge is increased by a minimum of 8 times number of limiting pedestrians according to equation (4.19) which is implemented in this simulation.

The response obtained using the small excitation force is smaller than the one obtained when Nakamura or Dallard’s load model was implemented. Dallard’s load model cannot be used to calculate the bridge response but can be used to evaluate the number of required limiting load and damping. Refer to section 4.4.1.1 for an explanation of why.
4.4.1.1 Damping Requirement and Limiting Number of Pedestrian To Avoid Instability

Since the lateral force is approximately proportional to the bridge response. There is a definite level of damping for which the damping force will exceed the excitation force [15], Assuming that the damping is linear, the modal damping force can be expressed as:

\[ \tilde{F}_D = \bar{c} \ddot{\phi} \]  \hspace{1cm} (4.15)

\[ \tilde{F}_D = C_{\text{critical}} c \ddot{\phi} \]  \hspace{1cm} (4.16)

\[ C_{\text{critical}} = \sqrt{MK} = 2\omega M \]  \hspace{1cm} (4.17)

According to Dallard et al, for the damping force to exceed the excitations force:

\[ c_c > \frac{nk_l}{8\pi fM} \]  \hspace{1cm} (4.18)

For a given level of damping, the limiting number of pedestrian to avoid instability is

\[ N_L = \frac{8\pi f c_c M}{k_l} \]  \hspace{1cm} (4.19)

This formula has been derived assuming the pedestrians are uniformly distributed over the whole span and mode shape is sinusoidal.

4.4.2 Nakamura’s Load Model

Nakamura also proposes a load model to obtain the pedestrian lateral dynamic forces. This load model is a modification of Dallard’s load model. Nakamura based his work on the observations and computations on the T-Bridge in Japan which experienced strong lateral vibration due to pedestrian’s excitation.
The basic equation in Nakamura’s model is the equation of motion.

See section 3.1.2

In accordance with Nakamura, the lateral dynamic force induced by pedestrians is modelled mathematically as:

\[
\ddot{f}(t) = k_1 k_2 \frac{\ddot{z}(t)}{k_3 + |\ddot{z}(t)|} G(f_B) M P g
\]

(4.20)

### 4.4.2.1 Simulation of Nakamura’s Load Model:

<table>
<thead>
<tr>
<th>Table 4.1. Values used in the simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 ) (Ratio of the force to the pedestrian’s weight)</td>
</tr>
<tr>
<td>( k_2 ) (percentage of pedestrians who synchronize to the girder vibration)</td>
</tr>
<tr>
<td>( G(f_B) ) (describes how pedestrians synchronize with the bridge’s natural frequency)</td>
</tr>
<tr>
<td>( k_3 ) (Limitation coefficient)</td>
</tr>
</tbody>
</table>

\( M P g \) is the modal self weight of pedestrians (During the load application it is assumed that the static vertical pedestrian load is \( Q = 752 \text{N} \) and the density of the pedestrians is \( 1.5\text{pers/m}^2 \)).

The modal self weight of the pedestrian is now calculated below

\[
M P g = \frac{144}{2} \times 1.5 \frac{\text{pers}}{m} \times 4m \times 75.2 \frac{\text{kg}}{\text{pers.g}}
\]

\( = 32486.4 \text{N} \)
Therefore \( \tilde{f}(t) = 428.82 \times \frac{\dot{z}(t)}{0.01 + |\ddot{z}(t)|} \) (4.22)

The initial velocity \( \dot{z}_0 \) is set to be 0.002. The initial displacement at \( u_0 = 0 \)

The velocity of the London millennium bridge was simulated using the dynamic load proposed by Nakamura. The equation of motion was solved using the central difference method the same way as shown in section 4.4 and then implemented in MATLAB. The simulated response is shown below. Also shown below in fig.4.6 is the lateral force versus velocity graph.

Nakamura assumes that the pedestrians synchronize proportionally with the girder velocity at low velocities. However, when the girder velocity becomes large, the pedestrians feel uncomfortable or unsafe and stop or decrease their walking pace. Therefore, the girder response does not increase infinitely but is limited at a certain level. This limitation depends on the coefficient \( k_3 \) [19].

![Figure 4.5. Displacement-time graph of the SDOF system](image-url)
As can be seen from fig.4.6, the force increases linearly with low velocities but the rate decreases at higher velocities. The response is limited at a certain level which depends on the limitation factor.

### 4.4.2.2 Comparison of Dallard’s and Nakamura’s load Model

Both models assume that the pedestrian force is a function of bridge velocity. Nevertheless, Dallard’s load model increases linearly with the bridge velocity and for this reason Dallard’s load model cannot be used to calculate bridge response. On the other hand, Nakamura’s load model increases linearly at low velocities but its increase rate becomes smaller at high velocities. Therefore Nakamura’s load model is used in evaluating the bridge response because the load model prevents the bridge response to go to infinity at high velocities, which is not the case with Dallard’s load model.
5 Suggestions for Design Solutions

5.1 General Overview of Possible Solutions

There are two suggested feasible options to solve the excessive vibration problems of pedestrian footbridges. Firstly, to increase the stiffness of the bridge to move the natural frequency of the bridge out of the range that could be excited by the pedestrian load. Secondly, to increase the damping of the bridge such that resonance response could be brought under control. Excessive vibrations have been caused by resonance between pedestrian’s loadings and natural frequency of footbridges.

Increase in stiffness affects both the resonant frequency as well as the system characteristic. A remedy could be to increase the natural frequency of the critical modes by increasing stiffness. However, in some cases increasing the stiffness might interfere with the beauty and structural taste of the footbridge. And for slender structures, increasing stiffness often implies a corresponding increase in mass which does not rhyme with the desired design taste of the structure. Therefore, the stiffening option is not favoured.

Additional damping is an effective way of controlling the resonance responses of excessive vibration problems of pedestrian footbridges. Damping is the energy dissipation properties of a material or system under cyclic stress. An extra damping system increases the amount of energy that is dissipated by the structure.

5.2 Additional Damping

Additional damping is used to increase the overall damping of a structure. Increasing the overall damping of pedestrian footbridge is an effective way of reducing the excessive dynamic response of pedestrian footbridges. Effects and modelling of two different additional dampers will be handled in the following sections:
5.2.1 Viscous Damper

Fluid viscous damper behaves like a shock absorber in an automobile schematic of a fluid viscous damper is shown in figure 5.1.

The damping fluid is silicone oil, which is inert, non-flammable, non-toxic, and stable for extremely long periods of time. The seals in the fluid viscous damper use a patented high technology design and provide totally leak free service.

The damping action is provided by the flow of fluid across the piston head. The piston head is made with a deliberate clearance between the inside of the cylinder and the outside of the piston head, which forms an annular orifice. The fluid flows through this orifice at high speed as the damper strokes. The shape of the piston head determines the damping characteristics.

The force/velocity relationship is given by the equation (5.1)

\[ F = c \cdot \ddot{x} \]  

(5.1)

\( F \) is the damping force
\( \dot{x} \) is the relative velocity across the damper
\( c_v \) is a constant proportionality related to the fluid viscosity. It is called the damping coefficient and has units of Ns/m, or kg/s.

When the fluid viscous damper strokes in compression, fluid flows from Chamber 2 to Chamber 1. When the fluid viscous damper strokes in tension, fluid flows from Chamber 1 to Chamber 2. The high pressure drop across the annular orifice produces a pressure differential across the piston head, which creates the damping force.

Consider a mechanical system with one degree of freedom. The system has a mass \( m \). Connected to the mass are a spring with stiffness \( k \) and a viscous damper with damping \( c_v \). The mass is driven by a dynamic force \( f(t) \). A horizontal force is chosen in order to avoid gravitational effect. This system is shown in figure 5.2.

![Mechanical system with one degree of freedom](image)

**Figure 5.2. Mechanical system with one degree of freedom [20]**

It is assumed that the mass can move without friction around its equilibrium position \( l_0 \). The spring and mass damper are assumed to be massless.

The spring which is subjected to a force \( f(t) \) gives a counter force which is proportional to the displacement from its equilibrium position. The constant of proportionality \( k \) is called the spring stiffness. The viscous damper comparably provides a counter force which is proportional to the velocity (Time derivative of position) \[9\]

Let \( c_v = c \) (for simplicity) \[5.2\]

From Newton’s second law,
\[ m\ddot{x} + c\dot{x} + kx = f(t) \]  \hspace{1cm} (5.3)

\[ \ddot{x} = \frac{d^2x}{dt} = \text{Acceleration} \]

\[ \dot{x} = \frac{dx}{dt} = \text{Velocity} \]

\[ x = \text{Displacement} \]

Taking the Laplace transform of both sides we have that;

\[ (ms^2 + cs + k)X(s) = F(s) \]  \hspace{1cm} (5.4)

Equation 4.3 leads to the dynamic transfer function, \( H(s) \), between \( F(s) \) and \( X(s) \)

\[ H(s) = \frac{X(s)}{F(s)} = \frac{1/m}{s^2 + 2s\zeta\omega_0 + \omega_0^2} \]  \hspace{1cm} (5.5)

\( \omega_0 \) is the undamped natural (angular) frequency in [rad/s]

The homogeneous solution to equation 5.3 is given below

\[ s_{1,2} = -\zeta\omega_0 \pm j\omega_0 \sqrt{1 - \zeta^2} \]  \hspace{1cm} (5.6)

Applying an inverse Laplace transform of equation 2.13, results in impulse response

\[ h(t) = C_1e^{ts_1} + C_2e^{ts_2} \]  \hspace{1cm} (5.7)
Making use of the fact that the two poles are complex conjugates. \( s_1 = s^*_2 \)
and that \( h(0) = 0 \), we have:

\[
h(t) = Ae^{-\zeta \omega_0 t} \sin(\omega_0 t \sqrt{1 - \zeta^2})
\]

(5.8)

Where \( A \) is a real constant. The frequency response \( H(f) \) which is the ratio between the spectrum of the output response and spectrum of the input (force) is obtained by the Laplace transform theory [9] by letting \( s = j\omega \).

\[
H(f) = \frac{X(f)}{F(f)} = \frac{1/K}{1 - \left[ \frac{f}{f_0} \right] + j2\zeta \left[ \frac{f}{f_0} \right]}
\]

(5.9)

The lower the viscous damping the greater the amplitude, for a given force. Horizontal damping of the London Millennium Bridge is provided primarily by viscous dampers. After the opening of the bridge, fluids viscous dampers were installed on the London Millennium Bridge mostly to suppress excessive vibrations in the lateral direction see Fig. 5.3. As a result, the damping ratio increased from 0.5% to 20% and near-resonant accelerations were reduced by about 40 times [20].

Most of the viscous dampers are situated on top of the transverse arms every 16 meters, beneath the deck, see Fig. 5.3. Each end of the viscous damper is connected to the apex of a steel V bracing. The apex of the bracing is supported on roller bearings that provide vertical support but allow sliding in all directions. The other ends of the bracing are fixed to the neighbouring transverse arms. In this way the horizontal modal movement over 16 meters is mobilised at each damper [20].
5.2.2 Tuned Mass Damper

Tuned mass damper can be analysed as a two degree of freedom system that can be used to reduce vibration. It is a device consisting of mass, a spring and a damper that is attached to a structure in order to reduce the dynamic response of the structure. The frequency of the damper is tuned to a particular structural frequency so that when that frequency is excited, the damper will resonate out of phase with structural motion. An example of a system with a tuned damper is shown in figure.5.4. Assume an SDOF system with a mass of $m$, a spring with stiffness $k$ and damping $c$. A second SDOF system $m_a$, $k_a$ and $c_a$ respectively. Assuming that the system is forced by a harmonic force, the displacement, $x$, of the mass that results is called forced vibration.
There is an angular frequency, \( \omega_a \), for which the mass \( m \) will not move at all. This is often called an antiresonance, by choosing the mass, \( m_a \) and the stiffness \( k_a \) in a way that \( \omega_a \) correspond to the operating frequency. The vibrations can be reduced substantially at that frequency:

\[
\omega_a = \sqrt{\frac{k_a}{m_a}} = \omega_r = \sqrt{\frac{k}{M}}
\]

(5.10)

The vibration absorption comes from the tuned mass damper. Values of \( m \), \( c \) and \( k \) (see section 4.3) of the millennium bridge were used. The tuned damper values were chosen so that the damping of the tuned damper was four times (Depending on how much attenuation you want to achieve) that
of the SDOF of the system and the frequency of the tuned damper was chosen equal to the resonance frequency of the SDOF system.

\[ 10^{-10} \]  10^{-9}  \]  10^{-8}  \]  10^{-7}  \]  10^{-6}  \]  10^{-5}  \]  10^{-4}  \]  10^{-3}  \]  10^{-2}  \]  10^{-1}  \]  10^{0}  \]  10^{1}  \]  10^{2}  \]  10^{3}  \]  10^{4}  \]  10^{5}  \]  10^{6}  \]  10^{7}  \]  10^{8}  \]  10^{9}  \]  10^{10}  \]

Effect of Tuned mass damper on SDOF

\[ \frac{\text{Dynamic flexibility}, m}{N} \]

\[ \text{Frequency ,Hz} \]

Without TMD: Relative damping = 0.0021
With TMD: Relative damping = 0.0083

As can be seen in Figure 5.5 after adding tuned mass damper, antiresonance occurs at the tuned frequency. The relative damping of the SDOF system with TMD is about four times the relative damping of the system without TMD.

5.2.2.1 Effects of a Tuned Mass Damper on Millennium Bridge.

A unit force is applied in the horizontal direction at the dotted position of the model shown in figure 5.6. TMDs were attached in the horizontal direction to the secondary beams in between the secondary beams’ point of contact to the deck beams.

Figure 5.5. Frequency response of the displacement of the SDOF model of the Millennium bridge before and after adding TMD
The FRF functions of first and second horizontal modes of the millennium bridge were extracted from ABAQUS (refer to section 4.1 for the nodes) and were used to show the effects of TMDs. The purpose was to reduce the vibration effects at the first and second horizontal modes. The real and imaginary parts of the FRF functions of the simulated bridge were imported into the MATLAB script. The FRFs were coupled using the MATLAB function ‘SUBSTRUCT’. This function is used for coupling structures together using FRF-coupling method [22]. The parameters used in tuning the TMDs to first and second horizontal mode of the bridge were shown in table 5.1.

Table 5.1. TMD parameters

<table>
<thead>
<tr>
<th></th>
<th>m(Kg)</th>
<th>c(N/m)</th>
<th>k(N/ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Horizontal Mode</td>
<td>500</td>
<td>40</td>
<td>3650</td>
</tr>
<tr>
<td>Second Horizontal mode</td>
<td>450</td>
<td>80</td>
<td>18E3</td>
</tr>
</tbody>
</table>

The TMD was tuned to 0.43Hz and 1.01Hz which correspond to the first and second horizontal mode of the bridge.
The effects of the tuned damper are shown in the figures below. For the first horizontal mode, the following effects were observed on the bridge model with and without damper:

Figure 5.7. Displacement-time graph of MDOF model of millennium bridge before attachment of TMD using Nakamura’s load model

Figure 5.8. Displacement-time graph of MDOF model of millennium bridge after attachment of TMD using Nakamura’s load model
As can be seen from figure 5.7 and 5.8, the displacement reduced from 0.043 to 0.0068 when the attached TMD was tuned to frequency of the first horizontal mode. For the second Horizontal mode, the following effects were observed on the bridge model with and without damper:

![Figure 5.9. Displacement-time graph of MDOF model of Millennium Bridge before attachment of TMD using Nakamura’s load model](image1)

![Figure 5.10. Displacement-time graph of MDOF model of Millennium Bridge after attachment of TMD using Nakamura’s load model](image2)
There was also a reduction of the displacement from 0.038 to 0.026 when the attached TMD was tuned to the second horizontal mode. This can be seen by comparing figures 5.9 and 5.10. The vibration attenuation of the two horizontal modes differed because their FRFs were extracted from the different nodes. Figures 5.11 and 5.12 show the curve fitting of the first and second horizontal mode after the addition of TMD.

Figure 5.11. Curve fitting of the first horizontal mode after attachment of TMD

Figure 5.12. Curve fitting of the second horizontal mode after attachment of TMD
As can be seen in figures 5.11 and 5.12, the curve fittings of the first and second horizontal mode of the bridge after attachment of TMD were perfect.

The simulation was carried out using Nakamura’s load model. A small excitation force is used to get the system going. Dallard’s load model cannot be used to calculate the bridge response (See section 4.4 for reason).
6 Conclusion

The main aim of this work was to simulate the effect mass tuned dampers have on reducing the excessive vibration caused by pedestrian loading on the London Millennium Bridge. A unit force load model to excite the bridge model.

A Theoretical study of structural dynamics and dynamic loads induced by pedestrians were also reviewed which can aid in understanding of dynamics in pedestrian footbridge.

The Effects of Nakamura and Dallard’s load model on the SDOF model of Millennium Bridge were also simulated. The simulations showed that Dallard’s load model increases linearly with the bridge velocity. On the other, Nakamura’s load model increases linearly with low velocities but the rate decreases at higher velocities. Nakamura’s load model prevents the excitation force to go to infinity at high velocities, which is not the case with Dallard’s load model.

The simulations showed that the vibrations reduced substantially when TMDs of suitable properties were added to the bridge. A good way to solve excessive vibrations in footbridges is to increase the overall damping of the system by installing additional damping system. This distinct approach is a possible solution to be considered in the design of footbridges in order to avoid the excessive vibration of pedestrian bridges.

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7 Description of functions in the toolbox

**Complexp** – Estimates the Poles with Complex Exponential Global Method

**Pol2resf** – Estimates Residues Using Frequency Domain Residuals.

**Substruct** - Adds the TMD to the Bridge Model

**Bridge_response_rap** - Solves the Forced Response of an MDOF System Using Both an External Excitation Force as well as Either Dallard’s or Nakamura’s load model.

**Timeresd_Rp** – Solves Forced Time Response for FRF Given by Residues and Poles

**Sigtrunc** – Does the Signal Truncation.

**Modeind1** - Mode Indicator Function.

**Maketime** – Creates the Time Vector.

**Filtercoeffcond** – Calculates the Filter Coefficient for the Given Input and Output.

**Impresp** – Calculates the Impulse Response From Frequency Response Function.

**Linearresp** – Calculates the Forced Linear Time Subroutine used for Timeresd.

**Htoeplitz** – Special Toeplitz Matrix Made up of an Impulse Response.

**RAP2FRFV_RES** – Used in Estimation of Residues, Poles and Residuals.

**RAP2FRFV** – Used in Estimation of Residues and Poles

**FRF TRUNC** - Does the FRF Matrix Truncation in Frequency
8 References


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