Design and Dynamic Analysis of a Space Frame for Acoustic Measurements

Tommi Paananen

Department of Mechanical Engineering
University of Karlskrona/Ronneby
Karlskrona, Sweden
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Thesis submitted for completion of Master of Science in Mechanical Engineering with emphasis on Structural Mechanics at the Department of Mechanical Engineering, University of Karlskrona/Ronneby, Karlskrona, Sweden.

Abstract:
Static and dynamic characteristics of a Space Frame were determined by using the Finite Element Method. Also experimental measurements were done on the manufactured frame resulting in the dynamic characteristics of the structure. Results from this Experimental Modal Analysis were compared with the results from FEM-calculations. Correlation between the models was obtained showing very good agreement between the FE-model and the manufactured Space Frame. Further, the FE-model was used to modify the manufactured frame so that the predetermined target values were fulfilled.

Keywords:
Structural Dynamics, Space Frame, Structural Stiffness, FE-method, Modal Analysis, Experimental verification, Correlation.
Acknowledgements

This work was carried out at the Department of Mechanical Engineering, at the University of Karlskrona/Ronneby, Karlskrona, Sweden, under the supervision of Ph.D. student Madeleine Hermann. Prof. Anders Ulfvarsson at the department of Naval Architecture and Ocean Engineering at Chalmers University of Technology has acted as a co-supervisor.

The project is financially supported by NUTEK, Volvo Car Corporation, SAAB Automobile and Collins & Aikman.

The aim of this project is to determine and control the static and dynamic characteristics of a Space Frame. The frame will be used in acoustic measurements further on, and therefore the structural characteristics will be thoroughly documented.

As can be understood, a lot of people have been involved in this project. I would like to thank all people at the involved institutes, companies and all students at the Master Programme for making this project possible.

Karlskrona, September 1998

Tommi Paananen
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1. Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>Stiffness matrix</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass matrix</td>
</tr>
<tr>
<td>$x$</td>
<td>Displacement vector</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Mode shape</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>$H$</td>
<td>Frequency response function</td>
</tr>
<tr>
<td>$I$</td>
<td>Area moment of inertia</td>
</tr>
<tr>
<td>$J$</td>
<td>Polar moment of inertia</td>
</tr>
<tr>
<td>$P$</td>
<td>Force</td>
</tr>
<tr>
<td>$R$</td>
<td>Rotational degree of freedom</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque</td>
</tr>
<tr>
<td>$F(\omega)$</td>
<td>Fourier transform of $f(t)$</td>
</tr>
<tr>
<td>$X(\omega)$</td>
<td>Fourier transform of $x(t)$</td>
</tr>
<tr>
<td>$S_{ff}$, $S_{xx}$</td>
<td>Auto-spectral density</td>
</tr>
<tr>
<td>$S_{fx}$, $S_{xf}$</td>
<td>Cross-spectral density</td>
</tr>
<tr>
<td>$b$</td>
<td>Width</td>
</tr>
<tr>
<td>$c$</td>
<td>Number</td>
</tr>
<tr>
<td>$h$</td>
<td>Height, response function</td>
</tr>
<tr>
<td>$i$</td>
<td>Number</td>
</tr>
<tr>
<td>$j$</td>
<td>Number</td>
</tr>
<tr>
<td>$k$</td>
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<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, thickness</td>
</tr>
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<td>$x$</td>
<td>Co-ordinate</td>
</tr>
<tr>
<td>$y$</td>
<td>Co-ordinate</td>
</tr>
<tr>
<td>$z$</td>
<td>Co-ordinate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Number</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coherence</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Natural frequency</td>
</tr>
</tbody>
</table>
Indices

\begin{align*}
B & \quad \text{Bending} \\
T & \quad \text{Torsional} \\
a & \quad \text{Analytic} \\
f & \quad \text{Input} \\
i & \quad \text{Number} \\
j & \quad \text{Number} \\
k & \quad \text{Number} \\
m & \quad \text{Measured, number} \\
n & \quad \text{Number} \\
v & \quad \text{Saint Venant} \\
w & \quad \text{Vlasov} \\
x & \quad \text{Co-ordinate, output} \\
y & \quad \text{Co-ordinate} \\
z & \quad \text{Co-ordinate}
\end{align*}

Abbreviations

\begin{align*}
\text{BIW} & \quad \text{Body In White} \\
\text{EMA} & \quad \text{Experimental Modal Analysis} \\
\text{FEM} & \quad \text{Finite Element Method} \\
\text{FRF} & \quad \text{Frequency Response Function} \\
\text{MAC} & \quad \text{Modal Assurance Criterion} \\
\text{MDOF} & \quad \text{Multi Degree of Freedom} \\
\text{MIMO} & \quad \text{Multi-Input-Multi-Output}
\end{align*}
2. Introduction

2.1 Background

In recent years a massive mass hunting has been present in car industry. When developing new car models the optimisation of the weight of the car has been increasingly important, due to the economic advantages of lighter cars. Cars that consume less fuel have also been more attractive for customers. To achieve this, use of new lighter materials as aluminium, magnesium, polymeric and composite materials, instead of steel, has been of interest.

However, different kinds of questions have appeared when studying the possibilities of using these new materials in cars. The cars still have to be safe in traffic, the performance should be as good as for the cars manufactured of conventional materials and the driving comfort should not be worse. The safety and the performance of the car are very dependent on the frame stiffness. Comfort is often related to the vibration and sound radiation problems. The question still remains; do these lightweight cars have the same performance as the cars of today?

When using steel plates vibration and sound radiation problems can occur. A lot of work has been done to solve these problems. Different kinds of sound insulation materials have been developed. Using these in cars as damping materials has been quite successful. The problem with using these materials, is that the weight of the car will increase. Recently, some efforts of manufacturing multi layer materials, in which the damping material is integrated in the plate material, have shed light on this area. Different institutes are investigating the possibilities of using these lightweight panels in cars.

2.2 Project Description

This work is a part of a larger project, carried out at Chalmers between 1996-1999, entitled "Heat and sound insulation, with lightweight materials; possibilities and consequences". The aim of the main project is to investigate the possibilities of using lightweight panels in cars. Departments involved at Chalmers are Applied Acoustics, Polymeric Materials and Naval Architecture & Ocean Engineering. Ingemansson Technology is also involved in the project as a sound and vibration consultant.
The project is financially supported by NUTEK, Volvo Car Corporation, SAAB Automobile and Collins & Aikman.

The first work on the project was done by Lenoir and Gallen [1] submitted at Chalmers. They have collected some of the texts available about current lightweight car projects and studies. Their work even investigates how different plates used in the car influence the stiffness of the car.

Next step in the project is to design and manufacture a space frame rig, similar to a car, to investigate acoustic and vibration characteristics of some lightweight panels. The panels will be fastened at the frame and the sound radiation from the panels will be measured when the frame is excited.

To investigate the results from the acoustic measurements the characteristics of the frame are important to know. To get a realistic picture of the panel behaviour, the characteristics of the frame should be as similar to the characteristics of a “real” car as possible. It can be said that the natural frequencies, mode shapes and stiffness of the frame should be similar to those of a real car. In this study the frame should behave as Body in White (BIW) of a smaller car. BIW is commonly used abbreviation in the car industry and describes a car skeleton, see figure 2.1.

![Figure 2.1 The Body In White of the 1991 GM Saturn](image)

The aim of this sub-project is to control and determine the static and dynamic characteristics of the space frame rig. The frame was originally
designed by Prof. Anders Ulfvarsson at Chalmers. The static characteristics are the bending and torsional stiffness, and the dynamic characteristics are the modal parameters: natural frequencies, mode shapes and damping ratios.

### 2.2.1 Method and Restrictions

The static and dynamic characteristics, except from the damping ratios, will be determined numerically by using Finite Element Method. The frequency range of interest is 0-400 Hz. The dynamic FE-model will be verified with an Experimental Modal Analysis (EMA) of the manufactured frame. The damping ratios will also be attained from the EMA. Finally a correlation between the FE-model and the experimental model will be done. If the correlation between the dynamic models is good, it is probably correct to assume that the static characteristics of the FE-model also have good correlation with the static characteristics of the manufactured frame. Further, the FE-model is used to modify the frame in order to fulfil the static and dynamic target values.

Target values for the Space Frame are obtained from the car manufactures and presented in table 2.1.

<table>
<thead>
<tr>
<th>Static Target value</th>
<th>Torsional Stiffness</th>
<th>~10 kNm/degree max defl. 1mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bending Stiffness</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dynamic Target Values</th>
<th>1:st Torsion mode</th>
<th>25-30 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1:st Bending mode</td>
<td>40-50 Hz</td>
</tr>
</tbody>
</table>

*Table 2.1 Target values for the Space Frame*

For the bending stiffness no target value is specified, but the maximum deflection of the transmission tunnel is of interest and should be less than 1 mm.
3 Frame Design

3.1 Frame Dimensions

The original frame was designed by Prof. Anders Ulfvarsson. The frame is supposed to represent a smaller car like VW Golf, MB A-class and Audi A3. The main dimensions of the frame were attained from investigations of cars like these. Another important dimension criterion was the spaces in the floor that are intended for the lightweight panels. From the point of manufacturing steel, with an E-modulus of $\approx 205$ GPa was chosen as material. The frame is manufactured of beams that are welded together, see figure 3.1. The length of the beams varies from 0.5 m to 1.5 m. The main dimensions of the frame are shown in Appendix A. See also Appendix B for the commonly used terms of the structural parts of a car skeleton.

![Figure 3.1 Space Frame design](image)

The beams are manufactured out of two 1.5 mm plates, one U-shaped and one L-shaped, which are spot-welded to each other. This results in a thin walled box beam with two flanges. The dimensions of the beam section at the bottom of the frame are 100x100 mm and 70x70 mm elsewhere. Figure 3.2 illustrates the beam sections.
Figure 3.2 Beam sections

The spot-welds have a distance of about 100-mm between each other. Figure 3.3 illustrates the beam construction and spot-welds of the manufactured frame.

Figure 3.3 Frame detail with spot-welded beams
4. Theories

4.1 Static Characteristics

4.1.1 Bending Stiffness

Parameters that influence the bending stiffness of the frame can be illustrated with a simple beam-bending example.

\[ \delta = \frac{Pl^3}{48EI} \]  
(4.1)

The deflection \( \delta \) can be determined from elementary cases of beam bending. In this case the deflection at the middle of the beam will be

The bending stiffness is defined as

\[ k_B = \frac{P}{\delta} \]  
(4.2)

Substituting equation 4.1 in 4.2 gives

\[ k_B = \frac{48EI}{l^3} \]  
(4.3)

The factor \( EI \) is often called flexural bending stiffness. It is easy to see that parameters affecting the bending stiffness of a beam are \( E \), Young’s modulus, \( I \), cross-sectional area moment of inertia and the length, \( l \), of the beam. \( I \) for a thin walled box beam cross section, with width, \( b \), height, \( h \), and wall thickness, \( t \), is
\[ I = \frac{th^2}{6} (3b + h) \] (4.4)

A closer look at equation 4.4 indicates that the \( k_B \) is linear dependent of \( b \) and \( t \). Equation 4.4 contains \( h^3 \) and therefor the \( k_B \) is not linear dependent of \( h \). As a conclusion a change in \( h \) will give more efficient change of \( k_B \) than a change in \( b \).

These simple relations can give some guidance of the bending characteristics of the frame. It is clear that the outer dimensions, like height and length of the frame largely affect the stiffness. Also, increasing beam section dimensions will result in a stiffer construction. A change in Young’s modulus will give a linear increase of stiffness, but in this work a change like this is not of interest.

### 4.1.2 Torsional Stiffness

The torsional stiffness parameters in the frame are illustrated with a simple beam model. A beam with length, \( l \), is exposed for a torque, \( T \), as seen in figure 4.2.

![Figure 4.2 Beam applied for torsion](image)

\[ T \] is causing an angular deflection, \( \varphi \). The torsional stiffness is defined as

\[ k_T = \frac{T}{\varphi} \] (4.5)

If the part is axisymmetric \( T \) contains only so called Saint-Venant torsion. If the section is not axisymmetric \( T \) will contain both Saint-Venant and Vlasov torsion, see e.g. [2]. In Saint-Venant torsion the warping of the beam is neglected, while in Vlasov torsion it is included. If the beam section is solid or closed it can be treated with good accuracy by using Saint-Venant torsion. If this is not the case a torsion parameter \( \alpha \) is
calculated. This parameter gives information about the amounts of the different types of torsion. Parameter $\alpha$ can be calculated with

$$\alpha = -\frac{GJ_v l^2}{\pi^2 EJ_w}$$

(4.6)

where $G$ is shear modulus, $J_v$ is Saint Venant moment of inertia, $l$ is the length of the beam, $E$ is Young’s modulus and $J_w$ is sectorial moment of inertia. The product $GJ_v$ is called torsional stiffness and the product $EJ_w$ is called warping stiffness. If the parameter $\alpha$ is close to 0, the case studied should be treated with Vlasov torsion, if the value is near 20 the case should be calculated as Saint-Venant torsion.

In this study the area of interest is what parameters, not the amount of them, that affect the stiffness of the frame. The two parameters affecting the torsional stiffness are the resistance against torsion and warping. As mentioned these are the two products $GJ_v$ and $EJ_w$. $G$ and $E$ are material parameters and not of interest in this study. $J_v$ and $J_w$ are geometry depending parameters, e.g. for a U-shaped beam section showed in figure 4.3 the geometry depending parameters will become

$$J_v = \frac{ct^3}{3}(2b + h - 2t) \quad \text{and} \quad J_w = \frac{b^3 h^2 t}{12}(3b + h)$$

(4.7)(4.8)

where $c$ is a shape factor and is $\approx 1.12$ for U-shaped sections.

![Figure 4.3 U-shaped beam section](image)

As is seen, the outer dimensions $b$ and $h$ of the section are the most important parameters for increasing stiffness. Clearly a change in $b$ will
give more efficient increase of torsion and warping stiffness in this case. Of course, the length of the beam also affects the stiffness of the frame. How parameter modifications affect the frame stiffness is difficult to say, due to the complexity of the geometry. One way to accomplish this is by doing a parameter study, but that is not carried out in this work.

4.1.3 Stress Distribution

However, Pagliarulo and Ugolini [3] have investigated the stress distribution in BIW. The results are presented in table 4.1.

<table>
<thead>
<tr>
<th>PART DESCRIPTION</th>
<th>TORSION WITHOUT GLASSES MAX STRESS</th>
<th>TORSION WITH GLASSES MAX STRESS</th>
<th>BENDING WITH GLASSES MAX STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DASH PANEL</td>
<td>0.77</td>
<td>0.48</td>
<td>0.10</td>
</tr>
<tr>
<td>FIREWALL (UPPER PART)</td>
<td>0.16</td>
<td>0.17</td>
<td>0.04</td>
</tr>
<tr>
<td>FIREWALL (LOWER PART)</td>
<td>0.22</td>
<td>0.19</td>
<td>0.09</td>
</tr>
<tr>
<td>INNER WING</td>
<td>0.23</td>
<td>0.29</td>
<td>0.16</td>
</tr>
<tr>
<td>LOWER A - PILLAR (INNER PART)</td>
<td>0.98</td>
<td>0.84</td>
<td>0.16</td>
</tr>
<tr>
<td>UPPER A - PILLAR (INNER PART)</td>
<td>1.00</td>
<td>1.00</td>
<td>0.56</td>
</tr>
<tr>
<td>SIDE - FRAME</td>
<td>0.92</td>
<td>0.93</td>
<td>0.39</td>
</tr>
<tr>
<td>LOWER WINDSHIELD CROSS-MEMBER</td>
<td>0.54</td>
<td>0.29</td>
<td>0.18</td>
</tr>
<tr>
<td>UPPER WINDSHIELD CROSS-MEMBER</td>
<td>0.78</td>
<td>0.40</td>
<td>0.09</td>
</tr>
<tr>
<td>ROOF RAIL</td>
<td>0.64</td>
<td>0.41</td>
<td>0.34</td>
</tr>
<tr>
<td>FLOOR (MID-PART)</td>
<td>0.13</td>
<td>0.13</td>
<td>0.20</td>
</tr>
<tr>
<td>SILL</td>
<td>0.27</td>
<td>0.26</td>
<td>0.68</td>
</tr>
<tr>
<td>FLOOR FRONT CROSS-MEMBER</td>
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<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>FLOOR MID CROSS-MEMBER</td>
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<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
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<td>1.00</td>
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<tr>
<td>REAR WHEEL-HOUSING</td>
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<td>0.16</td>
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<tr>
<td>REAR WALL</td>
<td>0.27</td>
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<td>0.05</td>
</tr>
<tr>
<td>REAR FENDER</td>
<td>0.34</td>
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<td>0.12</td>
</tr>
<tr>
<td>REAR GLASS UPPER CROSS-MEMBER</td>
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<td>0.03</td>
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<tr>
<td>ROOF</td>
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<td>0.14</td>
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<td>REAR SILL</td>
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<td>0.11</td>
</tr>
<tr>
<td>----------------------------</td>
<td>-------</td>
<td>------</td>
<td>-----</td>
</tr>
</tbody>
</table>

Table 4.1 Equivalent Von Mises stress in different load cases (scaled with maximum stress for the load case)

As can be read from the table, the stresses in the torsional load case are distributed widely in the BIW. In the bending load case the stresses are concentrated in the B-pillars, in the upper A-pillars, in the bottom and roof of the BIW.
4.2 Dynamics

4.2.1 Multiple-Degree-of-Freedom (MDOF) system

Figure 4.4 illustrates an undamped $n$-mass system with $n$ translational degrees of freedom, giving the system $n$ degrees of freedom. This simple system is used to illustrate the dynamic characteristics of the frame. A lot of literature is written on structural dynamics for example [4] and [5]. Here the frame is treated as undamped and the damping coefficients are determined further with experimental procedures. The number of degrees of freedom will increase a lot when the dynamic behaviour of the frame is calculated with FEM. Choice of element type and the number of elements will decide the number of degrees of freedom. Elements that are chosen for the determination of the natural frequencies and mode shapes of the frame are presented in section 5.1.3.

![Figure 4.4 Undamped MDOF system](image)

To determine the forces from the springs acting on each mass a free body diagram is made.

![Figure 4.5 Free body diagram of MDOF-system in Figure 4.4](image)
Using Newton’s second law equations of motion for each mass can be written as

\[
m_1\ddot{x}_1 + x_1(k_1 + k_2) - x_2 k_2 = 0 \\
m_2\ddot{x}_2 + x_2(k_2 + k_3) - k_2 x_1 - k_3 x_3 = 0 \\
\vdots \\
m_{n-1}\ddot{x}_{n-1} + x_{n-1}(k_{n-1} + k_n) - k_n x_n - k_{n-1} x_{n-1} = 0 \\
m_n\ddot{x}_n - k_n x_{n-1} + k_n x_n = 0
\]

Equation 4.9 consists \( n \) coupled second-order differential equations and can be written in matrix form as

\[
\begin{bmatrix}
m_1 & 0 & 0 & \cdots & 0 \\
0 & m_2 & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \ddots & \ddots \\
0 & \cdots & 0 & m_{n-1} & 0 \\
0 & \cdots & 0 & m_n & \ddots
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\ddots \\
\ddot{x}_{n-1} \\
\ddot{x}_n
\end{bmatrix}
= 
\begin{bmatrix}
k_1 + k_2 & -k_2 & 0 & \cdots & 0 \\
-k_2 & k_2 + k_3 & -k_3 & \cdots & \ddots \\
0 & \ddots & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & k_{n-1} + k_n & -k_n \\
0 & \cdots & 0 & -k_n & k_n
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\ddots \\
x_{n-1} \\
x_n
\end{bmatrix}
\]

or by using matrix notation as

\[
M\ddot{x} + Kx = 0 
\]  \hspace{1cm} (4.10)

where \( M \) is the mass matrix and \( K \) is the stiffness matrix. Note that both matrices are symmetric. These matrix equations can be solved as algebraic eigenvalue problem, resulting in structural natural frequencies and mode shapes of vibration.

### 4.2.2 Natural Frequencies and Mode Shapes

Assuming harmonic motion

\[
x(t) = \phi e^{i\omega t} 
\]  \hspace{1cm} (4.12)

where \( \phi \) is a \( nx1 \) vector of amplitudes and \( \omega \) is constant, the undamped natural frequency. Substituting equation 4.12 in 4.11 and the equation of motion becomes

\[
(K - \omega^2 M)\phi e^{i\omega t} = 0 
\]  \hspace{1cm} (4.13)

as \( e^{i\omega t} \neq 0 \)

\[
(K - \omega^2 M)\phi = 0 
\]  \hspace{1cm} (4.14)
Equation 4.14 is a generalised eigenvalue problem and has non trivial solutions as
\[ \det (K - \omega^2 M) = 0 \] (4.15)

Equation 4.15 is called characteristic equation of the system and yields \( n \) real solutions \( \omega_1^2, \omega_2^2, ..., \omega_n^2 \), known as the eigenvalues \( \lambda_1, \lambda_2, ..., \lambda_n \). Solving the eigenvalue problem for each eigenvalue yields the corresponding eigenvectors \( \phi_1, \phi_2, ..., \phi_n \). These eigenvectors are known as the mode shapes of the dynamic system. See e.g. [4].

Parameters affecting the natural frequencies of the frame are obviously the stiffness of the structure and the mass distribution in it. How the stiffness of the frame can be changed is discussed in chapter 4.1 and how the mass distribution will influence it is, due to the complexity of the frame, difficult to say exactly. This could be done in FE-programmes where different masses are located on chosen places of the FE-model. This trial-error method is not so efficient, but it can give some guidelines for the dynamic characteristics caused by mass distribution changes.

### 4.2.3 Strain Energy Distribution

Pagliarulo and Ugolini [3] have investigated strain energy distribution in BIW, the results are presented in table 4.2.

As seen in the table, the first bending mode strain energy is mainly distributed at the sides and roof of the car, nearly 44% of the total energy of the mode. Also some energy is received in the A-pillar and in the floor, while the rest of the car is not receiving so much energy. In the torsion mode the roof is not such a big energy receiver as in the bending mode, while the sides of the car are submitted to the greatest part of the total energy distribution. Also, the rear part of the car acts more like an energy absorber. These results/conclusions can give some guidance when determining what parts of the car should be changed if the frequencies should rise or go down.
Table 4.2 Strain energy distribution in different modes

<table>
<thead>
<tr>
<th>PART DESCRIPTION</th>
<th>FIRST TORSION</th>
<th>FIRST BENDING</th>
<th>SECOND BENDING</th>
</tr>
</thead>
<tbody>
<tr>
<td>DASH PANEL</td>
<td>0.8</td>
<td>2.2</td>
<td>0.3</td>
</tr>
<tr>
<td>FIREWALL (UPPER PART)</td>
<td>0.4</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>FIREWALL (LOWER PART)</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>INNER WING</td>
<td>2.4</td>
<td>4.6</td>
<td>0.1</td>
</tr>
<tr>
<td>LOWER A - PILLAR (INNER PART)</td>
<td>2.3</td>
<td>4.0</td>
<td>1.2</td>
</tr>
<tr>
<td>UPPER A - PILLAR (INNER PART)</td>
<td>5.5</td>
<td>6.8</td>
<td>1.2</td>
</tr>
<tr>
<td>SIDE - FRAME</td>
<td>28.1</td>
<td>21.3</td>
<td>8.4</td>
</tr>
<tr>
<td>LOWER WINDSHIELD CROSS-MEMBER</td>
<td>0.4</td>
<td>1.4</td>
<td>0.2</td>
</tr>
<tr>
<td>UPPER WINDSHIELD CROSS-MEMBER</td>
<td>0.8</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>ROOF RAIL</td>
<td>4.4</td>
<td>2.2</td>
<td>0.9</td>
</tr>
<tr>
<td>FLOOR (MIN-PART)</td>
<td>0.9</td>
<td>5.0</td>
<td>0.7</td>
</tr>
<tr>
<td>SILL</td>
<td>3.4</td>
<td>4.9</td>
<td>2.6</td>
</tr>
<tr>
<td>FLOOR FRONT CROSS-MEMBER</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>FLOOR MID CROSS-MEMBER</td>
<td>0.0</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>FLOOR REAR CROSS-MEMBER</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>B-PILLAR (INNER PART)</td>
<td>1.6</td>
<td>1.8</td>
<td>0.7</td>
</tr>
<tr>
<td>FLOOR (REAR PART)</td>
<td>2.3</td>
<td>1.3</td>
<td>22.3</td>
</tr>
<tr>
<td>REAR WHEEL-HOUSING</td>
<td>4.5</td>
<td>0.7</td>
<td>4.0</td>
</tr>
<tr>
<td>REAR WALL</td>
<td>3.2</td>
<td>0.3</td>
<td>5.0</td>
</tr>
<tr>
<td>REAR FENDER</td>
<td>4.9</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>REAR GLASS UPPER CROSS-MEMBER</td>
<td>0.5</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>ROOF</td>
<td>5.2</td>
<td>22.0</td>
<td>10.5</td>
</tr>
<tr>
<td>REAR SILL</td>
<td>0.8</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
</tbody>
</table>
| **Table 4.2 Strain energy distribution in different modes**

4.3 Experimental Modal Analysis (EMA)

In section 4.2.2 the modal parameters are calculated from a mathematical model. The modal parameters can also be found experimentally by doing an Experimental Modal Analysis, EMA. It is not possible to explain the theory and available equipment for an EMA in a few sentences. Readers who do not have any knowledge of EMA are recommended to read [6].

When the vibration measurements are performed, all data need to be processed to find the frequency response functions (FRFs). From the FRFs the modal parameters can be extracted. The procedure to find the FRFs is also treated in [6] and will be briefly described below.

The Fourier transforms are used to transform signals from time domain to frequency domain, and are very important steps in signal processing for vibrational sense. With this transformation it is possible to calculate the FRF of a structure. Figure 4.6 illustrates a simple measurement system with
one input signal $f(t)$ and one output signal $x(t)$. $F(\omega)$ and $X(\omega)$ are the Fourier transforms of $f(t)$ and of $x(t)$.

\[ f(t), F(\omega) \xrightarrow{h(t), H(\omega)} x(t), X(\omega) \]

\[ \text{Figure 4.6 Measurement-system model} \]

The response functions are defined as

\[ h_1(t) = \frac{x(t)}{f(t)} \quad \text{and} \quad h_2(t) = \frac{f(t)}{x(t)} \quad (4.16)(4.17) \]

and the FRFs are

\[ H_1(\omega) = \frac{X(\omega)}{F(\omega)} \quad \text{and} \quad H_2(\omega) = \frac{F(\omega)}{X(\omega)} \quad (4.18)(4.19) \]

When using signal processing tools the FRFs are defined as

\[ H_1(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)} \quad \text{and} \quad H_2(\omega) = \frac{S_{xx}(\omega)}{S_{xf}(\omega)} \quad (4.20)(4.21) \]

$S_{fy}(\omega)$ and $S_{xx}(\omega)$ are the input and output auto-spectrum, see e.g. [7], and are given by

\[ S_{xx}(\omega) = X(\omega)X^*(\omega) \quad \text{and} \quad S_{ff}(\omega) = F(\omega)F^*(\omega) \quad (4.22)(4.23) \]

where * denotes complex conjugate.

$S_{fy}(\omega)$ and $S_{xf}(\omega)$ are the cross input-output and cross output-input spectrums and are given by

\[ S_{fy}(\omega) = F(\omega)X^*(\omega) \quad \text{and} \quad S_{xf}(\omega) = X(\omega)F^*(\omega) \quad (4.24)(4.25) \]

$H_1(\omega)$ and $H_2(\omega)$ should give same results and may be used to determine the consistency of the measured data. This ratio between the two FRFs is called coherence function $\gamma^2$ and is defined
This value can vary between 0 and 1, where the values near 1 represent accurate data and low values indicate poor measurements.

When the FRFs have been measured and structural properties like natural frequencies, damping ratios and mode shapes can be computed, see for example [6]. In this work all these steps are integrated in the software/hardware used.

### 4.3.1 Multi-Input Multi-Output (MIMO) Analysis

When analysing large structures it is sometimes necessary to use more than one exciter. This is done to provide a better energy distribution in the test structure. Another advantage of using several shakers with different locations and excitation directions is the ability to excite all modes in a frequency range of interest. This will give better information about natural frequencies, mode shapes and damping loss factors.

In MIMO analysis the FRFs are defined in a similar way as for a single exciter/input analysis. Instead of getting a FRF vector a FRF matrix is obtained. Figure 4.7 below illustrates the MIMO measurement model.

\[
\begin{align*}
H_1(\omega) &= \frac{S_{fx}(\omega)S_{xf}(\omega)}{S_{ff}(\omega)S_{xx}(\omega)} = \left| \frac{S_{xf}(\omega)}{S_{ff}(\omega)} \right|^2 = \gamma^2 \\
\end{align*}
\]

\( (4.26) \)

Figure 4.7 MIMO measurement model

See e.g. [6] for deriving of the FRF matrix, which will become
\[
\begin{bmatrix}
H_{xf}(\omega)
\end{bmatrix}_{(mxn)} = \left[\begin{bmatrix} S_{ff}(\omega) \end{bmatrix}^{-1}
\begin{bmatrix} S_{fx}(\omega) \end{bmatrix}
\right]^T
\]  

(4.27)

One of the most important conditions for a successful MIMO analysis is that the input signals are not correlated. If the signals are correlated the FRF matrix may be singular and no reliable results will be obtained.

Another important property of linear conservative systems is that the measured FRF for a force at location \(j\) and a response at location \(i\) should correspond directly with the measured FRF for a force at location \(i\) and response at location \(j\). The FRF matrix is symmetric and this property is used to check the qualities of measurement set-up and measured data. The property described is called Maxwell’s Rule of Reciprocity.

### 4.4 Correlation

#### 4.4.1 Modal Assurance Criterion (MAC)

When the analytic and experimental vibrational characteristics of the investigated structure are obtained they are compared with each other. Several techniques are presented in [8]. The most widely used technique is the Modal Assurance Criterion, MAC, which estimates the degree of correlation between the calculated and measured mode shape vectors. The MAC between the \(j\):th measured mode, \(\phi_{mj}\), and the \(k\):th analytic mode, \(\phi_{ak}\), is defined as

\[
MAC_{jk} = \frac{\left| \begin{bmatrix} \phi_{mj}^T \phi_{ak}^* \end{bmatrix} \right|^2}{\left( \begin{bmatrix} \phi_{ak}^T \phi_{ak} \end{bmatrix} \begin{bmatrix} \phi_{mj}^T \phi_{mj} \end{bmatrix} \right)^{1/2}}
\]

(4.28)

MAC value can vary between 0 and 1. 1 means that the mode shape is a multiple of another.

The MAC values are often presented in a MAC matrix. If two identical mode shape matrixes are plotted against each other a clear diagonal with values of ones will be present. A simple example of this is given in figure 4.8.
In this example ten test frequencies with mode shapes respectively are plotted against each other. Note the obvious diagonal of ones and the symmetry of the matrix. Some very low MAC-values can be present in the off-diagonal regions due to the not totally ideal circumstances since two mode shapes can be partly identical.
5. Finite Element Model

5.1 Elements

The choice of elements in the FE-model will effect the accuracy of the results and the computation time. The reliability of the model is often dependent on the correct choice of elements. Elements suitable in this case are the beam and shell elements. Some advantages and disadvantages in using these are discussed in the sections below.

The theory of the Finite Element Method can be found for example in [9].

The programme used for the FEM is *I-DEAS Master Series 5.0*.

5.1.1 Beam Elements

A quick look at the structure indicates that the use of beam elements in the FE-model would be one to prefer. The cross-sections of the beams are well defined. How the spot-welds will affect the results must be investigated and if the spot-welds affect the results an equivalent beam section must be found.

One, and maybe the biggest advantage, with the beam elements are the short computation time due to the low number of elements in the models.

A beam element is defined primarily by quantities derived from the shape of the beam, rather than the beam geometry itself. Each node of the element has six (three translational and three rotational) degrees of freedom. Figure 5.1 illustrates a linear beam element, with 12 degrees of freedom.

![Figure 5.1 Linear beam element](image)
An often-used theory in FE-applications is the Timoshenko beam theory, which includes transverse shear deformation. It is known that the theory will give good results even for thin walled beams. When thin walled beams are present in the model the joining areas will be critical and may cause errors in the results. A simple example is given to illustrate the problem. Two thin walled beams are joined together forming a T-shaped structure. The length of each of the beams is 0.5 m and the box beam cross section has the dimensions of (A) 100x100x1.5 mm and (B) 50x50x1.5 mm. Beam B is joined to beam A as showed in figure 5.2.

![Figure 5.2 Beam model](image)

The FE-model has 19 linear beam elements and the beam A is clamped in both ends. To solve the first two natural frequencies of the structure a dynamic analysis is made. The results are presented in table 5.1. Modeshapes for the frequencies respectively are presented in figure 5.3.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>238.31</td>
</tr>
<tr>
<td>2</td>
<td>262.33</td>
</tr>
</tbody>
</table>

*Table 5.1 Two lowest natural frequencies for the beam model*
These results are further compared with the results from the analysis of the T-structure with shell elements. Hopefully, it will give a guidance of the choice of elements for the Frame FE-model.

### 5.1.2 Shell Elements

Shell elements are often used when thin walled constructions are analysed. The formulation of shell elements in I-DEAS is based on the Mindlin shell equations. Figure 5.4 illustrates a four node shell element with 24 (three translational and three rotational for each node) degrees of freedom.
The same beam structure as in figure 5.1 is modelled with linear quadrilateral and linear triangular elements. The model used includes 322 elements. Figure 5.5 shows the shell model.

The natural frequencies of the structure are calculated and two lowest frequencies are presented in table 5.2. Of course these values should be the same as for the beam model.
Mode Frequency [Hz]
\[
\begin{array}{|c|c|}
\hline
\text{Mode} & \text{Frequency [Hz]} \\
\hline
1 & 27.15 \\
2 & 163.46 \\
\hline
\end{array}
\]

*Table 5.2 Two lowest eigenfrequencies for the shell model*

Mode shapes for each natural frequency are presented in figure 5.6 below.

![Mode 1 27.15 Hz](image1)

![Mode 2 163.46 Hz](image2)

*Figure 5.6 Modeshapes for the shell model*

### 5.1.3 Choice of Elements

The results from the beam- and shell-model differ a lot. The higher frequencies of the beam model are probably due to a too stiff joining point. When the beams are thin walled, as in this example, the flexibility of the beam walls will influence the results. This will be most noticeable at the joining points. In this case, a local deformation in the wall of beam A, near the joining location, will be present when the structure is oscillating. The
beam element does not include this local deformation of the wall. Because of this the shell elements will give more correct results in this case.

The case studied can be related to the frame in several places, for example the joints between A-, B- and C-posts and the bottom frame. The disadvantage of using shell elements is the increasing size of FE-models, resulting in longer solution times. A use of beam elements for the space frame model is not advisable, due to the uncertain results. Instead the frame is modelled with shell elements.

5.2 Space Frame FE-model

5.2.1 Elements

The frame is modelled with eight node rectangular (parabolic quadrilateral) and six node triangular (parabolic triangle) elements. Rigid beam elements are used to model the spot welds. Due to the symmetry of the frame only half a model is needed for the calculations. When correlating the dynamic FE results with the test results the whole frame is also solved. The model of the half frame includes 5578 elements and 12 902 nodes. The weight of the whole frame is 142.54 kg. Figure 5.7 shows the frame FE-model.

Figure 5.7 Space frame FE-model
5.2.2 Boundary Conditions

The boundary conditions used in the symmetry plane of the frame are symmetric and anti-symmetric boundary conditions. These are listed in table 5.3. For the co-ordinates see figure 5.7.

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Symmetric</th>
<th>Anti Symmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>Free</td>
<td>Fixed</td>
</tr>
<tr>
<td>$y$</td>
<td>Fixed</td>
<td>Free</td>
</tr>
<tr>
<td>$z$</td>
<td>Free</td>
<td>Fixed</td>
</tr>
<tr>
<td>$R_x$</td>
<td>Fixed</td>
<td>Free</td>
</tr>
<tr>
<td>$R_y$</td>
<td>Free</td>
<td>Fixed</td>
</tr>
<tr>
<td>$R_z$</td>
<td>Fixed</td>
<td>Free</td>
</tr>
</tbody>
</table>

*Table 5.3 Boundary conditions for the symmetry plane*

The boundary conditions for the symmetry plane are used on the half frame in both static and dynamic analyses.

Except for the symmetry and anti-symmetry boundary conditions the FE-model is subjected to different loads and restraints depending of the type of analysis. These are explained below.

**Torsional Stiffness**

The antisymmetric restraints are used at nodes at the symmetry plane. In addition to the antisymmetric restraints some other restraints and loads are applied at the frame. These are listed in table 5.4. For the point locations see figure 5.8.

<table>
<thead>
<tr>
<th>Point</th>
<th>Restraint/Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Antisymmetry + $y$ fixed</td>
</tr>
<tr>
<td>2</td>
<td>$x, z$ fixed</td>
</tr>
<tr>
<td>3</td>
<td>$P_z = +1733$ N</td>
</tr>
</tbody>
</table>

*Table 5.4 Restraints/Loads in the torsional loadcase*
The points 2 and 3 are supposed to represent the rear- and front shock absorber locations at a car. The point 2 is located next to the C-pillar. The distance in x-direction from the A-pillar to point 3 is 0.4 m. Point 4 in figure 5.8 illustrates the “measurement” point, the x-co-ordinate is the same as for point 3 and the distance from the symmetry plane in y-direction is 0.34 m.

The load point 3 has a distance of 0.75 m in y-direction from the symmetry plane resulting in a torque of 2600 Nm on the whole frame.

**Bending stiffness**

When defining the bending stiffness of the frame the symmetry restraints are used for the nodes at the symmetry plane. Other restraints, loads and locations of these are presented in table 5.5 and figure 5.9.

<table>
<thead>
<tr>
<th>Point</th>
<th>Restraint/load</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$x, y, z$ fixed</td>
</tr>
<tr>
<td>6</td>
<td>$x, y, z$ fixed</td>
</tr>
<tr>
<td>7</td>
<td>$P_z = -1500$ N</td>
</tr>
</tbody>
</table>

*Table 5.5 Restraints/loads for the bending stiffness loadcase*
The load location 7 is supposed to represent the driver seat location in a car. The force is distributed to four points as shown in figure 5.9. Points 5 and 6 are the front and rear shock absorber locations. The value of interest is the maximum deflection, in z-direction, at the bottom of the frame.

![Figure 5.9 Load and restraint locations in the bending loadcase](image)

**Dynamic**

In the dynamic analysis the frame is allowed to vibrate freely. Even here the half model of the frame is used. If applying antisymmetric restraints, described in table 5.3, the antisymmetric mode shapes can be detected. For the symmetric modes the restraints for symmetry are used.

**5.2.3 Type of Analysis**

To the static analysis Linear Static Analysis is used in I-DEAS. A normal mode analysis is used when determining dynamic characteristics of the frame. Solution method used for the analysis is the Lanczos method. The method is an iterative method and is commonly used in structural analysis. See e.g. [10] for a deeper discussion of the method.
5.3 Results

5.3.1 Torsional Stiffness

The torsional stiffness is according to the equation 4.5

\[ k_T = \frac{T}{\varphi} \]  

(4.5)

The applied torque in this case is 2600 Nm. y-co-ordinate for the “measuring-point” is 0.34 m and the deflection in z-direction at the point is read to 1.335 mm giving a torsional stiffness of

\[ k_T = \frac{2600}{\arctan\left(\frac{0.001335}{0.34}\right)} = 11.55 \text{ kNm/degree} \]  

(5.1)

Figure 5.10 illustrates the deformation of the frame during the torsional load.

![Figure 5.10 Torsional deformation of the frame](image)

5.3.2 Bending Stiffness

The value of interest when determining the bending stiffness is the maximum deflection of the bottom of the frame. Figure 5.11 illustrates the bending deformation of the frame.
The deflection of the transmission tunnel, see Appendix B, at the bottom of the frame is shown in figure 5.12. The y-axis of the plot shows the deformation in z-direction and the x-axis shows the x-co-ordinate in the transmission tunnel.

**Figure 5.12 Deflection of the transmission tunnel, for x-direction see fig. 5.11**
The maximum deflection can be read to ~0.5 mm and is located in the front of the transmission tunnel.

### 5.3.3 Normal Modes

The twenty lowest natural frequencies of the frame predicted from the dynamic analyses are presented in table 5.6. Also, the type of mode is indicated with S for symmetric and AS for antisymmetric mode. The full list of natural frequencies up to 240 Hz is presented in Appendix C.

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQUENCY</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.6065</td>
<td>AS</td>
</tr>
<tr>
<td>2</td>
<td>47.0590</td>
<td>S</td>
</tr>
<tr>
<td>3</td>
<td>50.8811</td>
<td>AS</td>
</tr>
<tr>
<td>4</td>
<td>62.5112</td>
<td>S</td>
</tr>
<tr>
<td>5</td>
<td>67.9292</td>
<td>AS</td>
</tr>
<tr>
<td>6</td>
<td>71.7461</td>
<td>AS</td>
</tr>
<tr>
<td>7</td>
<td>76.0581</td>
<td>S</td>
</tr>
<tr>
<td>8</td>
<td>77.5722</td>
<td>AS</td>
</tr>
<tr>
<td>9</td>
<td>87.0570</td>
<td>AS</td>
</tr>
<tr>
<td>10</td>
<td>97.6469</td>
<td>AS</td>
</tr>
<tr>
<td>11</td>
<td>102.0848</td>
<td>S</td>
</tr>
<tr>
<td>12</td>
<td>109.4706</td>
<td>S</td>
</tr>
<tr>
<td>13</td>
<td>115.2485</td>
<td>AS</td>
</tr>
<tr>
<td>14</td>
<td>122.0415</td>
<td>S</td>
</tr>
<tr>
<td>15</td>
<td>126.6588</td>
<td>AS</td>
</tr>
<tr>
<td>16</td>
<td>137.3575</td>
<td>S</td>
</tr>
<tr>
<td>17</td>
<td>143.5154</td>
<td>AS</td>
</tr>
<tr>
<td>18</td>
<td>144.4089</td>
<td>S</td>
</tr>
<tr>
<td>19</td>
<td>149.2323</td>
<td>AS</td>
</tr>
<tr>
<td>20</td>
<td>149.6884</td>
<td>S</td>
</tr>
</tbody>
</table>

*Table 5.6 Twenty lowest calculated natural frequencies*

Figure 5.13 shows the first anti symmetric mode shape and figure 5.14 illustrates the first bending mode.
Figure 5.13 1:st Torsion mode 39.6 Hz

Figure 5.14 1:st Bending mode at 47.1 Hz

The lowest modes are global deformations, while the mode shapes of higher frequencies tend to be more local beam bending deformations. Figure 5.15 illustrates a typical local mode shape.
5.3.4 Discussion of Results

Results from the static analysis shows that the frame is a bit too stiff. The torsional stiffness is about 10% higher than the target value of 10 kNm/degree. For the static bending case the maximum deflection of the bottom is ~0.5 mm. This is less than the maximum specified value of 1 mm.

The dynamic analysis also shows a too stiff frame. The first torsion mode is ~10 Hz, 30 %, higher than the target value interval of 25-30 Hz, which is quite much. The first bending mode at 47.05 Hz hits the upper region of the bending target value interval of 40-50 Hz.

Both the static and dynamic analyses show a too stiff frame. A remaining question is if the FE-model can be used to represent the manufactured frame.
6. Measurements

6.1 Measurement Set-up

An experimental modal analysis of the frame was done at Chalmers, “Televerkstaden” in May 1998. PhD. Juha Plunt from Ingemansson Technology acted as supervisor for the measurements. To excite all modes of the frequency range a MIMO method was used. Two shakers were used to excite the structure at two different locations in two directions (y and z). The excitation locations were chosen so that an even energy distribution could be achieved. The excitation signal was a random noise of 0-600 Hz. The frame was placed on three air blabber isolators, allowing the frame to act as free-free. Figure 6.1 shows the measurement set-up of the frame and figure 6.2 shows the locations of excitation and support points.

Figure 6.1 Measurement set-up of the frame
The response of the structure was measured at 152 points on the frame with four three-axial accelerometers. The number of accelerometers was limited by the input ports in the front end, otherwise the number of accelerometers could be higher to reduce the measurement time. Numbers of measurement points and measured directions result in a test model with 456 degrees of freedom. Due to the number of accelerometers the measurement was run 38 times to be able to collect all FRFs between measurement points and shakers. Figure 6.3 illustrates the measurement chain, and the measurement equipment used is listed in Appendix D.
6.2 Measurement Set-up Verification

To make sure that the suspension of the frame does not affect the results the rigid body modes shall not be present in the natural frequency range of interest. The rigid body frequencies are presented in table 6.1.

<table>
<thead>
<tr>
<th>SHAPE</th>
<th>FREQUENCY (HERTZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>REC</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.740</td>
</tr>
<tr>
<td>2</td>
<td>4.048</td>
</tr>
<tr>
<td>3</td>
<td>6.350</td>
</tr>
<tr>
<td>4</td>
<td>8.854</td>
</tr>
<tr>
<td>5</td>
<td>10.500</td>
</tr>
<tr>
<td>6</td>
<td>23.792</td>
</tr>
</tbody>
</table>

*Table 6.1 Rigid body natural frequencies*

The lowest expected natural frequency according to the FE-calculation should be around 40 Hz. The six rigid body frequencies are all below this value and so the suspension of the frame should not affect the measurement results (frame will act as free-free above the rigid body frequencies).
Another way of checking the measurement conditions is the reciprocity test, presented in section 4.3.1. Reciprocity plot between the two excitation points is presented in figure 6.4.

![Figure 6.4 Reciprocity between the two excitation locations](image)

As can be seen, the two FRFs are almost identical up to 200 Hz and the peaks of the signals correlate quite well up to 240 Hz. Signals over 240 Hz tend to differ more, which may indicate poor results in the higher frequencies. Therefore it is not advisable to analyse frequencies over 240 Hz, due to the uncertain results.

In section 4.3 another method of checking the consistency of the measured data is presented. It is called coherence function. Coherence function for measurement point 143 is presented in figure 6.5.
The coherence is very close to 1 in almost all of the frequency range of 30-240 Hz. Yet the typical drop-offs just after the natural frequencies are present. The region below 30 Hz, with low coherence values, indicates of uncertain measurements and is not of interest in further studies.

As can be seen the measurement set-up seems to be good. The frame can be considered as free-free. The reciprocity test indicates excellent results up to 200 Hz and good results up to 240 Hz. Also, the coherence function shows that the measurement conditions are acceptable. If this had not been the case another suspension set-up should have been found out.

Figure 6.5 The coherence function
6.3 Measurement Results

The measurement results were extracted by using LMS CADA-X.

A typical FRF curve is presented in figure 6.6. Note that all of the natural frequencies are not present in the plot. Some of the six suspension (rigid body) frequencies are visible between 2 and 25 Hz.

![Frequency response function](image)

**Figure 6.6 A typical FRF**

The lowest twenty natural frequencies from the measurements are presented in table 6.2. A complete list of natural frequencies between 0 and 240 Hz is presented in Appendix E.
<table>
<thead>
<tr>
<th>SHAPE REC</th>
<th>FREQUENCY (HERTZ)</th>
<th>DAMPING (%)</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.796</td>
<td>0.322</td>
<td>AS</td>
</tr>
<tr>
<td>2</td>
<td>49.419</td>
<td>0.184</td>
<td>AS</td>
</tr>
<tr>
<td>3</td>
<td>50.246</td>
<td>0.181</td>
<td>S</td>
</tr>
<tr>
<td>4</td>
<td>62.715</td>
<td>0.319</td>
<td>S</td>
</tr>
<tr>
<td>5</td>
<td>66.426</td>
<td>0.155</td>
<td>AS</td>
</tr>
<tr>
<td>6</td>
<td>67.866</td>
<td>0.309</td>
<td>AS</td>
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<td>S</td>
</tr>
<tr>
<td>12</td>
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<td>0.131</td>
<td>AS</td>
</tr>
<tr>
<td>13</td>
<td>112.007</td>
<td>0.107</td>
<td>AS</td>
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<td>119.214</td>
<td>0.102</td>
<td>AS</td>
</tr>
<tr>
<td>15</td>
<td>130.434</td>
<td>0.183</td>
<td>S</td>
</tr>
<tr>
<td>16</td>
<td>134.975</td>
<td>0.246</td>
<td>AS</td>
</tr>
<tr>
<td>17</td>
<td>138.182</td>
<td>0.173</td>
<td>S</td>
</tr>
<tr>
<td>18</td>
<td>144.576</td>
<td>0.124</td>
<td>S</td>
</tr>
<tr>
<td>19</td>
<td>144.752</td>
<td>0.247</td>
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<tr>
<td>20</td>
<td>149.382</td>
<td>0.151</td>
<td>AS</td>
</tr>
</tbody>
</table>

*Table 6.2 Measurement results*

Mode shape of the measured 1:st torsion mode is presented in figure 6.7.

*Figure 6.7 Measured 1:st torsion mode at 39.79 Hz*
Figure 6.8 illustrates the measured 1:st bending mode.

Some of the measured mode shapes could be ill conditioned. This can depend on a noisy measurement or the wrong measurement location. These errors are not so obvious to detect when doing the measurements but are often found at a later stage of the test analysis. When studying the mode shapes some of the natural frequencies were shown to be noisy. These are indicated in the list in Appendix E. Overall the results from the measurements seem to be very good and most of the mode shapes are very clear.
7. FE- and Test-model Correlation

To evaluate the agreement of the FE-model with the manufactured frame it is compared with the test-model. If the FE-model differs from the test-model, the correlation results can be used to correct the FE-model.

The correlation analysis is done in I-DEAS application Test.

7.1 Mode Pairing

A good method to pair one test mode with one FE-mode is to use MAC-values, briefly presented in section 4.4.1. Frequencies, frequency difference, and the MAC value for the paired modes are presented in table 7.1. A complete list of modes up to 240 Hz is presented in Appendix F.

<table>
<thead>
<tr>
<th>Meas nr</th>
<th>FEM nr</th>
<th>Measured freq (Hz)</th>
<th>FEM freq (Hz)</th>
<th>Change %</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>39.606</td>
<td>-0.477</td>
<td>0.981</td>
</tr>
<tr>
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<td>3</td>
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<td>50.881</td>
<td>2.959</td>
<td>0.879</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>50.246</td>
<td>47.059</td>
<td>-6.343</td>
<td>0.876</td>
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<td>4</td>
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<td>62.511</td>
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<td>0.892</td>
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<tr>
<td>5</td>
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<td>6</td>
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<td>71.746</td>
<td>5.717</td>
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<tr>
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<td>7</td>
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<td>76.058</td>
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<td>0.700</td>
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<td>86.713</td>
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<td>0.947</td>
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<td>10</td>
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<td>97.647</td>
<td>2.138</td>
<td>0.805</td>
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<td>97.118</td>
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<td>5.114</td>
<td>0.719</td>
</tr>
<tr>
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<td>12</td>
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<td>109.471</td>
<td>3.590</td>
<td>0.909</td>
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<tr>
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<td>13</td>
<td>111.142</td>
<td>115.249</td>
<td>3.695</td>
<td>0.554</td>
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<tr>
<td>14</td>
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<td>119.214</td>
<td>126.659</td>
<td>6.245</td>
<td>0.758</td>
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<tr>
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<td>5.308</td>
<td>0.545</td>
</tr>
<tr>
<td>16</td>
<td>19</td>
<td>134.975</td>
<td>149.232</td>
<td>10.563</td>
<td>0.884</td>
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<td>17</td>
<td>18</td>
<td>138.182</td>
<td>144.409</td>
<td>4.506</td>
<td>0.777</td>
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<tr>
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<td>20</td>
<td>144.576</td>
<td>149.688</td>
<td>3.536</td>
<td>0.742</td>
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<td>19</td>
<td>17</td>
<td>144.752</td>
<td>143.515</td>
<td>-0.854</td>
<td>0.744</td>
</tr>
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<td>20</td>
<td>0</td>
<td>149.382</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>---</td>
<td>77.572</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>14</td>
<td>---</td>
<td>122.041</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Table 7.1 Comparision table of the modes

MAC values over 0.7 are said to be good, and as can be read in table 7.1 most of the values are over this value. Only two MAC-values are below.
Two of the measured modes in the table are not paired with any of the calculated modes. The measured mode at 112 Hz can be paired with the calculated mode at 122.04 Hz, giving a MAC-value of 0.336. The higher mode at 149.38 Hz is paired with a calculated mode in the table in Appendix F.

The calculated mode of 77.57 Hz could not be found in the measured results. Probably the measurement set-up is not able to identify this natural frequency.

The MAC-values are often illustrated graphically in a MAC-matrix. If the correlation is good and the same number of mode shapes is analysed a clear diagonal of staples is visible. Figure 7.1 shows the MAC-matrix with the twenty lowest modes.

Figure 7.1 MAC-matrix test-analysis
7.2 Natural Frequencies

One method to compare calculated and measured frequencies is to plot the measured value against the calculated value of each natural frequency. The points should lie on or close to a straight line of slope 1. If the points do lie close to the straight line, but in a different slope, almost certainly the material properties in the calculation model are erroneous. This kind of graph does not give information about the correlation between the mode shapes.

The twenty lowest natural frequencies from calculations and measurements are plotted against each other in figure 7.2. Only the paired frequencies in Appendix F are plotted in the figure.

![Figure 7.2 Comparison between the measured and calculated frequencies](image)

As can be seen in figure 7.2, the frequencies do correlate quite well until 100 Hz. The higher frequencies for the FE-model tend to be too high, which indicates a too stiff calculation model.
7.3 Conclusions

Some of the test modes could not be paired with an analytic mode. This probably depends on a noisy or ill conditioned measurement point. The fact that some of the calculated modes, do not pair with any test mode is not surprising. It is very difficult to find one measurement set-up that extracts all the natural frequencies of the analysed structure.

As can be seen in the results of the comparison, the FE-model represents the manufactured frame very well. The MAC-values are overall very good, while the natural frequencies over 100 Hz tend to differ slightly. The greatest difference in frequency is 10.5 %, which can be said to be acceptable. Overall the calculation model tend to be a little bit too stiff. This can depend on the wrong material parameters, but is probably due to a too stiff coupling at spot-welds. An updating process could be carried out to find and correct the error locations of the FE-model, but that is not done in this work, due to the acceptable correlation between the models.

The FE-model is now used to redesign the space frame so the target values are fulfilled.
8. Redesign

8.1 Modifications

As is seen from the calculations and measurements the frame seems to be too stiff. The static torsional stiffness is close to the target value but still a bit too high. No target value was specified for the static bending stiffness.

The natural frequencies of the frame are too high. The first bending mode is inside the target value limits, while the first torsion mode is around 10 Hz too high.

The final goal with this sub-project is to modify the existing space frame so that the target values are fulfilled.

As mentioned earlier, the natural frequencies depend on the stiffness of the frame and the mass distribution in it. The natural frequencies of the frame can consequently be driven down by reducing the stiffness or/and by increasing the mass of the frame. The stiffness reduction can be done in local and global level, modifying individual beams or modifying the main dimensions of the frame. Guidelines for the modifications of torsional and bending stiffnesses are discussed in chapter 4.1. Modifications on the sectional design of the beams in the rear of the frame should influence a lot to the static torsional stiffness of the frame. Some of the crossbeams have however unrealistic dimensions compared with corresponding beams in a real car. During the modification process it became clear that local modifications of the frame did not affect the natural frequencies as much as expected. To be able to force down the 1:st torsion mode, it is consequently necessary to modify the main dimensions of the frame. This process is expensive and time demanding, so the frequencies will be forced down with changes in mass distribution of the frame.

Due to the results it is advisable to concentrate on changes at the torsion mode. The simplest way of doing this is to add mass as far from the symmetry plane as possible. The amount of the mass necessary to add to the frame is difficult to say, but it can be solved with the well-known trial-error method. The locations where the mass can be added are presented in figure 8.1.
Figure 8.1 Mass adding locations

8.2 Results

After some attempts the frequencies were got close to the target values. The mass modifications done are presented in table 8.1.

<table>
<thead>
<tr>
<th>Region</th>
<th>Modification</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+5 kg</td>
</tr>
<tr>
<td>B</td>
<td>+5 kg</td>
</tr>
<tr>
<td>C</td>
<td>+1 kg</td>
</tr>
<tr>
<td>D</td>
<td>+1 kg</td>
</tr>
</tbody>
</table>

Table 8.1 Mass modifications

The results from this modification were shown to be acceptable. The results are presented in table 8.2 below.
Table 8.2 Results from the mass modification

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1:st Torsion mode</td>
<td>30.18 Hz</td>
</tr>
<tr>
<td>1:st Bending mode</td>
<td>42.13 Hz</td>
</tr>
<tr>
<td>Torsional stiffness</td>
<td>11.5 kNm/degree</td>
</tr>
</tbody>
</table>

The first torsion mode have been driven down ~10 Hz from 39.60 Hz to 30.18 Hz. The value is in the upper region of the target value interval of 25-30 Hz.

The first bending mode has also gone down around 5 Hz in frequency, from 47.06 Hz to 42.13 Hz. The target value interval for the lowest bending frequency was set to 40-50 Hz.

Note that the static torsional stiffness has not changed at all.

The modifications were done to decrease the first torsion mode but not the static stiffness of the frame. As can be seen from the results, the modifications were successful. The static stiffness can be decreased with some local modifications in the rear part of the frame. However this is not done in this work.
9. Conclusions

The aim of this sub-project was to design and control the static and dynamic characteristics of a space frame. Target values for the frame were obtained from car manufacturers and included the torsional stiffness and the 1:st torsion and bending frequency.

An analytic model of the space frame was modelled in FE-programme I-DEAS. This model was used to calculate the static stiffness and the natural frequencies of the frame. An experimental modal analysis was done on the manufactured frame giving the dynamic parameters of the “real” frame. The measurements gave good results up to 240 Hz and poorer for the frequencies over this. Therefor the frequency range for the analysis was limited to 0-240 Hz. The agreement between the analytic model and the real frame was checked by a comparison of the dynamic parameters of the two models. The results from this comparison were shown to be good. The greatest difference in frequency was 10 %, which is acceptable. To achieve even better correlation between the natural frequencies an updating process could be proceeded on the frame. This is however not done in this work, but is recommended if the analytic model is used in further analysis.

The results from the calculations and the tests showed that the frame did not fulfil the target values. It was mainly the 1:st torsion frequency that differed a lot from the target value. The modifications done on the frame were from then on concentrated to decrease this frequency. The easiest way of doing this was to add mass to the structure, which was done in the eight locations of the frame. The results of the modification were shown to be a success. After the modifications, the dynamic characteristics of frame are very close to the target values. The static characteristics were close to the target values before the modifications and therefor the changes in the frame were done in a way that the static characteristics did not change at all.

To be 100% sure of the modification results, an experimental modal analysis should be done on the modified frame to document the lowest natural frequencies. This is however not done in this work.
References


2. Ulfvarson A. and Åkesson B., 1984, “Teknisk Balkteori och Skrovstatik”, Course literature, Department of Naval Architecture and Ocean Engineering and Department of Solid Mechanics, Chalmers University of Technology


Appendices

Appendix A

Main dimensions of the frame are showed in figures A1 and A2.

Figure A1 Side view with main dimensions of the frame

Figure A2 Top view of the frame with main dimensions
Appendix B

Figure B1 Structural part terms in a space frame
Appendix C

Natural frequencies predicted from the FE-calculations. The type of mode is indicated with AS for antisymmetric and S for symmetric.

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQUENCY</th>
<th>TYPE</th>
</tr>
</thead>
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## Appendix D

### Measurement equipment

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<tr>
<td>Force Transducers (2)</td>
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</tr>
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<td>Preamplifiers (2)</td>
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<td>Accelerometers (4)</td>
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</tr>
<tr>
<td>Front-End</td>
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<tr>
<td>Software</td>
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<td>HP Apollo</td>
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### Appendix E

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<th>FREQUENCY (HZ)</th>
<th>DAMPING (%)</th>
<th>TYPE</th>
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<td>1</td>
<td>39.796</td>
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