Takagi-Sugeno and Mamdani Fuzzy
Control of a Resort Management System

Lujiao Tan

Thesis for the Degree Master of Science (two years)
in Mathematical Modeling and Simulation
30 credit points (30 ECTS credits)
November 2011

Blekinge Institute of Technology
School of Engineering
Department of Mathematics and Science
Supervisor: Elisabeth Rakus-Andersson
Abstract

By means of fuzzy set theory as well as Takagi-Sugeno and Mamdani fuzzy controller, this paper presents the investigation of a Resort Management System implemented by a combination of a T-S model and a Mamdani model. It demonstrates the procedure of the specific premise parameters identification and consequence parameters identification performed by regression knowledge in the T-S model, and the process of the fuzzification, the rule base creation and the defuzzification with COG technique in the Mamdani model. Therefore, an aggregation between T-S controller and Mamdani controller applied in the field of management by a novel angle is illustrated, which, as a result, devotes an improved management system that shares great convenience in the control process when combined with mathematics.

Moreover, a modification of the conventional Takagi-Sugeno and Mamdani controller is demonstrated in conjunction with fuzzy operations t-norms and OWA by adjusting the $\alpha$-value, which is used in the calculation of final outputs in the T-S model and the computation of rule consequences in the Mamdani model. The algebraic intersection, bounded intersection as well as the $\rho$-parameter t-norm are the t-norms which are going to be introduced. Besides, we have tested that t-norms generate the same alpha values when the membership degrees meet the boundary with the value of 1 or 0 while OWA can still yield a well-balanced result different from the one computing by minimum operation. Nevertheless both t-norms and OWA are able to shift the alpha-value in a well-adjusted way when the membership degrees lie in the interval[0,1]. A tendency has been shown that alpha-value tends to decrease by means of t-norms and OWA operations and consequently, the final outputs appear to be reduced.

Keywords: Fuzzy set theory; Takagi-Sugeno Controller; Mamdani Controller; Resort Management System; T-norms; OWA
Acknowledgement

It is a great honor for me to show my sincere gratitude to the people who have helped me with my studies.

Firstly, I would like to give my biggest appreciations to my dear supervisor Professor Elisabeth Rakus-Andersson who plays a vital role in my study of fuzzy world and teaches me a large number of useful techniques on how to implement the theory into practice. Her kindness, goodness and patience inspire me. Without her guidance, encouragement as well as support I would not have the opportunity to finish my master thesis in advance.

Afterwards, I would like to stretch my gratefulness to Nail Ibragimov, Claes Jogréus, Raisa Khamitova, Mattias Eriksson and other professors and managers who have helped me a lot in my master study of Mathematical Modeling and Simulation.

Last but not least, I also would like to present my thankfulness to my family and my friends who are always there offering me mental and physical support and helping me gain the strength and courage.
## Content

List of Figures .......................................................................................................................... 1
List of Tables ............................................................................................................................ 2

1 Takagi-Sugeno and Mamdani Fuzzy Control System ................................................................. 1
   1.1 Introduction ....................................................................................................................... 1
   1.2 Fundamental Concepts of Fuzzy Set Theory ................................................................. 1
      1.2.1 Fuzzy Set Theory and Basic Operations .............................................................. 1
      1.2.2 Introduction of Membership Functions ............................................................... 6
   1.3 Mamdani Controller ....................................................................................................... 7
   1.4 Takagi-Sugeno Controller ............................................................................................... 8
      1.4.1 Implication ............................................................................................................... 8
      1.4.2 Reasoning ............................................................................................................... 9
      1.4.3 Identification ........................................................................................................... 11
   1.5 Specific Method for Identification .................................................................................... 11
      1.5.1 Choice of Membership Functions in Premise Parameters Identification ................ 12
      1.5.2 Consequence Parameters Identification by Using Regression ............................... 14

2 Application of Takagi-Sugeno and Mamdani Fuzzy Control in a Resort Management System 15
   2.1 Takagi-Sugeno Fuzzy Control on the First-level Management ......................................... 15
      2.1.1 Control of Catering .............................................................................................. 16
      2.1.2 Control of Accommodation ............................................................................... 18
      2.1.3 Control of Leisure ............................................................................................... 21
   2.2 Mamdani Fuzzy Control on the Second-level Management ........................................ 23

3 Extensions of Fuzzy Operations .............................................................................................. 27
   3.1 T-norms ......................................................................................................................... 27
   3.2 S-norms ......................................................................................................................... 30
   3.3 OWA .............................................................................................................................. 32

4 Modification of Takagi-Sugeno and Mamdani Model by Shifting Alpha-value ..................... 35
   4.1 Selection of Alpha-value Based on T-norms in T-S model .............................................. 35
   4.2 Selection of Alpha-value by Means of OWA in T-S model ............................................. 36
4.3 Modification of T-S and Mamdani Resort Management System Based on Adjustment of Alpha-value ................................................................. 37
4.3.1 Modify the control of catering................................................................. 37
4.3.2 Modify the control of accommodation ..................................................... 38
4.3.3 Modify the control of leisure ................................................................... 40
4.3.4 Modify the control of RMS ...................................................................... 40
5 Conclusion........................................................................................................ 45
References ........................................................................................................... 46
List of Figures

Figure 1.1: Fuzzy set A="the integer close to 6"
Figure 1.2: Fuzzy set A="excellent" in X= [0,10]
Figure 1.3: Fuzzy set A="good" in X= [0,10]
Figure 1.4: Fuzzy set B="excellent" in X= [0,10]
Figure 1.5: The intersection of fuzzy set A and B
Figure 1.6: The triangular membership function T(u, 1,2,3)
Figure 1.7: The bell-shape membership function G(x, 2,2.5,3.3,5,4)
Figure 1.8: The trapezoidal membership function \( \Pi(x, 2,4,6,8) \)
Figure 1.9: Outline of Identification Algorithm
Figure 1.10: T-S Controller
Figure 1.11: The fuzzy constraints \( R_1 \rightarrow R_{11} \)
Figure 2.1: Resort Management System
Figure 2.2: Resort Management System with symbolic terms
Figure 2.3: Membership functions of premise variable “scale of health”
Figure 2.4: Membership functions of premise variable “scale of taste”
Figure 2.5: Membership functions of premise variable “level of comfort”
Figure 2.6: Membership functions of premise variable “level of price”
Figure 2.7: Membership functions of premise variable “grade of spa”
Figure 2.8: Membership functions of premise variable “grade of golf”
Figure 2.9: Membership functions of premise variable “grade of sports”
Figure 2.10: Membership functions of the independent variable “rank of catering”
Figure 2.11: Membership functions of the independent variable “rank of accommodation”
Figure 2.12: Membership functions of the independent variable “rank of leisure”
Figure 2.13: Membership functions of the dependent variable “quality of RMS”
Figure 2.14: Membership function \( \mu_1^{\text{conseq}}(z) \)
Figure 2.15: Membership function \( \mu_2^{\text{conseq}}(z) \)
Figure 2.16: Membership function of total consequence \( \mu^{\text{conseq}}(z) \)
Figure 3.1: Minimum t-norm
Figure 3.2: Algebraic t-norm
Figure 3.3: Bounded t-norm
Figure 3.4: Maximum s-norm
Figure 3.5: Algebraic s-norm
Figure 3.6: Bounded s-norm
Figure 4.1: Adjusted membership functions of \( \mu_1^{\text{conseq}}(z) \)
Figure 4.2: Adjusted membership functions of \( \mu_2^{\text{conseq}}(z) \)
Figure 4.3: Modified membership function of total consequence \( \mu^{\text{conseq}}_{\text{conseq}}(z) \)
Figure 4.4: Modified membership function of total consequence \( \mu^{\text{conseq}}_{\text{OWA}}(z) \)
List of Tables

Table 1.1: Parameters in the implication R
Table 1.2: Implications $R_i$
Table 1.3: An example of the reasoning process
Table 2.1: Statistical data of “Catering”
Table 2.2: Rule-based table of Catering
Table 2.3: Outputs of coefficients in $R_3$ of Catering
Table 2.4: Statistical data of “Accommodation”
Table 2.5: Rule-based table of Accommodation
Table 2.6: Rule-based table of Accommodation with Specific Outputs
Table 2.7: Statistical data of “Leisure”
Table 2.8: Rule-based Table of “RMS”
Table 3.1: The new sets $A_i$ with $a_i$ having the membership degrees in descending order
Table 3.2: The scheme of rules assigning the values of $b_{jk}$
Table 4.1: Modification of T-S model by shifting alpha-value based on t-norms
Table 4.2: Meanings for different operators
Table 4.3: Final outputs with different $\alpha$ values
Table 4.4: Modification of T-S model by shifting alpha-value by means of OWA
Table 4.5: Modified alpha values in the control of catering model
Table 4.6: Modified alpha values in the control of accommodation model
Table 4.7: Modified final outputs of control of accommodation
Table 4.8: Modified alpha values in the control of leisure model
Table 4.9: Modified alpha values in the control of Mamdani RMS model
1 Takagi-Sugeno and Mamdani Fuzzy Control System

1.1 Introduction

Control systems always play a very important role in the society, no matter in the field of engineering, management, medication or any others. Conventional control systems based strongly on an objective theoretical knowledge of the engineer, are mainly used in the control history. However, as the discovery of the fuzzy world becomes more and more popular and due to the interaction of mankind’s intuition and perspectives, fuzzy control systems are shown to be greatly active in the world of control area. Moreover, fuzzy control can still perform the control action even if the function \( f \) is unknown while the conventional control couldn’t make it.

Fuzzy controllers are rule-based models that link the input variables with the consequence variables by terms of linguistic variables, so the processing stage is determined by a collection of logic rules by means of IF-THEN statements [Zimmermann, 2001; http://www.faqs.org/docs/fuzzy/, 2003]. The most common and original fuzzy model is called the Mamdani controller discovered by E. Mamdani in 1973 [Mamdani and Assilian, 1973], then the modified models come out, such as the Sugeno controller [Sugeno, 1985], Adaptive fuzzy control [Zimmermann, 2001; Driankov, Hellendoorn and Reinfrank, 2010] and Takagi-Sugeno (T-S) fuzzy control system [Takagi and Sugeno, 1985]. In this paper, Takagi-Sugeno and Mamdani fuzzy controls are mainly discussed and a management application Resort Management System (RMS) will be illustrated to demonstrate the process of forming a T-S and a Mamdani model. Furthermore the modification of the Takagi-Sugeno and Mamdani model of RMS will be presented when combining it with the fuzzy operations which are going to be implemented in the computation of alpha-value in the final outputs and rules consequences of the model.

1.2 Fundamental Concepts of Fuzzy Set Theory

As a formal theory as well as an application-oriented “fuzzy technology”, fuzzy set theory leads a subjective phenomenon; however, there is nothing fuzzy about fuzzy set theory [Zimmermann, 2001]. Before we discuss the process of control systems, let us first introduce some fundamental concepts of fuzzy set theory [Zadeh, 1965; Zimmermann, 2001; Rakus-Andersson, 2007; Rutkowski, 2008; Gomide and Pedrycz, 1998; Dubois and Prade, 2000; Driankov, Hellendoorn and Reinfrank, 2010].

1.2.1 Fuzzy Set Theory and Basic Operations

**Definition 1.1**

A classical set is a collection of determined objects or elements defined by a certain property, such as \( A = \{5,6,7,8,9,10\} \) and \( A \subseteq [3,7] \).

**Definition 1.2**

A fuzzy set \( A \) is a collection of element pairs which includes the object members and the corresponding membership degrees which are less than or equal to 1, denoted by \( A = \)
\{(x, \mu_A(x), x \in X\}, \text{ where the universe } X \text{ is a collection of all element members } x; \mu_A: X \to [0,1] \text{ is called the membership function of a fuzzy set } A, \text{ and } \mu_A(x) \text{ is called the membership degree of } x.

**Example 1.1**

\(X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\};\)

\(A = \text{“the integer close to 6”};\)

\(A = \{(1,0), (2,0), (3,0.2), (4,0.6), (5,0.8), (6,1), (7,0.8), (8,0.6), (9,0.2), (10,0)\}, \text{ as in Fig. 1.1.}\)

![Figure 1.1: Fuzzy set A = “the integer close to 6”](image)

**Example 1.2**

\(X = \text{“exam grade”} = [0,10];\)

\(A = \text{“excellent”} \text{ is defined by the membership function}\)

\[
\mu_A(x) = \begin{cases} 
0 & \text{for } 0 \leq x < 8 \\
 x - 8 & \text{for } 8 \leq x < 9 \\
1 & \text{for } 9 \leq x \leq 10
\end{cases}
\]

plotted in Fig. 1.2.

![Figure 1.2: Fuzzy set A = “excellent” in X = [0,10]](image)

In what degree is 8.5 “excellent”? Answer: \(\mu_A(x = 8.5) = x - 8 = 8.5 - 8 = 0.5\).

**Definition 1.3**

When the universe X of a fuzzy set A is continuous, then the membership function \(\mu_A(x)\) will also be treated as a continuous membership function. And we have “Zadeh’s notation” of a fuzzy set.
\[ A = \sum_{x \in X} \mu_A(x)/x. \] (1.1)

in which the symbol sign \( \sum \) just denotes the combination of the element pairs instead of a real sum operation.

**Example 1.3**

\[ X = \{1,2,3,4,5,6,7,8,9,10\}; \]

\[ A = \text{"small integer in } X\"; \]

\[ A = \frac{1}{1} + \frac{0.9}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.4}{5} + \frac{0.1}{6} + \frac{0}{7} + \frac{0.8}{8} + \frac{0}{9} + \frac{0}{10}. \]

Also,

\[ A = \frac{1}{1} + \frac{0.9}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.4}{5} + \frac{0.1}{6}. \]

Normally, we omit the terms with the membership degrees equal to 0. And in this paper, “Zadeh’s notation” of a fuzzy set is mostly being used.

**Definition 1.4**

The support of a fuzzy set \( A \), denoted by \( \text{supp}(A) \) is a non-fuzzy set

\[ \text{supp}(A) = \{x: \mu_A(x) > 0\}. \] (1.2)

The \( \alpha \)-cut (or \( \alpha \)-intersection, \( \alpha \)-level cut) of the fuzzy set \( A \) is a non-fuzzy set denoted by \( A_\alpha \) and defined by

\[ A_\alpha = \{x: \mu_A(x) \geq \alpha\}, \alpha \in [0,1]. \] (1.3)

**Example 1.4**

If \( A = \frac{1}{1} + \frac{0.9}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.4}{5} + \frac{0.1}{6} \) then \( \text{supp}(A) = \{1,2,3,4,5,6\} \);

And for \( \alpha = \{0.1,0.4,0.6,0.8,0.9,1.0\} \), we decide

\[ A_{0.1} = \{1,2,3,4,5,6\}; \]
\[ A_{0.4} = \{1,2,3,4,5\}; \]
\[ A_{0.6} = \{1,2,3,4\}; \]
\[ A_{0.8} = \{1,2,3\}; \]
\[ A_{0.9} = \{1,2\}; \]
\[ A_{1.0} = \{1\}. \]

**Definition 1.5**

A linguistic variable is a variable which takes the values being linguistic terms, for instance, a
fuzzy set “age”= {“young”, ”middle-age”, ”old”}, where each term of the list “age” is now regarded as a fuzzy set.

**Definition 1.6**

If \( A = \{ (x, \mu_A(x)) \mid x \in X \} \) is a fuzzy set then the cardinality of \( A \) denoted by \( |A| \) is equal to

\[
|A| = \sum_{x \in X} \mu_A(x).
\]

The relative cardinality of \( A \) denoted by \( \|A\| \) is equal to

\[
\|A\| = \frac{|A|}{|X|}.
\]

**Example 1.5**

\( X = \{1,2,3,4,5,6,7,8,9,10\} \);

\( A \) = ”small integer in \( X \)”;

\[
A = \frac{1}{1} + \frac{0.9}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.4}{5} + \frac{0.1}{6};
\]

\[|A| = 1 + 0.9 + 0.8 + 0.6 + 0.4 + 0.1 = 3.8;\]

\[\|A\| = \frac{|A|}{|X|} = \frac{3.8}{10} = 0.38.\]

**Definition 1.7**

Let \( A = \{ (x, \mu_A(x)) \mid x \in X \} \) and \( B = \{ (x, \mu_B(x)) \mid x \in X \} \), then the topological union of two fuzzy sets \( A \) and \( B \) denoted by \( A \cup B \) is a fuzzy set \( A \cup B = \{ (x, \mu_{A \cup B}(x)) \mid x \in (\text{supp}(A) \cup \text{supp}(B)) \} \), given by the membership function

\[
\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)), \quad x \in (\text{supp}(A) \cup \text{supp}(B)).
\]

**Example 1.6**

\( X = \{1,2,3,4,5,6,7,8,9,10\} \);

\( A \) = ”the integer close to 6”;

\[
A = \frac{0.2}{3} + \frac{0.6}{4} + \frac{0.8}{5} + \frac{1.0}{6} + \frac{0.8}{7} + \frac{0.6}{8} + \frac{0.2}{9};
\]

\( B \) = ”small integer in \( X \)”;

\[
B = \frac{1}{1} + \frac{0.9}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.4}{5} + \frac{0.1}{6};
\]

\( \text{supp}(A) = \{3,4,5,6,7,8,9\} \);

\( \text{supp}(B) = \{1,2,3,4,5,6\} \);
\[ A \cup B = \max(0,1)/1 + \max(0,0.9)/2 + \max(0,2,0.8)/3 + \max(0,6,0.6)/4 + \max(0,8,0.4)/5 + \\max(1,0,0.1)/6 + \max(0,8,0)/7 + \max(0,6,0)/8 + \max(0,2,0)/9; \]

\[ A \cup B = 1/1 + 0.9/2 + 0.8/3 + 0.6/4 + 0.8/5 + 1/6 + 0.8/7 + 0.6/8 + 0.2/9. \]

**Definition 1.8**

Let \( A = \{(x, \mu_A(x)), x \in X\} \) and \( B = \{(x, \mu_B(x)), x \in X\} \), then the topological (hard) intersection of two fuzzy sets \( A \) and \( B \) denoted by \( A \cap B \) is a fuzzy set \( A \cap B = \{(x, \mu_{A \cap B}(x)), x \in \text{supp}(A) \cap \text{supp}(B)\} \), with the membership function

\[ \mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)), x \in \text{supp}(A) \cap \text{supp}(B) \]  \hspace{1cm} (1.7)

**Example 1.7**

\( X = \text{"exam mark"} = [0,10]; \)

\( A = \text{"good"} \) is defined by the membership function as in Fig. 1.3;

\[ \mu_A(x) \]

\[
\begin{array}{cccccccc}
0 & 2 & 4 & 6 & 8 & 10 \\
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1
\end{array}
\]

Figure 1.3: Fuzzy set \( A = \text{"good"} \) in \( X = [0,10] \)

\( B = \text{"excellent"} \) is defined by the membership function as in Fig. 1.4.

\[ \mu_B(x) \]

\[
\begin{array}{cccccccc}
0 & 2 & 4 & 6 & 8 & 10 \\
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1
\end{array}
\]

Figure 1.4: Fuzzy set \( B = \text{"excellent"} \) in \( X = [0,10] \)

\[ A \cap B = \begin{cases} 
  x - 8 & \text{for } 8 \leq x \leq 9 \\
  -x + 10 & \text{for } 9 \leq x \leq 10 
\end{cases} \]

shown in Fig. 1.5.

**Definition 1.9**

Let \( A = \{(x, \mu_A(x)), x \in X\} \). The complement of the fuzzy set \( A \) denoted by \( A' \) is also a fuzzy set.
Figure 1.5: The intersection of fuzzy set A and B

with the membership function

$$\mu_{A'}(x) = 1 - \mu_A(x), \text{for } x \epsilon X$$

(1.8)

**Example 1.8**

$X = \{1,2,3,4,5,6,7,8,9,10\}$;

$A =$ "small integer in $X$";

\[ A = \frac{1}{1} + \frac{0.9}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.4}{5} + \frac{0.1}{6}; \]

\[ A' = \frac{1}{0} + \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.4}{3} + \frac{0.6}{4} + \frac{0.9}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}. \]

**1.2.2 Introduction of Membership Functions**

**Definition 1.10**

The triangular membership function $T(u, \alpha, \beta, \gamma)$ where $\alpha \leq \beta \leq \gamma$ is defined as

$$T(u, \alpha, \beta, \gamma) = \begin{cases} 
0 & \text{for } u < \alpha, \\
\frac{u-\alpha}{\beta-\alpha} & \text{for } \alpha \leq u \leq \beta, \\
\frac{u-\gamma}{\beta-\gamma} & \text{for } \beta \leq u \leq \gamma, \\
0 & \text{for } u > \gamma.
\end{cases}$$

(1.9)

**Example 1.9**

A triangular membership function $T(u, 1,2,3)$, is seen as in Fig. 1.6.

Figure 1.6: The triangular membership function $T(u,1,2,3)$

**Definition 1.11**

A bell-shaped membership function (or a Gaussian membership function) contains an $s$-function and a $(1-s)$-function as defined by
Example 1.10
A Gaussian membership function \( \mu_{\text{Gauss}}(x) = G(x, 2,2.5,3,3.5,4) \), as is sketched in Fig. 1.7.

![Figure 1.7: The bell-shape membership function \( G(x, 2,2.5,3,3.5,4) \)](image1)

Definition 1.12
A trapezoidal membership function (also called the \( \prod \) function) \( \Pi(x, \alpha, \beta, \gamma, \delta) \) is defined as

\[
\Pi(x, \alpha, \beta, \gamma, \delta) = \begin{cases} 
0 & \text{for } x < \alpha, \\
\frac{x - \alpha}{\beta - \alpha} & \text{for } \alpha \leq x < \beta, \\
1 & \text{for } \beta \leq x < \gamma, \\
\frac{\delta - x}{\delta - \gamma} & \text{for } \gamma \leq x < \delta, \\
0 & \text{for } x \geq \delta.
\end{cases}
\] (1.11)

Example 1.11
A trapezoidal membership function \( \Pi(x, 2,4,6,8) \). See Fig. 1.8.

![Figure 1.8: The trapezoidal membership function \( \Pi(x, 2,4,6,8) \)](image2)

1.3 Mamdani Controller

The Mamdani controller consists of the following elements: fuzzification, rule base,
computational unit, defuzzification and process. The main idea is to use the linguistic terms to fuzzify the input variables which are put into use in the process stage based on the control rules. Therefore the computational core can be described as a three-step process consisting of [Zimmermann, 2001]

1) determination of the degree of membership of the input in the rule-antecedent,
2) computation of the rule consequences, and
3) aggregation of rule consequences to the fuzzy set “control action.”

We have a control system \( z = f(x, y) \) where \( x \) and \( y \) are independent variables, \( z \) is the dependent variable, \( f \) is unknown, and \( A_i, B_i \) and \( C_i \) are levels of fuzzy sets, according to the rule \( i: \) if \( x \) is \( A_i \) and \( y \) is \( B_i \) then \( z \) is \( C_i \) where \( i \) is the integer ranged from \([a, b]\). Therefore we have

1) \( \alpha_i = \min(\mu_i(x), \mu_i(y)) \), for all rules \( i \);
2) \( \mu_i^{\text{conseq}}(z) = \min(\alpha_i \text{, the level of } z \text{ corresponding to rule } i) = \min(\alpha_i, \mu_i(z)) \), for all rules \( i \);
3) \( \mu^{\text{conseq}}(z) = \max_i \left( \mu_i^{\text{conseq}}(z) \right) \).

In the defuzzification stage, there are certain strategies, such as Left of maximum (LOM), Right of maximum (ROM), Center of maximum (COM), Center of Area (COA) and the favorable one, the center of gravity (COG) which is defined as

\[
\text{COG } z = f(x, y) = \int_{\mu^{\text{conseq}}(z)dz} f \mu^{\text{conseq}}(z)dz 
\]

for the chosen \( x \) and \( y \).

1.4 Takagi-Sugeno Controller

Based on Mamdani controller, Takagi-Sugeno controller is a modified controller whose output is defined as a function of the inputs instead of fuzzy sets, which means that in each fuzzy subspace a linear input-output relation is formed. Tomohiro Takagi and Michio Sugeno introduced a mathematical tool to build a fuzzy model of a system where fuzzy implication and reasoning are used in their paper in the year of 1985 [Takagi and Sugeno, 1985].

1.4.1 Implication

Let us denote the membership function of a fuzzy set \( A \) as \( A(x), x \in X \). All the membership functions associated to the fuzzy sets are linear. It’s suggested that the format of a fuzzy implication \( R \) is written as

\[
R: \text{If } f(x_1 \text{ is } A_1, \ldots, x_k \text{ is } A_k) \text{ then } y = g(x_1, \ldots, x_k)
\]

where the parameters are explained in Table 1.1.
In the premise we shall only use logical “and” connectives and adopt a linear function in the consequence. So an implication is written as

\[
R: \text{If } x_1 \text{ is } A_1 \text{ and } \ldots \text{ and } x_k \text{ is } A_k \text{ then } y = p_0 + p_1 x_1 + \cdots + p_k x_k \quad (1.14)
\]

| \( y \) | Consequence variable whose value is inferred |
| \( x_1 \ldots x_k \) | Premise variables that also appear in the part of consequence |
| \( A_1 \ldots A_k \) | Fuzzy sets with linear membership functions representing a fuzzy subspace in which the implication \( R \) can be implemented for reasoning |
| \( f \) | Logical function connects the propositions in the premise |
| \( g \) | Function that implies the value of \( y \) when \( x_1, \ldots, x_k \) satisfy the premise |

**Table 1.1: Parameters in the implication \( R \)**

**Example 1.12**

If \( x_1 \) is small and \( x_2 \) is big then \( y = x_1 + x_2 \).

### 1.4.2 Reasoning

Suppose that we have implications \( R^i (i = 1, 2, \ldots, n) \), placed in Table 1.2.

<table>
<thead>
<tr>
<th>Implication</th>
<th>Premise</th>
<th>Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^1 )</td>
<td>( A_1^1 (x_1^0) ) ( A_2^1 (x_2^0) ) \ldots ( A_k^1 (x_k^0) )</td>
<td>( y^1 )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( R^n )</td>
<td>( A_1^n (x_1^0) ) ( A_2^n (x_2^0) ) \ldots ( A_k^n (x_k^0) )</td>
<td>( y^n )</td>
</tr>
</tbody>
</table>

where \( x_1^0, \ldots, x_k^0 \) are singletons and \( A_j^i \) stand for the fuzzy sets assigned to variables \( x_j \) in the \( i^{th} \) implication with \( 1 \leq i \leq n \) and \( 1 \leq j \leq k \).

Three steps to get the value of final output \( y \):

1. Calculate the value of \( y^i \) by
   \[
   y^i = g^i (x_1^0, x_2^0, \ldots, x_k^0) = p_0 + p_1 x_1^0 + \cdots + p_k x_k^0. \quad (1.15)
   \]
2. Find the truth value (also denoted by \( \alpha^i \)) of the proposition \( y = y^i \) by
   \[
   \alpha^i = |y = y^i| = |x_1^0 \text{ is } A_1^i \text{ and } \ldots \text{ and } x_k^0 \text{ is } A_k^i| = A_1^i (x_1^0) \land \ldots \land A_k^i (x_k^0), \quad (1.16)
   \]
   where \(|*|\) means the truth value of the proposition \( * \) and \( \land \) stands for minimum operation.
3. Calculate the final output \( y \) inferred from \( n \) implications by
   \[
   y = \frac{\sum |y = y^i| \alpha^i}{\sum |y = y^i|}. \quad (1.17)
   \]
Example 1.13
Suppose that we have the following three implications:

\[ R^1: \text{If } x_1 \text{ is low and } x_2 \text{ is low then } y = x_1 + x_2 \]
\[ R^2: \text{If } x_1 \text{ is high and } x_2 \text{ is low then } y = 2x_1 \]
\[ R^3: \text{If } x_2 \text{ is high then } y = 3x_2 \]

Table 1.3 shows the process of reasoning by each implication when we are given \( x_1 = 3, x_2 = 4 \). The column “Premise” presents the membership function of the fuzzy sets “low” and “high” in the premise while the column “Consequence” shows the values of \( y^i \) calculated by the function \( g^i \) of each consequence and “TV” shows the truth value of \( |y - y^i| \). For example, we have

\[ \alpha^1 = |y - y^1| = |x_1 = \text{low}_1 \land |x_2 = \text{low}_2| \cdot \text{where } \mu_{\text{low}_1}(x_1) = -\frac{7}{30}x_1 + 1 \text{ and } \mu_{\text{low}_2}(x_2) = -\frac{11}{60}x_1 + \frac{71}{60}, \text{ so } \alpha^1 = |x_1 = \text{low}_1 \land |x_2 = \text{low}_2| = \mu_{\text{low}_1}(3) \land \mu_{\text{low}_2}(4) = (-\frac{7}{30} \cdot 3 + 1) \land
\]

\[ (-\frac{11}{60} \cdot 4 + \frac{71}{60}) = 0.3 \land 0.45 = 0.3. \]

Similarly, \( \mu_{\text{high}_1}(x_1) = 0.4x_1 - 1 \text{ and } \mu_{\text{high}_2}(x_2) = 0.325x_2 - 0.95 \), so \( \alpha^2 = \mu_{\text{high}_1}(3) = 0.4 \cdot 3 - 1 = 0.2 \text{ and } \alpha^3 = \mu_{\text{high}_2}(4) = 0.325 \cdot 4 - 0.95 = 0.35. \)

The final output \( y \) inferred by the implications is obtained by

\[ y = \frac{0.3 \cdot 7 + 0.2 \cdot 6 + 0.35 \cdot 12}{0.3 + 0.2 + 0.35} = 8.82. \]

Table 1.3: An example of the reasoning process

<table>
<thead>
<tr>
<th>Implication</th>
<th>Premise</th>
<th>Consequence</th>
<th>TV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^1 )</td>
<td>( \text{low}_1 )</td>
<td>0.3 ( \text{low}_2 )</td>
<td>0.3 ( \land ) 0.45 = 0.3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>( \text{high}_1 )</td>
<td>0.2 ( \text{low}_2 )</td>
<td>y = 2 \cdot 3 = 6 0.2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>( R^3 )</td>
<td>( \text{low}_1 )</td>
<td>0.35 ( \text{high}_2 )</td>
<td>y = 3 \cdot 4 = 12 0.35</td>
</tr>
<tr>
<td></td>
<td>x_1 = 3</td>
<td>2</td>
<td>x_2 = 4</td>
</tr>
</tbody>
</table>

The reasoning method enables us to reduce the number of piecewise linear relations and therefore connect them smoothly. Moreover, with the fuzzy partition of input space, we can put
linguistic conditions to linear relations such as “$x_1$ is low and $x_2$ is high” so that we are able to use the subjective parameters that are observed by man.

1.4.3 Identification

Three items are taken into consideration when talking about the process of identification:

1. $x_1, \ldots, x_k$ Variables composing the premises of implications.
2. $A_1, \ldots, A_k$ Membership functions of fuzzy sets in the premises, denoted as premise parameters.
3. $p_0, \ldots, p_k$ Parameters in the consequence, denoted as consequence parameters.

Therefore, the algorithm of the identification of implications is divided into three steps corresponding to the three items that are mentioned.

1) Choice of Premise Variables: In this step, a combination of premise variables which will be presented as fuzzy sets is chosen out of the possible input variables we can consider.
2) Identification of Premise Parameters: The aim in this step is to choose the optimum premise parameters which hold the membership functions with the suitable shapes and determined according to the premise variables chosen at step 1).
3) Identification of Consequence Parameters: The consequence parameters that carry optimum values are obtained according to the least squares method for the given premise variables in step 1) and parameters in step 2).

From Fig. 1.9 the outline of the identification algorithm is presented.

![Outline of Identification Algorithm](image)

Thereafter, a flow chat of the Takagi-Sugeno (T-S) controller can be sketched in Fig1.10.

1.5 Specific Method for Identification

In this section, the specific method for choosing the proper membership functions in the premise parameters identification [Rakus-Andersson, 2007] and the concrete technique of using
regression in the consequence parameters identification [Hadi and Chatterjee, 2006] will be illustrated.

1.5.1 Choice of Membership Functions in Premise Parameters Identification

In order to demonstrate the method for premise parameters identification, first let us define the premise variables by means of linguistic terms named “level of comfort” = \{R_1 =”none”, R_2 =”almost none”, R_3 =”very low”, R_4 =”low”, R_5 =”rather low”, R_6 =”medium”, R_7 =”rather high”, R_8 =”high”, R_9 =”very high”, R_{10} =”almost perfect”, R_{11} =”perfect”\), whereas every single notion of this list represents a fuzzy set. Assume that all sets are defined in the space Z = [0,100] which is measured by the percentage of the satisfaction of the accommodation in the Resort. To guarantee the economic manipulation and avoid further complicated computations, the simple linear functions are suggested for the membership functions of the fuzzy sets from the list. Likewise

\[ L(z, \alpha, \beta) = \begin{cases} 0 & \text{for } z \leq \alpha, \\ \frac{z - \alpha}{\beta - \alpha} & \text{for } \alpha < z \leq \beta, \\ 1 & \text{for } z > \beta, \end{cases} \]

(1.18)
and

\[ \Pi(z, \alpha, \gamma, \beta) = \begin{cases} 
0 & \text{for } z \leq \alpha, \\
L(z, \alpha, \gamma) & \text{for } \alpha < z \leq \gamma, \\
1 - L(z, \gamma, \beta) & \text{for } \gamma < z \leq \beta, \\
0 & \text{for } z > \beta, 
\end{cases} \tag{1.19} \]

where \( z \) is an independent variable with proportion unit belonging to \([0,100]\) and \( \alpha, \beta, \gamma \) are some boundary parameters.

Thereafter, we can define

\[ \mu_{R_k}(z) = \begin{cases} 
1 - L(z, \alpha_k, \beta_k) & \text{for } k = 1, 2, 3, 4, 5, \\
L(z, \alpha_k, \beta_k) & \text{for } k = 7, 8, 9, 10, 11 
\end{cases} \tag{1.20} \]

and

\[ \mu_{R_6}(z) = \Pi(z, \alpha_6, \gamma, \beta_6) \tag{1.21} \]

in which \( z \in [0,100] \), while \( \alpha_k, \beta_k, \gamma \) belonging to the interval \([0,100]\) are borders for the fuzzy supports.

**Example 1.14**

Now we construct the parameters \( \alpha_k, \beta_k, \gamma \) to adopt the generation of the specific fuzzy sets, as the following.

\[ \mu_{R_1}(z) = \mu^{\text{none}}(z) = 1 - L(z, 0, 20), \]
\[ \mu_{R_2}(z) = \mu^{\text{almost none}}(z) = 1 - L(z, 10, 30), \]
\[ \mu_{R_3}(z) = \mu^{\text{very low}}(z) = 1 - L(z, 20, 40), \]
\[ \mu_{R_4}(z) = \mu^{\text{low}}(z) = 1 - L(z, 30, 50), \]
\[ \mu_{R_5}(z) = \mu^{\text{rather low}}(z) = 1 - L(z, 40, 60), \]
\[ \mu_{R_6}(z) = \mu^{\text{medium}}(z) = \Pi(z, 30, 50, 70), \]
\[ \mu_{R_7}(z) = \mu^{\text{rather high}}(z) = L(z, 40, 60), \]
\[ \mu_{R_8}(z) = \mu^{\text{high}}(z) = L(z, 50, 70), \]
\[ \mu_{R_9}(z) = \mu^{\text{very high}}(z) = L(z, 60, 80), \]
\[ \mu_{R_{10}}(z) = \mu^{\text{almost perfect}}(z) = L(z, 70, 90), \]
\[ \mu_{R_{11}}(z) = \mu^{\text{perfect}}(z) = L(z, 80, 100) \]

for \( z \in [0,100] \). A sketch of the membership functions is presented in Fig. 1.11.
1.5.2 Consequence Parameters Identification by Using Regression

In this paper the method of multiple linear regressions is adopted to calculate the consequence parameters $p_0, ..., p_k$ included in the formula given by

$$ y = p_0 + p_1 x_1 + p_2 x_2 + ... + p_k x_k + \varepsilon \quad (1.22) $$

in which $\varepsilon$ is the error and $\varepsilon \in N(0, \sigma)$. Nevertheless the error $\varepsilon$ is omitted later in this paper for the simplification of calculation. Therefore according to the Least Square method, the coefficients $p_0, ..., p_k$ are estimated by minimizing

$$ Q(p_0, p_1, ..., p_k) = \sum_{j=1}^{k} \left( y - (p_0 + p_1 x_1 + p_2 x_2 + ... + p_j x_j + ...) \right)^2. \quad (1.23) $$

Let us denote some definitions for further calculation:

$$ p = (p_0, p_1, ..., p_k)^T, \quad (1.24) $$
$$ y = (y_1, y_2, ..., y_n), \quad (1.25) $$
$$ X = \begin{pmatrix} 1 & x_{11} & x_{12} & ... & x_{1k} \\ 1 & x_{21} & x_{22} & ... & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & ... & x_{nk} \end{pmatrix} \quad (1.26) $$

where $x_{ij}$ is observation $i$ of the variable $x_j$. Thereafter the Least Square estimation is to minimize $Q(b) = (y - xb)^T (y - xb)$ by solving the system of equations

$$ \frac{\partial Q}{\partial p_0} = 0, \frac{\partial Q}{\partial p_1} = 0, ..., \frac{\partial Q}{\partial p_k} = 0. $$

Nevertheless, a Takagi-Sugeno and Mamdani fuzzy control of a resort management system will be presented in details in the next chapter.
2 Application of Takagi-Sugeno and Mamdani Fuzzy Control in a Resort Management System

Based on the knowledge which has been discussed above [Takagi and Sugeno, 1985; Zimmermann, 2001; Rakus-Andersson, 2007], this chapter will demonstrate how to implement the Resort Management System by means of Takagi-Sugeno (T-S) and Mamdani fuzzy controller [Piegat, 2001; Wang, Yu and Jing, 2010]. In Fig. 2.1, the main flow chart of the Resort Management System (RMS) is sketched and in Fig. 2.2 the corresponding symbolic variables are assigned for each linguistic variable.

Figure 2.1: Resort Management System

Figure 2.2: Resort Management System with symbolic terms

2.1 Takagi-Sugeno Fuzzy Control on the First-level Management

RMS is divided into two levels when performing the T-S control and Mamdani control. The first level performed by T-S model includes \textit{Rank of Catering} = f_1(\textit{Scale of Health, Scale of Taste}), \textit{Rank of Accommodation} = f_2(\textit{Level of Comfort, Level of Price}) and \textit{Rank of Leisure} = f_3(\textit{Grade of Spa, Grade of Golf, Grade of Sports}), while the second level implemented by Mamdani model is constructed by \textit{Quality of RMS} = f(\textit{Rank of Catering, Rank of Accommodation, Rank of Leisure}). Furthermore, all of these variables including the independent and dependent
ones are ranging from the interval with the unit of percentage $[0,100]$, representing the appreciation of all of these services offered by the resort.

2.1.1 Control of Catering

Perform T-S control on $\text{Rank of Catering} = f_1(\text{Scale of Health, Scale of Taste})$, which is denoted by $z' = f_1(x_1, y_1)$. Premise variables are $x_1$ = “scale of health” and $y_1$ = “scale of taste” as independent variables and consequence variable is $z'$ = “rank of catering” as dependent variable. We now regard the step of transforming the linguistic terms into numerical values in order to implement the identification. Therefore, the linguistic variables can be interpreted as the fuzzy sets with the independent variables ranged continuously in the scale holding the percentage values $[0,100]$. This interval with the same referring meanings is also involved in the rest of the Resort Management System for other linguistic variables, for instance “level of comfort” and “level of price”. According to the technique from 1.5.1, the premise parameters identification is constructed as in Fig. 2.3 and Fig. 2.4.

![Figure 2.3: Membership functions of premise variable “scale of health”](image)

![Figure 2.4: Membership functions of premise variable “scale of taste”](image)

The statistical data can be received by the feedback from the customers recorded monthly in the service management apartment. Among the operation data which are taken from percentage values, about 500 taken in June and July are used for the consequence parameters identification thus a small part of the data is stated as in table 2.1.
Table 2.1: Statistical data of “Catering”

<table>
<thead>
<tr>
<th>scale of health (%)</th>
<th>scale of taste (%)</th>
<th>rank of catering (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>89.6</td>
<td>73</td>
<td>75</td>
</tr>
<tr>
<td>89.78</td>
<td>78</td>
<td>79</td>
</tr>
<tr>
<td>89.45</td>
<td>81</td>
<td>82</td>
</tr>
<tr>
<td>89.89</td>
<td>87</td>
<td>87</td>
</tr>
<tr>
<td>89.6</td>
<td>91</td>
<td>90</td>
</tr>
<tr>
<td>89.7</td>
<td>96</td>
<td>92</td>
</tr>
<tr>
<td>89.56</td>
<td>94.5</td>
<td>92</td>
</tr>
<tr>
<td>89.6</td>
<td>96</td>
<td>93</td>
</tr>
<tr>
<td>89.5</td>
<td>96</td>
<td>93</td>
</tr>
<tr>
<td>89</td>
<td>98</td>
<td>94</td>
</tr>
<tr>
<td>89</td>
<td>99</td>
<td>95</td>
</tr>
<tr>
<td>89</td>
<td>94</td>
<td>93</td>
</tr>
<tr>
<td>89.6</td>
<td>98</td>
<td>95</td>
</tr>
</tbody>
</table>

The rule-based table is presented in Table 2.2:

Table 2.2: Rule-based table of Catering

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$y_1$</th>
<th>bad</th>
<th>good</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>$z_{11}$</td>
<td>$z'_{12}$</td>
<td></td>
</tr>
<tr>
<td>medium</td>
<td>$z_{21}$</td>
<td>$z'_{22}$</td>
<td></td>
</tr>
<tr>
<td>high</td>
<td>$z_{31}$</td>
<td>$z'_{32}$</td>
<td></td>
</tr>
</tbody>
</table>

Based on multiple linear regression method, consequence parameters identification will be implemented in SPSS by using the observation data. For each implication, we select the observation data according to the constraints from the corresponding fuzzy sets given from above to generate the coefficients $p_0, p_1, \ldots, p_k$ when performing in SPSS. For example, in implication $R^1$, $x_1$ (scale of health) is “low” and $y_1$ (scale of taste) is “bad” so we select the corresponding rows of data from the whole observation data with the constraint $0 \leq x_1 \leq 50$ and $0 \leq y_1 \leq 80$. Table 2.3 shows the output of the coefficients for the implication $R^1$, since the same method is performed for the other implications, so we omit the output tables for later sections in this paper.

Therefore, the rules can be constructed as the following.

$R^1$: If $x_1$ is “low” and $y_1$ is “bad” then $z'_{11} = 0.481 \times x_1 + 0.487 \times y_1 - 0.717$;

$R^2$: If $x_1$ is “low” and $y_1$ is “good” then $z'_{12} = 0.361 \times x_1 + 0.601 \times y_1 - 1.503$;

$R^3$: If $x_1$ is “medium” and $y_1$ is “bad” then $z'_{21} = 0.274 \times x_1 + 0.77 \times y_1 - 6.831$;

$R^4$: If $x_1$ is “medium” and $y_1$ is “good” then $z'_{22} = 0.278 \times x_1 + 0.758 \times y_1 - 6.529$;
Table 2.3: Outputs of coefficients in $R^1$ of Catering

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>Std. Error</td>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(Constant)</td>
<td>-.717</td>
<td>2.105</td>
<td>-.340</td>
</tr>
<tr>
<td></td>
<td>scale of health</td>
<td>.481</td>
<td>.057</td>
<td>.394</td>
</tr>
<tr>
<td></td>
<td>scale of taste</td>
<td>.487</td>
<td>.033</td>
<td>.675</td>
</tr>
</tbody>
</table>

a. Dependent Variable: rank of catering

$R^5$: If $x_1$ is “high” and $y_1$ is “bad” then $z_{11}' = 0.166 * x_1 + 0.89 * y_1 - 5.649$;

$R^6$: If $x_1$ is “high” and $y_1$ is “good” then $z_{32}' = 0.199 * x_1 + 0.843 * y_1 - 5.917$.

**Example 2.1**
Find the corresponding final output for the rank of catering with $z' = f_1(x_1, y_1) = f_1(70, 88)$ where the scale of health is 70 and the scale of taste is 88.

$x_1 = 70$ belongs to “medium” and “high”,
y$_1 = 88$ belongs to “good”.

So the rules can be obtained as

$R^1$: If $x_1$ is “medium” and $y_1$ is “good” then $z_{22}' = 0.278 * x_1 + 0.758 * y_1 - 6.529 = 0.278 * 70 + 0.758 * 88 - 6.529 = 79.635$;

$R^2$: If $x_1$ is “high” and $y_1$ is “good” then $z_{32}' = 0.199 * x_1 + 0.843 * y_1 - 5.917 = 0.199 * 70 + 0.843 * 88 - 5.917 = 82.197$.

$\mu$-medium\(^-(x_1 = 70) = 0.2$,

$\mu$-high\(^-(x_1 = 70) = 0.8$,

$\mu$-good\(^-(y_1 = 88) = 1$.

$\alpha_1' = \min(0.2, 1) = 0.2$,

$\alpha_2' = \min(0.8, 1) = 0.8$.

$z' = f_1(70, 88) = \frac{0.2 * 79.635 + 0.8 * 82.197}{0.2 + 0.8} = 81.68$.

So the final output for the rank of catering when $z' = f_1(70, 88)$ is equal to 81.68.

**2.1.2 Control of Accommodation**

Likewise, the T-S control of Rank of Accommodation $= f_2(Level of Comfort, Level of Price)$ can be denoted as $z'' = f_2(x_2, y_2)$, where the premise variables are $x_2$ and $y_2$, the consequence
parameter is $z''$ and $f_2$ is the logical function which connects the propositions in the premise (also denotes as control action). $x_2$ and $y_2$ belong to the scale $[0,100]$ where the values are measured by percentage from customers’ feedback, therefore the premise parameters identification can be sketched as in Fig. 2.5 and Fig. 2.6.

![Figure 2.5: Membership functions of premise variable “level of comfort”](image1)

![Figure 2.6: Membership functions of premise variable “level of price”](image2)

As in the case of catering, the operation data of accommodation can be obtained by the service feedback from the customers. We take the records from June and a part of them will be shown in the following table.

<table>
<thead>
<tr>
<th>level of comfort (%)</th>
<th>level of price (%)</th>
<th>level of accommodation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.5</td>
<td>83</td>
<td>85</td>
</tr>
<tr>
<td>84.67</td>
<td>84</td>
<td>85</td>
</tr>
<tr>
<td>84.8</td>
<td>88</td>
<td>85</td>
</tr>
<tr>
<td>84.7</td>
<td>86</td>
<td>85</td>
</tr>
<tr>
<td>84.5</td>
<td>88</td>
<td>84.6</td>
</tr>
<tr>
<td>85</td>
<td>87</td>
<td>86</td>
</tr>
<tr>
<td>85</td>
<td>89</td>
<td>85</td>
</tr>
<tr>
<td>85.35</td>
<td>89.6</td>
<td>86</td>
</tr>
<tr>
<td>85.56</td>
<td>91</td>
<td>85</td>
</tr>
<tr>
<td>86</td>
<td>92</td>
<td>86</td>
</tr>
<tr>
<td>86.2</td>
<td>93.5</td>
<td>86</td>
</tr>
<tr>
<td>86.35</td>
<td>94</td>
<td>86</td>
</tr>
<tr>
<td>86.68</td>
<td>96</td>
<td>87</td>
</tr>
</tbody>
</table>
The rule-based table can be sketched as in Table 2.5.

Table 2.5: Rule-based table of Accommodation

<table>
<thead>
<tr>
<th>$x_2$</th>
<th>$y_2$</th>
<th>cheap</th>
<th>moderate</th>
<th>expensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td></td>
<td>$z_{11}''$</td>
<td>$z_{12}''$</td>
<td>$z_{13}''$</td>
</tr>
<tr>
<td>high</td>
<td></td>
<td>$z_{21}''$</td>
<td>$z_{22}''$</td>
<td>$z_{23}''$</td>
</tr>
</tbody>
</table>

Thereafter, the rule-based table can be completed by constructing the data regression in SPSS, which is shown in Table 2.6.

Table 2.6: Rule-based table of Accommodation with Specific Outputs

<table>
<thead>
<tr>
<th>$x_2$</th>
<th>$y_2$</th>
<th>cheap</th>
<th>moderate</th>
<th>expensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td></td>
<td>$z_{11}'' = 0.975 \times x_2 - 0.044 \times y_2 + 3.934$</td>
<td>$z_{12}'' = 0.951 \times x_2 - 0.014 \times y_2 + 4.117$</td>
<td>$z_{13}'' = 0.983 \times x_2 - 0.067 \times y_2 + 5.376$</td>
</tr>
<tr>
<td>high</td>
<td></td>
<td>$z_{21}'' = 1.06 \times x_2 - 0.054 \times y_2 + 0.939$</td>
<td>$z_{22}'' = 1.008 \times x_2 - 0.024 \times y_2 + 1.24$</td>
<td>$z_{23}'' = 1.02 \times x_2 - 0.053 \times y_2 + 2.205$</td>
</tr>
</tbody>
</table>

**Example 2.2**

Find the corresponding final output for the rank of accommodation when $z'' = f_2(x_2, y_2) = f_2(88, 73)$ where the level of comfort is 88 and the level of price is 73.

$x_2 = 88$ belongs to “high”,
y2 = 73 belongs to “moderate” and “expensive”.

So the rules can be obtained as

$R^1$: If $x_2$ is “high” and $y_2$ is “moderate” then $z_{22}'' = 1.008 \times x_2 - 0.024 \times y_2 + 1.24 = 1.008 \times 88 - 0.024 \times 73 + 1.24 = 88.192$;

$R^2$: If $x_2$ is “high” and $y_2$ is “expensive” then $z_{23}'' = 1.02 \times x_2 - 0.053 \times y_2 + 2.205 = 1.02 \times 88 - 0.053 \times 73 + 2.205 = 88.096$.

$\mu\text{-high}(x_2 = 88) = 1$,

$\mu\text{-moderate}(y_2 = 73) = 0.08$,

$\mu\text{-expensive}(y_2 = 73) = 0.92$.

$\alpha_1'' = \min(1,0.08) = 0.08$,

$\alpha_2'' = \min(1,0.92) = 0.92$.

$z'' = f_2(88, 73) = \frac{0.08 \times 88.192 + 0.92 \times 88.096}{0.08 + 0.92} = 88.1$. 

20
So the final output for the rank of accommodation when $z'' = f_2(88, 73)$ is equal to 88.1.

2.1.3 Control of Leisure

T-S control on $Rank of Leisure = f_3(Grade of Spa, Grade of Golf, Grade of Sports)$ denoted by $z''' = f_3(x_3, y_3, s_3)$ can be presented in the same way as 2.1.1 and 2.1.2. Suppose the premise variables $x_3, y_3$ and $s_3$ ranged from the grades [0, 100] consistently and the premise parameters identification can be suggested as Fig. 2.7, Fig. 2.8 and Fig. 2.9.

![Figure 2.7: Membership functions of premise variable “grade of spa”](image)

![Figure 2.8: Membership functions of premise variable “grade of golf”](image)

![Figure 2.9: Membership functions of premise variable “grade of sports”](image)

A small part of the operation data which contains 200 records is shown in the following table.

Therefore, the control rules can be drawn as the following:

$R^1$: If $x_3$ is “bad”, $y_3$ is “poor” and $s_3$ is “inferior”,
then $z'''_{111} = 0.355 \times x_3 + 0.365 \times y_3 + 0.314 \times s_3 - 4.306$;
Table 2.7: Statistical data of “Leisure”

<table>
<thead>
<tr>
<th>grade of spa (%)</th>
<th>grade of golf (%)</th>
<th>grade of sports (%)</th>
<th>grade of leisure (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>62</td>
<td>92</td>
<td>81</td>
</tr>
<tr>
<td>89.5</td>
<td>73</td>
<td>84</td>
<td>79</td>
</tr>
<tr>
<td>89.6</td>
<td>85</td>
<td>86.5</td>
<td>87</td>
</tr>
<tr>
<td>89.65</td>
<td>87</td>
<td>91</td>
<td>88</td>
</tr>
<tr>
<td>90</td>
<td>83</td>
<td>96</td>
<td>89.6</td>
</tr>
<tr>
<td>90</td>
<td>88</td>
<td>78.56</td>
<td>86</td>
</tr>
<tr>
<td>91</td>
<td>86</td>
<td>82.5</td>
<td>86.5</td>
</tr>
<tr>
<td>91.5</td>
<td>88.5</td>
<td>81</td>
<td>86.3</td>
</tr>
<tr>
<td>92</td>
<td>91</td>
<td>80</td>
<td>86.8</td>
</tr>
<tr>
<td>93</td>
<td>96</td>
<td>83.5</td>
<td>90.5</td>
</tr>
<tr>
<td>94.5</td>
<td>97</td>
<td>86</td>
<td>92</td>
</tr>
<tr>
<td>96</td>
<td>98</td>
<td>88</td>
<td>94.5</td>
</tr>
<tr>
<td>97</td>
<td>98</td>
<td>96</td>
<td>97</td>
</tr>
</tbody>
</table>

\[ R^2: \text{If } x_3 \text{ is “bad”, } y_3 \text{ is “poor” and } s_3 \text{ is “superior”,} \]
\[ z'_{112}'' = 0.361 \times x_3 + 0.329 \times y_3 + 0.351 \times s_3 - 4.748; \]

\[ R^3: \text{If } x_3 \text{ is “bad”, } y_3 \text{ is “excellent” and } s_3 \text{ is “inferior”,} \]
\[ z'_{121}'' = 0.377 \times x_3 + 0.32 \times y_3 + 0.326 \times s_3 - 3.895; \]

\[ R^4: \text{If } x_3 \text{ is “bad”, } y_3 \text{ is “excellent” and } s_3 \text{ is “superior”,} \]
\[ z'_{122}'' = 0.37 \times x_3 + 0.304 \times y_3 + 0.323 \times s_3 - 2.447; \]

\[ R^5: \text{If } x_3 \text{ is “good”, } y_3 \text{ is “poor” and } s_3 \text{ is “inferior”,} \]
\[ z'_{211}'' = 0.383 \times x_3 + 0.352 \times y_3 + 0.317 \times s_3 - 5.558; \]

\[ R^6: \text{If } x_3 \text{ is “good”, } y_3 \text{ is “poor” and } s_3 \text{ is “superior”,} \]
\[ z'_{212}'' = 0.35 \times x_3 + 0.324 \times y_3 + 0.345 \times s_3 - 3.345; \]

\[ R^7: \text{If } x_3 \text{ is “good”, } y_3 \text{ is “excellent” and } s_3 \text{ is “inferior”,} \]
\[ z'_{221}'' = 0.361 \times x_3 + 0.317 \times y_3 + 0.323 \times s_3 - 2.419; \]

\[ R^8: \text{If } x_3 \text{ is “good”, } y_3 \text{ is “excellent” and } s_3 \text{ is “superior”,} \]
\[ z'_{222}'' = 0.369 \times x_3 + 0.324 \times y_3 + 0.324 \times s_3 - 3.389. \]

**Example 2.3**

Find the corresponding final output for the rank of leisure when \( z'' = f_3(x_3, y_3, s_3) = f_3(81, 70, 70) \) with which the grade of spa is 81, the grade of golf is 70 and the grade of sports is 70.

\[ x_3 = 81 \text{ belongs to “good”,} \]
\[ y_3 = 70 \text{ belongs to “excellent”,} \]
\[ s_3 = 70 \text{ belongs to “inferior” and “superior”.} \]
So the rules can be obtained as

\[ R^1: \text{If } x_3 \text{ is "good", } y_3 \text{ is "excellent" and } s_3 \text{ is "inferior",} \]
\[ \text{then } z''_{221} = 0.361 \times x_3 + 0.317 \times y_3 + 0.323 \times s_3 - 2.419 = 0.361 \times 81 + 0.317 \times 70 + 0.323 \times 70 - 2.419 = 71.622; \]

\[ R^2: \text{If } x_3 \text{ is "good", } y_3 \text{ is "excellent" and } s_3 \text{ is "superior"} \]
\[ \text{then } z''_{222} = 0.369 \times x_3 + 0.324 \times y_3 + 0.324 \times s_3 - 3.389 = 0.369 \times 81 + 0.324 \times 70 + 0.324 \times 70 - 3.389 = 71.86. \]

\[ \mu_{\text{good}}(x_3 = 81) = 1, \]
\[ \mu_{\text{excellent}}(y_3 = 70) = 1. \]
\[ \mu_{\text{inferior}}(s_3 = "70") = 0.1. \]
\[ \mu_{\text{superior}}(s_3 = 70) = 0.9. \]

\[ \alpha'_i = \min(1,1,0.1) = 0.1, \]
\[ \alpha''_i = \min(1,1,0.9) = 0.9. \]

\[ z''' = f_3(81,70,70) = \frac{0.1 \times 71.622 + 0.9 \times 71.86}{0.1 + 0.9} = 71.84. \]

So the final output for the rank of leisure when \( z''' = f_3(81,70,70) \) is equal to 71.84.

### 2.2 Mamdani Fuzzy Control on the Second-level Management

Mamdani fuzzy control model of the second-level management \( \text{Quality of RMS} = f(\text{Rank of Catering, Rank of Accommodation, Rank of Leisure}) \) can be denoted as \( z = f(z', z'', z''') \), where \( z \) is the dependent variable with percentage values ranged from \([0,100]\), \( f \) is the control action and \( z', z'' \) and \( z''' \) are independent variables with also percentage values belonging to \([0,100]\). The fuzzification process is shown by the following sketches.

![Membership functions of the independent variable “rank of catering”](image)
Therefore, the rule-based table can be obtained. See Table 2.8.

<table>
<thead>
<tr>
<th>$R^1$</th>
<th>$z'$</th>
<th>$z''$</th>
<th>$z'''$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>poor</td>
<td>ordinary</td>
<td>unsatisfactory</td>
<td></td>
<td></td>
</tr>
<tr>
<td>poor</td>
<td>ordinary</td>
<td>satisfactory</td>
<td>bad</td>
<td></td>
</tr>
<tr>
<td>poor</td>
<td>tremendous</td>
<td>unsatisfactory</td>
<td>bad</td>
<td></td>
</tr>
<tr>
<td>poor</td>
<td>tremendous</td>
<td>satisfactory</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>remarkable</td>
<td>ordinary</td>
<td>unsatisfactory</td>
<td>bad</td>
<td></td>
</tr>
<tr>
<td>remarkable</td>
<td>ordinary</td>
<td>satisfactory</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>remarkable</td>
<td>tremendous</td>
<td>unsatisfactory</td>
<td>medium</td>
<td></td>
</tr>
<tr>
<td>remarkable</td>
<td>tremendous</td>
<td>satisfactory</td>
<td>good</td>
<td></td>
</tr>
</tbody>
</table>
Example 2.4
Find the corresponding final output for the quality of RMS when \( z = f(z',z'',z''') = f(81.68,88.1,71.84) \). The input values are obtained from the outputs of example 2.1, 2.2 and 2.3 in the first-level management where the rank of catering is 81.68, the rank of accommodation is 88.1 and the rank of leisure is 71.84.

\( z' = 81.68 \) belongs to “remarkable”;
\( z'' = 88.1 \) belongs to “tremendous”;
\( z''' = 71.84 \) belongs to “unsatisfactory” and “satisfactory”.

Thus the rules can be yielded as

\[ R^1: \text{If } z' \text{ is “remarkable”, } z'' \text{ is “tremendous” and } z''' \text{ is “unsatisfactory”, then } z \text{ is “medium”}; \]
\[ R^2: \text{If } z' \text{ is “remarkable”, } z'' \text{ is “tremendous” and } z''' \text{ is “satisfactory”, then } z \text{ is “good”}. \]

\[
\begin{align*}
\mu_{\text{remarkable}}(z' = 81.68) & = 1, \\
\mu_{\text{tremendous}}(z'' = 88.1) & = 1, \\
\mu_{\text{unsatisfactory}}(z''' = 71.84) & = -\frac{1}{50} \times 71.84 + \frac{3}{2} = 0.06, \\
\mu_{\text{satisfactory}}(z''' = 71.84) & = \frac{1}{50} \times 71.84 - \frac{1}{2} = 0.94.
\end{align*}
\]

\( \alpha_1 = \min(1,1,0.06) = 0.06, \)
\( \alpha_2 = \min(1,1,0.94) = 0.94. \)

\( \mu_{\text{conseq}}^1(z) = \min\left(0.06, \mu_{\text{medium}}(z)\right). \) And the result is sketched in Fig. 2.14.

![Figure 2.14: Membership function \( \mu_{\text{conseq}}^1(z) \)](image)

\( \mu_{\text{conseq}}^2(z) = \min\left(0.94, \mu_{\text{good}}(z)\right). \) And the result is shown in Fig. 2.15.

A total consequence of the rules is given as a fuzzy set in \( z \) with the membership function

\[
\mu_{\text{conseq}}(z) = \max\left(\mu_{\text{conseq}}^1(z), \mu_{\text{conseq}}^2(z)\right) = \begin{cases} 
\frac{1}{25} z - 1 & \text{for } 25 \leq z < 26.5, \\
0.06 & \text{for } 26.5 \leq z < 51.5, \\
\frac{1}{25} z - 2 & \text{for } 51.5 \leq z < 73.5, \\
0.94 & \text{for } 73.5 \leq z \leq 100.
\end{cases}
\]
The total consequence is plotted in Fig. 2.16.

According to Center of Gravity, the total consequence can be defuzzified as

\[
z = f(81.68, 88.1, 71.84) = \frac{\int_{25}^{51.5} \left( \frac{1}{25} z - 1 \right) dz + \int_{51.5}^{73.5} 0.06 z + \int_{73.5}^{100} 0.94 z dz}{\int_{25}^{51.5} \left( \frac{1}{25} z - 1 \right) dz + \int_{51.5}^{73.5} 0.06 dz + \int_{73.5}^{100} 0.94 dz} = 78.59\%.
\]

Thus the quality of RMS when \( z = f(z', z'', z''') = f(81.68, 88.1, 71.84) \) is equal to 78.59%.

In the following chapter the extensions of fuzzy operations will be discussed before we come into further discussion of the modification of the Takagi-Sugeno and Mamdani fuzzy control of the Resort Management System.
3 Extensions of Fuzzy Operations

In this chapter, a further discussion on fuzzy operations such as the t-norm and s-norm (also noted as t-conorm) as well as OWA will be introduced [Zimmermann, 2001; Rakus-Andersson, 2007; Dubois and Prade, 2000; Saaty, 1978; Yager, 1988 and 2007].

Differing from conventional operations fuzzy operations are strongly based on the minimum and maximum operation since the corresponding intersection and union are suggested to be interpreted as logical “AND” and “OR” respectively. Therefore some definitions and examples will be shown to demonstrate the process of the operations. Furthermore, a new t-norm [Buckley and Siler, 1998] will also be introduced. The extensive fuzzy operations are going to be used to modify the T-S model by shifting the α-value (also known as the truth value) from each implication where \( \alpha^i = \mu^i(x^0_1) \land \ldots \land \mu^i(x^0_n) \), and to modify the Mamdani model by adjusting the α-value from each rule where \( \alpha_i = \min_i (\mu_i(x), \mu_i(y)) \). Hence, the modification which is performed by changing the α-value, will shift the outputs for the computation of \( y = \frac{\sum i \alpha^i y_i}{\sum \alpha^i} \) in the T-S model and \( \mu_{con}^i(z) = \min_i (\alpha^i, \mu_i(z)) \) in the Mamdani model.

3.1 T-norms

Definition 3.1
Let A, B, C and D be four fuzzy sets from the same universe \( X \). A t-norm can be described as

\[
\text{t}(\mu_A(x), \mu_B(x)) = \mu_{A \land B}(x) = \min(\mu_A(x), \mu_B(x))
\]  

(3.1)

with \( t: [0,1] \times [0,1] \to [0,1] \), and satisfies the following properties:

1) \( t(0,0) = 0, \ t(\mu_A(x), 1) = \mu_A(x) \);
2) \( t(\mu_A(x), \mu_B(x)) \leq t(\mu_C(x), \mu_D(x)) \) if and only if \( \mu_A(x) \leq \mu_C(x) \) and \( \mu_B(x) \leq \mu_D(x) \);
3) \( t(\mu_A(x), \mu_B(x)) = t(\mu_B(x), \mu_A(x)) \);
4) \( t(\mu_A(x), t(\mu_B(x), \mu_C(x))) = t(t(\mu_A(x), \mu_B(x)), \mu_C(x)) \)

where 2) is the monotonicity law, 3) is the commutative law and 4) is the associative law and t-norms should satisfy these laws. Also, note that each t-norm can be treated as the operation on membership functions of fuzzy sets A and B in the definition of intersection.

A test of the properties above:

1) \( \min(0,0) = 0, \ \min(\mu_A(x), 1) = \mu_A(x), \text{true} \);
2) \( \min(0.3, 0.4) \leq \min(0.5, 0.6) \text{ if } 0.3 \leq 0.5 \text{ and } 0.4 \leq 0.6, \text{true} \);
3) \( \min(0.5, 0.8) = \min(0.8, 0.5), \text{true} \);
4) \( \min(0.2, \min(0.3, 0.4)) = \min(\min(0.2, 0.3), 0.4) = 0.2, \text{ true} \).

Fig. 3.1 shows a minimum t-norm.
Definition 3.2 The algebraic t-norm
The algebraic intersection $A \cdot B$ where $A$ and $B$ are both fuzzy sets and have the same universe $X$, has another t-norm

$$\mu_{AB}(x) = \mu_A(x) \cdot \mu_B(x)$$  \hspace{1cm} (3.2)

as the membership function. It is shown in Fig. 3.2.

Definition 3.3 The bounded t-norm
A and B are defined as fuzzy sets from the same universe $X$. The bounded intersection $A \bigcirc B$ has another t-norm

$$\mu_{A\bigcirc B}(x) = \max(0, \mu_A(x) + \mu_B(x) - 1)$$  \hspace{1cm} (3.3)

as the membership function. It is plotted in Fig. 3.3.

Definition 3.4 $\rho$- parameter t-norm
Define fuzzy sets $A$ and $B$ in the same universe $X$ and given $\mu_A(x) = a$ and $\mu_B(x) = b$. The correlation coefficient between $a$ and $b$ is defined as $\rho$ and the standard deviation of $\sigma_a(\sigma_b)$ denoted by $\delta$ is equal to the square root of $a(1-a)(b(1-b))$. Then $\mu_A(x) \cap \mu_B(x)$ is a new t-norm [Buckley and Siler, 1998] which is denoted as

$$T(a, b) = ab - (\rho)(\delta).$$  \hspace{1cm} (3.4)
where \(-1 \leq \rho \leq 1\) and \((a, b)\) is in a restricted domain \(\Omega\) being a subset of\([0,1] \times [0,1]\).

Let \(\theta_1(x, y) = (xy^2)/[(1-x) + xy^2]\) and \(\theta_2(x, y) = x/[x + (1-x)y^2]\) for \(0 \leq x \leq 1, -1 \leq y \leq 1\) and \((x, y) \neq (1,0)\) in \(\theta_1\), \((x, y) \neq (0,0)\) in \(\theta_2\).

Thereafter, the restricted domain \(\Omega\) is given by:

1) If \(\rho = 1\) then \(\Omega = \{(a, b)|a = b, 0 \leq a \leq 1\}\) and \(T(a, b) = a = b\).

2) If \(0 < \rho < 1\) then \(\Omega = \{(a, b)|0 \leq a \leq 1, \theta_1(a, \rho) \leq b \leq \theta_2(a, \rho)\} = \{(a, b)|0 \leq b \leq 1, \theta_1(b, \rho) \leq a \leq \theta_2(b, \rho)\}\).

3) If \(\rho = 0\) then \(\Omega = [0,1] \times [0,1]\) and \(T(a, b) = ab\).

4) If \(-1 < \rho < 0\) then \(\Omega = \{(a, b)|0 \leq a \leq 1, \theta_1(1-a, \rho) \leq b \leq \theta_2(1-a, \rho)\} = \{(a, b)|0 \leq b \leq 1, \theta_1(1-b, \rho) \leq a \leq \theta_2(1-b, \rho)\}\).

5) If \(\rho = -1\) then \(\Omega = \{(a, b)|b = 1-a, 0 \leq a \leq 1\}\) and \(T(a, b) = 0\).

Denote the complement of \(A\) as \(A^c\) so the new t-norm satisfies:

1) \(A \cap A = A, \rho \) is 1

2) \(A \cap \emptyset = \emptyset, \) any \(\rho\).

3) \(A \cap A^c = \emptyset, \rho \) is -1.

4) \(A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \) appropriate \(\rho\).

5) \(A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \) appropriate \(\rho\).

The t-norm \(T\) depends on three variables \(a, b\) and \(\rho\). The fuzzy sets determine the value of \(\rho\) by satisfying the constraints of the restricted domain \(\Omega\) with the above conditions and the membership functions decide the value of \(a\) and \(b\). Once the value of \(\rho\) is determined we use it for all \(x\) when working with fuzzy sets \(A\) and \(B\) but another \(\rho\) value for fuzzy sets \(C\) and \(D\).

**Example 3.1**

Let \(X = \{2,4,6,8,10\}\) be a set of course hours per week, fuzzy sets \(A = \) “learning effects of classwork in hours per week” and \(B = \) “learning effects of homework in hours per week”, and the correlation coefficient between \(A\) and \(B\) is \(\rho = -0.95\).

\[
A = \frac{0.4}{2} + \frac{0.6}{4} + \frac{0.8}{6} + \frac{1}{8} + \frac{1}{10},
\]
B = 0.3/2 + 0.5/4 + 0.7/6 + 0.9/8 + 1/10.

$A \cap B = 0.3/2 + 0.5/4 + 0.7/6 + 0.9/8 + 1/10$;

$A \cdot B = 0.12/2 + 0.3/4 + 0.56/6 + 0.9/8 + 1/10$;

$A \bigcirc B = 0/2 + 0.1/4 + 0.5/6 + 0.9/8 + 1/10$;

$T(A,B) = 0.33/2 + 0.53/4 + 0.73/6 + 0.9/8 + 1/10$.

### 3.2 S-norms

**Definition 3.5**

Let A, B, C and D be four fuzzy sets from the same universe X. An s-norm (t-conorm) can be described as

$$s(\mu_A(x), \mu_B(x)) = \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

with $s: [0,1] \times [0,1] \to [0,1]$, shown in Fig. 3.4, and satisfies the following properties:

1) $s(1,1) = 1$, $s(\mu_A(x), 0) = \mu_A(x)$;
2) $s(\mu_A(x), \mu_B(x)) \leq s(\mu_C(x), \mu_D(x))$ if and only if $\mu_A(x) \leq \mu_C(x)$ and $\mu_B(x) \leq \mu_D(x)$;
3) $s(\mu_A(x), \mu_B(x)) = s(\mu_B(x), \mu_A(x))$;
4) $s(\mu_A(x), s(\mu_B(x), \mu_C(x))) = s(s(\mu_A(x), \mu_B(x)), \mu_C(x))$

where 2) is the monotonicity law, 3) is the commutative law and 4) is the associative law and t-norms should satisfy these laws. Yet each function $s$ satisfying the conditions 1) to 4) can be accepted as a new operation of union, performed on membership functions.

A test of the properties above:

1) $\max(1,1) = 1$, $\max(\mu_A(x), 0) = \mu_A(x)$,true;
2) $\max(0.3,0.4) \leq \max(0.5,0.6)$ if $0.3 \leq 0.5$ and $0.4 \leq 0.6$,true;
3) $\max(0.5,0.8) = \max(0.8,0.5)$,true;
4) $\max(0.2, \max(0.3,0.4)) = \max(\max(0.2,0.3), 0.4) = 0.4$, true.

**Definition 3.6 The algebraic s-norm**

The algebraic union $A + B$ where A and B are both fuzzy sets and have the same universe X, has another s-norm

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

as the membership function. It is sketched in Fig. 3.5.
Definition 3.7 The bounded s-norm

A and B are defined as fuzzy sets from the same universe X. The bounded union $A \oplus B$ generates another s-norm

$$\mu_{A \oplus B}(x) = \min\left(1, \mu_A(x) + \mu_B(x)\right)$$

(3.7)

as the membership function.
Example 3.2
Let $X = \{2, 4, 6, 8, 10\}$ be a set of course hours per week, fuzzy sets $A =$ “learning effects of classwork in hours per week” and $B =$ “learning effects of homework in hours per week”. We have

$$
A = \frac{0.4}{2} + \frac{0.6}{4} + \frac{0.8}{6} + \frac{1}{8} + \frac{1}{10};
$$

$$
B = \frac{0.3}{2} + \frac{0.5}{4} + \frac{0.7}{6} + \frac{0.9}{8} + \frac{1}{10};
$$

$$
A \cup B = \frac{0.4}{2} + \frac{0.6}{4} + \frac{0.8}{6} + \frac{1}{8} + \frac{1}{10};
$$

$$
A + B = \frac{0.58}{2} + \frac{0.8}{4} + \frac{0.94}{6} + \frac{1}{8} + \frac{1}{10};
$$

$$
A \oplus B = \frac{0.7}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}.
$$

3.3 OWA

OWA operator is the notation of Ordered Weighted Averaging operator introduced by Ronald R. Yager [Yager, 1988]. It provides a class of mean type aggregation operators, such as maximum, arithmetic average, intermediate and minimum.

Let $A = [a_1, a_2, ..., a_n]$. An $n$-dimensional OWA operator can be denoted as a mapping $F: R^n \rightarrow R$ which includes an associated collection of weights $W = [w_1, ..., w_n]$ provided that each $w_i$ belongs to the unit interval. A total sum of $w_i$ is equal to 1. Then $F(A)$ can be obtained as

$$
F(A) = \sum_{i=1}^{n} w_i b_i
$$

where $b_i$ is the $i^{th}$ largest of $a_i$ in A.

If $x_1, x_2, ..., x_n$ are some estimates of the same quantity $x$, then OWA has a type

$$
f(x_1, x_2, ..., x_n) = a_0 + a_1 \cdot x_{(1)} + a_2 \cdot x_{(2)} + \cdots + a_n \cdot x_{(n)}
$$

where $a_0, a_1, ..., a_n$ are constants.

The values $x_{(1)}, x_{(2)}, ..., x_{(n)}$ are described in minimum and maximum sense as

$$
x_{(1)} = \min(x_1, x_2, ..., x_n);
$$

$$
x_{(2)} = \max(x_1, x_2, ..., x_n),
$$

where $x(i)$ is the minimum of all the values except the $i^{th}$, for instance,

$$
x(1) = \min(x_2, x_3, x_4, ..., x_n),
$$

$$
x(2) = \min(x_1, x_3, x_4, ..., x_n),
$$

$$
x(n) = \min(x_1, x_2, ..., x_{n-1});
$$
Example 3.3
Let \( X = \{x_1, x_2, x_3, x_4\} \) be a space of quality parameters, \( A = \{a_1, a_2, a_3\} \) a space of services. We denote \( A_i \) being sets of appreciation of service \( a_i \) on all quality parameters for \( i = 1, 2, 3 \).

\[
A_1 = \frac{0.3}{x_1} + \frac{0.6}{x_2} + \frac{0.8}{x_3} + \frac{1}{x_4},
\]

\[
A_2 = \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{0.9}{x_3} + \frac{1}{x_4},
\]

\[
A_3 = \frac{1}{x_1} + \frac{0.8}{x_2} + \frac{0.6}{x_3} + \frac{0.4}{x_4},
\]

and the weights of importance assigned to services \( a_i \), are denoted by \( w_i \) where \( \sum_{i=1}^{3} w_i = 1 \) and \( 0 \leq w_i \leq 1 \) and here we have \( w_1 = 0.3, w_2 = 0.2 \) and \( w_3 = 0.5 \).

The common appreciation of \( a_1, a_2, a_3 \) on \( x_1, x_2, x_3, x_4 \) is equal to OWA \( (A_1, A_2, A_3) \). Now we rearrange the sets of \( A_i \) by \( a_i \) with the membership degrees in the descending order in set \( A_i \), see table 3.1.

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( w_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.9</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6</td>
<td>0.8</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.6</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Therefore the membership degrees of OWA \( (A_1, A_2, A_3) \) is calculated by

\[
\mu_{OWA}(x) = \sum \mu_{of \ membership \ degrees \ in \ the \ descending \ order \ in \ set \ A_i(x)} \ast w_i. \tag{3.10}
\]

\[
\mu_{OWA}(x_1) = 1 \ast 0.3 + 0.3 \ast 0.2 + 0.2 \ast 0.5 = 0.46,
\]

\[
\mu_{OWA}(x_2) = 0.8 \ast 0.3 + 0.6 \ast 0.2 + 0.5 \ast 0.5 = 0.61,
\]

\[
\mu_{OWA}(x_3) = 0.9 \ast 0.3 + 0.8 \ast 0.2 + 0.6 \ast 0.5 = 0.73,
\]

\[
\mu_{OWA}(x_4) = 1 \ast 0.3 + 1 \ast 0.2 + 0.4 \ast 0.5 = 0.7, \text{ so}
\]

\[
\text{OWA} \ (A_1, A_2, A_3) = 0.46/x_1 + 0.61/x_2 + 0.73/x_3 + 0.7/x_4.
\]

However, the weights which are used in the algorithm can be adopted by Saaty’s method [Rakus-Andersson, 2007]. If comparing \( x_j \) to \( x_k \) as to their importance, we use definitions of importance values \( b_{jk} \) and \( b_{kj} \) as follows

1) \( b_{kj} = \frac{1}{b_{jk}} \).
2) If \( x_j \) is more important than \( x_k \), then \( b_{jk} \) gets assigned a number according to the following scheme:

<table>
<thead>
<tr>
<th>Intensity of importance expressed by the value of ( b_{jk} )</th>
<th>Definition of importance of ( x_j ) over ( x_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance of ( x_j ) and ( x_k )</td>
</tr>
<tr>
<td>3</td>
<td>Weak importance of ( x_j ) over ( x_k )</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance of ( x_j ) over ( x_k )</td>
</tr>
<tr>
<td>7</td>
<td>Demonstrated importance of ( x_j ) over ( x_k )</td>
</tr>
<tr>
<td>9</td>
<td>Absolute importance of ( x_j ) over ( x_k )</td>
</tr>
<tr>
<td>2,4,6,8</td>
<td>Intermediate values</td>
</tr>
</tbody>
</table>

Thereafter, the matrix \( B \) is constructed by the values of \( b_{jk} \) and the weights are decided as components of the eigen vector which corresponds to the largest in magnitude eigen value of the matrix \( B \). Nevertheless, in order to keep the sum of the weights equal to 1, we divide each weight by their sum if the sum of the derived eigen vector is not equal to 1. As a result, we can get the final weights with their sum equal to 1.

**Example 3.4**

From example 3.3, the weights show priorities of services to concern. Let us say we treat \( a_1 \) most important, and then comes \( a_2 \) and in the end \( a_3 \). Therefore we obtain the matrix \( B \) with the contents of importance as

\[
B = \begin{bmatrix}
    a_1 & a_2 & a_3 \\
    \frac{1}{3} & 1 & 3 \\
    \frac{1}{5} & \frac{1}{5} & 1 \\
\end{bmatrix}
\]

where the maximum eigen value is \( \lambda = 3.0385 \), and the corresponding eigen vector \( V = [0.9161,0.3715,0.1506] \), since the sum of the elements in the vector exceeds 1, so we divide each entry by their sum and get a new vector \( V_{new} = [0.64,0.26,0.1] \). Thus the weights are interpreted as the vector \( W = [0.64,0.26,0.1] \), which means \( w_1 = 0.64, w_2 = 0.26, w_3 = 0.1 \).

As with the introduction of extensions of fuzzy operations, a modification of the T-S and Mamdani Resort Management System will be demonstrated in the next chapter.
4 Modification of Takagi-Sugeno and Mamdani Model by Shifting Alpha-value

In this chapter, the t-norms including the minimum t-norm, the algebraic t-norm, the bounded t-norm, and the ρ-parameter t-norm, as well as OWA (Ordered Weighted Averaging) [Saaty, 1978; Yager, 1988; Buckley and Siler, 1998; Zimmermann, 2001; Rakus-Andersson, 2007] will be performed to modify the T-S and Mamdani model. In the T-S model, we shift the α-value in each implication where \( \alpha^i = \mu^i(x_1^i) \land \ldots \land \mu^i(x_n^i) \) by replacing the minimum operator with other t-norms and OWA operations. Thus the estimate of the final output where \( y = \frac{\sum a_i y_i}{\sum a_i} \) will be modified. Likewise, in the Mamdani model, we adjust the α-value in each rule when calculating \( \alpha_i = \min_i(\mu_i(x), \mu_i(y)) \) with other t-norms and OWA operations instead of minimum t-norm, consequently the results in the computation of each rule consequence \( \mu_i\text{consed}(z) = \min_i(\alpha_i, \mu_i(z)) \) will be changed. Afterwards, a modification of Takagi-Sugeno and Mamdani Resort Management System will be illustrated by performing the same technique to shift the alpha values.

4.1 Selection of Alpha-value Based on T-norms in T-S model

The conventional Takagi-Sugeno model calculates the alpha-value with only minimum fuzzy operation in the computation of the final outputs. However, we try to modify the model by shifting the alpha-value with other t-norms, such as the algebraic t-norm, the bounded t-norm and the ρ-parameter t-norm which have been presented in the previous chapter.

In example 1.13, we have calculated the alpha-value by using minimum operation. Now we complete the Table 1.3 by modifying the alpha-values. See Table 4.1. Afterwards the corresponding final outputs will be computed.

In Table 4.1, the terms “TV” stands for truth value which is also interpreted as α-value and the operators show different meanings in Table 4.2.

Hence, the final outputs of the implications are constructed by the formula

\[
y = \frac{\sum a_i y_i}{\sum a_i}, 1 \leq i \leq 3
\]  

and the results differ as with α-value computed by different t-norms, see table 4.3.

<table>
<thead>
<tr>
<th></th>
<th>TV(min)</th>
<th>TV(\cdot)</th>
<th>TV(\odot)</th>
<th>TV(\rho = -0.85)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8.82</td>
<td>9.26</td>
<td>9.82</td>
<td>8.91</td>
</tr>
</tbody>
</table>

Table 4.3: Final outputs with different α values
Table 4.1: Modification of T-S model by shifting alpha-value based on t-norms

<table>
<thead>
<tr>
<th>Implication</th>
<th>Premise</th>
<th>Consequence</th>
<th>TV(min)</th>
<th>TV(·)</th>
<th>TV(⊙)</th>
<th>TV(ρ = -0.55)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^1$</td>
<td>low$_1$</td>
<td>$y = 3 + 4$</td>
<td>0.3 $\land$ 0.45 = 0.3</td>
<td>0.3 $\land$ 0.45 = 0.3</td>
<td>0.3 $\land$ 0.45 = 0.3</td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
<tr>
<td></td>
<td>low$_2$</td>
<td>$= 7$</td>
<td></td>
<td></td>
<td></td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
<tr>
<td></td>
<td>0.3 $\triangleright$ 0.45</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
<tr>
<td>$R^2$</td>
<td>high$_1$</td>
<td>$y = 2 \cdot 3$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 6$</td>
<td></td>
<td></td>
<td></td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
<tr>
<td>$R^3$</td>
<td>high$_2$</td>
<td>$y = 3 \cdot 4$</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 12$</td>
<td></td>
<td></td>
<td></td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
<tr>
<td></td>
<td>$x_1 = 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
<tr>
<td></td>
<td>$x_2 = 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3 $\land$ 0.45 = 0.3</td>
</tr>
</tbody>
</table>

Table 4.2: Meanings for different operators

<table>
<thead>
<tr>
<th>Operator</th>
<th>Meaning</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV(min)</td>
<td>Truth value calculated by minimum t-norm</td>
<td>$\min(a, b) = a$, where $a &lt; b$</td>
</tr>
<tr>
<td>TV(·)</td>
<td>Truth value calculated by algebraic t-norm</td>
<td>$a \cdot b = a \cdot b$</td>
</tr>
<tr>
<td>TV(⊙)</td>
<td>Truth value calculated by bounded t-norm</td>
<td>$a \odot b = \max(0, a + b - 1)$</td>
</tr>
<tr>
<td>TV(ρ = -0.85)</td>
<td>Truth value calculated by ρ-parameter t-norm</td>
<td>$T(a, b) = a \cdot b - \rho \delta$, where $\rho$ is defined by fuzzy sets and $\delta = \sqrt{a(1 - a)b(1 - b)}$</td>
</tr>
</tbody>
</table>

4.2 Selection of Alpha-value by Means of OWA in T-S model

In 4.1 the method of modifying alpha-value by means of t-norms has been discussed. Nevertheless another technique with all the membership degrees involved will be performed to adjust the alpha-value in the total consequence and this technique is called OWA, abbreviation for Ordered Weighted Averaging. Furthermore the weights which are presented in the technique are constructed by implementing Saaty’s method [Rakus-Andersson, 2007] which has been discussed in 3.3.

Now we use OWA to modify the T-S model by implementing it in the example 1.13. First of all, we build the matrix B with the contents of importance as

$$B = \begin{bmatrix} x_1 & x_2 \\ x_1 & 1 \\ x_2 & \frac{1}{3} \end{bmatrix}.$$

And the largest eigen value of matrix B is 2 with the corresponding eigen vector $V = [0.9487,0.3162]$. As a result the weights can be constructed as $W = [0.75,0.25]$, which
means \( w_1 = 0.75, w_2 = 0.25 \). Then we obtain the alpha-value by accomplishing the formula \( \alpha = w_1 \mu(x_1) + w_2 \mu(x_2) \). See table 4.4.

Table 4.4: Modification of T-S model by shifting alpha-value by means of OWA

<table>
<thead>
<tr>
<th>Implication</th>
<th>Premise</th>
<th>Consequence</th>
<th>TV(min)</th>
<th>TV(OWA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^1 )</td>
<td>( \text{low}_1 ) ( \text{low}_2 ) ( 0.3 ) ( 0.45 ) ( 5 ) ( 2 ) ( 6 )</td>
<td>( y = 3 + 4 = 7 )</td>
<td>( 0.3 \land 0.45 = 0.3 )</td>
<td>( 0.3 \times 0.75 + 0.45 \times 0.25 = 0.34 )</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>( \text{high}_1 ) ( \text{high}_2 ) ( 0.2 ) ( 0.35 ) ( 1 ) ( 2 ) ( 5 )</td>
<td>( y = 2 \times 3 = 6 )</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( R^3 )</td>
<td>( x_1 = 3 ) ( x_2 = 4 )</td>
<td>( y = 3 \times 4 = 12 )</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Consequently, the final outputs are computed by the equations:
\[
y = \frac{0.3 \times 7 + 0.2 \times 6 + 0.35 \times 12}{0.3 + 0.2 + 0.35} = 8.82,
\]
where the alpha-value is calculated by minimum t-norm;
\[
y = \frac{0.34 \times 7 + 0.2 \times 6 + 0.35 \times 12}{0.34 + 0.2 + 0.35} = 8.74,
\]
where the alpha-value is calculated by OWA.

4.3 Modification of T-S and Mamdani Resort Management System Based on Adjustment of Alpha-value

In chapter 2 we have discussed the conventional T-S and Mamdani fuzzy control of Resort Management System (denoted as RMS) which contains two levels of control stage and four control actions which are control of catering, control of accommodation, control of leisure and finally control of RMS. Now we modify the RMS model by changing the alpha values both in the T-S model and Mamdani model.

4.3.1 Modify the control of catering

First let us discuss the modification in control of catering in the T-S model where we use the case in example 2.1. We have computed the final output with alpha-value calculated by minimum t-norm, and we adjust the alpha-value by means of other t-norms and OWA.

In example 2.1, we need to find the corresponding final output for the rank of catering with \( z' = f_1(x_1, y_1) = f_1(70, 88) \) where the scale of health is 70 and the scale of taste is 88.
\(\mu_{\text{medium}}(x_1 = 70) = 0.2,\)
\(\mu_{\text{high}}(x_1 = 70) = 0.8,\)
\(\mu_{\text{good}}(y_1 = 88) = 1.\)

We construct the \(\alpha\) values computed by other \(t\)-norms and OWA in the following table.

<table>
<thead>
<tr>
<th>(R^1)</th>
<th>TV(min)</th>
<th>TV((\odot))</th>
<th>TV((\odot))</th>
<th>TV((\rho))</th>
<th>TV(OWA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

where \(\rho\) is undefined since \(\delta = \sqrt{a(1 - a)b(1 - b)} = \sqrt{0.2 \times 0.8 \times 1 \times 0} = 0\) and the weights \(w_1 = 0.5, w_2 = 0.5\) since both the parameters health and taste are equally important. Thus the final outputs only vary when alpha-value is computed with OWA.

When the alpha-value is calculated by minimum \(t\)-norm, algebraic \(t\)-norm, bounded \(t\)-norm and \(\rho\)-parameter \(t\)-norm, the final output for the rank of catering is calculated by

\[
z' = f_1(70, 88) = \frac{0.2 \times 79.635 + 0.8 \times 82.197}{0.2 + 0.8} = 81.68;
\]

When the alpha-value is calculated by OWA, the final output for the rank of catering is calculated by

\[
z' = f_1(70, 88) = \frac{0.6 \times 79.635 + 0.9 \times 82.197}{0.6 + 0.9} = 81.17.
\]

**4.3.2 Modify the control of accommodation**

Then we modify the control of accommodation. Let us try to find the corresponding final output for the rank of accommodation when \(z'' = f_2(x_2, y_2) = f_2(55, 60)\) where the level of comfort is 55 and the level of price is 60.

\(x_2 = 55\) belongs to “low” and “high”;
\(y_2 = 60\) belongs to “moderate” and “expensive”.

So the rules can be obtained as

\(R^1\): If \(x_2\) is “low” and \(y_2\) is “moderate”,
then \(z''_{12} = 0.951 \times x_2 - 0.014 \times y_2 + 4.117 = 0.951 \times 55 - 0.014 \times 60 + 4.117 = 55.582;\)

\(R^2\): If \(x_2\) is “low” and \(y_2\) is “expensive”,
then \(z''_{13} = 0.983 \times x_2 - 0.067 \times y_2 + 5.376 = 0.983 \times 55 - 0.067 \times 60 + 5.376 = 55.421;\)

\(R^3\): If \(x_2\) is “high” and \(y_2\) is “moderate”,
then \(z''_{22} = 1.008 \times x_2 - 0.024 \times y_2 + 1.24 = 1.008 \times 55 - 0.024 \times 60 + 1.24 = 55.24;\)
$R^4$: If $x_2$ is “high” and $y_2$ is “expensive”,
then $z''_{23} = 1.02 \cdot x_2 - 0.053 \cdot y_2 + 2.205 = 1.02 \cdot 55 - 0.053 \cdot 60 + 2.205 = 55.125$.

$\mu_{\text{low}^*}(x_2 = 55) = 0.625$,
$\mu_{\text{high}^*}(x_2 = 55) = 0.875$,
$\mu_{\text{moderate}^*}(y_2 = 60) = 0.6$,
$\mu_{\text{expensive}^*}(y_2 = 60) = 0.4$.

The modified alpha values are constructed in the following table.

Table 4.6: Modified alpha values in the control of accommodation model

<table>
<thead>
<tr>
<th>$R^1$</th>
<th>TV(min)</th>
<th>TV(∀)</th>
<th>TV(⊙)</th>
<th>TV(ρ)</th>
<th>TV(OWA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.375</td>
<td>0.225</td>
<td>0.519</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.25</td>
<td>0.025</td>
<td>0.406</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.525</td>
<td>0.475</td>
<td>0.461</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.35</td>
<td>0.275</td>
<td>0.31</td>
<td>0.76</td>
<td></td>
</tr>
</tbody>
</table>

where

1) in $\rho$-parameter t-norm

$\delta_1 = \sqrt{0.625 \cdot 0.375 \cdot 0.6 \cdot 0.4} = 0.24$,
$\delta_2 = \sqrt{0.625 \cdot 0.375 \cdot 0.4 \cdot 0.6} = 0.24$,
$\delta_3 = \sqrt{0.875 \cdot 0.125 \cdot 0.6 \cdot 0.4} = 0.16$,
$\delta_4 = \sqrt{0.875 \cdot 0.125 \cdot 0.4 \cdot 0.6} = 0.16$ and based on the conditions
If $0 < \rho < 1$ then $\Omega = \{(a, b)|0 \leq a \leq 1, \theta_1(a, \rho) \leq b \leq \theta_2(a, \rho)\}$ and
If $-1 < \rho < 0$ then $\Omega = \{(a, b)|0 \leq a \leq 1, \theta_1(1 - a, \rho) \leq b \leq \theta_2(1 - a, \rho)\}$ so
$\rho_1 = -0.6$, check $0.625 \in [0, 1]$, $0.6 \in [0.18, 0.625]$, true;
$\rho_2 = -0.65$, check $0.625 \in [0, 1]$, $0.4 \in [0.2, 0.59]$, true;
$\rho_3 = 0.4$, check $0.875 \in [0, 1]$, $0.6 \in [0.53, 0.98]$, true;
$\rho_4 = 0.25$, check $0.875 \in [0, 1]$, $0.4 \in [0.3, 0.99]$, true;
And the alpha value is calculated by the formula
$\alpha_i = \mu_i(x_2) \cdot \mu_i(y_2) - \rho_i \delta_i$. (4.2)

2) in OWA

$B = \begin{bmatrix} 1 & 1 \hline 3 & 1 \end{bmatrix}$ so the weights $w_1 = 0.75$, $w_2 = 0.25$;
And the alpha value is calculated by the formula
$\alpha_i = (\mu_i(x_2) * w_1 + \mu_i(y_2) * w_2)$. (4.3)

Hence the final outputs are given in Table 4.7.

Table 4.7: Modified final outputs of control of accommodation

<table>
<thead>
<tr>
<th>$z''$</th>
<th>TV(min)</th>
<th>TV(∀)</th>
<th>TV(⊙)</th>
<th>TV(ρ = -0.85)</th>
<th>TV(OWA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55.36</td>
<td>55.33</td>
<td>55.29</td>
<td>55.32</td>
<td>55.32</td>
<td>55.32</td>
</tr>
</tbody>
</table>
4.3.3 Modify the control of leisure

We discuss the modification of the leisure control model by altering the case in example 2.3. We have computed the final output with alpha-value calculated by minimum t-norm, so now we adjust the alpha-value by means of other t-norms and OWA.

In example 2.3, we need to find the corresponding final output for the rank of leisure when \( z''' = f_3(x_3, y_3, s_3) = f_3(81, 70, 70) \) with which the grade of spa is 81, the grade of golf is 70 and the grade of sports is 70.

\[
\begin{align*}
\mu_{\text{good}}(x_3 = 81) &= 1, \\
\mu_{\text{excellent}}(y_3 = 70) &= 1, \\
\mu_{\text{inferior}}(s_3 = 70) &= 0.1, \\
\mu_{\text{superior}}(s_3 = 70) &= 0.9.
\end{align*}
\]

We construct the adjusted \( \alpha \) values in the following table.

**Table 4.8: Modified alpha values in the control of leisure model**

<table>
<thead>
<tr>
<th></th>
<th>TV(min)</th>
<th>TV((\odot))</th>
<th>TV((\bigcirc))</th>
<th>TV((\rho))</th>
<th>TV(OWA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^1 )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.91</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.99</td>
</tr>
</tbody>
</table>

where \( \rho \) is undefined since all \( \delta \) are equal to 0 in this case, and the weights \( w_1 = 0.64, w_2 = 0.26, w_3 = 0.1 \) which are obtained from the corresponding eigen vector of the largest eigen value of the importance matrix \( B = \begin{bmatrix} 1 & 3 & 5 \\ \frac{1}{3} & 1 & 3 \\ \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix} \). And \( \alpha_1 = 0.64 * 1 + 0.26 * 1 + 0.1 * 1 = 0.91 \) and \( \alpha_2 = 0.64 * 1 + 0.26 * 1 + 0.1 * 0.9 = 0.99 \), consequently the final outputs only change when \( \alpha \)-value is computed by OWA.

When the alpha-value is calculated by minimum t-norm, algebraic t-norm, bounded t-norm and \( \rho \)-parameter t-norm, the final output for the rank of leisure is calculated by

\[
z''' = (81, 70, 70) = \frac{0.1 * 71.622 + 0.9 * 71.86}{0.1 + 0.9} = 71.84,
\]

When the alpha-value is calculated by OWA, the final output for the rank of leisure is calculated by

\[
z''' = (81, 70, 70) = \frac{0.91 * 71.622 + 0.99 * 71.86}{0.91 + 0.99} = 71.75,
\]

4.3.4 Modify the control of RMS

Last but not least, we derive the modification of the second-level Mamdani RMS model by modifying the example 2.4. We have computed the total consequence with the alpha-value computed by minimum operation as discussed in 2.2, so now we shift this alpha-value during the
computation of the rule consequences where $\mu_i^{\text{conseq}}(z) = \min_i(\alpha_i, \mu_i(z))$ based on other t-norms and OWA.

We want to evaluate the value of the quality of RMS when $z = f(z', z'', z''') = f(81.68, 88.1, 71.84)$, and the rules are given as

$R^1$: If $z'$ is “remarkable”, $z''$ is “tremendous” and $z'''$ is “unsatisfactory”, then $z$ is “medium”;

$R^2$: If $z'$ is “remarkable”, $z''$ is “tremendous” and $z'''$ is “satisfactory”, then $z$ is “good”.

\[
\begin{align*}
\mu^{\text{remarkable}}(z') &= 81.68 = 1, \\
\mu^{\text{tremendous}}(z'') &= 88.1 = 1, \\
\mu^{\text{unsatisfactory}}(z''') &= 71.84 = \frac{-1}{5} \times 71.84 + \frac{3}{2} = 0.06, \\
\mu^{\text{satisfactory}}(z''') &= 71.84 = \frac{1}{50} \times 71.84 - \frac{1}{2} = 0.94.
\end{align*}
\]

Thus the alpha values calculated by different t-norms and OWA are given in Table 4.9.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\text{TV(min)}$</th>
<th>$\text{TV(\odot)}$</th>
<th>$\text{TV(\bigcirc)}$</th>
<th>$\text{TV(\rho)}$</th>
<th>$\text{TV(OWA)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.87</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.99</td>
</tr>
</tbody>
</table>

where $\rho$ is undefined since all $\delta$ are equal to 0 in this case, and the weights $w_1 = 0.43, w_2 = 0.43, w_3 = 0.14$, which are obtained from the eigen vector by dividing each entry by their sum, where the vector $V = [0.69, 0.69, 0.23]$ is from the corresponding largest eigen value of the importance matrix $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix}$. Hence the rule consequences only differ when alpha-value is computed by means of OWA.

When $\alpha_1 = 0.06$ is calculated by minimum operator and other t-norms, the rule-1 consequence is computed by

$$
\mu_{1(\text{t-norms})}^{\text{conseq}}(z) = \min(0.06, \mu^{\text{medium}}(z)).
$$

When $\alpha_1 = 0.87$ is calculated by OWA, the rule 1 consequence is computed by

$$
\mu_{1(\text{OWA})}^{\text{conseq}}(z) = \min(0.87, \mu^{\text{medium}}(z)).
$$

The rule-1 consequences are sketched in Fig. 4.1.

When $\alpha_2 = 0.94$ is calculated by minimum operator and other t-norms, the rule-2 consequence is computed by

$$
\mu_{2(\text{t-norms})}^{\text{conseq}}(z) = \min(0.94, \mu^{\text{good}}(z)).
$$
When $\alpha_2 = 0.99$ is calculated by OWA, the rule-2 consequence is computed by

$$\mu_{2(OWA)}(z) = \min(0.99, \mu_{\text{medium}}(z)).$$

The rule-2 consequences are plotted in Fig. 4.2.

When alpha-value is calculated by t-norms, a total consequence of the rules is given as a fuzzy set in $z$ with the membership function

$$\mu_{t-norms}(z) = \max(\mu_{1(t-norms)}(z), \mu_{2(t-norms)}(z)) = \begin{cases} \frac{1}{25} z - 1 & \text{for } 25 \leq z < 26.5, \\ 0.06 & \text{for } 26.5 \leq z < 51.5, \\ \frac{1}{25} z - 2 & \text{for } 51.5 \leq z < 73.5, \\ 0.94 & \text{for } 73.5 \leq z \leq 100. \end{cases}$$

The result is plotted in Fig. 4.3.
When alpha-value is calculated by OWA, a total consequence of the rules is given as a fuzzy set in \( z \) with the membership function:

\[
\mu_{OWA}^{\text{conseq}}(z) = \max \left( \mu_{1(OWA)}^{\text{conseq}}(z), \mu_{2(OWA)}^{\text{conseq}}(z) \right) = \begin{cases} 
\frac{1}{25}z - 1 & \text{for } 25 \leq z < 46.75, \\
0.87 & \text{for } 46.75 \leq z < 53.25, \\
-\frac{1}{25}z + 3 & \text{for } 53.25 \leq z < 62.5 \\
\frac{1}{25}z - 2 & \text{for } 62.5 \leq z < 74.75, \\
0.99 & \text{for } 74.75 \leq z \leq 100.
\end{cases}
\]

The result is plotted in Fig. 4.4.

Figure 4.4: Modified membership function of total consequence \( \mu_{OWA}^{\text{conseq}}(z) \)

Therefore, according to Center of Gravity, when \( \alpha_i \) is calculated by t-norms, the final value of the quality of RMS when \( z = f(z', z'', z''') = f(81.68, 88.1, 71.84) \) is computed by the following equation:

\[
z = f(81.68, 88.1, 71.84) = \int_{25}^{26.5} \frac{1}{25}z - 1 dz + \int_{25}^{51.5} 0.06dz + \int_{51.5}^{73.5} \frac{73.5 - 2}{25}dz + \int_{73.5}^{100} 0.94dz = 78.59\%;
\]

However, according to Center of Gravity, when \( \alpha_i \) is calculated by OWA, the final value of the quality of RMS when \( z = f(z', z'', z''') = f(81.68, 88.1, 71.84) \) is calculated by the following equation:

\[
z = f(81.68, 88.1, 71.84) = \int_{25}^{46.75} \frac{46.75 - 1}{25}dz + \int_{46.75}^{53.25} 0.87dz + \int_{53.25}^{62.5} \frac{62.5 - 3}{25}dz + \int_{62.5}^{74.75} \frac{74.75 - 2}{25}dz + \int_{74.75}^{100} 0.99dz = 69.04\%.
\]

Overall, by comparing the results which have been modified by shifting the alpha values both in T-S model and in Mamdani model, we can observe that there’s a strong tendency that the final value decreases as the \( \alpha \)-value reduces. Instead of only taking the extreme values when applying minimum fuzzy operator for the computation of \( \alpha \)-value, the algebraic t-norm, the bounded t-norm, the \( \rho \)-parameter t-norm and OWA can combine the entire premise to yield the more balanced \( \alpha \)-value although it shows that most of the replaced operations produce smaller alpha values, which, as a result, derive the smoother outputs. Nevertheless, the process of the modification shows that OWA can still generate the well-adjusted alpha values even in the case
of some membership degrees reaching the value of 1, while all the t-norms that have been discussed in this paper stretches the same $\alpha$-value just as applying the intersection operation.
5 Conclusion

By introducing basic fuzzy set theory, this paper titled “Takagi-Sugeno and Mamdani Fuzzy Control of a Resort Management System” presents the process of forming a Takagi-Sugeno and Mamdani fuzzy model for the application in the field of management. Combined T-S controller on the first-level management and Mamdani controller on the second-level management with the extensive fuzzy operations, the Resort Management System (RMS) is able to perform a well-balanced control and management for the evaluation of the quality of the resort service.

Regarding Takagi-Sugeno controller, the scheme of implication, reasoning and identification is demonstrated with the explanation of examples. As for Mamdani controller, the process of the fuzzification, rule base and defuzzification has been discussed. After the introduction of the conventional T-S controller which has been implemented in the first level of the Resort Management System and the conventional Mamdani controller applied in the second level of the RMS, a further discussion of extended fuzzy operations where a new t-norm is presented has been illustrated to modify the Takagi-Sugeno model by means of shifting the alpha-value from each implication $\text{R}^i$ where $\alpha^i = \mu^i(x_1^0) \land \ldots \land \mu^i(x_k^0)$ and to adjust the Mamdani model based on changing the $\alpha$-value from each rule $i$ where $\alpha_i = \min_i(\mu_i(x), \mu_i(y))$. Therefore the modified RMS is generated. Besides, the comparison of the t-norms and OWA (Ordered Weighted Averaging) when computing the alpha values has been stretched, in which it has been shown that OWA plays a vital role in the computation of alpha values.

Furthermore, it is necessary to point out that Takagi-Sugeno fuzzy controller is preferable to be used in the circumstance where the observation data has been obtained, otherwise, Mamdani fuzzy controller is suggested to be considered since the control process can still be performed even without operation data due to the specialist’s knowledge and suggestion.

However, the study of T-S and Mamdani controller and its applications still goes on and needs to be improved since there are yet problems left to be solved in spite of the achievement gained so far. The error should be taken into consideration when retrieving the consequence identification with regression method; a more stable algorithm is needed when performing the premise parameters identification; a more completed technique should be founded to obtain the $\rho$ values in $\rho$-parameter t-norm; an enhanced t-norm should be drawn to fit the boundary situations where some of the membership degrees reach the value of 1.
References