Utveckling och utvärdering av en modifierad punktmassemodell för beräkning av projektilbanor med hjälp av riktiga skjutdata

Development and Evaluation of a Modified Point-Mass Trajectory Model using Firing Data

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Abstract

In this degree project a modified point-mass trajectory model has been developed and evaluated using real firing data. The model has been simulated using the simulation framework FLAMES where external conditions such as weather and terrain data are taken into account. The data used in the evaluation consists of a series of firings performed by the Swedish Armed Forces in year 2001. The data includes the locations of the gun, the target and the impact areas as well as the measured weather data. The results show that the model correlates well with the real data in general, and lengthwise in particular where most of the errors remain within 60 meters.
Referat

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Chapter 1

Introduction

Ever since the invention of gunpowder in China in the 9th century, the use of artillery in warfare has had a significant role. Early cannons used by the Chinese army against the Mongols were designed to fire flames, along with a cannonball or a spear, at the enemy forces. This form of early artillery could bring down enemy soldiers or horses in a single blow and would bring fear in the enemy ranks [1].

Today, artillery has taken an important role in modern warfare because of its long range and accuracy. Modern artillery can fire its projectiles several kilometres at distant targets. In order to guarantee the precision of the projectiles over such long distances, engineers have developed tools to calculate the trajectories in order to anticipate the location of the impact. Some of these tools are computer programs that can be fed with the coordinates of the starting and impact location, the initial velocity, atmospheric data such as wind and temperature, and aerodynamic properties of the projectile. Based on this information the programs calculate the elevation and azimuth of the gun barrel needed to hit the target location. Programs like these are crucial for modern high technological artillery systems, and are also used as simulation software as part of the education of artillery students.

1.1 Background

At the Swedish National Defence College (Försvarshögskolan, FHS) there are courses in artillery science where the students study the trajectories of different projectiles and which forces that affect them during their flight. The students are supplied with literature concerning the theory of the ballistics as well as simulation software that can plot the path of the trajectory.

The simulation software currently being used at FHS correlates well with existing firing data but lacks some key features that FHS needs. First and foremost there is no source code attached with the software which precludes any further development and understanding. The lack of source code also makes it impossible to fully describe unintuitive results which may appear during simulation. Another problem with the current software is the inability to manually disable different forces such as gravity.
or drag in order to fully understand how different forces affect the trajectory.

1.2 Purpose

The purpose of the work described herein has been to develop a trajectory model for a projectile with respect to outer forces such as gravity, drag and the Coriolis effect. The model should be structured in a way that allows for forces to be individually turned on or off in order to make it possible to analyse their effects.
Chapter 2

Theory

Numerous books and articles concerning digital simulation and the exterior ballistics of a projectile have been written throughout the years. These articles cover the use of different numerical integration methods and how to simulate the motion of the projectile in different situations. In this chapter a summary of the current state of research in these fields will be discussed.

2.1 Numerical integration

In order to simulate the motion of an object a numerical integration method is needed. During each time step in the simulation all exterior forces are calculated and then transformed into accelerations. The accelerations are used in order to determine the current velocities which can then be used to anticipate the new position of the object. There exist multiple numerical integration methods that can be used in digital simulation. Depending on the nature of the simulation and the needed accuracy, different methods can be utilized.

2.1.1 Euler’s forward method

The most basic approach of numerical integration is Euler’s forward method which takes the current state, derivatives and time step into consideration and is expressed as

\[ y_{n+1} = y_n + hf(t_n, y_n), \] (2.1)

where \( y_{n+1} \) is the next value, \( y_n \) is the current value, \( h \) is the time step, and \( f(t_n, y_n) \) is the current derivative \([2, p. 58]\). Hence, based on the current value, time step and derivatives, Euler’s forward method anticipates the next value by adding the product of the current derivative and the time step to the current value \([3]\). For short simulations with a small time step, Euler’s forward method can be sufficient. For simulation of long-range trajectories, Euler’s forward method is not accurate enough and a higher order method must be used \([3, p. 13][4, p. 166]\).
2.1.2 Runge-Kutta

One of the higher order methods that are currently widely used in digital simulation is the fourth order Runge-Kutta method (RK4) [5, p. 1]. RK4 is an extension of Euler’s forward method but in order to maintain high accuracy it uses a weighted average of four points instead of one and can be expressed as

\[ y_{n+1} = y_n + \frac{1}{6} h (k_1 + 2k_2 + 2k_3 + k_4), \]

\[ k_1 = f(t_n, y_n), \]
\[ k_2 = f(t_n + \frac{1}{2} h, y_n + \frac{1}{2} k_1), \]
\[ k_3 = f(t_n + \frac{1}{2} h, y_n + \frac{1}{2} k_2), \]
\[ k_4 = f(t_n + h, y_n + h k_3), \]

where \( y_{n+1} \) is the next value, \( y_n \) is the current value, \( h \) is the time step, and \( k_1, k_2, k_3, k_4 \) are the derivatives in the four points. RK4 is well suited for digital simulation because of its high accuracy and ability to self-start, i.e., it produces \( y_{n+1} \) in terms of \( y_n \) only [3, p. 6][6, p. 3]. Since RK4 is a fourth order method its local truncation error is in the order of \( O(h^5) \) and the total accumulated error is in the order of \( O(h^4) \).

2.1.3 Adams-Bashforth

The Adams-Bashforth methods are explicit multi-step methods, meaning that they are based on multiple previous values. The fifth order of Adams-Bashforth uses four previous values and can be expressed as

\[ y_{n+1} = y_n + h f(t_n, y_n), \]
\[ y_{n+2} = y_{n+1} + h \left( \frac{3}{2} f(t_{n+1}, y_{n+1}) - \frac{1}{2} f(t_n, y_n) \right), \]
\[ y_{n+3} = y_{n+2} + h \left( \frac{23}{12} f(t_{n+2}, y_{n+2}) - \frac{4}{3} f(t_{n+1}, y_{n+1}) + \frac{5}{12} f(t_n, y_n) \right), \]
\[ y_{n+4} = y_{n+3} + h \left( \frac{55}{24} f(t_{n+3}, y_{n+3}) - \frac{59}{24} f(t_{n+2}, y_{n+2}) + \frac{37}{24} f(t_{n+1}, y_{n+1}) - \frac{3}{8} f(t_n, y_n) \right), \]
\[ y_{n+5} = y_{n+4} + h \left( \frac{1901}{720} f(t_{n+4}, y_{n+4}) - \frac{1387}{360} f(t_{n+3}, y_{n+3}) + \frac{109}{30} f(t_{n+2}, y_{n+2}) - \frac{637}{360} f(t_{n+1}, y_{n+1}) + \frac{251}{720} f(t_n, y_n) \right), \]

(2.3)
2.2 REFERENCE FRAMES

Figure 2.1. A figure depicting the global reference frame and the local reference frame. The axes of the global reference frame are labeled as $x, y, z$ and the axes of the local reference frame are labeled as $x', y', z'$.

where $y_{n+5}$ is the next value, $y_n, \ldots, y_{n+4}$ are the previous and current values, $h$ is the time step and $f(t_n, y_n), \ldots, f(t_{n+4}, y_{n+4})$ are the previous and current derivatives. By relying on previous values, the Adams-Bashforth methods are not able to self-start like RK4 meaning that a lower order method has to be used in the initial steps of the simulation.

2.2 Reference frames

In order to simulate the actual motion of an object based on the previous calculations some kind of reference frame needs to be set up. A basic version of a reference frame is the two-dimensional coordinate system with two perpendicular axes, for example $x$ and $y$. In this basic reference frame the $x$ axis could correspond to an object’s horizontal position and the $y$ axis to its vertical. In such a reference frame motion of the object is easily simulated, but only as long as the motion is directed along any of the axes. For simulation of a new force, such as wind, that is acting from a different angle and not only along the $x$ axis or $y$ axis, the two-dimensional frame will not be sufficient and a third axis is needed.

2.2.1 Global reference frame

The most commonly used global reference frame is the Earth-Centered-Reference (ECR) frame, or Earth-Centered-Inertial (ECI) frame, which acts as a global reference frame in which all objects reside [5, p. 1][7, p. 3]. The ECR frame has its $z$ axis extending through the north pole, its $x$ axis extending through the equatorial meridian and its $y$ axis extending through the equatorial east.
2.2.2 Local body reference frame

The local body frame is a frame with its origin placed in the centre of gravity of the local object. The x axis of this frame extends through the front of the object, the y axis extends through the right side of the object and the z axis extends through the bottom of the object [2, p. 115].

2.2.3 Transformation between frames

The orientation of an object with respect to the ECR is defined by three angles, called Euler angles [2, p. 119]. These three angles define the orientation of one frame relative to the other but in a non-commutative manner, meaning rotations in different orders will produce different results. The operation needed to transform between the two frames can be expressed as a single transformation matrix, that multiplied by the coordinates in one frame produces the coordinates in the other frame. This matrix is often referred to as a direction cosine matrix (DCM) [2, p. 120].

2.3 Exterior ballistics

For long range projectile trajectories it is essential to take forces that affect the projectile into consideration. The exterior ballistics account for all of the forces and moments that affect the projectile during flight after it has left the muzzle of the cannon. Depending on the type of projectile, the intended trajectory and the needed accuracy these forces will be more or less significant meaning that some can be ignored while others cannot.

Similar to the different levels of reference frames there are different ballistic models that can be used when simulating the trajectory. These models differ in their degrees of freedom, how the mass is considered to be distributed and which forces that are taken into account. Current software, that for example is used to generate firing tables, utilizes either a point-mass model with two degrees of freedom or a modified point-mass model with four degrees of freedom [8].

2.3.1 Point-mass model

One of the most basic models is a point-mass model with two degrees of freedom (2-DOF) [8]. In this model the mass of the projectile is considered to be situated in a single point and the projectile can only move along two axes, usually horizontally along the x axis and vertically along the z axis. The forces that are simulated in this model are gravity and aerodynamic drag. For projectiles that are considered to be flying with predominantly small yaws this model will act as a good approximation of its trajectory [4, p. 165].
2.3. EXTERIOR BALLISTICS

2.3.2 Six degrees of freedom model

The six degrees of freedom (6-DOF) model is an advanced trajectory model which can adequately describe a projectile’s position and orientation at any time [9, p. 270]. The foundation of the model was laid in the early path of the twentieth century by the English scientists Fowler, Gallop, Lock and Richmond and has been refined by several others during the century [4, p. 187][10].

In the updated version of the model proposed by McCoy two differential equations need to be solved: one describing the linear motion of the projectile and one its angular motion. Given valid and accurate data describing the properties of the projectile, the 6-DOF model is the most accurate model available and it also allows for the yawing and pitching motions to be observed. For situations where some of the properties are missing or are not accurate enough the 6-DOF can, however, prove to be troublesome [4, p. 187].

2.3.3 Modified point-mass model

The modified point-mass model (MPMM), which was proposed by Robert F. Lieske and Mary L. Reiter in 1966, is an alteration of the 6-DOF model but with four degrees instead of six [11]. Instead of calculating all of the angular motions as in the 6-DOF model they are approximated as a total yaw of repose in the MPMM. The yaw of repose acts as the fourth degree of freedom and is used to calculate the effect of the drift producing forces, lift force and Magnus force, in the calculations [12].

Even though the MPMM utilises an approximation for the yaw of repose, the resulting trajectories have been reported to agree closely to those of the 6-DOF model [4, p. 216]. The MPMM has been standardised by NATO in the STANAG 4355 agreement and is also one of the models that are used while developing firing tables [13][8].

The complete equations for the MPMM consist of one equation explaining the linear acceleration of the projectile, one explaining the deceleration of the projectile’s axial spin and one to approximate the yaw of repose [4, p. 214]. The equations are the following:

\[
\frac{d\vec{V}}{dt} = -\rho S C_D \frac{v \vec{V}}{2m} + \rho S C_L \frac{v^2 \alpha_R}{2m} + \frac{\rho S d C_{N_m}}{2m} \rho (\vec{V} \times \alpha_R) + g + \vec{\Lambda}, \quad (2.4)
\]

\[
\frac{dp}{dt} = \frac{\rho S d v^2}{2I_x} \rho C_{F_s} + \frac{\rho S d v^2}{2I_x} \delta F C_{L_s}, \quad (2.5)
\]

\[
\alpha_R = \frac{-2I_s \rho (V \times \frac{d\vec{V}}{dt})}{\rho S d v^4 C_{Ma}}, \quad (2.6)
\]

where \( \vec{V} \) is the vector velocity, \( \rho \) (kg/m\(^3\)) is the density of the surrounding air, \( v \) (m/s) is the velocity of the projectile with respect to the surrounding air, \( S \) (m\(^2\)) is the projectile’s reference area, \( d \) (m) is the calibre of the projectile, \( m \) (kg) is the...
weight of the projectile, \( p \ (\text{rad/s}) \) is the axial rotational velocity of the projectile, \( \delta_F \ (\text{rad}) \) is the fin cant angle, \( I_x \ (\text{kg m}^2) \) is the projectile’s axial moment of inertia, \( \vec{v} \) is the vector velocity of the projectile with respect to the surrounding air, \( \vec{g} \) is the vector acceleration due to gravity, \( \vec{\Lambda} \) is the vector acceleration due to the Coriolis effect and \( \alpha_{\text{p}} \) is the yaw of repose.

The aerodynamic coefficients needed for the MPMM are as follows: \( C_D \) is the drag coefficient, \( C_{L_\alpha} \) is the lift force coefficient, \( C_{N_\alpha} \) is the Magnus force coefficient, \( C_{I_p} \) is the spin damping coefficient, \( C_{t_b} \) is the rolling moment due to the fin cant and \( C_{M_\alpha} \) is the overturning moment.

### 2.3.4 Forces and effects

#### Gravity

The common means to calculate gravitational force is by using Newton’s law of universal gravitation which takes the altitude of the projectile into account [14, p. 3]. Difference in gravity because of the ellipsoid shape of the earth is however usually not taken into account and the earth is considered to be a perfect sphere [14, p. 2].

Newton’s law of universal gravitation can be expressed as

\[
F_G = G \frac{m_1 m_2}{r^2},
\]

where \( G \ (\text{m}^2/\text{kg}^2) \) is the gravitational constant, \( m_1 \ (\text{kg}) \) is the mass of the first object, \( m_2 \ (\text{kg}) \) is the mass of the second object and \( r \ (\text{m}) \) is the distance between the centres of the two objects.

#### Centrifugal force

The centrifugal force is an effect of the spinning of the earth resulting in a force that wants to push an object away from the earth’s rotational axis. The centrifugal force is not included in the equations for the MPMM and compared to gravity the centrifugal force is relatively weak. For long range trajectories, however, some authors claim that it must be considered [14, p. 2]. The centrifugal force depends on the rotational speed of the earth as well as the mass and speed of the object and can be expressed as

\[
F_{sp} = -m \vec{\tilde{\Omega}} \times (\vec{\tilde{\Omega}} \times \vec{r}_{rel}),
\]

where \( m \ (\text{kg}) \) is the mass of the object, \( \vec{\tilde{\Omega}} \ (\text{rad/s}) \) is the rotational velocity and \( \vec{r}_{rel} \ (\text{m}) \) is the location of the object.

#### Coriolis effect

The Coriolis effect is another effect that applies in a situation with rotating reference frames [2, p. 150]. Similar to the centrifugal force, the Coriolis effect is rather weak.
2.4 SIMULATION SOFTWARE

but will have large impact for long range trajectories [14, p. 2]. The Coriolis effect depends on the rotational speed of the earth as well as the mass and velocity of the object and can be expressed as

\[ F_C = -2m\vec{\Omega} \times \vec{v}_{rel}, \]

(2.9)

where \( m \) (kg) is the mass of the object, \( \vec{\Omega} \) (rad/s) is the rotational velocity and \( \vec{v}_{rel} \) (m/s) is the relative velocity of the object [15, p. 173].

2.3.5 Aerodynamic forces

All of the aerodynamic forces in the MPMM are dependent on the density of the surrounding air and the relative velocity of the projectile. Some of the articles uses fixed values for the atmospheric data such as density and pressure [6, p. 2]. Other articles propose using an atmospheric model where the data depends on the altitude which would give a more precise simulation [14, p. 3]. Two articles use a three-phase trajectory where the projectile is considered being in free flight during the mid-course and thus is not affected by drag [14, p. 2][5, p. 3].

2.4 Simulation software

A simulation software is a program that allows a user to observe an operation through simulation without actually performing that operation. Usually a set of mathematical formulas and conditions are defined with the goal of modelling a real phenomenon with as high accuracy as possible. In trajectory simulation and related fields such as flight simulation the benefits of using a simulation software are many, even though the initial development of the simulation can be both costly and time consuming. Once a simulation software is fully operational it can be used over and over again and simulate scenarios that might be too costly or even impossible to exercise in real life. Safety is another aspect that can be guaranteed through using a simulation software, e.g., in the field of flight simulation, where pilots can rehearse life-threatening manoeuvres without risking their lives or the aircraft [2, p. 9]. A good simulation software may also allow the user to alter environmental data and input parameters to test different scenarios. For trajectory simulation, simulation software provides a controlled environment in which different forces, moments and coefficients can be applied and where the effects of the environment, such as weather, can be thoroughly tested.

Most of the work reported on in the scientific literature do not use a specific simulation software but instead manually sets up the reference frames and use a programming language, such as MATLAB, for the simulation [6, p. 1][16]. By using this approach the developer can fully control the environment in which the simulation is run. It could however be rather difficult if the simulation should include a more realistic earth model with topographic data, real time atmospheric data or 3D animations.
Dedicated simulation software, such as FLAMES, can provide the user with a predefined reference system, the ability to use realistic earth terrain data and atmospheric models [17]. These predefined reference systems can include transformation functions to easily go from one reference system to the other. By using a simulation software the process of accounting for different types of terrains can be simplified and local atmospheric changes which could increase the correctness of the simulation can be used.

2.5 Related fields

There are other areas within the field of simulation where aspects can be directly or indirectly applied to exterior ballistics simulation (EBS). Perhaps the most obvious areas are flight and small arms simulation.

2.5.1 Flight simulation

The similarities between flight simulation and EBS is that they both occur during a long range and that the same exterior forces affect them. Pretty much the same calculations that are used in flight simulation can be applied for EBS except for the obvious aerodynamic differences. One thing that makes flight simulation differ from EBS within this project is the forces, such as thrust and turning moments, that are induced by the operator of the aircraft. Within the scope of this work the projectile is considered dead, meaning that no new forces or moments can be applied by humans during its trajectory.

2.5.2 Small arms simulation

Small arms simulation can also be of interest for artillery EBS since these simulations are more or less the same thing, but on different scales. Most small arms projectiles are spin stabilized which will introduce the same aerodynamic effects as with spin stabilized artillery projectiles. Small arms fire is however usually shot at a much shorter distance, meaning forces such as the centrifugal force or the Coriolis effect will not have the same effect.
Chapter 3

Methodology

Based on the theory presented in the previous chapter, a trajectory model has been developed. This chapter describes the methodology and calculations used while developing the model.

3.1 Trajectory model

The development of the trajectory model was arguably the most crucial part of this project. The approach that was used while developing the trajectory model was to start with implementing a simple model with the most basic forces applied and then gradually work towards a more complex model.

The initial version of the model was a point-mass model with three degrees of freedom. In this model the forces of gravity, drag with respect to wind, centrifugal and the Coriolis effect were simulated. This model was built as the foundation for developing a more complex model and all of the forces were individually modelled and evaluated. The force of gravity was evaluated by comparing a trajectory only affected by gravity to a computed parabola. The effect of drag and wind was tested by adding wind with different magnitude and direction and observing the path of the trajectory. Since both the Coriolis effect and the centrifugal force originates from the earth’s rotation they could both be tested by firing one trajectory at the equator and one at the north or south pole. At the equator, where the earth’s rotation is the greatest, the Coriolis effect gave the trajectory a drift directed to the east and the centrifugal force increased the altitude of the trajectory. At the poles, where the rotation is essentially zero, the Coriolis effect and the centrifugal force had no effect on the trajectories.

Once the initial model was finished the work with developing a more complex model began. The two models that could be used to achieve this was either the 6-DOF or the MPMM. Because of the high accuracy and the possibility to observe the precessing motion of the projectile the 6-DOF model was initially chosen. During the development of the 6-DOF model problems did however arise because of the lack of different aerodynamic coefficients, which the model highly relies on. In
order to tackle this problem the MPMM was chosen since it does not rely on the aerodynamic coefficients as heavily as the 6-DOF model. According to McCoy, the MPMM provides the same accuracy of the trajectory as the 6-DOF model while also being easier to implement and calculate [4, p. 213, p. 216]. One of the downsides of using the MPMM instead of the 6-DOF model is that the yawing and pitching motions cannot be observed since they are approximated as a total yaw of repose. Another thing to note when using the MPMM is that it assumes that the epicyclic motion is small all along the trajectory and is therefore more suited for stable and symmetric projectiles [4, p. 213].

3.1.1 Simulation framework

In order to simulate the model a simulation framework called FLAMES has been used. FLAMES (FLexible Analysis, Modeling, and Exercise System) is a family of commercial off-the-shelf software products that provide a framework for developing constructive simulations and interfaces between live, virtual, and constructive simulations. The reason for choosing FLAMES is that it provides both a global and a local reference frame, an earth model with terrain data and the possibility to add atmospheric data. FLAMES also provides two views, a top-down two-dimensional view and a three-dimensional view which allows for real-time observation of the simulation.

The global reference frame in FLAMES is defined as an ECR coordinate frame which keeps track of all objects by a location vector that points from the origin of the earth to the origin of the object. Each object also stores vectors defining their velocity, acceleration and rotation in the global reference frame. The local reference frame for an object is found by transforming its global vectors with a transformation matrix. The transformation matrix is provided by FLAMES and is calculated from the object’s location and rotation in the global reference frame. Centered in the ECR coordinate frame is a spherical globe which represents the earth with a given radius to mimic the size of the real earth. At the surface of the globe a terrain data map can be placed in order to simulate height and depth.

The atmospheric data can be added to FLAMES by utilising its atmospheric model. Out of the box FLAMES provides a standard model where pressure, temperature and wind are defined. These values can however be modified in order to simulate the weather conditions recorded by a weather station or balloon.

To simulate motion, FLAMES run a simulation loop at a user-given time step. During these time steps, every object in FLAMES is allowed to update its position in the global coordinate frame.

3.1.2 Numerical integration

The integration method chosen was the 5-step Adams-Bashforth method (5-sABM). The reason for using this method is because it provides a high order of accuracy while also being easy to implement. The 5-sABM has an error of order $h^5$ which is
3.1. TRAJECTORY MODEL

considered to be good enough for the purpose of this project. Since a fast simulation
time is not one of the priorities and a fast computer can be used while running the
simulation, a small time step can also be used in order to improve the accuracy
even more. By using FLAMES the previous results used in 5-sABM are also easily
stored as variables. Since 5-sABM is unable to self-start, the first four steps of
the simulation are integrated using Euler’s forward method. At the subsequent
iterations the previous values are then utilised by 5-sABM.

3.1.3 Calculations

To run the model in FLAMES, Equations 2.4, 2.5 and 2.6 were implemented. In
addition to the accelerations in Equation 2.4 the centrifugal force is also included.
For simplicity, all of the forces are defined in the local reference frame where they
can be expressed in a more intuitive manner. The complete calculations for the
motion of the projectile are defined below. For readability, pseudo code has been
used.

MPMM

The force of gravity in the current location is found by using Newton’s law of grav-
itation:

\[ \vec{F}_g = -\left(6.6742 \times 10^{-11} \frac{5.9736 \times 10^{24} m}{|\vec{L}_G|^2}\right) \hat{e}, \]  

where \( m \) (kg) is the weight of the projectile, \( \vec{L}_G \) is the location vector of the projectile
in the global reference system and \( \hat{e} \) is a normalised vector pointing from the centre
of the earth towards the origin of the projectile in the local reference system. Since
the location vector is defined in the global reference frame and FLAMES uses meters
as the vector units, the magnitude of the location vector corresponds to the distance
between the centre of the earth and the position of the projectile. Since \( \hat{e} \) is directed
towards the projectile the gravitational magnitude has been negated in order to
reverse the vector and instead have it pointing towards the earth.

The aerodynamic drag is calculated using the formula

\[ \vec{F}_D = -\left(\frac{C_D S \rho v}{2}\right) \vec{v}_L, \]  

where \( C_D \) is the drag force coefficient, \( S \) (m\(^2\)) is the reference area of the projectile
and is defined as \( S = \pi d^2/4 \) where \( d \) (m) is the calibre, \( \rho \) (kg/m\(^3\)) is the current
density, \( v \) (m/s) is the relative velocity and \( \vec{v}_L \) is the velocity vector in the local
reference frame. Since the force of drag only acts in the direction opposite of the
velocity, the drag is negated and multiplied to the current velocity vector in order
to achieve the correct direction.
CHAPTER 3. METHODOLOGY

The lift force, which in the MPMM will cause the drift of the projectile, is defined as

\[ \vec{F}_L = \rho S C_{L\alpha} \frac{v^2}{2} \vec{\alpha}_R, \]  
(3.3)

where \( C_{L\alpha} \) is the lift force coefficient and \( \vec{\alpha}_R \) is the yaw of repose. The lift force coefficient is an approximation based on the normal force coefficient and the drag coefficient and is calculated as

\[ C_{L\alpha} = C_{N\alpha} - C_{D0}, \]  
(3.4)

where \( C_{D0} \) is the zero yaw drag coefficient [4, p. 35].

For a right-hand spin stabilised projectile the yaw of repose will be directed to the right of the trajectory. Depending on the value of \( C_{L\alpha} \), this force will make the projectile depart from its straight trajectory.

The effect from the Magnus force is calculated as

\[ \vec{F}_N = \rho S d C_{Nps} \frac{v^2}{2} p (\vec{v}_L \times \vec{\alpha}_R), \]  
(3.5)

where \( d \) (m) is the calibre of the projectile and \( p \) (rad/s) is the spin of the projectile. As with the lift force, the Magnus force will produce a drift of the projectile. Because of the cross product the vector is however mostly located along the z axis which will produce a longer or shorter trajectory depending on the value of \( C_{Nps} \).

Both the Coriolis effect and the centrifugal force depend on the spinning velocity of the earth and are defined as

\[ \vec{F}_C = -2m \vec{\Omega}_L \times \vec{v}_L, \]  
(3.6)

\[ \vec{F}_{sp} = -m \vec{\Omega}_L \times (\vec{\Omega}_L \times \vec{L}_L), \]  
(3.7)

where \( m \) (kg) is the mass of the object, \( \vec{\Omega}_L \) (rad/s) is the rotational velocity of the earth in the local reference frame and \( \vec{L}_L \) (m) is the location of the projectile in the local reference frame.

By summing all of the forces a complete force vector can be defined:

\[ \vec{F}_{tot} = \vec{F}_g + \vec{F}_D + \vec{F}_L + \vec{F}_N + \vec{F}_C + \vec{F}_{sp}. \]  
(3.8)

By dividing \( \vec{F}_{tot} \) with the weight of the projectile the acceleration of the projectile in the local reference frame can be found:

\[ \vec{A}_L = \frac{\vec{F}_{tot}}{m}. \]  
(3.9)

The yaw of repose is calculated as

\[ \vec{\alpha}_R = \frac{-2 I_z p (\vec{v}_L \times \vec{A}_L)}{\rho S d v^4 C_{Ma}}, \]  
(3.10)
3.1. TRAJECTORY MODEL

Figure 3.1. Vector representation of the current yaw of repose, velocity and the forces affecting the projectile. All of the vectors have been normalised in order to see their directions more clearly.

\[ \vec{\alpha} \]
\[ \vec{F}_g \]
\[ \vec{F}_L \]
\[ \vec{F}_D \]
\[ \vec{F}_{sp} \]
\[ \vec{v} \]
\[ x \]
\[ y \]
\[ z \]

where \( I_x \) (kg \( m^2 \)) is the projectile’s axial moment of inertia and \( C_{Ma} \) is the overturning moment. This form of approximation of the yaw of repose is the classical form. Further simplifications can be made in order to remove the use of the acceleration vector in the approximation. In the calculations performed within this project the acceleration vector is first calculated using the yaw of repose from the previous time step and then the new yaw of repose is calculated using the new acceleration vector.

By using the acceleration vector the new velocity vector can now be determined through using the 5-sABM. Then the velocity vector is used to determine the new local positions of the projectile in the same way. The new local position vector is then transformed to its corresponding global vector and registered into FLAMES. The directions of all of the forces, the yaw of repose and the velocity are shown in Figure 3.1.

The only part of the MPMM which is not included in the calculation of the projectile’s linear motion is the spin acceleration. A spin stabilised projectile starts off with an axial spin that is approximated as

\[ p_0 = \frac{2\pi v_0}{n_d} \]  
(3.11)
where $p_0$ (rad/s) is the initial spin, $v_0$ (m/s) is the initial velocity, and $n$ (calibres/turn) is the rifling twist rate at the gun muzzle [4, p. 192]. The acceleration of the spin can then be calculated as

$$
\vec{A}_p = \frac{\rho S d^2 v}{2 I_x} - p C_{lp},
$$

(3.12)

where $C_{lp}$ is the spin damping coefficient of the projectile.

Using $\vec{A}_p$ the new value of $p$ is then integrated by using Euler’s forward method.

### Aerodynamic coefficients

The value of the different aerodynamic coefficients used in the calculations have all been approximated using splines. The reason for using splines is that they can provide a smooth approximation for the coefficients based on only a few measurements. All of the coefficients rely on the current Mach value and some also relies on the yaw of repose. For $C_D$ and $C_{lp}$ two linear splines have been made and are evaluated at every time step based on the current Mach number. For $C_{M_\alpha}$, $C_{N_{\alpha p}}$ and $C_{L_{\alpha}}$, which all rely on both the Mach number and the angle of attack, three surface splines were made and are also evaluated at each time step. The Mach number is calculated by dividing the total velocity by the speed of sound in the current location. The complete formula for determining the Mach number is

$$
Mach = \frac{v}{\sqrt{\frac{1.4 p}{\rho}}},
$$

(3.13)

where $v$ (m/s) is the current velocity, $p$ (Pa) is the pressure in the current location, $\rho$ (kg/m$^3$) is the current density and 1.4 is the adiabatic index of air [15, p. 25].

The angle of attack ($\alpha_t$) is based on the yaw of repose and is approximated by $\alpha_t = \arcsin |\vec{\alpha}|$ [4, p. 213].

### Atmospheric coefficients

As with the aerodynamic coefficients, the atmospheric coefficients have also been evaluated using splines. The atmospheric coefficients used in the above calculations are the density and the wind. According to the gas law, the density can be calculated from the surrounding temperature and pressure using the following equation:

$$
\rho = \frac{P}{T} \frac{28.9644}{8314.32},
$$

(3.14)

where $P$ is the pressure, 28.9644 (kg/mol) is the mean molecular weight of air, $T$ (K) is the temperature and 8314.32 (Nm/(kmolK)) is the gas constant [2, p. 99].

In order to calculate the density one spline for the temperature was made based on the data presented in the next section. Measurements of the pressure was however
3.2. DATA

Table 3.1. Constant values describing the projectile.

<table>
<thead>
<tr>
<th>d (m)</th>
<th>$I_x$ (kg m$^2$)</th>
<th>n (calibre/turn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.155</td>
<td>0.146</td>
<td>20.0</td>
</tr>
</tbody>
</table>

absent in the data so the standard atmosphere model (U.S. Standard Atmosphere 1976) in FLAMES was used to retrieve the pressure.

For the wind, two splines dependent on the current altitude was made. The first spline contains the wind speed and the second spline the direction of the wind.

3.2 Data

The data used for calculating the different coefficients of the projectile and in the evaluation stages was provided by ArtSS, which is the Swedish Army Artillery School. ArtSS is located in Boden in the northern part of Sweden and is responsible for the development of methods and equipment to be used by the artillery regiment as well as some parts of educating future artillery officers. The data contains information about the SGR77 which is an explosive projectile shell that is used in artillery systems such as Haubits 77A and Archer [19]. The data also contains detailed information from a series of firings that were performed by ArtSS in the middle of 2001. During this event multiple rounds of SGR77 were fired by a Haubits 77 from two locations aiming at two different targets. Included in the data are also atmospheric conditions such as wind speed, direction and temperature which was measured at the time. The data provided by ArtSS suits this project very well since it covers all of the needed coefficients while also providing trustworthy data to be used for the evaluation.

3.2.1 Projectile data

The data about the projectile that can be regarded as constant during the whole trajectory were the values for the calibre, axial moment of inertia and rifling twist rate. All of the constant values were delivered in their correct units and are listed in Table 3.1.

The linear and bilinear coefficients were measured at different Mach values and yaws of repose. Some of the coefficients can be expressed as the sum of two parts, one linear or bilinear and one cubic part. For example, the total drag coefficient is defined as

$$C_D = C_{D_0} + C_{D_\delta^2} \sin^2 \alpha_t,$$

where $C_{D_0}$ is the zero-yaw drag coefficient, $C_{D_\delta^2}$ is the yaw drag coefficient and $\alpha_t$ is the angle of attack. The data used in this project only consists of the linear or bilinear parts. Since the cubic part of the coefficients vary with the squared sine of
angle of attack the cubic part should be rather small, especially when using MPMM where the angle of attack is assumed to be small throughout the whole trajectory.

3.2.2 Atmospheric conditions

The atmospheric conditions contained in the data consist of six measurements that were made during the day of firing. Each measurement contains the wind direction, the wind speed and the temperature at eight different altitudes ranging from 0 to 4000 meters.

All of the angles in the data were expressed in the term “streck” which was previously used by the Swedish Armed Forces (SwAF) before converting to NATO mil [20]. One streck is $\frac{1}{6300}$ of a circle so in order to convert the angles into degrees the following formula was used:

$$\theta_{\text{deg}} = \frac{\theta_{\text{streck}} 360}{6300},$$

where $\theta_{\text{deg}}$ is the angle in degrees and $\theta_{\text{streck}}$ is the angle in streck.

Conversions

The wind direction contained in the data was defined as hundreds of streck and was first converted into degrees using Equation 3.16. Wind direction in FLAMES is defined as the direction in which the wind is blowing. Therefore the original wind direction, which was defined as the angle from which the wind is blowing, was directed in the opposite direction using the following equation

$$\text{wind}_\beta = \text{wind}_\alpha + 180,$$

where $\text{wind}_\beta (\text{deg})$ is the direction in which the wind is blowing and $\text{wind}_\alpha (\text{deg})$ is the direction from which the wind is blowing.

The altitude was written as hundreds of meters and was therefore multiplied by 100 to equal meters. The value of the altitude was also relative to where the measuring equipment, probably a weather balloon, was released, rather than the sea level as in FLAMES. 640 meters, which was the altitude of the weather station, was therefore added to the initial altitudes. In order to account for altitudes below 640 meters a new entry was manually added at sea level with the same atmospheric conditions as the one at 640 meters.

SwAF defines degrees below zero to be the absolute value of the temperature plus 50, e.g., minus five degrees is expressed as 55 and minus 15 degrees is expressed as 65 [21, p. 113]. All of the temperatures in the data above 50 were therefore reduced by 50 and negated.

Reference data

In Table 3.2 reference data after the conversions are shown. The altitude has been multiplied by a factor of 100 in order to express it in meters. The wind direction
3.3. EVALUATION

Table 3.2. Atmospheric record 1 after conversion.

<table>
<thead>
<tr>
<th>altitude (m)</th>
<th>$\text{wind}_\beta$ (deg)</th>
<th>wind vel (m/s)</th>
<th>temp (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>157.1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>640</td>
<td>157.1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>940</td>
<td>151.4</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>1140</td>
<td>180</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>1640</td>
<td>174.3</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>2140</td>
<td>151.4</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>2640</td>
<td>145.7</td>
<td>7</td>
<td>-3</td>
</tr>
<tr>
<td>3640</td>
<td>151.4</td>
<td>13</td>
<td>-7</td>
</tr>
<tr>
<td>4640</td>
<td>145.7</td>
<td>14</td>
<td>-13</td>
</tr>
</tbody>
</table>

has been expressed in angles and inverted, and the temperatures are now expressed with negative values.

3.2.3 Firing data

The firing data consisted of a series of test firings that was conducted in July 2001. In the seven series used in this project five shells were fired in each series from two different locations aiming at two different targets.

Conversions

The coordinates of the guns, targets and impact areas were all given in the RT90 coordinate system which was previously used for map production by the SwAF [22]. In order to fit the coordinates into the longitude/latitude system used by FLAMES all of the coordinates were translated using an on-line tool [23].

The initial rotational velocity of the projectile was expressed in rotations per second and was converted into radians per second.

Reference data

In Tables 3.3, 3.4, 3.5 and 3.6 reference data before and after conversion are shown. All of the coordinates have been translated from RT90 to latitude and longitude. The angles have been converted from radians to degrees and the rotational velocity is expressed in radians per second. The projectile weight has also been multiplied by a factor of 1000 in order to convert it from grams into kilograms which is used in FLAMES.

3.3 Evaluation

The goal of the evaluation was to see how well the developed model could predict the outcome of the firing data. In order to perform the evaluation a number of datasets
CHAPTER 3. METHODOLOGY

Table 3.3. Location of gun 1 and target 1 before conversion.

<table>
<thead>
<tr>
<th>Gun x</th>
<th>Gun y</th>
<th>Target x</th>
<th>Target y</th>
</tr>
</thead>
<tbody>
<tr>
<td>12487</td>
<td>94026</td>
<td>27360</td>
<td>91150</td>
</tr>
</tbody>
</table>

Table 3.4. First shot before conversion.

<table>
<thead>
<tr>
<th>$v_0$ (m/s)</th>
<th>$x$</th>
<th>$y$</th>
<th>$w$ (g)</th>
<th>az (str)</th>
<th>el (str)</th>
<th>$\omega$ (rot/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>577.4</td>
<td>27198</td>
<td>91169</td>
<td>42570</td>
<td>6082</td>
<td>648</td>
<td>186.3</td>
</tr>
</tbody>
</table>

Table 3.5. Location of gun 1 and target 1 after conversion.

<table>
<thead>
<tr>
<th>Gun lat</th>
<th>Gun lng</th>
<th>Target lat</th>
<th>Target lng</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.412888</td>
<td>13.820484</td>
<td>61.545472</td>
<td>13.757918</td>
</tr>
</tbody>
</table>

Table 3.6. First shot after conversion.

<table>
<thead>
<tr>
<th>$v_0$ (m/s)</th>
<th>lat</th>
<th>lng</th>
<th>$w$ (kg)</th>
<th>az (deg)</th>
<th>el (deg)</th>
<th>$\omega$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>577.4</td>
<td>61.544024</td>
<td>13.758371</td>
<td>42.570</td>
<td>13.76</td>
<td>37.03</td>
<td>1170.56</td>
</tr>
</tbody>
</table>

were created in FLAMES to mimic the data contained in the series of firings. Each dataset consists of five artillery units, five impact areas and a target. Each unit was loaded with their respective values for initial velocity, azimuth, elevation and the weight of the projectile and placed at the position of the gun. The impact areas and target were placed at their corresponding positions. The atmospheric conditions during the firings were also loaded into FLAMES using a custom atmosphere model.

The simulation was then run and the locations of the simulated impact areas were compared against the corresponding locations in the firing data. The comparison was done both visually and numerically. The visual part consists of graphs where the coordinates have been plotted. The numerical comparison consists of tables where the distances from the simulated impact areas to their corresponding real impact area as well as to the target are calculated. The distances are calculated using the Haversine formula which calculates the great-circle distance between two points from their longitude and latitudes [24].

For some of the rounds, values for initial velocity were absent and were therefore not used in the evaluation process. The last round from the third series was also left out because it was recorded to have landed a couple of kilometres off target, probably due to a malfunctioning projectile, extreme weather conditions or error in the data.
3.3. EVALUATION

Table 3.7. Time-step comparison.

<table>
<thead>
<tr>
<th>time-step (s)</th>
<th>distance (m)</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11653.76</td>
<td>4.73 %</td>
</tr>
<tr>
<td>0.1</td>
<td>12134.15</td>
<td>0.59 %</td>
</tr>
<tr>
<td>0.01</td>
<td>12199.09</td>
<td>0.05 %</td>
</tr>
<tr>
<td>0.001</td>
<td>12204.48</td>
<td>0.01 %</td>
</tr>
<tr>
<td>0.0001</td>
<td>12205.11</td>
<td>0.00 %</td>
</tr>
<tr>
<td>0.00001</td>
<td>12205.15</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 3.2. Plotted time-step comparison.

3.3.1 Time-step

In order to determine the time-step needed for the 5-sABM to converge a number of test shots with different time-steps were fired. The shots were fired using the projectile data of the first round in the sixth series. The results are displayed in Table 3.7 and Figure 3.2.

By assuming the resulting distance of the simulation with time-step 0.00001 to be the correct value, a relative error of the simulations with a longer time-step can be determined. Based on the results shown in Table 3.7 a time-step of 0.001 was used during the evaluation of the model. With a time-step of 0.001 the simulations can be run fairly quick using a regular laptop computer while still maintaining a good accuracy.
Chapter 4

Results

This chapter contains the results from the evaluation process. The presented graphs and tables consist of the results from each of the simulated series.

4.1 Shoot data comparison

After running each dataset in FLAMES the coordinates of the target, impact areas and simulated impact areas were compiled into a table. Below are the results from series 1, 3 and 6 presented as plotted values. These series were chosen because they give somewhat different results and were therefore interesting to observe. In Figures 4.1, 4.2 and 4.3 the coordinates of the simulated and real impact areas as well as the targets have been projected to x and y coordinates using a converter provided by MATLAB [25]. The same values are provided as tabular data in Tables A.1, A.2 and A.3. The \( \text{lat}_r, \text{lng}_r \) and \( \text{dist}_{rt} \) represent the latitude, the longitude and the distance from impact to target for the real values of each round, while \( \text{lat}_s, \text{lng}_s, \text{dist}_{sr}, \text{dist}_{st} \) represent the latitude, the longitude, the distance from impact to real impact and impact to target for the simulated rounds.

4.2 Error distributions

By plotting the distances between each simulated impact area and the real impact area as a histogram the distribution of the error can be observed. In Figures 4.4, 4.5 and 4.6 such distributions containing all valid rounds from the series are presented. Figure 4.4 contains the distances between each simulated impact area and the target. Figure 4.5 contains the distances between each real impact area and the target. Figure 4.6 contains the distances between each simulated impact area and its corresponding real impact area. Figures 4.7 and 4.8 show the distances between each simulated impact area and its corresponding real impact area along the x and y axes separately.
CHAPTER 4. RESULTS

Figure 4.1. Results from simulation of the first series.

Figure 4.2. Results from simulation of the third series.
### 4.2. ERROR DISTRIBUTIONS

**Figure 4.3.** Results from simulation of the sixth series.

![Graph showing real impact area, simulated impact area, and target](image)

**Figure 4.4.** Distances between the simulated impact areas and the target.

![Histogram showing number of impacts at different distances](image)
CHAPTER 4. RESULTS

Figure 4.5. Distances between the real impact areas and the target.

Figure 4.6. Distances between the simulated impact areas and the real impact areas.
4.2. ERROR DISTRIBUTIONS

Figure 4.7. Distances between the simulated impact areas and the real impact areas along the x axis.

Figure 4.8. Distances between the simulated impact areas and the real impact areas along the y axis.
Chapter 5

Discussion

The results presented in Chapter 4 both show the validity of the model and indicate areas for future improvement. In this chapter the results are discussed and future work is proposed.

5.1 The results

5.1.1 Shoot data comparison

Although similar, Figures 4.1, 4.2 and 4.3 represent three different outcomes of the simulations. In Figure 4.1 the simulated impact areas have a similar distribution to that of the real impact areas, while in Figures 4.2 and 4.3 the distributions of the real impact areas are scattered horizontally. The reason for the small horizontal deviation for the simulated impact areas is because the only difference between each shot is the initial velocity and weight. The differences will produce a small deviation sideways because of the Coriolis effect but also a greater deviation in terms of range. One of the reasons for the side drift of the real impact areas seen in Figures 4.2 and 4.3 could be changes in weather. For the simulated impact areas the same weather parameters were used for each shot which most likely was not the case for the real shots. Changes in wind speed, direction and temperature can greatly influence the trajectories of projectiles, especially over great distances [21, p. 118].

The three figures also show that the location of the simulated impact areas compared to the real impact areas differ from time to time. In Figure 4.1 the simulated shots landed about 150 meters to the left of the real impact areas while being pretty similar in range. In Figure 4.2 the sideways difference is about 100 meters while the range is almost identical. The results in Figure 4.3, on the other hand, instead shows a deviation to the right. These differences could also be explained by changes in weather or imperfections in the data. The accuracy of the coordinates in the firing data is unknown and errors may also have been introduced when converting the coordinates into FLAMES.
5.1.2 Error distributions

Figures 4.4, 4.5 and 4.6 are interesting to observe because they give a good overview of how the differences between the real and the simulated impact areas as well as the target are distributed. Figures 4.4 and 4.5 show that the maximum distance from the simulated and real impact areas to the target are about 250 meters and that most of the impact areas are located about 150 meters from the target.

Figure 4.6 shows that most of the distances between the simulated impact areas and the corresponding real impact areas are below 150 meters. The closer the simulated impact areas are to their corresponding real impact areas the better the model would be able to predict the outcome of the firing.

Figures 4.7 and 4.8 show how the errors between the simulated and real impact areas are distributed along the x and y axes. Most of the shots have an error of about 80-150 meters in side and about 20-50 meters in length. These results show that the model simulates the outcome rather well, especially when looking at the errors in length.

One thing to take into consideration is the precondition behind the distribution of the real impact areas. Since neither the elevation nor the azimuth changes throughout a series the author concluded that the purpose was to hit the same location with each shot. If the angles would have changed one could assume that the simulation tool being used during the firing was recalibrated based on the previous shot.

Another thing to take into consideration is the uncertainty of the initial velocity. For the simulation tool used during the firing the exact initial velocity was not known. As seen in Figure 5.1 the initial velocity could vary with about 3 meters per second which will effect the trajectory over long distances. For the simulations done in this project these exact velocities were known which would give the simulation the upper hand.

5.2 Future work

During the work on this project a couple of areas where improvements can be made have been found. These areas include improvements to the simulation model as well as the data and how the data is used.

5.2.1 Randomness

As indicated by the distributions of initial velocity and projectile weight in Figures 5.1 and 5.2, these values vary by some degree. One improvement to the simulation model could be to calculate the initial velocity and weight based on these distributions instead of using fixed values, which would result in a more realistic simulation. The same reasoning can probably be applied when it comes to weather data as well. As seen in Table 3.2 the wind speed and direction were measured at eight different altitudes before the firing. These measurements are most likely very exact but local
5.2. FUTURE WORK

Figure 5.1. Distribution of initial velocity in the sixth series.

Figure 5.2. Distribution of the projectile weight in all series.
variations in weather may occur between these measurements. By adding slight variations in direction and speed the resulting simulation may be more realistic and a scattered distribution similar to the real impact areas could be achieved.

For introducing randomness it is important to notice that, e.g., initial velocity and projectile weight are not independent variables. It will therefore be important to develop a probabilistic model using, e.g., a Bayesian network [26], in order to take the dependencies between the random variables into account.

### 5.2.2 Data accuracy

Even though the data provided by ArtSS and used in this project was of high quality, improvements could probably be made. For example, the weather data could be measured at multiple locations along the intended trajectory and at several altitudes. By measuring at different locations along the trajectory a more accurate model with local deviations would be achieved. A greater number of altitudes would also improve the exactness of the splines that were used in this project. Figures 5.3 and 5.4 represent the wind speed and direction where the dots are values measured at different altitudes and the curve is a spline fitted to those dots. As seen in the figures the splines are fitted to move along the measured values without running through the exact values but rather providing an averaged approximation. By using additional measurements these splines could be improved to more precisely fit the values and thus give a more realistic representation.
5.2. FUTURE WORK

Figure 5.4. Wind direction spline.
Chapter 6

Conclusions

Based on the results presented in this degree project the author concludes that the MPMM is a good model to use for trajectory simulation. The evaluation shows that the model, when simulated with realistic weather and terrain conditions, correlates well with real firing data. Some differences between the simulated and the real impact areas are present and could be explained by changes in weather conditions or imperfections in the data.

Using a simulation software such as FLAMES has proven to be helpful during the development, simulation and evaluation of the model. By providing a controlled environment where things such as atmospheric conditions and terrain data are included the author has been able to fully focus on the development of the model. This has also proven useful during the simulation and evaluation since it allows for more realistic scenarios to be simulated. By adding the appropriate terrain and weather data that was present during the real firings a realistic simulation environment could be achieved, which was crucial for the evaluation process.

Depending on the future use of the simulation model the MPMM can be more or less sufficient. The intended use of the model at FHS is for educational purposes where a good correlation with real firing data and high modularity is important. The MPMM allows for different forces to be individually turned on and off while still being fairly easy to implement. By only utilizing four degrees of freedom all of the external forces are however not present. For future development of a model where all aerodynamic coefficients are available a 6-DOF could be used which allows for all external forces and moments as well as the epicyclic and pitching motion to be observed.
Bibliography


Appendix A

Results appendix

Table A.1. Tabular results from simulation of the first series.

<table>
<thead>
<tr>
<th>lat_r</th>
<th>lng_r</th>
<th>dist_{rt} (m)</th>
<th>lat_s</th>
<th>lng_s</th>
<th>dist_{sr} (m)</th>
<th>dist_{st} (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.544024</td>
<td>13.758371</td>
<td>162.8</td>
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<td>13.755603</td>
<td>160.4</td>
<td>155.8</td>
</tr>
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Table A.2. Tabular results from simulation of the third series.

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<th>dist_{rt} (m)</th>
<th>lat_s</th>
<th>lng_s</th>
<th>dist_{sr} (m)</th>
<th>dist_{st} (m)</th>
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</thead>
<tbody>
<tr>
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<td>13.758371</td>
<td>162.8</td>
<td>61.544608</td>
<td>13.755603</td>
<td>160.4</td>
<td>155.8</td>
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<td>102.8</td>
<td>61.544985</td>
<td>13.755403</td>
<td>147.7</td>
<td>143.8</td>
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<td>147.2</td>
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Table A.3. Tabular results from simulation of the sixth series.

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<th>lng_s</th>
<th>dist_{sr} (m)</th>
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</thead>
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