Design of power supply system in DC electrified transit railways - Influence of the high voltage network

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Abstract

Urban rail systems such as subways and trams transport millions of people every day, offering a high level of service. Most of these systems are fed by direct current (DC). The design of the power supply network of DC electrified transit railways is of great importance and requires the use of simulation models. The power supply network is composed of a high voltage network (fed with AC) linked to a traction network (fed with DC) by traction substations. Many simulation models ignore the high voltage network in the design process whereas it has a significant influence on the results.

A Newton-Raphson algorithm is implemented to solve the AC load flow in the high voltage network, and coupled to the existing simulation software Symphonie.

Three different high voltage network architectures are simulated, and the simulation results are analyzed. The results show that the voltage drop at the AC side of traction substations and the load sharing between them varies significantly from one architecture to another. In particular, when several traction substations are connected to the same high voltage loop, voltage drops can be significant for some traction substations.

In conclusion, the design of the power supply network of DC electrified transit railways requires the simulation of the high voltage network when several substations are connected to the same high voltage loop.

Key words: railway simulation, electric traction, AC/DC load flow, high voltage network
Sammanfattning


I detta arbete har en belastningsfördelningsalgoritm baserad på Newton-Raphsons metod implementerats för att skapa en bättre modellering av högspänningsdelen i den befintliga programvaran Symphonie.

Tre olika högspänningsstrukturer har bearbetats och resultatet visar att spänningsfall och belastningsfördelning varierar mellan de olika strukturerna. Speciellt när flera transformatorstationer är kopplade till samma högspänningssystem, kan spänningsfall vara påtagligt för vissa transformatorstationer.

Sammanfattningsvis måste utformningen av högspänningsystemet till likströmsförsörjda järnvägar utformas med hjälp av anpassade matematiska modeller som tar hänsyn till placering av transformatorstationer liksom högspänningsnätets kapacitet.
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Introduction

Nowadays, almost all metropolitan areas are equipped with subway systems or trams. Subway systems are commonly referred to as mass rapid transit systems (MRT), whereas trams are usually called light transit systems. They have been implemented to solve two problems: the congestion due to cars and the pollution, also due to cars. Urban rail systems usually offer a very good capacity, compared to other modes, and very good performances.

Up to now, electricity has always been the main type of energy for urban rail systems\(^1\) because it is the only energy which has all the qualities required to operate such systems. Indeed the pollution is very limited in the area where the energy is consumed. Vibrations and noise due to electric motors transforming electricity into mechanical energy are also limited. In case of underground systems, electricity is a good solution because the primary energy does not need to be carried by the train, thus avoiding serious safety issues that would be raised with diesel trains. Electric motors are also a lot more energy efficient and less noisy than diesel motors. Without electricity urban rail systems would be unable to offer such a high level of service, in terms of capacity, performance, availability and quality. This shows that electricity is a major component of every urban rail system.

The characteristics of urban rail system operation (frequent stops, short headway, short dwell times) and environment (tunnels, shared infrastructure) calls for specific solutions regarding the power supply system. The power supply system must provide a high commercial speed (i.e. short travel times), a reasonable energy consumption, a good reliability (no interruption of service in case of failures in the power supply system) and high safety standards.

The components of the power supply system are numerous, the electricity flowing from bulk substations to the motors of the train through many elements such as rectifier substation, the catenary or the third rail, the pantograph, etc. With so many elements, a lot of issues can be raised.

The work carried out in this master thesis focuses on the design of the AC network of DC electrified transit railways. A simulation model has been built to study the influence of the architecture of the AC network on the sizing of the power supply system.

The 1\(^{st}\) part of this report provides the basic principles of electric traction and a review of some existing simulation models for railway systems. The 2\(^{nd}\) part introduces the simulation model built for the purpose of the study. In the 3\(^{rd}\) and 4\(^{th}\) parts, the simulation results are presented and then analyzed and discussed.

\(^1\)The first line of Paris Subway system, build in 1900, was already powered with electricity
Chapter 1

Context and state of the art

1.1 Electric traction in DC electrified transit systems

1.1.1 Railway dynamics

In this section, the equations used to calculate the motion of the train are introduced. This motion is dependent on the track geometry and the traction equipment characteristics. These equations are relatively simple which does not mean that railway dynamics is a simple subject. It just means that simple models are well suited to the study of running times and power consumption. Subjects such as ride stability, ride comfort, wear or derailment require more advanced models, that can be found in [10].

Equation of motion

The motion of the train can be described by the one-dimensional Newton’s equation. Indeed the train can be modeled as a point following the track.

\[
\sum_{i=1}^{n} F_i = m^* a
\]  

(1.1)

where \( F_i \) are the different forces acting on the train, \( m^* \) is the mass of the train, corrected to take into account the inertia of the rotating mass \( (m^* = \xi m) \) and \( a \) is the acceleration of the train along the track.
The different forces acting on the train can be split in two categories:

- $F_{in}$: Forces produced by the train, positive in traction, negative in braking
- $F_{ex}$: Forces mainly against train motion

The equation 1.1 can therefore be written:

$$F_{in} - F_{ex} = ma \quad (1.2)$$

The force opposing train movement can be written:

$$F_{ex} = F_r + F_{gr} + F_c \quad (1.3)$$

where $F_r$ is the train resistance (resistance to forward motion), $F_{gr}$ is the force due to gradients, and $F_c$ is the resistance due to the curves.

$F_r$ is always modeled as follows:

$$F_r = A + Bv + Cv^2 \quad (1.4)$$

The coefficient $A$ is related to the axle load, the coefficient $B$ takes into account the quality of the track and the stability of the train, while the coefficient $C$ accounts for the aerodynamic resistance. The part $A + Bv$ is generally referred to as the rolling resistance, while $Cv^2$ is the aerodynamic resistance. Several formulas exist to derive $A$, $B$ and $C$, many of them can be found in [1]. The main parameters are the mass of the train, the number of axles and the geometric characteristics of the train (front size area for example). The Davis formula is widely used in railway:

$$F_r = 6.4 \cdot m + 130 \cdot n + 0.14 \cdot m \cdot V_t + \beta[0.046 + 0.0065(N - 1)] \cdot A \cdot V_t^2 \quad (1.5)$$

with:

- $V_t$ speed of the train in $km/h$
- $m$ the static mass of the train
- $n$ the number of axles
- $N$ the number of cars
- $A$ the front section area of the train
- $\beta$ a coefficient which is equal to 1 if the train is outside, and 2 or 3 if in tunnels
The gradient force is simply:

\[ F_{gr} = m g \sin(\alpha) \]  

(1.6)

where \( \alpha \) is the gradient angle (Fig. 1.1). The gradient is usually defined by the tangent of this angle in \( mm/m \) or \( \% \). For gradients below 120 \( mm/m \), that is to say in almost every cases in railways, it is common to assume that the sinus and the tangent are similar. The gradient resistance is therefore given by:

\[ F_{gr} = i \times 10^{-3} m g \]  

(1.7)

where \( i \) is the gradient in \( mm/m \) or \( \% \). By convention, \( i \) is positive if the train is going up, and negative if the train is going down.

A train running in a curve is subjected to an additional rolling resistance, due to the friction between the wheels and the rail. This friction phenomenon is quite complicated and depends very much on the type of the bogies, and the shape of the rails. For travel time calculation purposes there is no need to derive complicated models. The resisting force produced by the curve is modeled by the following equation:

\[ F_c = \frac{k_e}{r} \times 10^{-3} m g \]  

(1.8)

where \( k_e \) (\( m \)) is a coefficient depending on the track gauge (Table 1.1) and \( r \) is the radius of the curve. The tighter the curve is, the higher is the resisting force. This equation is empirically derived and is widely adopted when performing running time and power consumption calculations.

### Table 1.1: Track gauge coefficient for curve resistance (Source [1])

<table>
<thead>
<tr>
<th>( k_e ) [( m )]</th>
<th>Track gauge [( mm )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>1435</td>
</tr>
<tr>
<td>530</td>
<td>1000</td>
</tr>
<tr>
<td>400</td>
<td>750</td>
</tr>
<tr>
<td>325</td>
<td>600</td>
</tr>
</tbody>
</table>

#### Speed limitation in curves

In curves the train is exposed to the centrifugal acceleration. This centrifugal acceleration is responsible for a track plane acceleration which can be unacceptable from the passenger comfort point of view. Therefore this track plane acceleration must be compensated. This is done by elevating one of the two rails. The height difference between the two rails is called the cant.

For a curve radius \( R \), a speed \( v \) and a distance between the two rails \( 2b_0 \), there is an equilibrium cant \( h_{eq} \) for which the track plane acceleration is zero:

\[ h_{eq} = \frac{2b_0}{g} \times \frac{v^2}{R} \]  

(1.9)

The equilibrium cant can not be achieved in all cases (different trains have different speeds !). Therefore each train has to adapt its speed to the curve radius and the real cant \( h_t \). To have no track plane acceleration the speed must be:

\[ v = \sqrt{\frac{R g h_t}{2b_0}} \]  

(1.10)
Usually the requirement on the resulting track plane acceleration is to be lower than 1 \( m/s^2 \). A common value in metro projects is 0.883 \( m/s^2 \).

**Tractive force and speed**

The tractive effort that can be produced by the engine(s) of the train is speed-dependent. It is always represented in a force-vs-speed diagram (1.2). The tractive effort is limited by:

- the maximum torque
- the maximum power

In operation, the tractive effort vs speed diagram can also be limited by the adhesion and the comfort. The tractive effort vs speed diagram represented here doesn’t represent the performances of the traction motor(s), but the tractive effort that is developed in operation.

![Figure 1.2: Tractive effort vs speed diagram of a suburban train](image)

The horizontal part of curve correspond to the range of speed in which the tractive effort is limited either by the maximum torque, the adhesion or the maximum acceleration allowed for comfort reasons.

The next part of the curve correspond to maximum power of the traction motor(s). The product \( Z(v) \times v \) remains constant (\( Power = Z(v) \times v \)), and thus the curve has the shape of the function \( x \mapsto \frac{1}{x} \).

Sometimes force-vs-speed diagram presents an additional range of speed in which the tractive effort \( Z(v) \) decreases proportionally to \( \frac{1}{v^2} \).

Each rolling stock has its own force-vs-speed diagram, this diagram is provided the rolling stock manufacturer.
This diagram is given for traction and braking. The shape of the diagram representing the braking effort is quite different from the traction curve because the braking effort is the result of both mechanical and electrical braking.

A particular rolling stock has two different tractive effort vs speed diagrams:

- a continuous diagram. It represents the effort that the train is able to develop in continuous operation. This diagram is used when calculating speed profiles of metro trains. Indeed the motors of metro trains don’t have much time to cool down because accelerations and braking phases follow each other rapidly.

- a diagram representing the maximum tractive effort. The maximum tractive effort can be used for example during the acceleration phase of a high-speed train, but cannot be kept for long periods (in a long uphill for example) because the motor would otherwise overheat.

**Adhesion**

The force developed by the traction motor is transmitted by the wheel-rail contact. The transmitted force is limited by adhesion. The maximal force that can be transmitted depends on the adhesive weight \( F_{ne} \) and the adhesion coefficient \( \mu \):

\[
F_{\text{max}} = \mu F_{ne}
\]

The adhesive weight for a loco with \( n \) powered axle is given by:

\[
F_{ne} = \frac{m}{n} g \cos(\alpha)
\]

For gradients higher than 90 \(^{\circ}\), adhesion can not be used (unless the train is equipped with cogwheel drives), therefore it is considered that \( \tan(\alpha) < 90 \(^{\circ}\) \), and the approximation \( \cos(\alpha) \approx 1 \) can be made. From eq. 1.11 and 1.12 it comes:

\[
F_{\text{max}} = \mu \frac{m}{n} g
\]

In 1943 Curtius and Kniffler derived an adhesion curve for a dry rail as a function of the vehicle speed (fig. 1.3).

The adhesion coefficient follows the equation:

\[
\mu = 0.161 + \frac{7.5}{3.6v + 44}
\]

with \( v \) the speed in \( \text{km/h} \).

Actually the adhesion depends on many other parameters such as the sliding speed, moisture on the rails, load transfers (neglected in the previous formulas). The trains can achieve adhesion coefficient higher than 0.3, however for safety considerations adhesion coefficient is always assumed to be lower than 0.25 (this is important when calculating safety distances for example).

In DC electrified transit railways (mainly metros and trams), the maximal acceleration is not determined with respect to adhesion but to comfort. Nowadays traction control system can achieve accelerations higher than 1.5 \( m/s^2 \) but for comfort purposes the acceleration is usually limited to approximately 1.1 \( m/s^2 \).
The following table displays the adhesion coefficient between different surfaces (used by vehicles in public transportation).

<table>
<thead>
<tr>
<th>Surfaces</th>
<th>Adhesion coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel/Steel (Metro, Tram, conventional railway systems)</td>
<td>0.3</td>
</tr>
<tr>
<td>Rubber tyre/Asphalt (Trolleybus)</td>
<td>0.55</td>
</tr>
<tr>
<td>Rubber tyre/Concrete (Metro and Trolleybus)</td>
<td>0.62</td>
</tr>
<tr>
<td>Rubber tyre/Steel (Metro)</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Resolution of the equation of motion

Let’s rewrite the equation of motion:

\[
\begin{align*}
    m^* a &= Z(v) - (A + B v + C v^2) + i(s) 10^{-3} m g + \frac{k_v}{r(s)} 10^{-3} m g \\
    a &= \frac{dv}{dt} \\
    v &= \frac{ds}{dt}
\end{align*}
\]  

(1.15)

where \( s \) is the curvilinear abscissa.
The equation of motion (eq. 1.1) is now transformed into a non-linear differential equation. It calls for time-step integration.

1.1.2 Power supply system

The power is distributed to the train through the substations (connected to the distributor network), the high voltage network, the rectifier substations and the catenary or third rail.

Choice criteria

The characteristics of the power supply system have to be defined in order to fulfill the requirements of railway operation:

- safety
- high level of service
- adapted to environmental constraints

Safety is mainly related to the risk of fire and electrification hazard for people. This explains for example while trams are not fed with a third rail lying on the ground but with an overhead system.

To ensure a good quality of service, the power system must supply the trains with a voltage as stable as possible. It has also to be designed to cope with failures. For example, the loss of a substation should have only little influence on the operation, the headway and the running time should not be affected.

The electrification system must also be compatible with the other components of the railway system. The rolling stock is of course compatible, but when a new fleet is introduced, the ability of the infrastructure to feed the trains has to be checked (current collection, electromagnetic compatibility, power levels, voltage levels). The compatibility with communication and signaling system is also of high importance and interference phenomena have to be investigated.

Electrification system and power distribution

Different electrical supply systems are used all over the world. The reasons for so many different systems are mainly historical. In the beginning of electric traction, electric motors where somehow limited, they could not operate in any range of frequency and voltage. Nowadays a lot of old systems are still used because it’s very expensive to change an electrification system and because each of them has its advantages. More information about the history of electrification systems can be found in [1].

The main electrification systems currently used are:

- 15 kV AC 16 2/3 Hz, used in Sweden and Germany for example
- 25 kV AC 50 Hz, used in France for example
- 2x25 kV AC 50 Hz, used in France on high speed lines
- 750 V DC
• 1500 V DC
• 3000 V DC

In subway systems, which are generally in tunnels, the electrification system is very often a 750 V DC system (sometimes 600 V DC). With such a system, a third rail is used which limits the diameter of the tunnel. The rolling stock can also be lighter because of a reduced electrical equipment, it means that the train can carry more passengers.

1500 V DC and 3000 V DC are not often used in urban railways, they are typical electrification systems for suburban railways and intercity lines.

AC systems come with higher voltage. Their main advantages is that traction substations can be located further from each other, because voltage drop in the catenary is lower (because the current flowing through the catenary is lower), and the catenary is much cheaper than with DC systems. The drawback is that traction substations need more space (that’s why it’s very difficult to build them in urban areas), and electrification hazards are more significant.

It must be noted that a lot of trains are able to run with different systems. It happens a lot when trains cross the border between two countries with different electrification systems. It’s also the case in France, where the electrification system of suburban lines is a mix of 1500 V DC and 25 kV AC 50 Hz.

The distribution of electric power from the converter station to the trains can be made in two ways:

• 3rd rail system, where a conductor rail is located between the two rails
• Overhead system, where an overhead wire is used as electric conductor. This system with wire conductors is called the catenary system.

Because of potential electrical hazards, the voltage level of third rail is usually lower than 750 V. Voltage is low, the current is high, which means that losses are quite significant. Therefore the traction substations must be close to each other. The main advantage of third rail systems is that they are cheap (compared to an overhead catenary system).

For speeds higher than 100 km/h, the third rail is more difficult to use, leading to the use of overhead systems.

Different types of overhead catenary systems exist, depending on the speed of the train and the voltage level. An overhead catenary system is composed of one or several wire conductors hung over the track. One of them is the contact wire, it is in contact with the train’s current collector, called the pantograph. In order for this contact to be maintained, the contact wire should have an even elasticity distribution and therefore has to be hold by a messenger wire and droppers.

Figure 1.4 represents different types of catenaries.
Typical suburban railways (RER in Paris, Pendeltåg in Stockholm) are fed with catenaries. In case of 1500 V DC, the catenary is quite complex and heavy because the current is high, this leads to the use of a special catenary, called compound, which has two contact wires. In case of AC (15 kV or higher), the catenary is simpler.

Another kind of catenary exists: the rigid catenary (Fig 1.5). It is a rigid conductor hung over the track. Its main advantage over a classic catenary is that it requires less space and is therefore very convenient in tunnels. Moreover, it often has a large equivalent copper section, allowing high currents. A rigid catenary is mainly used in metros.
The power is distributed to the third rail or the catenary by the substations. In a substation the voltage from the public grid (or a high voltage network) is transformed to the supply voltage of the trains (that is to say the one of the electrification systems mentioned above). In AC electrified railways, traction substations are basically composed of one or several transformers. In DC electrified railways, traction substations are made of one or several transformers and one or several rectifiers, therefore traction substations for DC are also referred to as rectifier substations.

**Description of DC electrified systems**

The power is distributed to the train through the bulk substations (connected to the distributor network), the high voltage network, the rectifier (or traction) substations and the catenary or third rail. Figure 1.6 represents the power supply system of a metro/tram line.

![Figure 1.6: Cartoon of the power supply system of a DC electrified transit railway](image)
High voltage network

The high voltage network connects the traction substations to the public grid. Several architectures exist. The goal of this project is to analyze the influence of the architecture of the high voltage network on the sizing of the power supply system. Three different high voltage infrastructures are presented and studied in this project.

The voltage level of the high voltage network is usually in the range of 15 kV to 22 kV.

Traction substations

Traction substations connect the AC high voltage network and the DC traction network. A traction substation is composed of a transformer, and a rectifier. The two catenaries (or third rails) are connected to the traction substation in parallel as represented in Figure 1.7.

In DC electrified transit railways, traction substation powers are usually in the range of 1.5 MW to 5.5 MW.

Traction network

The traction network brings the power to the pantograph. The current flows through the catenary or third rail and back through the rails.

The following simple example (Figure 1.8) explains the basic theory of DC traction.
In the example only one train is running. The power required at its pantograph is $P$. The line is entirely supplied by one rectifier substation. The rectifier substation is modeled by a voltage source with an internal resistance $r$. $U_0$ is the no-load output voltage of rectifier substation. The current $I$ flows through the overhead line (or third rail), the train, and returns to the rectifier substation by the rails. The resistance of the whole circuit $R$ (excluding the internal resistance of the rectifier substation) is the sum of the overhead line/third rail resistance and the resistance of the two rails in parallel.

The train voltage is simply:

$$U_{\text{train}} = \frac{P}{I} \quad (1.16)$$

Ohm’s law gives:

$$U_0 = rI + RI + \frac{P}{I} \quad (1.17)$$

Equation 1.17 can be easily solved for $I$. With several trains, several conductors and several rectifier substations, the principle remains the same, but it is impossible to solve the equation by hand.

1.2 Simulation models

Progress in computer hardware and software have enabled engineers to develop simulation models very useful when calculating train movement or for dimensioning of the power supply network. Simulation softwares were first used in the seventies. Nowadays operational and traction studies are almost always carried out with the help of a simulation software. At the beginning computers were only used to perform complicated calculations (such as matrix manipulation), but later integrated softwares, simulating the whole railway system, have been developed [5].

1.2.1 The need for models

Equation 1.1 presented in 1.1.1 is relatively simple, so one could ask why simulation models are needed. But when parameters such as changing speed limitations, tunnels (affecting the
aerodynamic resistance) or changing rate of electric braking as a function of speed come into play, then it becomes difficult to perform fast hand calculations.

The need for simulation models is even greater when it comes to power network solutions because several trains must be considered together, and that they ”affect” each other by exchanging energy for example. In order to design a power supply network, many alternatives have to be considered, without the simulation models it would be impossible to converge toward the optimal network.

Many interactions exist between train movement, traction drives and power supply. These interactions can be described precisely but they are so numerous that they can’t be well apprehended without the use of computers.

1.2.2 General structure of simulation models

The general structure of simulation models has been studied in [5] and is summarized in this subsection. A railway simulation software usually has two simulators: the train motion simulator and the traction power simulator.

Train motion

Calculating train motion means calculating the speed profile of a train between two points, according to track geometry (curves, gradients), environment (tunnels, wind), speed limitations, and sometimes signaling system. This calculation is position dependent, it is therefore important to arrange the input data related to track geometry and speed restrictions in a smart way. It is common to provide the value of the parameters such curve radius, gradient for every meter or 10 meters and to interpolate between these points.

Track representation varies from one software to another. Some softwares, like OpenTrack, represents the railway network with links and nodes. Nodes are typically stations or signal points along the track. Links represent parts of the track and are described with curve radii, gradients, etc. Some other softwares represent the track meter by meter. The stations are placed along the line thanks to a kilometric point.

The train motion is calculated over several intervals (of 1 s for example). This is called a time-based approach. Another approach, used by softwares taking the signaling system into account, is event-based.

Traction power supply

There is a very wide range of possibilities when designing the power supply system of a railway. It means that for engineers there are a lot of alternatives to study. This calls for time-efficient algorithms to solve the power supply network equations. The system of equations is not linear, which calls for iterative techniques.

As for the train motion calculation, the solving of the power supply network is time-based. It means that the network is solved at each time instant. The topology of the network is constantly changing because the trains are running on it. For urban transit railways, the top speed is rarely higher than 120 km/h. At this speed a train covers 33 m in one second. This distance is short
compared to the characteristic size of the network, which means that a time step of one second is a totally acceptable value to run the simulation.

The first step is to determine the state of the electrical objects along the line: which trains are consuming power, which trains are regenerating power, which substations are giving power, are there any substations out of service?

Then the load flow (or power flow) is calculated. There are two major iterative techniques used in railway simulations:

- the Gauss-Seidel method, known to be easy to implement but with rather poor convergence
- the Newton-Raphson methods, more difficult to implement but providing faster convergence

The particularity of load flow calculation in railways is that the network may contain both an AC and a DC part. The solving of the network can be either sequential (the AC and DC part are alternatively solved until the final solution is obtained), or unified (the AC and DC part are solved simultaneously).

Whatever iterative technique (Gauss-Seidel or Newton-Raphson) and the AC/DC solving technique (sequential or unified), a matrix of the traction power network needs to be derived. This matrix is basically a representation of the nodes and the links in the network. In electrical networks, the nodes are seldom connected to more than 3 other nodes. It means that the matrix contains a lot of zeros. This calls for an efficient ordering of the matrix in order to speed up the calculations. Nowadays softwares like Matlab already have efficient ordering algorithms which make the work easier for the developers.

Up to now, the link between the train movement simulator and the traction power network simulator hasn’t been tackled. One of the outputs of the train motion simulator is the power required by the train for each time step. This power is the input to the traction power network simulator. The thing is that the performances of traction motors decrease with reduced pantograph voltage, this reduction being due to voltage drop in the overhead line or the third rail. It means that the pantograph voltage has an influence on the power developed by the train. If the voltage drop is limited then the two simulators can work uncoupled (i.e. the train movement is entirely calculated and then the network is solved), but if voltage drop are too important, both the train movement and the power network solutions may be false. In that case, the two simulators must be coupled: at time snapshot $t$, the train motion over the period $\Delta t$ is calculated from the train state at $t$ and the pantograph voltage between $t - \Delta t$ and $t$, then the traction power network is solved for time period between $t$ and $t + \Delta t$ with the train power as input, the output is a new pantograph voltage which is used for the next time step.
CHAPTER 1. CONTEXT AND STATE OF THE ART

Other features

Some softwares provide additional features:

- Traction drive modeling
- Signaling system
- Advanced graphical representation/ User friendly interface
- Import and export tools

Traction drive modeling usually comes down to the diagram tractive effort vs. train speed. At the early engineering stages, it is not necessary and usually not possible to go further in the modeling of the traction drives, because the rolling stock is not yet precisely defined. However, train operators running simulations with existing rolling stock may be interested in a detailed modeling of the traction drives.

A few simulation softwares take into account the signaling system. This kind of tool is essential for the operators, because they need to know if the timetables they plan may be achieved in reality. It’s also a very useful tool to study the effects of incidents (train failure, signal failure, track failure).

User-friendly interfaces can make the work considerably faster and easier for engineers. Track, train and software edition may take a very long time if no efforts have been put on the user interface.

Some softwares can import files from track design programs such as railML. This is very useful to avoid spending several days on track descriptions for example. The ability to quickly export results for post processing (in Excel for example) is also very convenient.

1.2.3 Existing softwares

There are several railway simulation softwares. However only a few of them are commercially available. Most of the simulation softwares are developed by railway engineering companies and rail consultancies. Table 1.3 presents a few softwares (some of them might not exist anymore):
Table 1.3: A few simulation softwares

<table>
<thead>
<tr>
<th>Software</th>
<th>Commercially available</th>
<th>Developed by</th>
</tr>
</thead>
<tbody>
<tr>
<td>OpenTrack (and Open Power Net)</td>
<td>Yes</td>
<td>ETH (Technical university of Zurich)</td>
</tr>
<tr>
<td>Railsim</td>
<td>Yes</td>
<td>University of Hannover</td>
</tr>
<tr>
<td>Esmeralda</td>
<td>Yes</td>
<td>SNCF (the French railway operator)</td>
</tr>
<tr>
<td>ELBAS-SINANET</td>
<td>Yes</td>
<td>ELBAS GmbH</td>
</tr>
<tr>
<td>ENS (Electric Network Simulator)</td>
<td>Not anymore</td>
<td>?</td>
</tr>
<tr>
<td>EMM (Energy Management Model)</td>
<td>Not anymore</td>
<td>?</td>
</tr>
<tr>
<td>Marcadet</td>
<td>No</td>
<td>RATP (the operator of the Paris subway)</td>
</tr>
<tr>
<td>Simalim</td>
<td>No</td>
<td>EGIS Rail (a French rail consultancy)</td>
</tr>
<tr>
<td>DLTA</td>
<td>No</td>
<td>Artelia (a French rail consultancy)</td>
</tr>
</tbody>
</table>

Many universities also develop their own software.

A general comment that can be made about existing simulation softwares is that most of them are not user-friendly at all.

### 1.2.4 Validation of simulation models

The validation of a simulation model must be the first thing to do after the development of the model. However there are some validation techniques that are not easy to carry out with railway simulation models. For example it is generally very difficult to validate a model through an experiment. The reason is that it would require a railway track, a rolling stock, and the train should run exactly like planned in the timetable used in the simulation. It’s possible with existing lines, for which data from several years exist. However getting access to the data may be much more difficult.

The goal of simulation softwares is to size an electrical infrastructure, therefore it is commonly accepted that a software is validated if the infrastructure built on the basis of the simulation softwares, actually gives satisfactory results in operation.

A first validation step is to run a simulation on a very simple case (one train, very simple power network) that can be solved "by hand".

To ensure the quality of railway simulation models. The results of three simulations (one for a commuter train, one for a high-speed train and one for a freight train) done with a validated software are provided for comparison with the software to validate. If the software gives results within a certain percentage of the reference simulation, then the model is validated.
1.3 Load flow analysis at AC level

It is a common practice not to simulate the AC high voltage network when designing the power supply system of a DC electrified transit railway.

However simulations of the DC traction network show that the results are very sensitive to the input voltage of traction substations which depends very much on the high voltage network and its infrastructure.

The goal of this project is to analyze the influence of the high voltage network infrastructure on the power supply system. This analysis will also determine for which kind of high voltage infrastructure it is necessary to model and simulate the high voltage network and for which kind of high voltage infrastructure a simulation at DC level is good enough.
Chapter 2

Method

2.1 Different architectures

2.2 Model of the AC distribution network

2.2.1 Bulk substations

Bulk substations are represented as perfect voltage sources. This representation would be more accurate if an impedance representing the impedance of the bulk substation and the impedance of the network above the bulk substation, was added to the voltage source. This improvement will be implemented later on.

\[ V \]

Figure 2.1: Model of a bulk substation

2.2.2 Lines

The AC distribution network is a three-phase network. In the power flow calculations, the AC supplies are considered to be balanced and purely sinusoidal. Therefore a single-phase equivalent circuit can be used.

Given the length of the AC lines (usually below 20 km), and the voltage level (between 15 kV and 22 kV), the lines can be modeled as a short line with a resistance and an inductance. The capacitive part of the impedance can be ignored. Each piece of line is thus modeled according to Figure 2.2.
The lines are characterized by a resistivity (in $\Omega/m$) and an inductance per unit length (in $H/m$). Complex representation is used to solve the power flow. The line between nodes $i$ and $k$ is represented in Figure 2.3.

The impedance of the line is given by:

$$z_{ik} = r_{ik} + jx_{ik}$$  \hspace{1cm} (2.1)

where $r_{ik}$ is the resistance and the $x_{ik}$ the reactance.

In the load flow analysis, the admittance is used rather than the impedance. The admittance $y_{ik}$ is given by:

$$y_{ik} = \frac{1}{z_{ik}} = g_{ik} + jh_{ik}$$  \hspace{1cm} (2.2)

with

$$g_{ik} = \frac{r_{ik}}{r_{ik}^2 + x_{ik}^2}$$

$$h_{ik} = -\frac{x_{ik}}{r_{ik}^2 + x_{ik}^2}$$  \hspace{1cm} (2.3)

### 2.2.3 Traction substations

Harmonic currents are neglected. The output voltage of the rectifier is assumed to be proportional to the input voltage of the rectifier.

From the AC side, traction substations are modeled by power sources with a fixed power factor. This power factor is fixed to $\cos(\phi) = 0.9$, which is the targeted value in operation.

The active power consumed by traction substations is determined by the simulation at the DC side. $P_{DC}$ is the sum of the power of the traction substation on the DC side and the power losses in the traction substation. The active power of the traction substation, noted $P_{SST}$, is equal to $P_{DC}$:

$$P_{SST} = P_{DC}$$  \hspace{1cm} (2.4)

The reactive power, noted $Q_{SST}$ is calculated from the power factor ($\cos(\phi) = 0.9$) and the active power:

$$Q_{SST} = \tan(\phi) = \sqrt{\frac{1 - \cos^2(\phi)}{\cos^2(\phi)}} P_{SST} = 0.484 P_{SST}$$  \hspace{1cm} (2.5)
2.2.4 Other electrical objects

Depending on the architecture of the high voltage network, other electrical devices may be linked to the high voltage network. These devices are usually located in stations (light, escalators, elevators, air conditioning) and sometimes along the track (ventilation system, smoke extraction). All these devices are linked to the high voltage network through station substation.

In the program station substations are modeled as consumer nodes in the network. The position and the active power are the only parameters to be indicated by the user. The power factor is also assumed to be $\cos(\phi) = 0.9$.

2.3 Power network solution

This section describes how the power network solution is derived.

2.3.1 DC power flow calculation

The DC power flow calculation is performed by an existing software which has been validated over the past few years. The validation process has gone through comparisons with results given by Esmeralda, the simulation software used by the SNCF since the 70s.

2.3.2 AC power flow calculation

The high voltage network consists of several nodes linked by lines. The nodes can be traction substations, bulk substations or station substations. The number of nodes is noted $n$.

Each node $i$ is characterized by:

- a voltage $V_i$
- a phase $\theta_i$
- an active power $P_i$
- a reactive power $Q_i$

$P_i$ and $Q_i$ are algebraic values, they represent the active and reactive power injected at node $i$.

Nodes indexing

The nodes of the high voltage network are indexed by a depth first search (DFS) algorithm. Figure 2.4 represents an example of high voltage network and the order in which the nodes are visited, following a DFS algorithm.
Vectors and matrices ordering is directly linked to the nodes indexing. An efficient ordering of the matrices and vectors speed up the calculation, this is therefore important. The DFS algorithm is the best way of ordering the matrix in this case.

**Derivation of the system of equation**

Complex numbers are used to derive the system of equations:

\[ E_i = V_i \cos(\theta_i) + V_i \sin(\theta_i) = V_i e^{j\theta_i} \quad (2.6) \]

\( E \) is the matrix of complex voltages:

\[
E = \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix} 
\quad (2.7)
\]

\( I \) is the matrix of injected currents:

\[
I = \begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix} 
\quad (2.8)
\]
For each node $i$, the injected current at this node $I_i$ is:

$$ I_i = \sum_{k \neq i} [y_{ik}(E_i - E_k)] $$

$$ = [\sum_{k \neq i} y_{ik}]E_i - \sum_{k \neq i} [y_{ik}E_k] \quad (2.9) $$

Setting $Y_{ii} = \sum_{k \neq i} y_{ik}$ and $Y_{ik} = -y_{ik}$, it comes

$$ I_i = Y_{ii}E_i + \sum_{k \neq i} [Y_{ik}E_k] \quad (2.10) $$

And the following equation holds:

$$ I = YE \quad (2.11) $$

with $Y$ the bus admittance matrix:

$$ Y = \begin{bmatrix} Y_{11} & Y_{12} & \ldots & Y_{1n} \\ Y_{21} & Y_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots \\ Y_{n1} & Y_{n2} & \ldots & Y_{nn} \end{bmatrix} \quad (2.12) $$

$G$ and $H$ are respectively the real and the imaginary part of the bus admittance matrix:

$$ Y = G + jH \quad (2.13) $$

To find the network solution it is necessary to get a system of equations with powers. The complex power injected at node $i$ is:

$$ \Pi_i = P_i + jQ_i = E_iI_i^* = E_i \sum_k Y_{ik}^*E_k^* $$

$$ = Y_{ii}^*E_iE_i^* + \sum_{k \neq i} Y_{ik}^*E_kE_k^* \quad (2.14) $$

Identifying the active and the reactive part, it comes:

$$ \begin{cases} 
  P_i = \Re(\Pi_i) = G_{ii}V_i^2 + \sum_{k \neq i} V_iV_k[G_{ik}\cos(\theta_i - \theta_k) + H_{ik}\sin(\theta_i - \theta_k)] \\
  Q_i = \Im(\Pi_i) = -H_{ii}V_i^2 + \sum_{k \neq i} V_iV_k[G_{ik}\sin(\theta_i - \theta_k) - H_{ik}\cos(\theta_i - \theta_k)] \quad (2.15) 
\end{cases} $$

Two equations per node are available, which leads to a non linear system with $2n$ equations. In order for the system to be solved there should be no more than $2n$ unknowns. It means that for each node two values among $P_i$, $Q_i$, $V_i$ and $\theta_i$ should be known.
For traction substations and station substations $P_i$ and $Q_i$ are known from the DC power flow calculation. For the bulk substations, $V_i$ is known and $\theta_i$ is set to 0. It is important to note that two bulk substations cannot be connected (this is the reason why the phase can be set to zero in several points of the network).

This kind of system can be solved by two algorithms:
- Gauss-Seidel algorithm
- Newton-Raphson algorithm

As stated in chapter 1.2.2, Newton-Raphson algorithm provides better convergence, therefore this method is implemented. The method is described in the following section.

**Newton-Raphson method**

The Newton-Raphson algorithm is an iterative method in several steps:

1. Initial values are chosen for $V_i(0)$ and $\theta_i(0)$. For bulk substations $V_i$ and $\theta_i$ are already known. For other nodes (traction substations and station substations), $V_i(0)$ and $\theta_i(0)$ are set respectively to the nominal voltage and 0. From initial values, $P_i(0)$ and $Q_i(0)$ are calculated thanks to the following equations:

\[
\begin{align*}
P_i(0) &= G_{ii}V_i(0)^2 + \sum_{k \neq i} V_i(0)V_k(0)[G_{ik}\cos(\theta_i(0) - \theta_k(0)) + H_{ik}\sin(\theta_i(0) - \theta_k(0))] \\
Q_i(0) &= -H_{ii}V_i(0)^2 + \sum_{k \neq i} V_i(0)V_k(0)[G_{ik}\sin(\theta_i(0) - \theta_k(0)) - H_{ik}\cos(\theta_i(0) - \theta_k(0))] \\
\end{align*}
\]

(2.16)

2. For each node $i$ the quantities $\Delta P$ and $\Delta Q$ are calculated, this is the convergence test:

\[
\begin{align*}
\Delta P &= \max_i(|\Delta P_i|) = \max_i(P_i - P_i(0)) \\
\Delta Q &= \max_i(|\Delta Q_i|) = \max_i(Q_i - Q_i(0))
\end{align*}
\]

(2.17)

where $P_i$ and $Q_i$ are the specified values of active and reactive power (the ones determined by the DC power flow calculation). For bulk substations, $P_i$ and $Q_i$ are unknown, therefore $P_i - P_i(0)$ and $Q_i - Q_i(0)$ cannot be evaluated. These quantities are set to zero, in this way they do not affect the convergence test.

If $\Delta P < \epsilon_P$ and $\Delta Q < \epsilon_Q$ then $(P(0), Q(0), V(0), \theta(0))$ is the solution of the system.

$\epsilon_P$ and $\epsilon_Q$ reflects the precision. The lower they are, the better is the precision.

If not take step 3.

3. $P_i$ and $Q_i$ are developed at the first order:

\[
P_i = P_i(V_k(0) + \Delta V_k, \ldots, \theta_k(0) + \Delta \theta_k, \ldots)
= P_i(0) + \sum_{k=1}^{n} \frac{\partial P_i}{\partial V_k} \Delta V_k + \sum_{k=1}^{n} \frac{\partial P_i}{\partial \theta_k} \Delta \theta_k
\]

(2.18)
The same equation holds for \( Q_i \), replacing \( P \) by \( Q \).

It comes:

\[
\Delta P^{(1)}_i = \Delta P_i = \sum_{k=1}^{n} \frac{\partial P_i}{\partial V_k} \Delta V_k + \sum_{k=1}^{n} \frac{\partial P_i}{\partial \theta_k} \Delta \theta_k
\]

\[
\Delta Q^{(1)}_i = \Delta Q_i = \sum_{k=1}^{n} \frac{\partial Q_i}{\partial V_k} \Delta V_k + \sum_{k=1}^{n} \frac{\partial Q_i}{\partial \theta_k} \Delta \theta_k
\]  

(2.19)

These two equations can be obtained for every nodes, which gives the following matrix system:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = J
\begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix}
\]  

(2.20)

where \( J \) is the Jacobian matrix of the system:

\[
J = \begin{bmatrix}
\frac{\partial P}{\partial V} & \frac{\partial P}{\partial \theta} \\
\frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial \theta}
\end{bmatrix}
\]  

(2.21)

The derivation of the Jacobian matrix is presented in Appendix A.

Then \( \Delta V \) and \( \theta \) are determined by:

\[
\begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix} = J^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]  

(2.22)

Note that before solving Equation 2.22 it is necessary to remove the rows corresponding to bulk substations in \( \Delta V \), \( \Delta \theta \), \( \Delta P \), \( \Delta Q \) and for \( J \) the rows and the columns corresponding to bulk substations. Without this operation, \( J \) is a singular matrix, and the system cannot be solved.

4. Update \( V_i \) and \( \theta_i \) for all nodes except bulk substations:

\[
V^{(1)}_i = V^{(0)}_i + \Delta V_i \\
\theta^{(1)}_i = \theta^{(0)}_i + \Delta \theta_i
\]  

(2.23)

Go back to step 1 incrementing all the indices

### 2.3.3 AC/DC power flow calculation

The two previous sections have described how to find the power flow solution of the DC traction network and the AC high voltage network. This section describes how the AC/DC power flow is performed.

As stated in section 1.2.2, the solving of the network can be either sequential or unified. A unified method provides faster convergence, but in this case the DC load flow algorithm was already implemented in the software, so it was easier to only develop the AC load flow algorithm and couple it with the DC one.

The principle of the AC/DC power flow algorithm is the following:
1. The DC network is solved. One of the inputs of the DC network power calculation is the voltage of traction substations on the AC side. In this first step, the voltage of the traction substations on the AC side is considered to be the nominal voltage of the high voltage network. One of the outputs of the DC network power calculation is the power needed at each traction substation.

2. The AC network is solved, taking the power needed at each traction substation as input. The output of the AC network solving is the voltage at each node of the high voltage network.

3. The DC network is solved, taking as input the traction substations voltage derived at the previous step. The results of the DC network power flow calculation are compared to the results of the previous DC network power flow calculation. If these results are sufficiently close to each other, then the whole network is solved. If not, step 2 is taken again.

The meaning of sufficiently close depends on the accuracy of the calculation. The precision $\epsilon$ is a parameter of the simulation and is usually set to 1 V. Each node of the DC network has a voltage $U_j, j \in \text{(DC network nodes)}$. If $i$ is the number of the iteration, the condition for the AC/DC power flow algorithm to stop is:

$$\max_{j \in \text{(DC network nodes)}} (U_j^i - U_j^{i-1}) < \epsilon$$  (2.24)
Chapter 3

Simulation results

In this work three different high voltage infrastructures have been simulated. These infrastructures are presented in section 3.2. The input data of the simulation consist of:

- The line characteristics. The chosen line doesn’t correspond to an existing line, but it has curves and gradients that are typically found on subway lines.
- The train characteristics
- The courses and the timetable used in the simulation
- The infrastructure parameters (resistivity, reactance, internal impedance, etc)

The results consist of traction substation voltage and current, and train voltage. The analysis of the results is carried out in the following chapter (4).

3.1 Input data

3.1.1 Line

The chosen line has curves and gradients that are typical of subway lines. The line is 10 km long and has five stations. Figure 3.1 presents the line characteristics.
CHAPTER 3. SIMULATION RESULTS

Figure 3.1: Line characteristics
Figures 3.2a and 3.2b show the line profile respectively in 2D and 3D.

3.1.2 Train

The dynamic parameters of the train are described in Table 3.1. The parameters of the train have been derived from existing rolling stocks. They do not correspond to a particular train but are in the range of what exists in terms of length, capacity, weight, acceleration and power. The tractive force vs speed curve is presented on Figure 3.3.
CHAPTER 3. SIMULATION RESULTS

Table 3.1: Dynamic parameters of the train

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>54 m</td>
</tr>
<tr>
<td>Weight</td>
<td>154 t</td>
</tr>
<tr>
<td>Max speed</td>
<td>115 km/h</td>
</tr>
<tr>
<td><strong>Train resistance</strong></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2541 N</td>
</tr>
<tr>
<td>B</td>
<td>77.616 N.s/m</td>
</tr>
<tr>
<td>C</td>
<td>14.5696 N.s^2/m^2</td>
</tr>
<tr>
<td>Curve resistance factor</td>
<td>800 m</td>
</tr>
<tr>
<td><strong>Dynamic parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Max acceleration</td>
<td>1 m/s^2</td>
</tr>
<tr>
<td>Max power</td>
<td>1700 kW</td>
</tr>
<tr>
<td>Max deceleration</td>
<td>1.1 m/s^2</td>
</tr>
<tr>
<td>Max braking power</td>
<td>3300 kW</td>
</tr>
</tbody>
</table>

Figure 3.3: Tractive force vs speed curve of the train used in simulations

The electric parameters of the train are described in Table 3.2
Table 3.2: Electric parameters of the train

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Voltage</strong></td>
<td></td>
</tr>
<tr>
<td>Minimal voltage</td>
<td>1000 V</td>
</tr>
<tr>
<td>Nominal voltage</td>
<td>1500 V</td>
</tr>
<tr>
<td>Maximal voltage</td>
<td>1800 V</td>
</tr>
<tr>
<td><strong>Powertrain efficiency</strong></td>
<td></td>
</tr>
<tr>
<td>In traction</td>
<td>0.9</td>
</tr>
<tr>
<td>In braking</td>
<td>0.85</td>
</tr>
<tr>
<td><strong>Other parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Auxiliary power</td>
<td>120 kW</td>
</tr>
<tr>
<td>Electric braking rate</td>
<td>100 %</td>
</tr>
</tbody>
</table>

### 3.1.3 Courses and Timetable

Two services have been defined:

- Station 1 → Station 5 with a stop of 30 seconds in every intermediate station
- Station 5 → Station 1 with a stop of 30 seconds in every intermediate station

Figure 3.4 shows the two courses:

![Figure 3.4: Services used in the simulation](image-url)
A course is the association of a service and a train. In the simulation there are two courses that use the same train. In course 1 the train carries out service 1 and in course 2 the train carries out service 2.

The speed profiles of the two courses are obtained after simulation. They are presented in Figures 3.5 and 3.6:
CHAPTER 3. SIMULATION RESULTS

Figure 3.5: Course 1

Course 1 – loaded
Mission: Service 1
Time: 00:09:08
Travelled distance: 10.000 km
Average speed: 65.60 km/h
Consumed energy: 132.6 kWh
Braked energy: −83.1 kWh
CHAPTER 3. SIMULATION RESULTS

Figure 3.6: Course 2

- Speed
- Instruction speed
- Time
- Line max speed
- Curves
- Gradient
- Elevation

- Speed
- Instruction speed
- Time
- Line max speed
- Curves
- Gradient
- Elevation

Course 2
Train: Train - max load
Mission: Service 2
Time: 00:09:11
Travelled distance: 10.000 km
Average speed: 65.31 km/h
Consumed energy: 119.6 kWh
Braked energy: −82.7 kWh
CHAPTER 3. SIMULATION RESULTS

The timetable used in the simulations is a roster with courses 1 and 2. The headway (i.e. the
time between two following trains) is 120 s, which is a regular headway during rush hours. Figure
3.7 represents the timetable used in the simulation.

3.1.4 Infrastructure parameters

The characteristics of the infrastructure are presented in Table 3.3.

Table 3.3: Infrastructure parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC network</td>
<td></td>
</tr>
<tr>
<td>Nominal voltage</td>
<td>1500 V</td>
</tr>
<tr>
<td>Resistance of the catenary</td>
<td>0.021 Ω/km</td>
</tr>
<tr>
<td>Resistance of two rails in parallel</td>
<td>0.015 Ω/km</td>
</tr>
<tr>
<td>High voltage network</td>
<td></td>
</tr>
<tr>
<td>Nominal voltage</td>
<td>15 kV</td>
</tr>
<tr>
<td>Line resistance</td>
<td>0.72 Ω/km</td>
</tr>
<tr>
<td>Line reactance</td>
<td>0.08 Ω/km</td>
</tr>
<tr>
<td>Traction substations</td>
<td></td>
</tr>
<tr>
<td>Internal resistance</td>
<td>0.02 Ω</td>
</tr>
<tr>
<td>No-load voltage</td>
<td>1500 V</td>
</tr>
</tbody>
</table>
Figure 3.8 represents the DC side of the infrastructure used in the simulation.

Horizontal lines represent the conductors, while vertical lines represent connectors between the conductors. The catenaries and the rails are connected at traction substations. The substations are represented in purple. They are positioned at kilometric points (kp) 0, 3, 7 and 10. In these simulations traction substations are reversible, it means that they can feed back current on the high voltage network.

### 3.1.5 Simulation parameters

The simulation parameters are:

- time step: 1 s. It means that the load flow is calculated every second
- precision (DC network): 1 V.
- precision (high voltage network): 1 %. The AC iteration stops when power residuals represent less than 1 % of actual powers.
- duration of the simulation : 120 s of operation is simulated. The timetable is periodic (with a period of 120 s), so there is no need to simulate on a longer period.
CHAPTER 3. SIMULATION RESULTS

3.2 Three different high voltage infrastructures

This section presents the three different high voltage networks that have been simulated. Infrastructure 1 and 2 are very different infrastructures, infrastructure 3 is a tradeoff between infrastructures 1 and 2.

3.2.1 Infrastructure lay-out 1

In infrastructure 1, each traction substation is linked to a bulk substation. Each line has a length of 1 km. With this kind of high voltage infrastructure, traction substations are independent.

![Diagram of Infrastructure lay-out 1](image-url)

Figure 3.9: Infrastructure lay-out 1
The purple diamonds represent traction substations. The black squares represent bulk substations. The figure comes from the graphical user interface of the software.

### 3.2.2 Infrastructure lay-out 2

In infrastructure 2, traction substations are linked together in a loop. The line between the bulk substation and traction substation 1 is 1 km long. The lines linking the traction substations are given by the kilometric point differences. With this kind of infrastructure, the behavior of each substation is influenced by the behavior of the other substations. It is necessary to calculate the load flow at the AC level.

![Figure 3.10: Infrastructure lay-out 2](image-url)
3.2.3 Infrastructure lay-out 3

Infrastructure 3 is a tradeoff between infrastructure 1 and infrastructure 2. Again the lines linking bulk substations to traction substations are 1 km long. The length of other lines are given by kilometric point differences.

Figure 3.11: Infrastructure lay-out 3

3.3 Output data

In the first set of simulations, all the traction substations have the same settings:

- same internal impedance
• same no-load transformer ratio. If V (the voltage on the AC side) equals 15 kV, then U (the voltage on the DC side) equals 1500 V.

This section presents the results of the first set of simulations. The goal of this first set of simulations is to study the influence of the high voltage network on the voltage levels at DC side, and the load share between traction substations. The train voltages are also studied because they may affect train performances.

3.3.1 Substations

In this subsection, the evolution of traction substations voltage, current and power (at DC side) are presented in several graphics.

voltage

Figures 3.12, 3.13, 3.14 and 3.15 represents the evolution of traction substations output voltage during the period of simulation.

![Figure 3.12: Evolution of substation 1 output DC voltage](image)

Figure 3.12: Evolution of substation 1 output DC voltage
CHAPTER 3. SIMULATION RESULTS

Figure 3.13: Evolution of substation 2 output DC voltage

Figure 3.14: Evolution of substation 3 output DC voltage

Figure 3.15: Evolution of substation 4 output DC voltage
Current

Figures 3.16, 3.17, 3.18 and 3.19 represents the evolution DC current through traction substations during the period of simulation.

Figure 3.16: Evolution of current flowing through substation 1

Figure 3.17: Evolution of current flowing through substation 2
CHAPTER 3. SIMULATION RESULTS

Figure 3.18: Evolution of current flowing through substation 3

Figure 3.19: Evolution of current flowing through substation 4

Power

Figures 3.20, 3.21, 3.22 and 3.23 represent the evolution of traction substations output power during the period of simulation. The output power is the product of the output DC voltage and the current flowing through the substation.
CHAPTER 3. SIMULATION RESULTS

Figure 3.20: Evolution of SST 1 output power

Figure 3.21: Evolution of SST 2 output power

Figure 3.22: Evolution of SST 3 output power
When it comes to power it’s also interesting to study:

- maximum power, because a traction substation has a power limit than cannot be exceeded,
- root mean square power (RMS power), because it represents the heating of the traction substation,
- mean power, because it represents where the consumed energy comes from.

Figures 3.24, 3.25 and 3.26 compares maximum, RMS and mean power of each infrastructure. The four traction substations are represented on the same graphic in order to see the load share.
3.3.2 Trains

Train voltage is an important parameter as the train can only run if the voltage remains in a certain range. Moreover, train performances can be affected by a too low voltage. Figure 3.27 represents the voltage of the train running along the track from Station 1 to Station 5 (Course 1) for the three infrastructures.
Figure 3.27: Voltage of a train running from Station 1 to Station 5 as a function of distance
Chapter 4

Analysis and discussion

In this chapter, the results of the simulations carried out and presented in the previous chapter are analyzed with regard to the sizing of the infrastructure. When designing an electrical infrastructure it is necessary to, among else, check the voltage levels and the power required for each substations. The following analysis assesses the quality of each infrastructure.

4.1 Analysis of the simulation results

4.1.1 Voltage drop

The voltage drop experienced by the train comes from:

- the voltage drop in the high voltage network
- the voltage drop in the traction substation
- the voltage drop in the catenaries and the rails

The output voltage of the traction substations is due to the voltage drop in the high voltage network and the traction substations.

It can be seen in the graphs that the voltage drop at the traction substations is limited in the case of infrastructure 1. The output voltage remains higher than 1400 V. In the case of infrastructure 1, the voltage drop is maximum for SST2, which indeed is the most loaded traction substation.

In the case of infrastructure 2, the voltage drop increases from SST1 to SST4. This is because the current flowing in the high voltage network is much higher than in the case of infrastructure 1 where the current is split in the four lines. Due to this significant voltage drop the traction substations at the end of the line are less loaded. The current required at SST 4 flows through the whole line linking the bulk substation to SST 1, SST 2, SST 3 and SST 4. And the same thing happen for SST 2 and 3. The line between the bulk substation and SST 1 has to carry the total current required at the traction substations.

In case of infrastructure 3, the voltage drop is significant for SST 2 and SST 4, which are at the end of respective high voltage line. The voltage drop is however more limited than in the case
of infrastructure 2.

The train voltage remains above 1300 V in the case of infrastructure 1 and 3, but reaches value lower than 1150 V in the case of infrastructure 2. Such low voltage is problematic for the train performances. Usually the performances of the trains decrease slowly between 1500 and 1200 V and then more significantly below 1200 V.

The voltage drop results were expected. Just looking at the high voltage infrastructure it was possible to say which infrastructure would have the biggest voltage drop. The simulation enables us to quantify this voltage drop.

4.1.2 Load sharing

The load sharing is an important parameter when designing a power supply system. A power supply system in which the load is equally balanced between traction substations will require less maintenance. Therefore it is interesting to study the influence of the high voltage infrastructure on the load sharing. The load at a traction substation is well represented by the RMS power. Figure 3.25 shows the load sharing for the three infrastructures.

In infrastructure 1, the load is quite well balanced except for SST 4. The most loaded substation is SST 2.

In infrastructure 2, the load is very badly balanced, with SST 1 having almost twice the load of SST 2 and 3, and SST 4 almost not loaded at all.

In infrastructure 3, the loads are also badly balanced, SST 1 and 3 being a lot more loaded than SST 2 and 4.

The load sharing is linked to the voltage. A traction substation with a low output voltage will not be able to feed trains that are distant from the substation, and thus they will not be very much loaded.

The conclusion that can be drawn from these results is that infrastructures 2 and 3 are not appropriate. In the case of infrastructure 2 both voltage drop and load sharing are not appropriate, while for infrastructure 3 the load sharing could be improved.

4.2 Improvement of the load sharing

There are several ways to improve the load sharing between the traction substations:

- Finding the right location for each traction substation. For a given operation (timetable), there is always a set of best locations for traction substations. It is not an easy task to find these best locations because there are a lot of parameters to take into account. This is not achievable without simulations. In practice finding the best locations is not interesting because the constraints on the location of traction substations are very high. In the urban environment, it is hard to find space. In metro projects, traction substations are often located in the stations, or just next to the stations to limit civil works.

- Finding the right transformer ratio for each traction substation. If a traction substation is not loaded like the others, changing the transformer ratio enables to have a better
balance. Changing the transformer ratio changes the no-load output voltage of the traction substation.

In this project, it is assumed that the positions of traction substations are fixed. The best location for traction substations will therefore not be studied.

The improvement of the load sharing will be achieved by changing the transformer ratio of traction substations. The method used is iterative. The iteration stops when the loads are balanced or the transformer ratios cannot be further increased or decreased.

It is assumed that the no-load output voltage of traction substations must remain in the range $[1500 \text{ V}; 1650 \text{ V}]$. Below 1500 V, the voltage offered to the trains is likely to be too low (after the voltage drop in the catenaries and the rails). Above 1650 V energy exchanges between trains (regenerative braking) cannot be done efficiently, resulting in the loss of braking energy, higher energy consumption, and more wear of the brakes.

### 4.2.1 Infrastructure 2

In infrastructure 2, the no-load output voltage of SST 4 is directly set to the maximum value: 1650 V. The no-load output voltage of infrastructure remains at 1500 V. After a few iterations, the load sharing presented in Figure 4.1 is obtained with the following set of no-load output voltages:

- SST 1 : 1500 V
- SST 2 : 1550 V
- SST 3 : 1620 V
- SST 4 : 1650 V

![Figure 4.1: Infrastructure 2 - Comparison of SST RMS power before and after improvement](image)

The improvement is limited. The load sharing cannot be significantly improved as the whole range of possible no-load output voltages is already used. The conclusion is that infrastructure 2 is not appropriate here.
4.2.2 Infrastructure 3

In infrastructure 3, SST 2 and 4 are less loaded than the two other ones. Therefore the no-load output voltages of SST 2 and 4 are increased. After a few iterations, a well balanced load is achieved with the following no-load output voltages:

- SST 1 : 1500 V
- SST 2 : 1550 V
- SST 3 : 1550 V
- SST 4 : 1625 V

Figure 4.2 shows a comparison of the load sharing before and after improvement. It can be seen the load is better balanced with the new parameters.

![Figure 4.2: Infrastructure 3 - Comparison of SST RMS power before and after improvement](image)

4.3 Which infrastructure to choose ?

Designing the high voltage network infrastructure is an important task in the design of a power supply system for DC electrified railways. As shown in this project the high voltage network infrastructure has a big influence on the voltage drop and the load sharing that are two very important criteria to study in the design process.

Infrastructure 1 is not often chosen. The reason is that the number of bulk substations is usually not as high as the number of traction substation.

Usually one bulk substation feeds a group of traction substations (infrastructure 2 and 3). The size of the group depends on the power required at traction substations, the distance between traction substations and the power available at the bulk substation. In this work, the bulk substation of infrastructure 2 feeds too many traction substations resulting in a significant voltage drop and bad load sharing especially if the transformer ratio are not set to their optimal value.

A simulation of the same line, with the same parameters, but without the high voltage network, has also been carried out in order to evaluate the impact of the high voltage network.
Traction substation output voltages are higher than when the high voltage network is simulated because the resistance of the high voltage network is not taken into account any more. Therefore it is not relevant to compare the voltage drops.

It’s however interesting to compare the load sharing. Figure 4.3 represents the load sharing when the high voltage network (HVN) is not simulated and when it is simulated. It shows that the simulation without the high voltage network gives a load sharing rather close to the load sharing obtained with infrastructure 1.

Figure 4.3: SST RMS Power with and without high voltage network simulation

This implies that in case of high voltage infrastructure of type 1, it is not necessary to simulate the high voltage network. In order to still have the right voltage drops, the impedance of the high voltage network has to be taken into account at the level of the traction substation impedance.

In case of infrastructure 2 and 3, it is necessary to simulate the high voltage network, otherwise the results obtained can be quite far from reality.
Conclusion

The design of the power supply network of a DC electrified transit railway is a complex task requiring the use of simulations models. The power supply network is composed of a high voltage network (AC) linked to a traction network (DC) by traction substations (equipped with rectifiers). Many simulation models ignore the high voltage network in the design process. The goal of this project was to study the influence of the high voltage network on this design process, with special focus on the voltage drop and the load sharing between traction substations.

Starting from an existing simulation software, an AC load flow algorithm has been implemented and coupled to the existing DC load flow algorithm. The solving method is based on a Newton-Raphson algorithm.

The voltage drop and load sharing have been analyzed by simulations. Significant discrepancies between the different architectures of high voltage network have been observed, showing that for some kind of architectures simulating the high voltage network is necessary. When all the traction substations are independent of each other on the AC side, it is not necessary to simulate the high voltage network. However if some traction substations are on the same high voltage loop then it will be necessary to simulate it. Improvement of the load sharing can be achieved by setting the transformer ratios of the traction substations to precise values.

Further work could be carried out on the post processing of simulation output data. Voltages and currents in the high voltage network could be analyzed, in order to study for example the losses in the high voltage network or the power required at each bulk substation. In the model used in this project the output voltage of bulk substations is constant. However this is not always the case in reality, this is something that could be implemented in a later version of the model. Finally a way to speed up the calculation would be to implement a unified AC/DC load flow algorithm instead of alternatively solving the AC and the DC network.
Appendix A

Derivation of the Jacobian Matrix

In this appendix, the derivation of the Jacobian matrix is presented. The general equations are reminded below:

\[
\begin{align*}
\mathbf{P}_i &= G_{ii} V_i^2 + \sum_{k \neq i} V_i V_k [G_{ik} \cos(\theta_i - \theta_k) + H_{ik} \sin(\theta_i - \theta_k)] \\
\mathbf{Q}_i &= -H_{ii} V_i^2 + \sum_{k \neq i} V_i V_k [G_{ik} \sin(\theta_i - \theta_k) - H_{ik} \cos(\theta_i - \theta_k)]
\end{align*}
\]  
(A.1)

\[
\mathbf{J} = \left[ \begin{array}{c} \frac{\partial \mathbf{P}_i}{\partial V_i} \\
\frac{\partial \mathbf{P}_i}{\partial \theta_i} \\
\frac{\partial \mathbf{Q}_i}{\partial V_i} \\
\frac{\partial \mathbf{Q}_i}{\partial \theta_i} \end{array} \right]
\]  
(A.2)

\(\frac{\partial \mathbf{P}}{\partial \mathbf{V}}\) is a \(n \times n\) matrix, with the following elements:

\[
\frac{\partial \mathbf{P}_i}{\partial \mathbf{V}_i} = 2 G_{ii} V_i + \sum_{k \neq i} V_k [G_{ik} \cos(\theta_i - \theta_k) + H_{ik} \sin(\theta_i - \theta_k)]
\]  
(A.3)

\[
\frac{\partial \mathbf{P}_i}{\partial \mathbf{V}_j} = V_i [G_{ij} \cos(\theta_i - \theta_j) + H_{ij} \sin(\theta_i - \theta_j)]
\]

\(\frac{\partial \mathbf{P}}{\partial \mathbf{\theta}}\) is a \(n \times n\) matrix, with the following elements:

\[
\frac{\partial \mathbf{P}_i}{\partial \theta_i} = \sum_{k \neq i} V_i V_k [-G_{ik} \sin(\theta_i - \theta_k) + H_{ik} \cos(\theta_i - \theta_k)]
\]  
(A.4)

\[
\frac{\partial \mathbf{P}_i}{\partial \theta_j} = V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - H_{ij} \cos(\theta_i - \theta_j)]
\]

\(\frac{\partial \mathbf{Q}}{\partial \mathbf{V}}\) is a \(n \times n\) matrix, with the following elements:

\[
\frac{\partial \mathbf{Q}_i}{\partial \mathbf{V}_i} = -2 H_{ii} V_i + \sum_{k \neq i} V_k [G_{ik} \sin(\theta_i - \theta_k) - H_{ik} \cos(\theta_i - \theta_k)]
\]  
(A.5)

\[
\frac{\partial \mathbf{Q}_i}{\partial \mathbf{V}_j} = V_i [G_{ij} \sin(\theta_i - \theta_j) - H_{ij} \cos(\theta_i - \theta_j)]
\]
\( \frac{\partial Q}{\partial \theta} \) is a \( n \times n \) matrix, with the following elements:

\[
\frac{\partial Q_i}{\partial \theta_i} = \sum_{k \neq i} V_i V_k \left[ G_{ik} \cos(\theta_i - \theta_k) + H_{ik} \sin(\theta_i - \theta_k) \right]
\]

\[
\frac{\partial Q_i}{\partial \theta_j} = -V_i V_j \left[ G_{ij} \cos(\theta_i - \theta_j) + H_{ij} \sin(\theta_i - \theta_j) \right]
\]  

(A.6)
Appendix B

Matlab equations

The Newton Raphson algorithm is solved with Matlab. In order to use efficiently the computing capacities of Matlab, the equations have to be written with their matrix forms. It requires a little work which is presented in this appendix.

The following notations are used:

\[ \Theta = \begin{bmatrix} \theta_1 & \theta_1 & \ldots & \theta_1 \\ \theta_2 & \theta_2 & \ldots & \theta_2 \\ \vdots & \vdots & \ddots & \vdots \\ \theta_n & \theta_n & \ldots & \theta_n \end{bmatrix} \]  \hfill (B.1)

The \( \cdot \) indicates a multiplication term by term between two vectors or matrices of the same size. The \( \times \) indicates the regular matrix product.

If \( A = (A_{ij})_{i,j \in [1,n]} \),

\[
\begin{align*}
\cos(A) &= (\cos(a_{ij}))_{i,j \in [1,n]} \\
\sin(A) &= (\sin(a_{ij}))_{i,j \in [1,n]}
\end{align*} \hfill (B.2)

The matrix \( (\theta_i - \theta_j)_{i,j \in [1,n]} \) is obtained by:

\[
(\theta_i - \theta_j)_{i,j \in [1,n]} = \begin{bmatrix} \theta_1 - \theta_1 & \theta_1 - \theta_2 & \ldots & \theta_1 - \theta_n \\ \theta_2 - \theta_1 & \theta_2 - \theta_2 & \ldots & \theta_2 - \theta_n \\ \vdots & \vdots & \ddots & \vdots \\ \theta_n - \theta_1 & \theta_n - \theta_2 & \ldots & \theta_n - \theta_n \end{bmatrix} = \Theta - \cdot \Theta \hfill (B.3)
\]

The function \( \text{diag} \) takes a vector as argument and is defined as follows:

\[
V^{\text{diag}} \rightarrow \text{diag}(V) = \begin{cases} 
V_i & \text{if } i = j \\
0 & \text{if } i \neq j 
\end{cases}, (i,j) \in [1,n]^2 
\]  \hfill (B.4)

Equations reminded in A.1 are written below in their matrix form:
$P = V \cdot [G \cdot \cos(\Theta - t\Theta) + H \cdot \sin(\Theta - t\Theta)] \times V$ \hspace{1cm} (B.5)

$Q = V \cdot [G \cdot \sin(\Theta - t\Theta) - H \cdot \cos(\Theta - t\Theta)] \times V$ \hspace{1cm} (B.6)

The matrix form of the elements of the Jacobian matrix are presented below:

\[
\frac{\partial P}{\partial V} = diag(V) \times [G \cdot \cos(\Theta - t\Theta) + H \cdot \sin(\Theta - t\Theta)] + diag([G \cdot \cos(\Theta - t\Theta) + H \cdot \sin(\Theta - t\Theta)] \times V)
\]

\[
\frac{\partial P}{\partial \theta} = diag(V) \times [G \cdot \sin(\Theta - t\Theta) - H \cdot \cos(\Theta - t\Theta)] \times diag(V) + diag(V \cdot [\neg G \cdot \sin(\Theta - t\Theta) + H \cdot \cos(\Theta - t\Theta)] \times V)
\]

\[
\frac{\partial Q}{\partial V} = diag(V) \times [G \cdot \sin(\Theta - t\Theta) - H \cdot \cos(\Theta - t\Theta)] + diag([G \cdot \sin(\Theta - t\Theta) - H \cdot \cos(\Theta - t\Theta)] \times V)
\]

\[
\frac{\partial Q}{\partial \theta} = diag(V) \times [-G \cdot \cos(\Theta - t\Theta) - H \cdot \sin(\Theta - t\Theta)] \times diag(V) + diag(V \cdot [G \cdot \cos(\Theta - t\Theta) + H \cdot \sin(\Theta - t\Theta)] \times V)
\]
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