A comparison of the IRB approach and the Standard Approach under CRR for purchased defaulted retail exposures

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Abstract

We investigate under what circumstances the IRB approach under Regulation (EU) no 575/2013 (Capital Requirements Regulation) renders a lower capital requirement for purchased defaulted retail exposures than under the Standard Approach of the same regulation. We also discuss some alternative approaches to calculating the capital requirement for the mentioned exposures. The results show that it is only beneficial in a few cases to use the IRB approach compared to the Standard Approach. We can also see that the IRB approach in some cases renders a capital requirement that is clearly not reflecting the actual risk. Of the alternative approaches investigated, a Value-at-Risk approach seems most promising for further development.
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Acronyms

AV Accounting Value.

BCBS Basel Committee on Banking Supervision.

BIS Bank of International Settlements.

CF Conversion Factor.

CIU Collective Investment Undertaking.

CRD IV Capital Requirements Directive IV.

CRM Credit Risk Mitigation.

CRR Capital Requirements Regulation.

DRO Debt Relief Order.

EL Expected Loss.

EL_{BE} Best Estimate of EL for Defaulted Exposures.

EV Exposure Value.

EV_{IRB} EV in the IRB approach.

EV_{SA} EV in the SA.

FI Finansinspektionen.

HD Hard Default.

ICAAP Internal Capital Adequacy Assessment.

IRB Internal Ratings Based.
LGD  Loss Given Default.

M  Maturity.

NV  Nominal Value.

PD  Probability of Default.

PHD  Probability of HD.

PSD  Probability of SD.

RW  Risk Weight.

RWEA  Risk-weighted Exposure Amount.

SA  Standard Approach.

SCRA  Specific Credit Risk Adjustment.

SD  Soft Default.

UL  Unexpected Loss.

VaR  Value-at-Risk.
Chapter 1

Introduction

Credit institutions and investment firms (hereafter “institutions”) have to deal with many different types of risks, e.g. credit risk, market risk, operational risk, concentration risk, liquidity risk, legal risk, etc. It is in the interest of both the institutions and the financial supervisors to have good risk management for these types of risk; for the profitability of and survival of the institution, but also for the stability of the financial system. In the credit business, credit losses occur all the time. While it is never possible to know in advance how much that will be lost, the institution can forecast on average how much it can expect to lose.

These losses are referred to as Expected Loss (EL). They are considered a cost component by institutions and are managed for example by pricing credit exposures and Specific Credit Risk Adjustments (SCRAs) such as impairments. Sometimes the institutions do experience peak losses, so called Unexpected Losses (ULs). The institutions know that they will occur, but cannot forecast the timing and the severity of these losses.

One of the uses of an institution’s capital is to absorb these types of losses where the risk premiums of credit exposures etc. cannot. However, if it (as an extreme example) holds capital sufficient to cover a loss of the whole credit portfolio, the rest of the institution’s business will suffer as there will be less capital available for other types of investments. This will in turn slow down the economy and be a disadvantage for everyone. If it holds too little capital, it might not be able to cover its debt obligations and will become insolvent, which will also hurt the economy. The goal is to find the balance where the level of the capital is most favorable for the economy as a whole.
1.1 The Basel Accords

1.1.1 Basel I

In 1974, the Basel Committee on Banking Supervision (BCBS) was formed as a response to a messy liquidation of the German bank Herstatt where banks outside Germany took heavy losses on their unsettled trades with Herstatt, adding an international dimension to the debacle [6]. The incident caused the central bank governors of the G-10 countries to form BCBS under the direction of Bank of International Settlements (BIS) located in Basel, Switzerland. In 1988 the BCBS published a set of minimal capital requirements known as the 1988 Basel Accord, today known as Basel I [1]. It focused mainly on credit risk and classified the exposures of institutes into five different categories which had different Risk Weights (RWs): 0, 10, 20, 50 and 100%.

The Risk-weighted Exposure Amount (RWEA) is, basically, obtained by multiplying the RW with the Exposure Value (EV):

\[ \text{RWEA} = \text{RW} \cdot \text{EV} \]  

The EV is in most cases the Accounting Value (AV) net of SCRA and Credit Risk Mitigation (CRM) techniques such as collateral and guarantees. Off-balance sheet exposures can in some cases also be included. The adequate capital to hold for UL was chosen as 8% of the RWEA, the so called Cooke ratio, named after the chairman at the time, Peter Cooke.

1.1.2 Basel II

In June 2004, BCBS published a new set of rules called Basel II [3]. The aim with Basel II was to ensure that capital allocation was more risk sensitive, to separate operational risk and credit risk and to align economic and regulatory capital more closely to reduce the regulatory arbitrage.

The RWEA for credit risks was combined with risk exposure amounts for market risk and operational risk to sum up to a total risk exposure amount. The capital requirement was as before 8%, but now in regard to the total risk exposure amount.

The concept of Three Pillars were introduced, where Pillar 1 deals with minimal capital requirements, Pillar 2 deals with supervisory review, where the most important part is a review of the institution’s Internal Capital Adequacy Assessment (ICAAP) of all risks they are exposed to, and Pillar 3
deals with market discipline in terms of public disclosure of capital requirements and risk management. Basel II also added the 150% RW to be used for low credit rating borrowers.

1.1.3 Basel III

In June 2011, as a direct consequence of the 2007-2008 financial crisis, a third set of rules called Basel III [4] were published by BCBS. The aim of Basel III was to require institutions to hold more capital, and to hold capital of higher quality. The capital requirement of the RWEA was increased from 8% to between 10,5-21,5% by introducing capital buffer requirements. The capital buffer requirements depend on the type of institution, the size of the institution and the state of the economy in the countries where the institution has its exposures. Other requirements were also introduced, as the leverage ratio requirement which sets a requirement of the institutions leverage. In January 2013, an amendment [5] to Basel III concerning liquidity requirements was published.

1.2 Implementation of the Basel rules in EU regulation

The Basel rules have since January 2014 been implemented in the Capital Requirements Directive IV (CRD IV) [10] and the Capital Requirements Regulation (CRR) [11] at an EU level. CRR is directly applicable in all member states, and CRD IV has to be implemented in the respective member state’s national law. The rules governing capital requirements are included in CRR and thus uniform (except for a few national options for waivers) in the whole EU.

Pillar 1 of the Basel rules states that institutes should calculate capital requirements to cover credit risks for UL in the asset side of the balance sheet and some off-balance sheet posts. In CRR the capital requirement for credit risks, that are the risks that are covered in this master thesis, can be calculated by one or a combination of two approaches; the Standard Approach (SA) and the Internal Ratings Based (IRB) approach. The incentive to move from the SA to the more demanding IRB approach is that the capital requirement for credit risks can be decreased if the company shows that its actual risk level is lower than the SA indicates.
1.3 The SA in CRR

In the SA in CRR, all assets and off-balance sheet exposures are classified into 17 exposure classes. The exposure classes have several predetermined RWs assigned to them that depends on one or several of the factors; maturity, credit rating, level of diversification, type of collateral and underlying exposures (in the case of Collective Investment Undertakings (CIUs)). The RWEA is then calculated by multiplying the EV in the SA (EV$_{SA}$), with the RW:

\[
\text{RWEA} = \text{RW} \cdot \text{EV}_{SA}. \tag{1.2}
\]

The EV$_{SA}$ for assets is the Nominal Value (NV) net of SCRA and CRM:

\[
\begin{align*}
\text{EV}_{SA} &= \text{NV} - (\text{SCRA} + \text{CRM}) \\
&= \text{AV} - \text{CRM},
\end{align*} \tag{1.3}
\]

where

\[
\text{AV} = \text{NV} - \text{SCRA}. \tag{1.4}
\]

The EV$_{SA}$ for off-balance sheet items is the NV net of SCRA times the Conversion Factor (CF) minus CRM:

\[
\begin{align*}
\text{EV}_{SA} &= (\text{NV} - \text{SCRA}) \cdot \text{CF} - \text{CRM} \\
&= \text{AV} \cdot \text{CF} - \text{CRM},
\end{align*} \tag{1.5}
\]

The CF is 0, 20, 50 or 100 % depending on the level of risk of the off-balance sheet item.

1.4 The IRB approach in CRR

1.4.1 Exposure classes and EV$_{IRB}$

Under the IRB approach the exposures are classified into seven exposure classes:

- central governments and central banks
Figure 1.1: Probability distribution of losses

- institutions
- corporates
- retail
- equity
- securitisations, and
- other non credit-obligation assets.

The RWEA is calculated the same way as in the SA:

$$ \text{RWEA} = \text{RW} \cdot \text{EV}_{\text{IRB}}. $$  \hfill (1.6)

The EV in the IRB approach ($\text{EV}_{\text{IRB}}$) is calculated a little different than the $\text{EV}_{\text{SA}}$. The $\text{EV}_{\text{IRB}}$ for assets is calculated gross of SCRA:

$$ \text{EV}_{\text{IRB}} = \text{NV} - \text{CRM}. $$  \hfill (1.7)

Thus the following always hold:

$$ \text{EV}_{\text{SA}} \leq \text{EV}_{\text{IRB}}. $$  \hfill (1.8)

1.4.2 The IRB RW function

Basics

The IRB RW function is a function dependant on three parameters, Probability of Default (PD), Loss Given Default (LGD) and the parameter $R$. 


R is in some exposure classes a function of PD, so in those cases it reduced down to two dependencies. For sovereign, corporate and institutional exposures there is a fourth risk parameter, Maturity (M). As this master thesis only covers retail exposures, an analysis of the effect of M on the capital requirement is not included. To be eligible for an IRB approach, the company must demonstrate to the competent authorities, in Sweden Finansinspektionen (FI), that it meets a set of minimum requirements, of which access to reliable data on which to base its calculations is a cornerstone.

The IRB approach serves to solve the bank insolvency problem by using a stochastic credit portfolio model and making the bank hold enough capital so the the losses will exceed this capital only with a small predetermined probability $1 - \alpha$ set by supervisors i.e. the Value-at-Risk (VaR) at the confidence level $\alpha$. In CRR, $\alpha = 99.9\%$. As can be seen in Figure 1.1 the VaR is the sum of EL and UL. The RW functions estimates the UL by taking the difference between the VaR and EL:

\[ UL \equiv \text{VaR}_\alpha - \text{EL}. \quad (1.9) \]

The IRB RW function thus captures the UL which is used to calculate the RWEA.

The formula given in CRR is

\[ \text{RW} = \left( \text{LGD} \cdot \Phi \left( \frac{\Phi^{-1}(\text{PD}) + \sqrt{R} \Phi^{-1}(0.999)}{\sqrt{1 - R}} \right) - \text{LGD} \cdot \text{PD} \right) \cdot 12.5 \cdot 1.06, \quad (1.10) \]

where $\Phi$ is the cumulative standard normal distribution function, $\Phi^{-1}$ is the quantile function and $R$ is the correlation with the systemic risk factor and depends on the exposure class.

The parameter PD used in the formula is the average probability of default that reflects expected default rates during normal business conditions. The formula uses the PD together with the LGD to calculate the EL:

\[ \text{EL} \equiv \text{PD} \cdot \text{LGD}. \quad (1.11) \]

The LGD should reflect an economic downturn condition where losses are expected to be proportionally higher than during normal business conditions.

It also maps PD to a conditional PD: $\Phi(F(\text{PD}, R))$. This conditional PD reflects the probability of default given a conservative value of the systematic
risk factor. The conditional PD together with the (downturn) LGD gives the VaR of that exposure:

\[ \Phi(\Phi(PD, R)) \cdot LGD = \text{VaR}_\alpha. \]  

(1.12)

The formula then yields

\[ \Phi(\Phi(PD, R)) \cdot LGD - PD \cdot LGD = \text{VaR}_\alpha - \text{EL} = \text{UL}, \]

(1.13)

using Formulas (1.9) & (1.11).

The factor 12.5 used in the formula is the reciprocal of 8%, used to derive a RW from the calculated capital requirement. The factor 1.06 was introduced by BCBS in 2003 to offset the decrease in capital requirement which resulted from switching from a VaR formula to a UL only formula.

**Development of the Basel IRB credit risk model**

The IRB RW function used in CRR is taken from the Basel accords where it was developed by M. B. Gordy [7], based on a working paper of a later published article by O. A. Vasicek [12], which in its turn was built on a model by R. C. Merton [9].

The model specification was subject to an important restriction to fit supervisory needs. The model needed to be portfolio invariant, meaning that the risk weight should only depend on the asset, and not on the portfolio it was part of. This was a choice based on simplifying the task of calculating the capital requirement for institutions, and for the supervisors to verify them.

This criteria makes the diversification effects more difficult to take in account, but as a solution for this, the model is calibrated to a well diversified portfolio, where a deviation for this should be accounted for in the institution’s ICAAP process. The calibration of the diversification factor (correlation factor) differs between exposure classes, as empirical evidence [2] supports that larger firms (corporates) are more closely linked to the systematic risk factor than retail customers. Also, the model is calibrated such that lower PDs has a higher correlation factor than higher PDs. This is also based on empirical evidence, shown by the BCBS [2], and can be explained by the fact that with a higher PD, the specific risk components is higher, as the default depends more on the individual risk drivers than on the overall state of the economy.

The Merton model assumes that a default occurs for a counterparty when the value of its assets is less than of its debts.
Let the value $A_i$ of asset $i$ of a portfolio be represented as

$$A_i = \sqrt{\rho_i Y} + \sqrt{1 - \rho_i}\epsilon_i,$$  \hspace{1cm} (1.14)  

where $\rho_i$ is the correlation between asset $i$ and the systemic risk factor $Y$, and $\epsilon_i$ is the specific risk factor of asset $i$.

$Y$ and $\epsilon_i$ are i.i.d. $\mathcal{N}(0, 1)$, meaning that $A_i$ is normally distributed.

If we define a binomial variable $Z_i$ for each asset that takes the value 1 (meaning a default) with probability $p_i$ and the value 0 with probability $1 - p_i$ and remember that according to Merton [9] an asset defaults when its value goes below the value of its debts $D_i$ we can write that as

$$p_i = P(Z_i = 1) = P(A_i \leq D_i),$$  \hspace{1cm} (1.15)  

and as $A_i$ is normally distributed we get

$$p_i = \Phi(D_i),$$

or

$$D_i = \Phi^{-1}(p_i),$$  \hspace{1cm} (1.16)  

where using Formulas (1.15) & (1.16) gives us

$$P(Z_i = 1) = P(A_i \leq \Phi^{-1}(p_i)).$$  \hspace{1cm} (1.17)  

The threshold $D_i$ is thus a function of the default probability of an asset.

The value of an asset $A_i$ depends on the state of the economy $Y$. If we realize the systemic risk factor $Y = y$ we get

$$P(Z_i = 1 | Y = y) = P(\sqrt{\rho_i}Y + \sqrt{1 - \rho_i}\epsilon_i \leq \Phi^{-1}(p_i) | Y = y),$$

$$P(Z_i = 1 | Y = y) = \Phi\left(\epsilon_i \leq \frac{\Phi^{-1}(p_i) - \sqrt{\rho_i}y}{\sqrt{1 - \rho_i}}\right),$$

and knowing the distribution of $\epsilon_i$ we get
\[ P(Z_i = 1|Y = y) = \Phi \left( \frac{\Phi^{-1}(p_i) - \sqrt{\rho_i}y}{\sqrt{1 - \rho_i}} \right). \] (1.18)

To get a model for the fraction loss of a portfolio we define the fraction loss of a portfolio as

\[ L = \sum \text{LGD}_i Z_i. \] (1.19)

Assuming that the portfolio is infinitely granular and no loans are significantly larger than the rest, Vasicek [12] showed that if we set \( y = -\Phi^{-1}(\alpha) \) we can then get the \( \alpha \)-quantile of the fraction loss distribution as

\[ q_\alpha = \text{LGD}_i \Phi \left( \frac{\Phi^{-1}(p_i) + \sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1 - \rho_i}} \right), \] (1.20)

as \( Y \) is a standard normal distributed variable and we want \( y \) to take the value of the worst-case loss scenario with a confidence level \( \alpha \).

If we set \( \alpha = 0.999 \), subtract the EL and multiply it with 12.5 and 1.06 we get the IRB RW formula (1.10):

\[ \text{RW}_i = \left( \text{LGD}_i \Phi \left( \frac{\Phi^{-1}(p_i) + \sqrt{\rho_i}\Phi^{-1}(0.999)}{\sqrt{1 - \rho_i}} \right) - \text{EL}_i \right) \cdot 12.5 \cdot 1.06. \] (1.21)

**Retail Exposures**

For retail exposures, the IRB RW formula (1.10) is used together with the following formula for the parameter \( R \):

\[ R = 0.03 \cdot \frac{1 - e^{-35 \cdot \text{PD}}}{1 - e^{-35}} + 0.16 \cdot \left( 1 - \frac{1 - e^{-35 \cdot \text{PD}}}{1 - e^{-35}} \right). \] (1.22)

The formula gives a correlation factor between 0.03 and 0.16 depending on the PD.
**Defaulted Retail Exposures**

For defaulted retail exposures PD has to be set to 100%. If the ordinary formula for retail exposures is used, the RW goes to zero when PD goes to one:

$$\lim_{PD \to 1} RW = \lim_{PD \to 1} \left( \frac{\Phi^{-1}(PD) + \sqrt{R} \Phi^{-1}(0.999)}{\sqrt{1 - R}} \right) - LGD \cdot PD \cdot 12.5 \cdot 1.06$$

$$= [LGD - LGD] \cdot 12.5 \cdot 1.06 = 0.$$  \hspace{1cm} (1.23)

It does, though, exist systematic uncertainty in realized recovery rates for these exposures, so a RW of zero is not desired. The formula given by CRR for defaulted retail exposures is

$$RW = \max(0, LGD - EL_{BE}) \cdot 12.5.$$.  \hspace{1cm} (1.24)

Here $EL_{BE}$ reflects the best estimation of EL conditioned on that the exposure has defaulted.

If we use Formula (1.12) and let $PD \to 1$ we get that

$$\lim_{PD \to 1} VaR_\alpha = \lim_{PD \to 1} \Phi(PD, R) \cdot LGD = LGD.$$ 

By using Formula (1.9) and the condition to use $EL_{BE}$ instead of EL when the asset has defaulted we get

$$\lim_{PD \to 1} UL = \lim_{PD \to 1} VaR_\alpha - EL = LGD - EL_{BE}.$$ \hspace{1cm} (1.25)

Which means that Formula (1.23) aims to estimate the UL on a defaulted exposure.
Chapter 2

Problem Formulation

2.1 RWEA for purchased defaulted retail exposures

The problem addressed in this master-thesis is to find under which circumstances the SA gives a lower capital requirement than the IRB approach for purchased defaulted retail exposures (non-performing loans). It will also be investigated how well the two methods within the current CRR regulation captures the risk, as well as finding alternative methods to the SA and IRB approach of CRR.

The assumption is that an institution has holdings in pools of past-due retail loans that have been purchased from other institutions for a fraction (1-10%) of the face-value of the original loans. The business-case is that the institution can recover a much larger fraction of the face value of the pools of retail loans than the price it has paid for them. CRR states

“1. The unsecured part of any item where the obligor has defaulted in accordance with Article 178, or in the case of retail exposures, the unsecured part of any credit facility which has defaulted in accordance with Article 178 shall be assigned a risk weight of:

(a) 150%, where specific credit risk adjustments are less than 20% of the unsecured part of the exposure value if these specific credit risk adjustments were not applied;

(b) 100%, where specific credit risk adjustments are no less than 20% of the unsecured part of the exposure value if these specific credit risk adjustments were not applied.” [11, Article 127(1)].
CRR thus consider these holdings to be high-risk exposures in the SA, which means that the institution must assign a 150% or 100% RW to these holdings when it calculates the RWEA. In this master thesis it is assumed that the SCRAs are more than 20% of the NV and thus a RW of 100% is used when calculating the RWEA according to the SA.

First it will be investigated under what conditions the IRB approach can accomplish a lower RWEA than the SA. The different factors to take in account are: the definition of default, the estimation of LGD and EL, the SCRA made on the exposures versus the EL amount and the inclusion/non-inclusion of dilution risk. Second, an alternative method will be investigated.

In the following sections it will be explained how the different factors will affect the capital requirement.

### 2.2 The definition of default

The definition of default will affect what percentage of the total exposures that are considered to be in default. CRR gives the following definition of default:

“1. A default shall be considered to have occurred with regard to a particular obligor when either or both of the following have taken place:

(a) the institution considers that the obligor is unlikely to pay its credit obligations to the institution, the parent undertaking or any of its subsidiaries in full, without recourse by the institution to actions such as realising security;

(b) the obligor is past due more than 90 days on any material credit obligation to the institution, the parent undertaking or any of its subsidiaries. Competent authorities may replace the 90 days with 180 days for exposures secured by residential property or SME commercial immovable property in the retail exposure class, as well as exposures to public sector entities. The 180 days shall not apply for the purposes of Article 127.” [11, Article 178(1)].

As all the exposures purchased by the company are past due more than 90 days, they are considered to be in default when they acquire them. Furthermore, CRR states:

“5. If the institution considers that a previously defaulted exposure is such that no trigger of default continues to apply, the institution shall rate the obligor or facility as they would for a
non-defaulted exposure. Where the definition of default is subsequently triggered, another default would be deemed to have occurred.” [11, Article 178(5)].

This means that if the counterparty starts to pay according to the original payment plan, or to a new agreed plan, the exposure will no longer be considered to be in default, and will thus be treated as a performing retail exposure.

2.3 Estimation of LGD and EL$_{BE}$ for defaulted exposures

According to CRR the LGD and EL$_{BE}$ are calculated as follows:

“(i) if PD = 1, i.e. , for defaulted exposures, RW shall be

\[ RW = \max\{0, 12.5 \cdot (\text{LGD} - \text{EL}_{BE})\}; \]

where EL$_{BE}$ shall be the institution’s best estimate of expected loss for the defaulted exposure in accordance with Article 181(1)(h);

” [11, Article 154(1)].

This implies that the LGD is the LGD for the exposure before the default occurred. Article 181(1)(h) of CRR states:

“(h) for the specific case of exposures already in default, the institution shall use the sum of its best estimate of expected loss for each exposure given current economic circumstances and exposure status and its estimate of the increase of loss rate caused by possible additional unexpected losses during the recovery period, i.e. between date of default and final liquidation of the exposure;” [11, Article 181(1)(h)].

A big problem here is that if the institution purchased the exposure when it was already in default it means that they don’t have data to estimate the LGD, i.e. when the exposure was still performing. The institutions selling the exposures usually don’t give out this data, and if the didn’t use the IRB approach, they would only have calculated the EL, and it wouldn’t be possible to derive LGD from this.
2.4 Provisions made on the exposures versus the EL amount

The EL amount (EL \cdot EV_{IRB}) affects the own funds in the following manner.

“Institutions shall subtract the expected loss amounts calculated in accordance with Article 158 (5), (6) and (10) from the general and specific credit risk adjustments and additional value adjustments in accordance with Articles 34 and 110 and other own funds reductions related to these exposures. Discounts on balance sheet exposures purchased when in default in accordance with Article 166(1) shall be treated in the same manner as specific credit risk adjustments. Specific credit risk adjustments on exposures in default shall not be used to cover expected loss amounts on other exposures. Expected loss amounts for securitised exposures and general and specific credit risk adjustments related to these exposures shall not be included in this calculation.” [11, Article 159].

Article 166(1) of CRR states:

“1. Unless noted otherwise, the exposure value of on-balance sheet exposures shall be the accounting value measured without taking into account any credit risk adjustments made.

This rule also applies to assets purchased at a price different than the amount owed.

For purchased assets, the difference between the amount owed and the accounting value remaining after specific credit risk adjustments have been applied that has been recorded on the balance-sheet of the institutions when purchasing the asset is denoted discount if the amount owed is larger, and premium if it is smaller.” [11, Article 166(1)].

This means, in the case of purchased defaulted exposures, that the discount value, which is the difference NV and the AV of the exposure, should be treated as SCRA. Article 158(5) of CRR states

“5. The expected loss (EL) and expected loss amounts for exposures to corporates, institutions, central governments and central banks and retail exposures shall be calculated in accordance with the following formulae:

\[
\text{Expected loss(EL)} = \text{PD} \times \text{LGD}
\]
Expected loss amount = EL [multiplied by] exposure value.

For defaulted exposures (PD = 100%) where institutions use own estimates of LGDs, EL shall be $EL_{BE}$, the institution’s best estimate of expected loss for the defaulted exposure in accordance with Article 181(1)(h).” [11, Article 158(5)].

Further, Article 36(1) of CRR states

“1. Institutions shall deduct the following from Common Equity Tier 1 items:[...]

[...](d) for institutions calculating risk-weighted exposure amounts using the Internal Ratings Based Approach (the IRB Approach), negative amounts resulting from the calculation of expected loss amounts laid down in Articles 158 and 159[...];” [11, Article 36(1)].

This means that if the discount of the asset, i.e.

$$SCRA = NV - AV$$

is less than the $EL_{BE}$ amount, the difference, i.e.

$$\max(0, EL_{BE} \cdot EV_{IRB} - SCRA)$$

shall be deducted from the Common Equity Tier 1 items. This has the same effect as adding

$$\max(0, EL_{BE} \cdot EV_{IRB} - SCRA) \cdot 12.5$$

(2.1)

to the RWEA. In CRR you also have the following statement

“Tier 2 items shall consist of the following:[...]

(d) for institutions calculating risk-weighted exposure amounts under Chapter 3 of Title II of Part Three, positive amounts, gross of tax effects, resulting from the calculation laid down in Articles 158 and 159 up to 0.6% of risk-weighted exposure amounts calculated under Chapter 3 of Title II of Part Three.”[11, Article 62].

which means that if the SCRA is higher than the $EL_{BE}$ amount, the difference up to 0.6% of the RWEA should be added to the Tier 2 items, i.e.
\[
\min(\max(0, \text{SCRA} - \text{EL}_{BE} \cdot \text{EV}_{IRB}), \text{RWEA} \cdot 0.006)).
\]

In this master thesis, this effect is ignored, as it won’t affect the ratio of Common Equity Tier 1 items to own funds, and thus won’t affect the RWEA in the same way as when deducting an amount from the Common Equity Tier 1 items.

## 2.5 Inclusion/Non-inclusion of Dilution Risk

The dilution risk is defined in CRR as:

“(53) ‘dilution risk’ means the risk that an amount receivable is reduced through cash or non-cash credits to the obligor;” [11, Article 4(1)(53)].

This means basically that the exposure amount is reduced fully or partially, but is not considered to be in default. An example is when a purchased good or merchandise doesn’t live up to its standard, and has to be compensated for, as stated in the Basel II accords:

“Examples include offsets or allowances arising from returns of goods sold, disputes regarding product quality, possible debts of the borrower to a receivables obligor, and any payment or promotional discounts offered by the borrower (e.g. a credit for cash payments within 30 days).” [3, 85].

The dilution risk should thus not be applicable in this case, as the company is not responsible for the quality of any products bought. The effect of the dilution risk will thus be ignored.

It is also stated in CRR that:

“1. Institutions shall calculate the risk-weighted exposure amounts for dilution risk of purchased corporate and retail receivables in accordance with the formula set out in Article 153(1). [...]”

“5. The competent authorities shall exempt an institution from calculating and recognising risk-weighted exposure amounts for dilution risk of a type of exposures caused by purchased corporate or retail receivables where the institution has demonstrated to the satisfaction of the competent authority that dilution risk for that institution is immaterial for this type of exposures.”[11, Article 157(1) & 157(5)].

i.e. if the company can show that the dilution risk is negligible, it is not needed to calculate it.
Chapter 3

Method

3.1 Different calculation methods for RWEA

In this chapter we will investigate what RWEA the IRB approach would render for the exposures. We will also discuss alternative methods for calculating the RWEA. As a precondition for all methods, there is assumed that no dilution risk exists, or that is negligible, so no RWEA will be calculated for dilution risk. It is also assumed that no CRM is used, so

\[ EV_{IRB} = NV, \]

and

\[ EV_{SA} = AV. \]

If we combine Formulas (1.24) & (2.1) we get the following relation for RWEA for defaulted retail exposures in the IRB approach:

\[ RWEA = \left( \max(0, LGD - EL_{BE}) + \max(0, EL_{BE} - \left( \frac{NV - AV}{NV} \right) \right) \cdot 12.5 \cdot NV. \]  

(3.1)

The formula for RWEA in the SA is simply

\[ RWEA = AV \cdot RW = AV \cdot 1 = AV, \]  

(3.2)

as the RW for defaulted retail exposures, where the SCRA is more than 20% of the NV, is 100% in the SA.
To get a better picture on under what conditions the IRB approach will get a lower RWEA than the SA we need to look at methods to estimate ELBE and set up a few basic scenarios to see what the effect is on the RWEA.

3.2 The IRB approach

3.2.1 Estimation of LGD and ELBE

A problem with LGD for purchased defaulted retail exposures is that this parameter is not available for the institution, as the exposures are already in default when they are acquired. Therefore, in practice, this approach cannot be used. We will however look at a hypothetical case where this parameter exists for all loans.

3.2.2 Scenario 1

Under the first scenario, these conditions apply.

- All exposures are considered to be in default.
- The provisions made on the exposure are higher than the ELBE amount.
- The ELBE is higher than the LGD.

Under this scenario, the RWEA will be exactly 0 as suggested by Formula (3.1).

3.2.3 Scenario 2

Under the second scenario, these conditions apply.

- All exposures are considered to be in default.
- The provisions made on the exposure are higher than the ELBE amount.
- The ELBE is lower than the LGD.

Under this scenario Formula (3.1) reduces down to

\[ \text{RWEA} = \max(0, \text{LGD} - \text{EL}_{BE}) \cdot 12.5 \cdot NV. \]  

(3.3)

If we want to see when the IRB approach gets a lower or equal RWEA than the SA we get the relation
Figure 3.1: Max value of LGD-EL\textsubscript{BE} as a function of \( c \)

\[
\text{max}(0, \text{LGD} - \text{EL}_{BE}) \cdot 12.5 \cdot NV \leq AV; \tag{3.4}
\]

\[
\text{max}(0, \text{LGD} - \text{EL}_{BE}) \leq \frac{AV}{NV} \cdot 0.08. \tag{3.5}
\]

If we set \( AV \) to be a fraction of \( NV \) i.e. some constant \( c \) times \( NV \):

\[
AV = c \cdot NV, \quad c \in [0, 1],
\]

we get

\[
\text{max}(0, \text{LGD} - \text{EL}_{BE}) \leq c \cdot 0.08, \quad c \in [0, 1]. \tag{3.6}
\]

As can be seen in Figure 3.1, the positive difference \( \text{max}(0, \text{LGD} - \text{EL}_{BE}) \) can be at most 8% of the fraction \( c \), e.g. 0.8% when \( c = AV/NV = 10\% \).

The RWEA is very unstable, and only allows a small difference between \( EL_{BE} \) and LGD for all values of \( c \). As an example of an extreme effect of Formula (3.3) we set LGD to be 95%, \( EL_{BE} \) to be 0% and \( AV \) to be 5% of
NV ($c = AV/NV = 5\%$). We then divide Formula (3.3) by AV to see the relation between RWEA and AV:

$$\frac{\text{RWEA}}{\text{AV}} = \max(0, \text{LGD} - \text{EL}_{BE}) \cdot 12.5 \cdot \frac{1}{c} \quad (3.7)$$

$$= \max(0, 0.95) \cdot 12.5 \cdot \frac{1}{0.05} = 237.5.$$ 

This means that the RWEA is 237.5 times AV. To translate this into a capital requirement, which can be between 10.5-20.5\% of RWEA in CRR as stated earlier, we get a capital requirement between 24.9 and 48.7 times the AV, which does not make sense, as the maximum capital that can be lost on AV is simply AV.

### 3.2.4 Scenario 3

Under the third scenario, these conditions apply.

- All exposures are considered to be in default.
- The provisions currently made on the exposure are lower than the EL\(_{BE}\) amount.
- The EL\(_{BE}\) is higher than the LGD.

Under this scenario the whole difference between the provisions made, and the EL\(_{BE}\) amount becomes the RWEA:

$$\text{RWEA} = \max \left(0, \text{EL}_{BE} - \left(\frac{NV - AV}{NV}\right)\right) \cdot 12.5 \cdot NV. \quad (3.8)$$

If we want to see when the IRB approach gets a lower or equal RWEA than the SA we get the relation

$$\max \left(0, \text{EL}_{BE} - \left(\frac{NV - AV}{NV}\right)\right) \cdot 12.5 \cdot NV \leq AV,$$

$$\max(0, \text{EL}_{BE} \cdot NV - NV + AV) \leq AV \cdot 0.08,$$

$$\max(0, \text{EL}_{BE} \cdot NV - NV) \leq -0.92 \cdot AV,$$

$$\text{AV} \lesssim \max(0, NV - \text{EL}_{BE} \cdot NV) \cdot 1.087.$$
Since AV can’t take negative values we have the final result:

\[
AV \lesssim (NV - EL_{BE} \cdot NV) \cdot 1.087, \quad AV \geq 0.
\]  

(3.9)

The AV must thus be \(\lesssim 108.7\%\) of NV minus the EL\(_{BE}\) amount to have a smaller or equal RWEA as in the SA.

If we set AV to be a fraction c of NV:

\[
AV = c \cdot NV, \quad c \in [0, 1],
\]

we get

\[
c \lesssim (1 - EL_{BE}) \cdot 1.087,
\]

\[
0.92 \cdot c \leq 1 - EL_{BE},
\]

\[
EL_{BE} \leq 1 - 0.92 \cdot c.
\]

(3.10)

If we take the same example as in Scenario 2, where \(c=10\%\), we get from Figure 3.2 that \(EL_{BE} \lesssim 90.8\%\) to get a lower RWEA than in the SA.

### 3.3 Alternative approaches for calculating the RWEA

#### 3.3.1 Reasons for an alternative approach

The reasons for suggesting an alternative approach to calculate the RWEA for purchased defaulted retail exposures in the IRB approach are:

1. CRR is not clear on how to treat exposures that are in default when purchased,
2. Retail exposures don’t default in the same way as corporate exposures, for which the Merton model was developed, and thus not correctly captures the risks of the defaulted exposures.
3. The IRB model is very unstable for defaulted exposures.
Figure 3.2: Max value of $EL_{BE}$ as a function of $c$

Reason 1 have been treated in earlier sections. When the institution don’t have access to the LGD of the original exposure (before default), it is not possible to use the IRB approach for such exposures.

Reason 2 is based on the fact that a retail customers rarely default the way that a corporate customer does. A normal reason for a retail default is that the customers economy is temporarily distressed, and it is not unusual that the full debt is paid back at a later time. The only ways the a retail customer can be legally relieved from their debt is either by death, getting a Debt Relief Order (DRO) from the government or debt prescription, if the creditor hasn’t canceled the prescription period of the debt. The relief from debt by death of the customer can only be granted after all the remaining assets in the debtors estate are sold and used to pay off the debt. The default of a retail customer based on the 90 day past due criteria can thus be seen as a Soft Default (SD), and the case of the death of the customer, a DRO, prescription of the debt, or the fact that the customer cannot be found can be seen as a Hard Default (HD).

Reason 3 is based on the fact that only small changes in the level of LGD, $EL_{BE}$ or SCRA for the defaulted exposure gives large differences in the RWEA. In some cases the capital requirement resulting from the RWEA can be higher than the AV of the exposure, which was shown in Subsection 3.2.3.
3.3.2 The SD/HD approach

Jiří Witzany [13] showed that a full-loss-default can be modeled as

\[
\text{Loss} = \Phi \left( \frac{\Phi^{-1}(\text{PD} \cdot \text{LGD}) + \sqrt{R} \Phi^{-1}(0.999)}{\sqrt{1-R}} \right) \cdot NV. \quad (3.11)
\]

When he derives the formula, he models LGD as a binomial variable that either takes the value 1 or 0. This is a simplification, but Jiří Witzany [14] showed that the LGD for retail loans either take values close to 100% or values close to 0%.

We can think of LGD as a Probability of HD (PHD), remembering that LGD takes values close to 1 and 0. Consequently we think of the PD as a Probability of SD (PSD). Based on these changes, the formula becomes

\[
\text{Loss} = \Phi \left( \frac{\Phi^{-1} (\text{PSD} \cdot \text{PHD}) + \sqrt{R} \Phi^{-1}(0.999)}{\sqrt{1-R}} \right) \cdot NV. \quad (3.12)
\]

If we then let PSD converge to 1, and use the supervisory factors for calculating the RWEA, we get a model for an exposure that has made an SD:

\[
\text{RWEA} = \left( \Phi \left( \frac{\Phi^{-1} (\text{PHD}) + \sqrt{R} \Phi^{-1}(0.999)}{\sqrt{1-R}} \right) \right) \cdot NV \cdot 12.5 \cdot 1.06.
\]

In lack of a better model for calculating \( R \), which is out of scope of this master thesis, we use Formula (1.22) but replace PD with PHD:

\[
R = 0.03 \cdot \frac{1 - e^{-35 \cdot \text{PHD}}}{1 - e^{-35}} + 0.16 \cdot \left( 1 - \frac{1 - e^{-35 \cdot \text{PHD}}}{1 - e^{-35}} \right). \quad (3.14)
\]

If we as in Subsection 3.2.3 divide the RWEA with AV and set

\[
\text{AV} = c \cdot NV, \quad c \in [0, 1],
\]

we get the ratio between the two as

\[
\frac{\text{RWEA}}{\text{AV}} = \Phi \left( \frac{\Phi^{-1} (\text{PHD}) + \sqrt{R} \Phi^{-1}(0.999)}{\sqrt{1-R}} \right) \cdot \frac{12.5 \cdot 1.06}{c}. \quad (3.15)
\]
Figure 3.3: The ratio RWEA/AV

The result can be seen in Figure 3.3, where $c$ begins at 0.1, meaning that $AV = 0.1 \cdot NV$ as it otherwise explodes when $c$ approaches 0.

What can be seen is that at medium PHDs and low fractions, $c$, the RWEA gets many times higher than AV, meaning that the capital requirement will be higher than in the SA. If we assume that AV always will reflect $NV - SCRA$, i.e. $c = 1 - PHD$, we get the graph in Figure 3.4.

As can be seen in Figure 3.4, the ratio $\frac{RWEA}{AV}$ is an increasing function of PHD, so in the region where $c=1-10\%$ which is the business case of the institution, the SD/HD formula gives 9-10 times as as high RWEA as the SA.

### 3.3.3 The VaR approach

Another approach for calculating the RWEA can be calculating a VaR for the portfolio of purchased defaulted exposures. Here, the relative returns can be calculated for all loans and then the empirical distribution or a fitted distribution can be used.

A full VaR approach lies outside the scope of this master thesis, but to see what RWEA a portfolio would get with this approach we make 100
Figure 3.4: The ratio RWEA/AV where $c = (1 - \text{PHD})$

simulations of different portfolios with $m = 500, 1000, 1500, \ldots, 7500$ loans and $n = 12$ monthly i.i.d. $\mathcal{N}(0, 1)$ returns per loan.

An $m \times m$ variance-covariance matrix $\Sigma$ is constructed as

$$\Sigma = \frac{(\mathbf{R} - \mathbf{1}/n)^T(\mathbf{R} - \mathbf{1}/n)}{n - 1},$$

where $\mathbf{R}$ is an $n \times m$ matrix of monthly returns, and $\mathbf{1}$ is an $n \times n$ matrix of ones.

We are using $n - 1$ rather than $n$ in the denominator. This correction, known as Bessel’s correction, gives the unbiased covariance. This was shown by Friedrich Bessel and was described by Kenney and Keeping [8].

Then the monthly VaR with a confidence interval of 99.9% is calculated and then scaled to a yearly VaR by multiplying with $\sqrt{12}$, as

$$\text{VaR}_{0.999} = \Phi^{-1}(0.999)\sqrt{\mathbf{w}^T \Sigma \mathbf{w} \cdot \sqrt{12}}, \quad (3.16)$$

where $\mathbf{w}$ is a column vector of length $m$ with the relative weights $w_1, w_2, w_3, \ldots, w_m$ of the loans in the portfolio. The weights are i.i.d. $\ln \mathcal{N}(0, 1)$ and
Figure 3.5: The mean VaR of 100 simulations as a function of the number of loans in the portfolio

\[
\mathbf{w}^T \mathbf{1} \mathbf{w} = 1, \quad w_i \geq 0.
\]

We then take the mean of the VaR values calculated for each \( m \) to see the dependence of VaR on the diversification of the portfolio.

As we can see in Figure 3.5, the VaR decreases with the number of loans in the portfolio as is expected, because of diversification effects.

We can see that mean of the VaR is approximately proportional to \( 1/m \):

\[
\text{mean VaR} \propto \frac{1}{m}.
\]

If we then look at the converging value of VaR as \( 1/m \) converges to 0 we get that a well diversified VaR with a monthly standard deviation of 1 is approximately 0.2. To compare this to the RWEA in the SA we multiply it with 12.5 and AV:

\[
\text{RWEA} = \text{AV} \cdot \text{VaR} \cdot 12.5 = \text{AV} \cdot 0.2 \cdot 12.5 = \text{AV} \cdot 2.5.
\]
The RWEA in the SA is AV, so the RWEA in the VaR approach with a monthly standard deviation of 1 is 2.5 times larger than the RWEA in the SA. As the standard deviation is proportional to the VaR, the monthly standard deviation must be less than
\[ \frac{1}{2.5} = 0.4 \]
to get a lower RWEA in the VaR approach than in the SA. This is a huge monthly standard deviation.

The reason for using AV and not NV above is that the VaR is calculated relative to the AV and not the NV. We take consideration of the EL in assuming that the EL amount always is equal to NV-AV i.e.

\[ AV = NV(1 - EL) = NV - SCRA. \]

Note that this approach covers for both EL and UL on the AV, but still gives a smaller RWEA for relatively large standard deviations.
Chapter 4

Results and Discussion

4.1 IRB approach capital requirement

The problem to solve in this thesis was to investigate under what conditions the IRB approach renders a lower capital requirement than the SA. What was found is that in only a few cases the IRB approach is beneficial in regards to a low capital requirement. It was also shown that in some extreme cases the model gives such results that the capital that should be held to cover risk in the asset exceeds the value of the asset itself. This is of course nonsense for a non-leveraged asset and shows the need of alternative models to calculate the capital requirement for purchased defaulted retail loans if the SA shows to not reflect the risk in the assets in a proper way.

4.2 Alternative approaches

In addition to investigating under what conditions the IRB approach would render a lower capital requirement than the SA some alternative approaches were considered. The SD/HD approach showed to render a higher capital requirement in most cases that coincided with the business model of the institution. The VaR approach on the other hand showed to give lower capital requirements on a well diversified portfolio with rather high standard deviations. The VaR approach adds complexity on the task of calculating a capital requirement, and will be more complex for supervisors to verify, but nonetheless is an approach that is interesting to further develop. A complete VaR approach goes outside the scope of this thesis, an investigation was merely done to find if the approach would be beneficial for any portfolio at all.
4.3 Final words

In general, it was found that the IRB approach is not beneficial for defaulted retail exposures. Of the alternative approaches, only the VaR approach can be more beneficial than the SA. Beneficial both in the sense of the institution getting a lower capital requirement, but also in the sense of being more risk sensitive. The regulations are clearly in need of an update to be more risk sensitive and properly handle these type of exposures in the IRB approach.
Bibliography


[10] The European Parliament and the Council of the European Union. Directive 2013/36/EU of the european parliament and of the council of 26 june 2013 on access to the activity of credit institutions and the prudential supervision of credit institutions and investment firms,


