Optimising a small satellite for hard X-ray polarisation studies of gamma ray Bursts

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Abstract

Gamma ray bursts (GRBs) originate from extremely energetic extra-galactic events, and much is still unknown about them. Whereas the energy and time structure of GRBs have been studied extensively the past years, only a few polarisation measurements have been made on their initial, prompt emission. Determining the polarisation of GRBs will therefore provide a means to test proposed emission models. A small satellite (SPHiNX) has been proposed to measure the polarisation of X-rays from GRBs.

The performance of the detector is optimised using Monte Carlo simulations of a typical GRB source, in order to produce a more efficient design. First, the effect on the performance when varying individual parameters is studied, and then when using several changes in conjunction for the optimisation. The new design improves the primary figure of merit, the minimum detectable polarisation (MDP), by 7% over the initial design, and improves the secondary figure of merit, the effective area, by 22%.

Sammanfattning

Gammablixtar (GRBs) härrör från extremt energiska skeenden i andra galaxer, och ännu är mycket okänt kring dem. Emedan energi- och tidsstrukturen hos GRBs har studerats utförligt under senare år, har endast ett fåtal mätningar genomförts på polarisationen hos den initiella strålningen. Att fastställa hur polarisationen hos GRBs ser ut är därför ett sätt att testa föreslagna utsträlningsmodeller. En liten satellit (SPHiNX) har föreslagits för att mäta polarisationen i röntgenstrålar från GRBs.

Detektorns prestanda opimeras genom att simulera GRB-strålning, så att en effektivare design kan tas fram. Först undersöks hur de individuella parametrarna påverkar prestandan, och sedan används sammantagna ändringar för att optimera utseendet. Den slutgiltiga designen förbättrar det primära godhetstalet, den minsta detekterbara polarisationen (MDP), med 7% jämfört med den ursprungliga designen, och förbättrar det sekundära godhetstalen, den effektiva arean, med 22%. 
Acknowledgments

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Chapter 1

Introduction

Gamma ray bursts (GRBs) are exotic occurrences in other galaxies. Much is unknown about them, and one way in which to increase the knowledge of them is to measure the polarisation of gamma rays received from them. To do this, the Segmented Polarimeter for High Energy X-rays (SPHiNX) [1], which is a small satellite mission carrying a detector for this purpose, has been proposed. This thesis is concerned with the optimisation of said detector with respect to certain parameters, as to maximise the yield of trustworthy data. Monte Carlo simulations are carried out and analysed, in order to propose an optimised design.

In this section, cursory information is presented on the physical processes involved in GRBs, how they might produce polarised radiation, and how this may be detected. In chapter 2, a description of the data analysis is given, including details on the simulations, and which figures of merit are to be used for the optimisation. This optimisation is performed in chapter 3, where the results of simulating different designs are presented. Chapter 4 discusses these results, and the proposed design.

1.1 Gamma ray bursts

GRBs are brief, extremely energetic events that momentarily outshine most sources of high-energy radiation in the sky. As GRB events are distributed all over the celestial sphere and not confined to the galactic plane, their sources are held to be extragalactic, something which has also been confirmed by redshift measurements. Studies during recent decades have shed light on some aspects of these events, but the mechanisms behind GRBs are still not well understood. They are believed to originate from highly energetic cosmological events such as the formation of stellar black holes (BH), or the merger of binary systems of two neutron stars (NS), or a NS-BH binary, ultimately leaving a compact object [2][3].

As the driving mechanisms behind GRBs are poorly understood, one way in which to classify GRBs is instead by the duration of their radiative emission. Short GRBs last from several milliseconds to a few seconds. It has been theorised that these events might be caused by the collapse of rapidly rotating binary systems of two NS, or a NS and BH
binary. However, a number of GRBs have been confirmed to originate from the same site as supernova events, suggesting the stellar BH formation theory is more accurate in explaining them. These are typically long GRBs, and last from a few seconds to several thousands of seconds [3]. The classification is a crude one, and as the appearance of GRBs vary wildly within each category, there is reason to believe their underlying mechanisms are not easily delimited.

The intensity and time variability of GRB emissions have been thoroughly studied. A distinction is commonly made between the high-intensity, high-energy prompt emission in the beginning of a burst, and the decaying intensity and energies of the so-called afterglow which follows and ranges from X-rays to radio waves. The cause of the afterglow is more agreed upon than the cause of the prompt emission, and this prompt emission is what might contain more information on the inner processes of GRBs [4].

There are several proposed models for GRBs, and some contain different expectations on the two new data figures provided by polarisation measurements; the degree of polarisation (\(\Pi\)) and the angle of polarisation. The hope is that these polarisation data will narrow down the possible models, or eliminate some of them. This would help explain the processes responsible for GRBs. Some measurements of GRB polarisation have already been made [5][6][7], however analysing them gave no significant results [8][9].

This demonstrates the need for more measurements with dedicated GRB polarimeters in order to further the understanding of GRBs.

1.1.1 Emission models

The energy in the emission of GRBs come from the immense gravitational energy released in either the collapse of the core of a massive star, or the merging of two compact objects. Between these events and radiation being emitted, the newly formed compact object is thought to direct infalling material along the axis of rotation in highly relativistic jets of material, moving at speeds exceeding 99.999% of the speed of light, spread over an angle of 5-10 degrees. The radiation is thought to be produced in these jets [3]. A key factor in the emission mechanism is the magnetic field structure of the jet [10]. The GRB can be detected only if the jet is aligned with the line of sight to the observer.

The inner parts of the jets are opaque to photons, so photons do not escape until they reach the so-called photosphere. Here, the distinction between different models considering the degree of polarisation of the emitted radiation becomes relevant [10]. Below, a brief introduction to some popular models is given.

**Synchrotron model:** These emission models predict the emitted high-energy photons to be synchrotron radiation. This entails that electrons emit polarised photons when accelerated in magnetic fields present in the jet. Depending on the field structure, different models predict different angles of polarisation and \(\Pi\), ranging from high to low [9][10].

**Compton drag model:** This emission model suggests that inverse Compton scattering is the origin of radiation. This involves relativistic electrons colliding with
photons and imparting them with higher energy, as well as polarisation [11]. It has been suggested this model could give $\Pi \gg 40\%$ [10].

**Photospheric model:** This emission model suggests that part of the radiation originates from quasi-thermal radiation in the photosphere. If coupled with for example Compton drag, polarisation is expected, and could take values $\Pi \leq 40\%$ [8][9].

There are however a number of factors to consider when evaluating the significance of a polarisation measurement, including the time variability of the polarisation degree as well as the angle of polarisation, the velocity of the jet, and the off-axis angle [9]. Because these parameters create confines for the different models, novel polarisation measurements should aid in determining how GRBs work.

Though all these considerations are important for the analyses of actual GRBs, most lie beyond the scope of this report and will thus not be considered further. Of the two polarisation parameters, only the degree of polarisation will be considered when evaluating the performance of the detector, since the angle of polarisation is only interesting in relation to the geometric properties of an actual jet.

### 1.2 X-ray and gamma ray polarimetry

Polarimetry is the measurement and study of polarisation in waves, or electromagnetic radiation. In the case of gamma rays and X-rays, observations cannot be made on the surface of the Earth as the atmosphere is opaque to the relevant wavelengths.

Possible photon-matter interactions are the photoelectric effect, Compton scattering, and pair production. All can theoretically be used for polarisation detection. In the chosen detection process, Compton scattering is used, with the photoelectric effect for photon absorption. Both are described below.

#### 1.2.1 Photoelectric effect

When a photon interacts with an atom, the photon may be absorbed by one of the electrons of the atom. If the photon energy is larger than the binding energy of one of the electrons, the atom will be ionised and emit the electron. This effect, discovered by Albert Einstein in 1905, is called the photoelectric effect. The kinetic energy of the emitted electron is given by

$$E_k = h\nu - E_B,$$  \hspace{1cm} (1.1)

where $E_k$ is the kinetic energy of the emitted electron, $h$ is the Planck constant, $\nu$ is the frequency of the incident photon and $E_B$ is the binding energy of the emitted electron [12].
Figure 1.1: Geometric view of a Compton event. The incident photon has momentum $\vec{k}_0$ parallel to the z-axis and is polarised in the x-direction. It scatters at the origin and continues with momentum $\vec{k}$ at an angle $\theta$ relative to the z-axis, so that its projection in the xy-plane makes an angle $\phi$ relative to the x-axis.

In Compton interactions [13] the incident photon scatters off a free charged particle, usually an electron. Some of the photon energy is converted into kinetic energy of the electron, so the scattered photon has a lower energy than before the interaction.

The energy of the scattered photon depends on the polar scattering angle on the form

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos \theta)},$$

where $E'_\gamma$ is the energy of the scattered photon, $E_\gamma$ is the energy of the incident photon, $m_e$ is the electron mass and $c$ is the speed of light in vacuum [12]. The differential cross-section of Compton scattering, which is a measure of the probability of a scattering to occur, is dependent on the polar angle, $\theta$, and the azimuthal angle $\phi$ relative to the polarisation direction of the incident photon (see Figure 1.1). It is given by the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \frac{k^2}{k_0^2} \left( \frac{k}{k_0} + \frac{k_0}{k} - 2 \sin^2 \theta \cos^2 \phi \right).$$

where $r_e$ is the classical electron radius, $r_e$ is the ratio between the energy of the scattered photon and that of the incident photon $[11]$.

As the energy of a photon is proportional to its momentum ($E \sim k$), the Klein-Nishina formula can be rewritten as

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \epsilon^2 (\epsilon + \epsilon^{-1} - 2 \sin^2 \theta \cos^2 \phi),$$

where $\epsilon$ is the ratio between the energy of the scattered photon and that of the incident photon $[11]$;
Figure 1.2: Visualisation of the scattering distribution described by the Klein-Nishina formula with $\theta = \pi/2$ and energies ranging from 50 keV to 4 MeV. The view is along the initial propagation direction, with a vertical polarisation. A significantly higher probability to scatter at angles perpendicular to the polarisation axis for lower energies is seen.

$$\epsilon = \frac{E'_\gamma}{E_\gamma} = \frac{1}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos \theta)}.$$  \hspace{1cm} (1.5)

Equation 1.4 shows that the azimuthal scattering angle depends on the polarisation angle of the incoming photon. If a beam of light were to be observed, its polarisation degree $\Pi$ could therefore be determined by observing the anisotropy in the scattering angles, an example of which is shown in Figure 1.2. By making a histogram of the observed scattering angles as seen in Figure 1.3, the so-called modulation factor

$$M = \frac{C_{\text{max}} - C_{\text{min}}}{C_{\text{max}} + C_{\text{min}}}$$ \hspace{1cm} (1.6)

can be determined, where $C_{\text{max}}$ and $C_{\text{min}}$ are the maximum and minimum values of the fitted curve. The polarisation degree can then be found using

$$\Pi = \frac{M}{M_{100}},$$ \hspace{1cm} (1.7)

The angle of polarisation can also be extracted from the modulation curve, as the phase of the curve; the minima of the curve occurs at angles which coincide with the polarisation angle.
Figure 1.3: An example of a modulation curve. The histogram shows the 12 angle bins used. The curve is assumed to be sinusoidal when fitting. $C_{\text{max}}$ and $C_{\text{min}}$ are the values used in Equation 1.6.

1.3 SPHiNX

For the SPHiNX mission, a detector launched into orbit has been proposed to the Swedish National Space Board. If approved, this would put the instrument in orbit around the Earth so that proper polarisation measurements of X-rays in the approximate range of 30 - 300 keV can be made.

Because of limited dimensions, a weight limit of 15 kg has been set on the entirety of the detector. The parts that will not be varied in the simulations are considered fixed, and the weight budget allotted is therefore approximately 9 kg. This includes the plastic and BGO scintillators, the shielding, and the carbon fiber, all of which will be explained below.

1.3.1 Detector array

A schematic overview of the detector to be used in the SPHiNX mission is seen in Figure 1.4. The main component of the seven hexagonal cells is the plastic scintillator in the middle of each one. The choice of material should be directed by the goal of producing a large number of Compton events as well as relegating other interactions which might interfere. As such, a plastic has been selected which predominantly produces Compton events in the relevant energy range of 30 - 300 keV, see Figure 1.5, left. Surrounding each cell is a lining of bismuth germanium oxide (BGO) crystals, chosen in order to intercept the scattered Compton photons from the central plastic scintillator. This is done through the photoelectric effect, and the cross-section of these events can be seen in Figure 1.5, right.
Figure 1.4: View of the detector model. Seen here are the blue plastic scintillators, the red BGO crystals, the grey shield, and the white carbon fibres.

The plastic scintillator cells have further been divided into four smaller units, and each hexagonal side has two BGO crystal units. This has been done in order to increase the possible angles between units and thus the resolution of the measurements, as they will be used to plot polarimetric data, described in section 1.2.1.
Figure 1.5: Mass attenuation as dependent on energy and type of interaction, taken from [15]. Plastic scintillator to the left, BGO to the right. The energy range of the SPHiNX mission is 30 - 300 keV, shown with violet lines.

Surrounding the detector array is a shield, for blocking some of the interfering background radiation. It is made from three layers; lead, tin and copper, with lead on the outside and copper on the inside. The function of the lead is to halt incoming background radiation, which includes cosmic X-rays, since a reduction in their detection would yield cleaner measurements. The tin is used to absorb emitted photons from the lead after absorbing the background radiation, and the copper to absorb emissions from the tin. Though no shielding is present on the face of the detector, background coming from directions in which the instrument is not pointed will still be mitigated this way.
Both the plastic and the BGO crystals are scintillators, meaning they emit optical radiation after absorbing the X-ray radiation. These new photons, more numerous than the ones being absorbed, are then detected by PMTs (photomultiplier tubes) and APDs (avalanche photodiodes) underneath the components, as seen in Figure 1.6, left. The PMTs are rectangular, so the plastic scintillators, which have a different shape, are tapered down to this form further down in the detector. The shield extends 6 cm below the scintillators, covering the PMTs, APDs and electronics.

1.3.2 Detection principle

Given the selective spread of polarised photons in Compton scattering (Figure 1.2), all that is needed to produce a modulation curve is to distinguish in what direction the photons scatter. This is accomplished with the use of the PMTs affixed to the plastic scintillators and APDs to the BGO scintillators. When an interaction occurs in any scintillator, the light produced is detected and translated by the corresponding PMT or APD. Given a high enough time resolution, this can then be used to select the photons that produce exactly two events: optimally, one in a plastic component, and one in a BGO crystal. These should predominantly consist of photons undergoing Compton scattering and subsequent photoelectric absorption. The reversed case is possible, but unlikely, given the cross-sections of the interactions. Using the positions of the units involved, an angle can be obtained, given a pre-defined zero-point, see Figure 1.6, right. These discrete angles will only match the mean of all scatterings between two units, but the measurable polarisation degree should be unaffected, introduced errors notwithstanding.
Chapter 2

Methods

In this chapter, details are first given on the simulations and how relevant data are extracted, as well as some error corrections. Then the primary figures of merit in the optimisation are defined.

The performance analysis of the detector is based on Monte Carlo simulations developed by Dr. Maxime Chauvin at the Department of Physics using the Geant4 framework [17]. In the simulations, a model of the detector is constructed, and subsequently irradiated with photons one at a time. The resulting energy deposits of incoming X-rays in the detector are then recorded and used in the analysis.

The data analysis was performed using MATLAB [18]. As part of this project a complete library for processing data from the Geant4 simulations has been constructed.

2.1 Sources of simulated photons

In the simulation the GRB is represented by a disc with a radius of 20 cm emitting photons towards the detector. The energy distribution of the source photons is given by the power law \( P(E) \propto E^\alpha \), in the energy range 32 keV - 1 MeV. The lower limit is chosen to be 32 keV as this is the lowest possible photon energy that can produce a valid double event. To represent a realistic GRB emission, a photon flux of 10 cm\(^{-2}\)s\(^{-1}\) and the power law index \( \alpha = -1.05 \) are used [19].

The background radiation is assumed to only consist of cosmic diffuse gamma rays, the main contributor. It is represented by a radiating hemisphere with a radius of 50 cm, centred on the detector. The energy distribution of the background photons is a broken power law for cosmic diffuse gamma rays, and is given by

\[
P(E) \propto \left[ \left( \frac{E}{E_b} \right)^{\alpha_1} + \left( \frac{E}{E_b} \right)^{\alpha_2} \right]^{-1},
\]

where the break energy \( E_b \) and \( \alpha_{1,2} \) are constants. In this thesis an energy range of 5 keV - 1 MeV with a photon flux of 6.71 cm\(^{-2}\)s\(^{-1}\) is used.
2.2 Analysing simulation data

The simulation will simulate all the interactions of each photon individually. Therefore the output data will be a set of discrete photon interactions. The interactions are represented in the output data by the energies deposited in each scintillator. By examining these energies, the photon trajectories can be reconstructed.

When modelling a real scenario, each detected interaction is assumed to occur at the geometric center of the smallest parallelepiped enclosing the scintillator. For the BGO this corresponds to the scintillator itself. Using these positions, the scattering angle relative to a fixed axis of any Compton event can be calculated. By dividing the angle distribution into 12 bins, each spanning 30°, and adding each valid event to the appropriate bin, a histogram as seen in figure 1.3 is produced.

2.2.1 Event selection

Practical limitations set by the performance of the materials in the detector makes it impossible to detect too small energy deposits. Thus the adoption of detection thresholds is required. For the SPHiNX project, three different thresholds for individual events will be considered.

**Trigger threshold:** This is the minimum energy required to trigger the data acquisition in the detector electronics. These thresholds are set to 30 keV for plastic scintillator cells and to 50 keV for BGO. In principle this threshold should be exceeded by photoelectric absorption, especially in the BGO, as these events deposit more energy than Compton events.

**Hit threshold:** This is the minimum energy required for the event to be considered a hit; at lower energies the light yield from the scintillators is too low. These thresholds are set to 2 keV for plastic scintillator units and to 50 keV for BGO units. The levels are set to be low enough to detect Compton events, but still reject noise caused by the electronics.

**Upper discriminator:** This is the maximum energy allowed for the event to be considered a hit. This threshold is set to 300 keV for all units. If the upper discriminator is exceeded, all other events associated with that photon are discarded. A large part of these events are caused by background radiation and are therefore not relevant.

2.2.2 Eliminating systematic errors

The detector array is designed for maximum axisymmetry. The fact that the detector is finite in size will however introduce systematic errors in the measurements due to the detector geometry. Two such effects, and how they are accounted for, are described below.
Errors associated with the orientation of detector cells

The detector array consist of 7 different hexagonal detector cells, of which 3 are oriented in a certain direction, 2 more are rotated by an angle of 60° from this orientation, and another 2 rotated by an angle of 60° in the other direction (see Figure 1.4). Each orientation will produce a unique angle distribution for each cell. This is accounted for by weighting the contribution of each cell by

\[ w_{0°} = \frac{1}{3} \cdot \frac{7}{3} \quad w_{\pm60°} = \frac{1}{3} \cdot \frac{7}{2}. \]  

(2.2)

The factor \( \frac{1}{3} \) norms the weights so that the average weight applied is 1. The application of these weights can be corroborated by considering the modulation curve of an unpolarised beam (Figure 2.1). In the case of an unpolarised beam and a perfect detector, this curve should be constant for all angles.

![Figure 2.1: Comparison of the modulation curve before (left) and after (right) the application of weights correcting for systematic errors due to the rotation of detector cells. For clarity, only events going from plastic to BGO are regarded.](image)

Bias in Plastic to Plastic Events

When considering events that Compton scatter in a plastic scintillator unit and are then photoelectrically absorbed in another plastic scintillator unit, we observe a large bias in certain directions. This is due to the fact that the angle distribution is not uniform, and will depend on the geometry of the detector. This effect is shown in Figure 2.2.
The exact corrections needed to account for this are dependent on the geometry of the detector, and the weights will have to be recalculated for each change in the geometry. Therefore these events will not be considered when simulating series where the detector geometry changes, and will only be applied to the initial and final designs.

This bias can be eliminated by weighting each bin in such a way that an unpolarised beam will produce a flat modulation curve. Figure 2.3 shows the effect of these corrections.
2.3 Measuring performance

The minimum detectable polarisation (MDP) is defined as the lowest detectable degree of polarisation for which there is only a 1\% probability that it is detected by chance [16]. It is given by

\[
\text{MDP} = \frac{4.29}{M_{100}} \cdot S \sqrt{\frac{S + B}{T}},
\]

where \(M_{100}\) is the modulation factor for a completely polarised beam (see section 1.2.1), \(S\) and \(B\) are the source and background counting rates per second, and \(T\) is the observation time in seconds. This means that the MDP is proportional to \(T^{-1/2}\). The MDP is the primary figure of merit considered in this thesis.

It is assumed that optimising the MDP for on-axis incidence also optimises it for other angles. Therefore, only on-axis simulations will be performed, except when producing plots for the field of view, which are described below.

The effective area is the cross-section of producing a double event from which polarisation measurements can be made, and a high value is important for producing high-resolution measurements. It is a measure of the efficiency of the detector, and is defined as

\[
\text{Effective area} = \frac{\text{No. of valid events} \times \text{Area of source}}{\text{No. of photons emitted}}.
\]

Using the effective area, it is possible to define the instrument’s field of view, a measure of how much of the sky the instrument can observe in which it can make significant detections. It is defined as the opening angle of a cone within which the effective area is at least 50\% of the maximum value. The maximum value typically occurs at on-axis incidence. Examples of two effective areas as functions of the angle of incidence can be seen in Figure 2.4.

![Figure 2.4: The effective area plotted against the angle of incidence for two cases. The field of view is defined as twice the angle at which the effective area assumes half its maximum value. These angles are represented by the dashed lines. Although the blue curve would have a lower field of view, it has a consistently higher effective area.](image-url)
Chapter 3

Results

In this chapter, the performance of the detector is presented by applying the developed data analysis on the simulation data, and showing how the figures of merit vary over different parameters with respect to the initial design proposal.

First, the initial design proposal is analysed. Then, changes to individual parameters are studied, and lastly an optimisation is performed by changing individual parameters separately, and in one case by changing two parameters simultaneously.

All simulations are for a fully polarised GRB source with a polarisation angle of 90°, assumed to shine for 10 seconds and with a flux of 10 cm$^{-2}$s$^{-1}$, using the methods described in chapter 2. Excepting the simulations for producing field of view plots, all sources are located on-axis.

3.1 Analysis of initial design proposal

The initial design proposal is a preliminary optimisation, developed before this optimisation was undertaken. It is used as a default setting for the continued optimisation presented in this chapter. The initial design is given as

**Plastic scintillator:** Outer diameter of 11.0 cm and a height of 6.0 cm. The diameter is defined as the diameter of the smallest cylinder enclosing a hexagonal cell.

**BGO scintillator:** Thickness of 4 mm, a height of 6.0 cm and a width of 27.5 mm. The width is determined by the size of the plastic, and scaled accordingly in order to cover the cells.

**Shield:** Total height of 12.0 cm, and layers of 0.25 mm copper, 0.5 mm tin, 1.0 mm lead and 3.0 mm of carbon fiber. Additionally, 1.0 mm of carbon fiber is placed on the face of the detector.

The performance analysis of this design uses both the weightings described in section 2.2.2. Figure 1.3 shows the modulation curve of this design, and the red curve in Figure 2.4 shows the plot for the field of view. The results are shown in table 3.1. Note that
the mass is well below the specified mission limit of 15 kg. This is to allow for other equipment, such as electronics. The value 9078 g is therefore considered the maximum allowed mass of the detector.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDP</td>
<td>4.23% ± 0.20%</td>
</tr>
<tr>
<td>Effective area</td>
<td>84.55 cm² ± 0.17 cm²</td>
</tr>
<tr>
<td>Modulation factor</td>
<td>31.34% ± 1.43%</td>
</tr>
<tr>
<td>Source rate</td>
<td>(3.38 \times 10^4) s⁻¹ ± 39 s⁻¹</td>
</tr>
<tr>
<td>Background rate</td>
<td>(1.82 \times 10^4) s⁻¹ ± 57 s⁻¹</td>
</tr>
<tr>
<td>Polarisation angle</td>
<td>(89.77° ± 0.34°)</td>
</tr>
<tr>
<td>Field of view</td>
<td>122°</td>
</tr>
<tr>
<td>Mass</td>
<td>9078 g</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of the performance of the initial design proposal.

### 3.2 Analysis of individual parameters

In this section the effect of individual parameters on the performance will be considered. When one parameter is varied the others are kept at the values of the initial design proposal, unless a change is required to accommodate the changes in the studied parameter. For all parameters a 10 second GRB source with a flux of \(10\) cm\(^{-2}\)s\(^{-1}\) is used. Only the weighting of plastic to BGO events is used, described in section 2.2.2.

#### 3.2.1 BGO scintillator thickness

Here the the effect of changing the thickness of the BGO scintillator is shown. A range of \(1 - 15\) mm was studied. However a minimum limit of \(4\) mm is set due to the fact that below this limit not enough scintillating light will reach the photodiode, thus making real measurements unreliable. The effect on the performance of this parameter is shown in figure 3.1.

![Figure 3.1: Mass (left), effective area (middle) and MDP (right) varying over the thickness of the BGO. The green lines represent the values used in the initial proposal (Table 3.1), and the black line in the mass plot shows the mass limit.](image)
As BGO is a very dense material, increasing this parameter above the initial design proposal will increase the overall mass significantly.

### 3.2.2 BGO scintillator height

Here the effect of reducing the BGO scintillator height is shown. The height is measured from the face of the detector, and as such shortening the BGO will remove material from the bottom of the detector. The effect of this is shown in Figure 3.2. Although the BGO is very dense, the small changes in volume results in only modest mass reductions.

![Figure 3.2: Mass (left), effective area (middle) and MDP (right) when decreasing the height of the BGO from the default value at 60 mm. The shortening is made on the bottom part of the BGO crystals. The green lines represent the values used in the initial proposal (Table 3.1), and the black line in the mass plot shows the mass limit.](image)

### 3.2.3 Alternate scintillator material

Replacing the BGO scintillators with a new type of scintillator, GAGG (Gd$_3$Al$_2$Ga$_3$O$_{13}$) could be a possibility. This material has a superior light yield, but lacks the space heritage of BGO scintillators. A test performed under the same conditions as in section 3.1 found an effective area of $80.14 \text{ cm}^2 \pm 0.51 \text{ cm}^2$. This represents a less effective detector. However the superior light yield of the GAGG allows for a reduction of the trigger and event thresholds. These are selected to be 20 keV, compared to 50 keV for the BGO. When this is taken into account an effective area of $101.40 \text{ cm}^2 \pm 0.54 \text{ cm}^2$ is found, which is higher than that of the initial design.

In all other simulations, only BGO will be used.
3.2.4 Plastic scintillator width

Here the effect of changing the outer diameter of the plastic scintillator is shown. The width of the BGO and diameter of the shielding will change in order to accommodate the changes made to the plastic scintillator.

![Plastic scintillator width graphs](image)

Figure 3.3: Mass (left), effective area (middle) and MDP (right) when varying the diameter of the plastic scintillator cells. The green lines represent the values used in the initial proposal (Table 3.1), and the black line in the mass plot shows the mass limit.

3.2.5 Total detector height

Here the effect of changing the height of the detector is shown. In this context the total height is considered to be the height of both the plastic scintillators and BGO. The shielding will still extend an additional 6 cm beyond the scintillators.

![Total detector height graphs](image)

Figure 3.4: Mass (left), effective area (middle) and MDP (right) when varying the height of the scintillators. The green lines represent the values used in the initial proposal (Table 3.1), and the black line in the mass plot shows the mass limit.
3.2.6 Shield thickness

Here the effect of changing the shield thickness is shown. As the only purpose of the shielding is to reduce the effect of the background on measurements only background radiation will be considered.

Removing the shielding completely shows no appreciable negative effects on the measured performance of the detector. This is because the outer ring of BGO scintillators will absorb most of these photons. The single event count rate must however be studied as the electronics cannot handle too frequent events. Details of how the electronics process these events are beyond the scope of this project.

Two different shield designs will be tested. The first design is simply one layer of lead, the second a structure consisting of layers of lead, tin and copper. The tin and copper thicknesses will be held constant at 0.5 mm and 0.25 mm respectively. The initial design proposal utilises the second design with a lead thickness of 1.0 mm. In both cases only the lead thickness will be varied. The carbon fiber shielding will change size only to accommodate changes in the lead thickness.

![Graphs showing background rate and mass changes with varying lead thicknesses](image_url)

Figure 3.5: The upper plots show the total number of detected interactions with background photons, as the shield thickness is varied. In the left plot, only lead is used. In the right plot, lead, tin and copper are used, but only the thickness of the lead is varied. The lower plots display the mass changes. The green lines represent the values of the initial design proposal, and the black lines in the mass plots represent the mass limit.

The inner layers of tin and copper have a mass of 712 g. Thus for the same thickness of lead a mass reduction of 712 g is achieved if the tin and copper layers are removed.

Effects of possible radioactive activation of the lead from prolonged exposure to space conditions have not been considered.
3.3 Optimisation of width and height ratio

In this section, the performance as a function both of the total height of the detector and the diameter of the plastic scintillators is shown. In Figure 3.6 the MDP has been plotted against these parameters. For this test, a single layer of lead shielding with a thickness of 0.5 mm was used. As can be seen in Figure 3.5, this gives only a slight increase in the number of background-produced events, while saving 1406 g. This increases the margin for increments to the width and height. Note that the irregularities in the MDP are due to statistical fluctuations, as the standard error in the MDP is ≈ 0.2% in this range.

Since using more mass for increasing the size of the detector seems to generally decrease the MDP, points close to the line corresponding to the mass limit of 9000 g are of interest. As the lines corresponding to constant effective area closely follow the lines of constant mass, choosing optimal data points is mainly guided by the MDP. From...
Figure 3.6 two promising points were chosen for further tests, shown as red dots in the figure. One corresponds to a height of 66 mm and a width of 116 mm, and the other to a height of 63 mm and a width of 119 mm. Both weightings described in section 2.2.2 were used in the analysis. The calculated MDP of these designs was found to be 3.94%±0.15% and 4.05%±0.19%, respectively. Thus, no statistically significant difference in the MDP between these two possible designs was found. However, the effective area for the first point was found to be slightly better, 103.22 cm² ± 0.18 cm² compared to 102.68 cm² ± 0.18 cm² of the other point. The blue curve in Figure 2.4 shows the plot of the field of view for the first point, and a summary of the performance of this design is shown in Table 3.2. Although the field of view is lower than for the initial design, using the definition in section 2.3, the effective area is consistently higher for all angles, as seen in Figure 2.4. The field of view angle in parentheses is the angle at which the new design assumes half the maximum effective area of the initial design.

<table>
<thead>
<tr>
<th>MDP</th>
<th>3.94% ± 0.15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective area</td>
<td>103.22 cm² ± 0.18 cm²</td>
</tr>
<tr>
<td>Modulation factor</td>
<td>30.47% ± 1.09%</td>
</tr>
<tr>
<td>Source rate</td>
<td>3.98 · 10⁴ s⁻¹ ± 42 s⁻¹</td>
</tr>
<tr>
<td>Background rate</td>
<td>2.21 · 10⁴ s⁻¹ ± 62 s⁻¹</td>
</tr>
<tr>
<td>Polarisation angle</td>
<td>90.20° ± 0.40°</td>
</tr>
<tr>
<td>Field of view</td>
<td>116° (129°)</td>
</tr>
<tr>
<td>Mass</td>
<td>9074 g</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of the performance of the new design proposal.
Chapter 4

Discussion

When varying the BGO crystals’ thickness, the reason for the diminishing returns of increasing the thickness beyond 4 mm is likely because the material intercepts most of the scattered photons at this thickness. An increased thickness therefore does not markedly improve this interception, which is the basis of the performance change. This means the figures of merit remain unchanged, whereas the mass increases, suggesting this parameter is best left unchanged.

For the reduction of BGO height, the figures of merit worsen as the valid events are lessened. If not considering the mass decrease, it was not clear beforehand if this change would instead increase the modulation factor enough to instead improve the MDP, since removing interactions low in the detector also removes a more smeared angle distribution resulting from smaller $\theta$ angles (Equation 1.4). These correspond to photons hitting the BGO at more oblique angles as seen in Figure 1.6. Worth noting is that part of the relevant interactions may come from backwards scatterings of photons upwards in the detector, meaning the smearing effect these may produce remains unmitigated by removing BGO in the lower parts. Increasing the height of the BGO instead of decreasing it seems to not be able to produce better results either, as the curves both for the MDP and the effective area (Figure 3.2) flatten out towards the initial height, giving small improvements versus an almost linear increase in mass if trying this.

The effect of extending the BGO beyond the face of the detector has not been analysed. This could be favorable depending on how much back-scattering takes place in the relevant energy range, since this behavior is dependent on energy (Equation 1.4). If there is a significant amount of backwards scattering, a small increase in height above the detector might help catch a larger part of the clearly defined angle distribution, since the highest parts of the plastic scatter the most photons. This follows from the fact that scatterings higher up decrease the number of photons available for scattering in the lower parts.

When analyzing the results for using GAGG as the border scintillator material, it should be noted that given the material’s novelty, its qualities may not be adequately determined yet, leading to possible errors in the results. Further tests with more accurate parameters need to be performed in order to assess the material’s viability, given that it is also shown to withstand extended spaceflight. Even though the effective area would seem to increase, BGO is still used for all other simulations.
When considering the background interactions of shield variations, the limited effect on the total number of events may be because of the limited solid angle the shield covers when keeping it the same height as the rest of the detector. This would mean that most background-produced events come from radiation incident from above, and the shield not absorbing it.

Increasing the shield height is something to possibly test in future simulations. Since GRBs detected at greater angles of incidence will produce less relevant results, a small increase in shield height would thus hinder more background radiation, and only suboptimal GRB detections. This would improve the signal to noise ratio of GRBs located more favorably.

Furthermore, because of limitations in Geant4, the simulations do not properly account for radioactive decay in the shielding, which could interfere with the results. More comprehensive studies should be undertaken to assess the shield’s ability to reduce the number of realistic background-produced events, including varying the thickness of the different layers with respect to each other, to keep the mass of the shield minimal without significantly worsened results.

Varying the width of the plastic, the modulation factor decreases with increasing width, since increasing the area in which photons may scatter smears out the anisotropic angle distribution; having a very small area would lead to clearer groupings of scattered photons. The effective area improves as expected, since increasing the detector area increases the number of interactions. This also seems to overcome the worse modulation factor in the MDP calculation, as it improves as well. Since the mass grows quickly along with these improvements, there is no clear optimal preference.

Increasing the height of the detector leads to more photons scattering at low $\theta$ angles producing valid events. This corresponds to photons hitting the BGO at more oblique angles as seen in Figure 1.6. However, these photons have a less clearly defined angle distribution as prescribed by Equation 1.4, and therefore may worsen the overall modulation factor. The effective area improves, since increasing the length of distance traveled in the plastic increases the probability of interaction. This increase overcomes the worsening of the modulation factor, so that the MDP also improves.

Contrary to what was expected because of the rapid mass increase, increasing the width of the detector with respect to the increase in height seemed to improve the MDP when keeping the mass approximately constant. The final proposal is based on this result and less on how the effective area varies with these parameters, since it seems most dependent on mass, as evidenced by the similar shapes of the black and white curves in Figure 3.6. The more favorable of the two chosen points modestly improves the MDP by 7% over the initial design, with a significance level marginally lower than 1$\sigma$, but increases the effective area by 22%. The increase in effective area is important, as more detected photons allow better studies of time or energy evolutions of GRBs. As a consequence, the number of background events increase. This apparently does not adversely affect the MDP, but should be tested further in a realistic context to ensure saturation does not occur in the electronics.
Chapter 5

Summary and conclusions

In this thesis, the performance of a small polarimeter which has been proposed as a satellite mission is studied. Because of the small nature of the satellite, a mass restriction requires the detector to be of an optimal design. The detector uses plastic and BGO scintillators to measure polarization by detecting Compton scattering angles, and has an outer shield to block incoming background radiation.

The performance of the detector was studied using an existing model based on the Geant4 framework. A complete data analysis was developed in MATLAB to analyse the output data of the simulations.

The analysis of the detector was performed in two parts; firstly varying a single parameter to determine its effect on the performance, and secondly varying the total height of the detector in conjunction with the width. It was shown that a reduction in shielding is possible without a significant effect on the performance. Removing the layers of tin and copper, and halving the lead thickness increased the background count rate by 5% with a mass reduction of 1406 g. This extra mass was used to increase the total height and width of the detector. The performance of this new proposed detector compared to the initial design proposal is shown in Table 5.1.

<p>| | | |</p>
<table>
<thead>
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<td><strong>MDP</strong></td>
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<td>−6.9%</td>
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<td>103.22 cm² ± 0.18 cm²</td>
<td>+22.1%</td>
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</tr>
<tr>
<td><strong>Field of view</strong></td>
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<td>+5.7%</td>
</tr>
<tr>
<td><strong>Mass</strong></td>
<td>9074 g</td>
<td>±0.0%</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of the performance of the developed design proposal compared to the initial design proposal.
Bibliography

[1] M. Pearce et al. (The SPHiNX Collaboration), "SPHiNX: Segmented Polarimeter for High eNergy X-rays". Submitted to Swedish National Space Board funding call 2013-IS.