Experimental Demonstrator of the Uncertainty Principle
Degree Project in Engineering Physics, First Level (SA104X)

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Abstract

The goal of this project was to create an intuitive and clear demonstrator of the defining properties of quantum mechanics using single slit diffraction of light, which has quantum mechanical properties because of light’s wave-particle duality. In this report we will describe the process and thoughts behind our project of creating a portable demonstration of the uncertainty principle. By designing and building both a physical setup with a laser, a slit, mirrors, lenses, beam-splitters, attenuators and cameras, and developing software which displays images from the cameras in a clear user interface with calculations we hope that students from high schools and gymnasiums that visit Vetenskapens Hus at Alba Nova will learn something new while using the demonstrator.
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1 Introduction

After James Maxwell formulated his equations that combined electricity and magnetism into a single theory at the end of the 19th century, the general consensus became that light travelled as an electromagnetic wave. It was not until Albert Einstein posted his theory on the photoelectric effect that the idea that light behaves both as a wave and as particles gained momentum again.

Our demonstration is based on photon diffraction, the natural phenomena that allows photons to diffract in a single slit just like a wave to create a diffraction pattern. Combining this with the uncertainty principle of quantum mechanics, we can analyse our results and compare it to the inequality $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ derived by Earle Hesse Kennard in 1927 [1]. The basis for this principle came from the groundbreaking work in quantum mechanics by German physicist Werner Heisenberg who earlier that same year gave the argument that such a limit for the deviation in position and momentum should exist.

1.1 Objective

The primary goal of this project is to create a portable demonstrator of the uncertainty principle of quantum mechanics, $\sigma_x \sigma_p \geq \frac{\hbar}{2}$, with the help of single slit diffraction of light. This demonstration is meant for showing high school students real quantum effects and educating the basic principles of quantum mechanics, which are visually clear in the diffraction pattern from the single slit experiment.

To show these effects, the demonstration is divided into two parts. First we create the optical experiment, with a Helium-Neon laser, a slit, mirrors, lenses, attenuators and detectors. This setup shows the diffraction pattern on a white paper. Connected to the detectors we also have a computer with software capable of displaying the detected images and calculating the essential properties from these patterns which can then be related to the uncertainty principle.

The tasks in this project were divided as follows. Philip was in charge of the optical experiment and writing the theoretical background, while Anders handled the programming of the software and the measurement analysis. It should be noted that we have worked together on this project and helped each other with our different tasks.
2 Theory

2.1 The Uncertainty Principle

The uncertainty principle of quantum mechanics describes the inherent uncertainties in a particle’s properties such as position and momentum due to particles’ wave nature. What the principle states is that it is impossible to know or prepare precisely both a particle’s position $x$ and its momentum $p_x$ along the same axis. There is a strict lower limit for the product of the uncertainties of these two properties: $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ with $\hbar$ being the reduced Planck constant $\frac{\hbar}{2\pi} = 1.055 \cdot 10^{-34}$ Js. In physics, this uncertainty $\Delta$ is defined as the standard deviation $\sigma$, which is the notation we will use in this report [2].

This uncertainty relation can be seen in a single slit diffraction experiment. Sending light through a narrow slit, the light is diffracted and a diffraction pattern is seen on a screen on the other side. Making the slit less wide, thus reducing the uncertainty of the photons’ position, makes the diffraction pattern wider indicating an increased uncertainty of their momentum in the direction of the slit width. We would expect this diffraction to occur due to the wave nature of light. The interesting thing is this: If you would send each photon by itself through the slit, it would yield the same result. The photons hitting the screen would create a diffraction pattern anyway if you would accumulate their impact positions. This shows that photons (and particles in general) have an inherent uncertainty in position and momentum.

2.2 Theoretical Value of Uncertainty

The light intensity of the laser used in the project can be seen as having the form of a Gaussian function. If you open the slit completely so that none of the light is blocked, we can approximate the intensity function over the horizontal axis ($x$) as a Gaussian if we average the intensity value of the beam over the vertical axis ($y$). Suppose that this Gaussian intensity function (which is proportional to the probability distribution of the light) has a standard deviation of $\sigma_x$. From this follows that

$$I(x) \propto e^{-\frac{x^2}{2\sigma_x^2}}.$$ 

Since the complex amplitude of the wave $A(x)$ is proportional to the square root of the intensity we get:

$$A(x) \propto \sqrt{I(x)} \propto e^{-\frac{x^2}{4\sigma_x^2}}.$$ 

Theory [3] shows that the far field intensity function can be found by applying the Fourier transform to this complex amplitude function of the source,
2.2 Theoretical Value of Uncertainty

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with the frequency variable equalling $\xi = \frac{x'}{\lambda}$, with $z$ being the distance from the source to the measuring point on the $z$-axis and $x'$ being the distance from the $z$-axis at this point.

$$A(x) \propto e^{-\frac{x'^2}{4\sigma_x^2}} \xrightarrow{\mathcal{F}} U(\xi) \propto e^{-4\sigma_x^2(\pi\xi)^2} \rightarrow I(\xi) \propto U(\xi)^2 \propto e^{-8\sigma_x^2(\pi\xi)^2},$$

where $U(\xi)$ is the complex amplitude of the diffracted wave depending on $\xi$ and $I(\xi)$ is the related intensity which both also have the form of a Gaussian function. Comparing this to the original Gaussian function, we can see that $\xi$ has a standard deviation of $\sigma_\xi = \frac{1}{4\pi\sigma_x}$.

A photon has the linear momentum

$$p = \hbar k,$$

with $k$ being the wave vector $\frac{2\pi}{\lambda}$. The momentum along the $x$-axis, $p_x$, for the diffracted beam can be calculated using

$$p_x = p \sin \theta = \hbar \frac{2\pi}{\lambda} \sin \theta$$

with the average being zero, see Figure 1. Since $x' << z$ for the far field,

$$\frac{x'}{z} \approx \sin \theta.$$

This means that $\xi$ can be rewritten as

$$\xi \approx \frac{\sin \theta}{\lambda} \rightarrow p_x = 2\hbar \xi \pi \rightarrow \sigma_{p_x} = \frac{\hbar}{2\sigma_x}.$$  

As mentioned in section 2.1 we are only interested in the deviation of the momentum along the same axis as the deviation of the position, which is along the $x$-axis. Using the values for the standard deviations in the uncertainty inequality $\sigma_x \sigma_{p_x} \geq \frac{\hbar}{2}$ we find that this result represents the equality. This means that using a Gaussian beam one saturates the uncertainty inequality.

If we instead close the slit, the intensity function at the slit looks more like a rectangular function. This would return a diffraction pattern on the form of a sinc function squared,

$$I \propto \text{sinc}^2(w\xi) = \left(\frac{\sin \pi w\xi}{\pi w\xi}\right)^2$$

by Fourier transforming, where $w$ is the width of the slit. Calculating the standard deviation of the normalised $I(\xi)$ one will find that the integral does not converge. This is because $\xi$ is not limited and

$$\left(\frac{\sin \pi w\xi}{\pi w\xi}\right)^2 \xi^2 \propto \sin^2 \pi w\xi.$$
2.3 Uncertainty of Laser Source

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in the integral does not converge when \( \xi \) goes toward infinity. What you need to do is look at the angular distribution of the intensity (since \( \theta \) is limited between \( -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \)), which you can get by integrating the complex amplitude of the light over the slit width.

2.3 Uncertainty of Laser Source

In an ideal case the beams from a light source would be parallel, but in reality, due to the uncertainty principle, any finite source has a divergence. The specification of the laser has two interesting parameters for the uncertainty; beam diameter (\( \frac{1}{\sigma_x} \)) of 0.65 mm and beam divergence (\( \frac{1}{\sigma_\theta} \)) of 1.24 mrad. The beam radius (0.325 mm) \( r_0 \) is half of the beam diameter and the beam divergence in one direction (0.62 mrad) from the z-axis is half of the stated beam divergence. The description (\( \frac{1}{\sigma_x} \)) means that where the absolute value of \( x \) (or \( r \) in the two dimensional case) equals the beam radius, the intensity of the beam is \( e^{-2} \) times the maximum value which occurs at the center of the beam. Remember that a Gaussian function with a standard deviation of \( \sigma_x \) is proportional to \( e^{-\frac{x^2}{2\sigma_x^2}} \). We can use this to derive \( \sigma_x \):

\[
e^{-2} = e^{-\frac{r_0^2}{\sigma_x^2}} \rightarrow 2 = \frac{0.325^2 \text{ mm}^2}{2\sigma_x^2} \rightarrow \sigma_x = 0.1625 \text{ mm}
\]

We can find \( \sigma_{p_x} \) in the same way by looking at the beam divergence. First we can derive \( \sigma_\theta \) in the same way as above, which yields \( \sigma_\theta = 0.31 \text{ mrad} \). In a similar manner as in the previous section we can find \( \sigma_{p_x} \) by approximating \( \theta \approx \sin \theta \rightarrow \sigma_\theta \approx \sigma_{\sin \theta} \). The deviation in momentum \( \sigma_{p_x} = \sigma_\theta p = \sigma_\theta \hbar \frac{2\pi}{\lambda} = 3078\hbar \). Multiplying this by \( \sigma_x \) we get the uncertainty value \( \sigma_x \sigma_{p_x} = 0.50018\hbar \approx 0.5\hbar \), which is the best result the laser can have. This value can be compared to our measured value \( \sigma_{p_x} = 3300\hbar \) for a fully opened slit that is reported in the Results section below.
3 Method

Our demonstrator consists of two parts. An optical setup which consists of the laser and optical components and will provide the results, and a software that will do calculations and display clear images of the diffraction pattern and the slit.

![Photograph of the optical setup.](image)

3.1 Optical Setup

The optical setup of our demonstration consists of a Helium-Neon laser with a wavelength of 632.8 nm, an adjustable slit, two lenses with focal lengths of 15.3 mm and 200 mm, several mirrors and beam splitters to direct the light, and two CMOS-cameras. These are all placed and mounted onto a portable optical board, see Figure 2. The purpose of this setup is to diffract the light beam from the laser and direct the light to the two cameras. One camera will show the near field image of the slit (with next to no diffraction) to provide a clear representation of the slit width while the other camera will display the diffraction pattern. We also have the pattern displayed on a white paper/board. To see a general schematic of how the different objects are placed, see Figure 3. For distances and more specific details, see the next section. Here is a list of the different objects in the demonstration:
3.1 Optical Setup

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Helium-Neon Laser
A laser that emits a TEM_{00} mode of light of wavelength \( \lambda = 632.8 \) nm. Has a beam diameter of 0.65 mm (1/2) and a beam divergence of 1.24 mm (1/2).

Adjustable slit
A slit whose width can be adjusted. Used range (affected by size of light source) = 0 – 1000 \( \mu \)m.

Beam splitters
Objects of glass that reflect parts of the light and transmits the rest to divide the beam into two. 1% – 10% reflection at 45\(^\circ\) angle of incidence depending on the polarization of the incoming light.

Mirrors
Reflects light.

Attenuators
Reduces the intensity of the beam by factors 10\(^2\) or 10\(^4\) so the cameras are not saturated, see Figure 3.

Biconvex lenses
Focuses the light as wanted.

CMOS-Cameras
Captures images. 1280 \( \times \) 1024 pixels, 6.66 mm \( \times \) 5.32 mm.

Figure 3: Schematic over the optical setup.
3.1 Optical Setup

3.1.1 Projecting Images to the Cameras

As mentioned above the demonstration includes two cameras. The two cameras display two different images; one displays an image of the slit, while the other displays an image of the far field diffraction pattern. To project the slit image to the first camera we used a small lens with a focal length $f = 15.3$ mm to magnify the image. Due to the difficulty in measuring the distance between the slit and the lens we first placed those two and adjusted the camera until the image was in focus. The distance between the source (slit) and lens, $S_1$ and the distance between the lens and the image (camera) $S_2$ have to satisfy the formula for thin lenses:

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f},$$

with the magnification being $S_2 / S_1$. When the slit is very narrow it is possible to notice some diffraction in the image of the slit. This is due to light at high diffraction angles that does not go through the lens since it is small, 9.24 mm in diameter, which means that some spatial harmonics of the slit image are missing. This occurs less when the slit is wider since the light is less diffracted. The schematic for this can be seen in Figure 4.

Figure 4: Schematic over the light travelling to the near field camera. Does not include beam splitters, mirrors or attenuators.

Even though the exact values for $S_1$ and $S_2$ are not known the approximate magnification can be derived by looking at the width in pixels on the image, the pixel width specification of the camera and the slit width that can be read from the control on the slit. The error in this method comes from the slit control, which from testing and comparing to the image does not seem to be increasing the width linearly as it should be. The difference is small though. More on this in the Software section.

The second camera will display an image of the diffraction pattern. The diffraction phenomena is called far field diffraction or *Fraunhofer diffraction*.
(after German physicist Joseph von Fraunhofer) if the beams from the slit to the screen (in this case the camera) can be seen as parallel [4]. In the setup there is a lens to help projecting the image since Fraunhofer diffraction also occurs in the focal plane of a biconvex lens, see figure 5. We place the slit approximately at the focal point of a lens with a focal length of 200 mm. As one can see in figure 5, this ensures that the beams will be parallel if diffraction occur. If the slit is wide and the beams are parallel from the start (ideal case), they will project an image at the focal point of the lens where the camera is placed to emulate a far field (since there is next to no diffraction and the angle is zero). Since the distance between the slit and the lens is so much greater than the deviation $x'$ for the diffracted beams the beams can be seen as parallel and our pattern can be seen as far field diffraction. This also means that we can approximate the deviation angle $\theta$ with $\theta \approx \sin \theta \approx \tan \theta = \frac{x'}{d}$ where $d = f$ is the distance between the slit and the lens.

Figure 5: Projection of the far field diffraction pattern.
3.2 Software

A large part of the project was the development of the software to acquire the data from the images provided by the cameras and displaying this data in a simple way for the user. This was done with the help of the programming language LabView, because it is very useful for image processing and user friendly. LabView is a graphical programming language that works with icons that act as functions with different input and output and carries out specified operations. For more information we recommend the National Instrument’s webpage [5]. This section provides an explanation of the user environment and then goes into more detail on how the calculations of the targeted values were done from the provided images.

\[
\sigma_x = \sqrt{\langle \Delta x \rangle^2} = \frac{72 \mu m}{100 \mu m} \quad \sigma_p = \sqrt{\langle \Delta p \rangle^2} = \frac{16954 h}{N s}
\]

Figure 6: Image of the user environment.

3.2.1 Description of the User Environment

Figure 6 shows what the user sees on the screen while running the program. On the left side the image of the near field camera is shown and below it is a plot of the light intensity in that image, this will be described in more detail under the section Calculating the Standard Deviation of the Position. Above the image is the calculated value for the standard deviation of the position, \( \sigma_x \). To the left is the "Stop"-button, that stops the program, and the "Options"-button that shows or hides several control functions, such as the ability to change the wavelength \( \lambda \) used in the calculations, moving the region for measuring the light intensity in the near field image and an
indicator of the measured slit size, given in μm. On the right side is the far field camera image with the related intensity plot, which is explained in the section Calculating the Standard Deviation of the Momentum and above it is the value of the standard deviation of the momentum, \( \sigma_{p_x} \). At the top is the product \( \sigma_x \cdot \sigma_{p_x} \), which is the Heisenberg uncertainty and this value is compared to \( \frac{\hbar}{2} \).

![Acquired image of the slit with the defined region of interest.](image)

Figure 7: Acquired image of the slit with the defined region of interest. The rings in the image are caused by dust on the camera and other components in the setup.

### 3.2.2 Calculating the Standard Deviation of the Position

In the near-field camera an image of the slit with characteristic sharp edges is shown, see Figure 7. We wish to measure the amount of light passing through the slit and use this information to determine the width of the slit and also the uncertainty of the light’s position, which is described by the standard deviation. This is done by measuring the intensity in a rectangular region of interest, characterised by the red lines, along horizontal pixel lines and then added and averaged vertically, which gives sharp edges. By determining the distance between the sharp edges we can translate this to the width of the slit and compare it with the value of the slits own meter. The proportional constant was determined by testing a large value, 700 μm, for the slit width, adjusted with the manual slit size control, and comparing it to the slit width in pixels on the image, a large value was chosen to minimize the error. To determine the standard deviation Equation (1) was used.

\[
\sigma_x = \sqrt{\sum w(x_i)(x_i - x_0)^2}, \quad \text{and} \quad (1)
\]
3.2 Software

\[ w(x_i) = \frac{I(x_i)}{\sum I(x_i)} \tag{2} \]

where \( x_i \) represents each pixel in the area of interest, \( w(x_i) \) is the probability weight function and \( x_0 \) is the average position of the light calculated with the help of Equation (3).

\[ x_0 = \sum w(x_i)x_i. \tag{3} \]

3.2.3 Calculating the Standard Deviation of the Momentum

![Acquired image of the diffraction pattern with the defined region of interest.](image)

The second detector provides an image of the diffraction pattern, Figure 8 and this is used to measure the uncertainty of the momentum. By studying the image a value of the standard deviation of the momentum is acquired through a very similar method as for the near-field. The program finds the image of the diffraction pattern by scanning the image for high intensities and creates a region of interest around it and calculates the intensity the same way as before. Figure 9 displays the average intensity in the region of interest and in it the diffraction pattern is clearly visible even though it might not be in the direct image of the diffraction pattern. In the image the \( y \)-axis is in logarithmic scale so that the high orders would be easier to spot, but it gives the impression that their intensity is much higher compared to the 0:th order than it actually is. From the average intensity the standard deviation can be derived, but on the original image the light has a position in pixels and to change the position unit from pixels to micrometers the position value was multiplied by the pixel size of 5.2 \( \mu \)m. This translates into the momentum of the light, along the same axis as the position, by using Equation (4), where \( \lambda \) is the wavelength of the light from the laser and \( \theta \) is the diffraction angle.
Figure 9: Average intensity graph for the diffraction image.

\[ p_i = \frac{2\pi \hbar}{\lambda} \sin \theta_i. \] \hfill (4)

For the same reasons as in the example with the Gaussian wave we know that the angles present are very small and that \( \sin \theta \approx \frac{x'}{d} \), where \( x' \) is the same as the example and \( d \) is the distance between the far field lens and the slit. Equation (5) gives us the average momentum of the light, \( p_0 \) and this is then used in Equation (6) to get the standard deviation of the momentum in the far field, \( \sigma_{p_x} \).

\[ p_0 = \sum p(x'_i)p_i, \] \hfill (5)

\[ \sigma_{p_x} = \sqrt{\sum w_p(p_i)(p_i - p_0)^2}, \text{ and} \] \hfill (6)

\[ w_p(p_i) = \frac{I(p_i)}{\sum I(p_i)}. \] \hfill (7)
4 Results

The software provides us with four measurable parameters, the slit size, the standard deviation in the position and the momentum and their product. To test the demonstration we measured these parameters for different slit sizes to see if the behaviour of the measured values corresponds to theory. The results are displayed in Table 1.

Table 1: Measurements of $\sigma_x$, $\sigma_p$, and the Heisenberg Uncertainty.

<table>
<thead>
<tr>
<th>Slit size [µm]</th>
<th>$\sigma_x$ [µm]</th>
<th>$\sigma_p \hbar^{-1}$ [Ns]</th>
<th>$\sigma_x \cdot \sigma_p \hbar^{-1}$ [Js]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully opened</td>
<td>214</td>
<td>3300</td>
<td>0.71</td>
</tr>
<tr>
<td>900</td>
<td>195</td>
<td>4100</td>
<td>0.80</td>
</tr>
<tr>
<td>800</td>
<td>183</td>
<td>4700</td>
<td>0.86</td>
</tr>
<tr>
<td>700</td>
<td>167</td>
<td>5200</td>
<td>0.87</td>
</tr>
<tr>
<td>600</td>
<td>150</td>
<td>6200</td>
<td>0.93</td>
</tr>
<tr>
<td>500</td>
<td>129</td>
<td>8200</td>
<td>1.06</td>
</tr>
<tr>
<td>400</td>
<td>106</td>
<td>11600</td>
<td>1.23</td>
</tr>
<tr>
<td>300</td>
<td>79</td>
<td>16500</td>
<td>1.30</td>
</tr>
<tr>
<td>200</td>
<td>54</td>
<td>24600</td>
<td>1.32</td>
</tr>
<tr>
<td>100</td>
<td>24</td>
<td>39200</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The results will be discussed under section 5.3 and an explanation of the odd value for the smallest slit size will be given there.

5 Discussion

Under this section we will discuss and motivate our choices during the process of making this demonstration, starting with the design of the demonstration, both of the physical design and the design of the software. Subsequently we discuss and try to draw conclusions from the acquired results.

5.1 Physical Design

Setting up and creating the demonstration took a lot of time and work since everything needed to be aligned and much thought went into the design of the setup. Since the objective was to show quantum mechanical effects for high school students, the focus of the design was making it clear, understandable and interesting, but also easy to operate. For example: In the beginning we thought only one camera was sufficient for making the demonstration, but after some consideration and for reasons such as less user interactions, the setup was changed to use two cameras instead. This change made it easier to display all the information and thereby it is probably easier for the students to understand and study this phenomenon. At the start the
intensity of the light from the laser was too strong. This problem was solved with the help of attenuators and by trial and error the appropriate value for the attenuators was found. Accidentally we also found out that a rotation of the laser, in the axis direction of the light, the intensity of the light could be changed, probably because of the polarization of the light, which affects the amount of reflected light in the beam splitters.

The sensitivity in the alignment of the components made it important to make sure that everything was secured properly, since the setup is supposed to be portable. While detaching the portable optical board from the table we found that the alignment was disturbed by a small amount, but the images were still centred on the cameras.

Because of the sensitivity of the components it was of great importance that the demonstration required minimal manual handling from the user. In the current form, the only thing that needs to be touched by the user (excluding the computer) is the control for the slit width.

5.2 Software Design

In the design of the software the three focus points, clear, understandable and interesting, become very apparent. Important figures and values, such as $\sigma_x$, $\sigma_p$ and their product, are centred and large. Only relevant information is displayed and in the standard view no user input is available, all to make it clear and easy to understand. Other interesting, but less important, information and useful controls are hidden under “Options”. To make it easier for the demonstrator, an automatic definer of the region of interest and also an automatic adjuster of the gain for the far field camera were implemented. This will give the demonstrator free hands to focus on explaining the problem and phenomenon showed on the screen.

The way of calculating the standard deviation in both position and momentum, averaging over several horizontal lines in a region of interest, instead of for example only choosing one line, was preferred since then all the light could be studied, which gives a more correct image of the distribution of the light, and also the error in for example measuring the slit size gets smaller. On the other hand all these calculations causes a decrease in performance speed, but this was not very noticeable and thereby acceptable.

For the slit we needed to compensate for the background noise, because even though the dark pixels in the image (those outside the slit image) had a very small intensity, their impact on the standard deviation of the lights position seemed significant. Since we knew that in reality these areas outside of the slit should have been dark we could neglect them by considering values un-
5.3 Measured Results

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Since the intensity of the light in the diffraction image is dependent on the slit size, a program for automatically adjusting the gain, light intensity amplifier, of the camera was implemented, but since this also increases the background noise, a way of neglecting the background noise was tried. This was done by finding the minimum average intensity and multiplying it by 1.5 and then subtract it from the whole intensity vector and all negative values were set to 0. To keep the gain low, so that the amplification of the noise were kept at a minimum another way of adjusting the intensity was implemented, automatic change of the exposure time of the camera. This would have been preferable to be used as the only changing factor of the intensity, but since small changes in the exposure time for some unknown reason caused huge changes in the intensity, this was not possible. This way of handling the noise seems to give reasonable values and nice intensity plots such as the one in Figure 9 where the diffraction pattern is easy to see in a lin-log plot even though it is difficult to see by eye in the original image of the diffraction pattern.

5.3 Measured Results

In the results we could clearly see that the product, which is to be compared with the Heisenberg limit, was increasing with decreasing slit sizes and that we got the lowest value for the fully opened slit, which was expected because when the slit is fully opened the laser’s light intensity distribution looks almost like a Gaussian wave which should then provide a result with the product exactly equal to $\frac{\hbar}{2}$ which was shown under the Theory section. There are several reason why we do not get such a results, for example the light distribution in not a perfect Gaussian, aberrations from the lenses and other imperfections with the setup. The product increase with decreasing slit sizes because the slit disrupts the light’s path, which gives us the diffraction pattern, and this causes a decrease in the standard deviation in the position, because the light gets more focused and starts looking like a sinc-function rather than a Gaussian, but also a large increase of the standard deviation of the momentum, because of the higher orders of diffraction. For the last measurement with the slit size around 100 µm the diffraction pattern was so spread out that only the 0:th order and half of the 1:st order actually hit the camera which could explain the unexpected value acquired for that measurement.
6 Conclusion

During this project, we found that the task which consumed the most time by far was the software programming and more specifically, finding a good way to auto-adjust the intensity of the pattern.

In our results we find no reason to believe that the Heisenberg uncertainty principle should be untrue, instead we find a strong coherence with theory and another implication that the principle holds. Since our results are close to the theoretical result given by the example of the Gaussian wave, we feel assured that our methods of calculation are correct and our results solid.

We believe and hope that our demonstration will be able to educate young students that visit Vetenskapens Hus, interest them in an otherwise, for most people, spooky and distant theory as quantum mechanics and that this is done in a clear and understandable way.

7 Acknowledgements

We would like to thank our supervisors Marcin Swillo and Gunnar Björk for all their help and support during this project. We would also like to thank the Department of Applied Physics for letting us do our project at their institution.
References


