Abstract

Today the armed forces of many countries need to strike accurately potential enemies, wherever they might be, from a safe place. Since naval units can be deployed almost everywhere in the open sea, the idea of a naval cruise missile emerged in the 70’s. These missiles are designed to be launched from various naval vehicles such as frigates or submarines and strike deeply in the enemy territory. A program called *Missile de Croisière Naval* (MdCN Naval Cruise Missile) was therefore launched in 2006 by the DGA, the French procurement agency. MBDA is the industrial company appointed by the DGA to design and build the missile.

Control aspects on a cruise missile are of primary interest since they impact the reliability, performance and availability of the weapon. In the aeronautics and weapon industry, gain scheduled controllers are used in most cases. However, many non-linear techniques have been developed in the literature and might improve the behaviour of the missile.

The main objective of the present thesis is to apply non-linear techniques on the control and guidance loops of the MdCN too see whether of not they can improve such a system. Based on this report it should be easy for the engineers of the DGA to compare the controllers of the thesis and the classical gain scheduled controllers used in the industry. To achieve this task some basic knowledge of flight dynamics are recalled and a model of the MdCN is computed and divided into the control loop and the guidance loop. Then a non-linear controller for the launch phase using a Lyapunov based technique called back-stepping is designed and tested through a statistic analysis. During the cruise phase different anti-windup strategies are applied on the propulsion control loop of the missile and compared. Finally a software interface with a navigation-dedicated tool is coded and implemented in Simulink to analyse the complete Guidance-Navigation-Control loop and to see how navigation errors impact the control algorithms.

The main contributions of this thesis are the controllers designed for the launch phase and the propulsion loop that will be compared with the controller that MBDA is going to deliver next year to see whether or not the non-linear techniques used in the thesis should be used on the missile. Furthermore, all the tools and procedure set up to interface the control and guidance laws with the navigation models and filters will give the possibility to the DGA to have a deeper understanding of the algorithms used by MBDA and to make sure that navigation and estimation issues are properly taken in account when designing the control and guidance laws.
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Introduction

Today the armed forces of many countries need to strike accurately from a safe place potential enemies wherever they might be. Since naval units can be deployed almost everywhere in the open sea, the idea of a naval cruise missile emerged in the 70’s. These missiles are designed to be launched from various naval vehicles such as frigates or submarines and strike deeply in the enemy territory. This solution was indeed a good answer to the challenge that many countries are facing.

However the launch from a naval carrier is much more complicated than the launch from an aircraft or a ground station. First of all since the carrier might be in patrol in an hostile environment, his position, i.e. the initial position of the missile might be subject to uncertainties, due to GPS jamming for example. Furthermore naval missiles are launch from a more or less vertical position without any initial vertical velocity. So during the same phase they need to accelerate enough to start the turbojet propeller, swing up to a horizontal position and reach their cruise trajectory and of course they shouldn’t be a hazard for their carrier! The main goal of this thesis will be to investigate how non-linear techniques might improve the performance and availability of such weapons. A non-linear controller for the launch phase will be designed and tested. In addition to this study some anti-windup techniques are compared and tested on the propulsion loop. And finally an interface between a navigation software, called Oceani and Matlab/Simulink is designed to investigate navigation issues and their influence on the controller.

The thesis outlines as follow:

- Chapter I introduces the context of the thesis with a brief presentation of the Direction Générale pour l’Armement and a description of the missile.
- Chapter II presents some basic knowledge in aerodynamics that is then used to derive a model of the missile and its booster.
- Chapter III describes the design of the non-linear controller for the launch phase
- Chapter IV deals with the cruise phase, it describes the control-guidance strategy but emphasis on the anti-windup techniques and the propulsion loop.
- Chapter V introduces the navigation issues and the interface designed between Oceani and Simulink and the influence of navigation errors on the control loops.
- Appendix A describes the toolbox used to model the missile under Simulink and all the implementation issues on the simulator.
- Appendix B presents the so-called three-loops autopilot, a classical linear controller widely used in the industry that will be used here during the cruise phase.
I. Context of the thesis

I.1 Presentation of the Délégation Générale pour l’Armement

I.1.a The Délégation Générale pour l’Armement

Founded in 1961 by General De Gaulle the DGA (Direction Générale pour l’Armement) is the French procurement agency. Defence system architect and technical expert serving the Armed Forces, the DGA controls the design and development of weapons systems, and monitors the availability of the necessary industrial and technological capacities. It works closely with military staff from identification of future needs through to user satisfaction.

The DGA has three main missions:

- Preparing the future of defence systems
- Equipping the armed forces
- Promoting defence equipment export

The DGA employs 18 000, of which 80% civilians and 20% military, in many different establishments on the French territory but it has also representations in 20 countries all over the world. The DGA is divided in several directorates. This project will take place in one of them: the Direction de l’Expertise Technique (DET Direction of the technical expertise). The DET consists of several entities among them the technical centres and especially the LRBA.
I.1.b Laboratoire de Recherches Balistiques et Aérodynamiques

The Laboratoire de Recherches Balistiques et Aérodynamiques (LRBA technical centre for research in ballistic and aerodynamics) is one of the technical centres of the DET. Installed on a field of 545 ha, the LRBA is located near the city of Vernon on a plateau above the Seine about 50 minutes North West from Paris.

![Figure 1 Air Picture of the LRBA](image)

Since the 1950’s it provides a strong expertise in missile systems and in many armament programs, a brief history of the site is shown below:

- 1928: First mortars ammunition factory
- 17th May 1946: LRBA's foundation, expertise centre on missiles and rockets based on the German knowledge of that time.
- 1946-1970: Many activities in several fields of study: studies on the inertial systems built by the industry, observation satellites, several ballistic works from 1958, studies on ergol propulsion, rockets trajectories: design of the first French rockets VERONIQUE (1952), VESTA (1964) propellers VEXIN (1963) and VALOIS (1966) which will be used on the space rockets DIAMANT. One of them launched the first French satellite Astérix on 26th November 1965.
- 1971: Transfer of the propulsion activities to the European Propulsion Company (SNECMA). Military activities in LRBA became a part of the missiles directorate
- 1973: LRBA is entrusted with an interdirectorate mission for inertial techniques and navigation systems.
- 1985: Mission for the standardization of environmental tests and techniques.
I.1.c Performance de Systèmes Missiles

The LRBA consists in several divisions and departments, the thesis will be held in a department called PSM (Performances de Systèmes Missiles, Performances of Missiles Systems) of the MAN division (Missiles et Armes Nucléaires, Missiles and Nuclear Weapons).

The PSMD department provides a technical expertise of the global performances of the Missiles systems from the planning of the mission to the integration of the weapon systems into a bigger one. Generally speaking the work consists of:

- Leading performances studies
- Evaluations through numerical simulations
- Specifications and needs of weapons systems
- Analysis of the technical documentations of the armaments programs
- Technical support during negotiations with the industry
- Proposition of early studies subjects.

I.2 Presentation of the Missile de Croisière Naval

I.2.a Context

During the First Gulf War 1991 the US Army used massively the Tomahawk to strike deeply in the enemy territory. The French Forces realized at that time the benefit they could get from such a cruise missile able to hit any objective in the depth from a safe area by night or by day whatever the weather.

To match this need and guarantee its independence toward the US technology, France decided in 1993 to develop its own tactical cruise missiles launched from aircrafts, ships or submarines. A first air-ground anti runway missile called APACHE was designed by Matra Defence (MBDA France today) and is operational in the French Air force since 2002. Based on the work done on the APACHE Matra developed together with British Aerospace another air-ground missile dedicated to the destruction of military or logistic infrastructures. This missile called SCALP-EG in France and Storm Shadow in the United Kingdom was used with success in operation for the first time by the British army in Iraq during the Second Gulf War; the missile is also a commercial success since it has been sold to many countries like Italy, Greece or the United Arab Emirates.

Concerning the naval version of the missile called Naval SCALP or MdCN (Missile de Croisière Naval Naval Cruise Missile) the first preliminary studies started in 2002 and are leaded by the DGA while MBDA is the industrial in charge of the program. The department PSM from the LRBA is responsible for the evaluation of the global performances of the
missile that includes the performances of the control laws of the guidance and autopilot devices.

**I.2.b General characteristics of the missile**

The MdCN is to be launched by the future FREMM (multi missions European frigate) and the future submarines SNA/SSN from the class Barracuda. The main components of the missile are:

- Large wingspan wings: for high range and thin airframe.
- Turbojet propeller TR 50: This guarantees a good kilometric consumption.
- Spreadable wings, deflectors and air entry: To store the missile in NATO standard tubes of 533mm. The wings, deflector and air entry spread once the missile has been fired and has left his launch device.
- Booster: called SAB (Système d’Accélération et de Basculement acceleration and « tip over » system) is a solid propellant rocket motor with two swivelling nozzles. Since the MdCN is launch from a vertical position, it needs an initial propeller to leave its carrier and move up to a horizontal position where the booster is dropped and the turbojet turned on.

![MdCN prototype with its SAB at the bottom](image)

**Figure 2 MdCN prototype with its SAB at the bottom**

*Note the two swivelling nozzles*

- DCM (Dispositif de Changement de Milieu Environment change device): Submarine version only. The DCM is the waterproof device used to protect the missile when the MdCN is launched from a submarine and has to go from the water into the air. The DCM is dropped as soon as the Missile has left the water.
I.2.c Chronology of a launch

The pictures below describe a standard launch and flight of a MdCN:

Figure 3 General structure of the MdCN

Figure 4 Launch from a FREMM
II. Modelling of the missile

The purpose of this chapter is to derive a model for the missile, i.e. to get a state space model of the system. This model will be derived using a physical modelling based on Newton’s laws.

The forces and torques applied on the system are:
- The aerodynamic forces
- The mass and inertial torques
- The thrust produced by the SAB, the booster of the missile.

In part II.1 the reference systems where this forces and torques are expressed are presented, introducing all the angles used to describe the position and attitude of the missile during a flight. Then the usual model of the aerodynamic force is presented and the aerodynamic coefficients are briefly discussed.
Part II.2 deals with the mass and inertial effects. Since the booster is burning some propellant it will lose some weight thus the mass, inertial matrix and center of mass of the missile will evolve with time. The purpose of this part is to describe this evolution.
Part II.3 and II.4 deal with the thrust produced by the SAB during the flight. In part II.3 the thrust is derived and then expressed in a proper reference frame (the body reference system...
see II.1.a) while part II.4 models the actuators (here the swivelling nozzles) and their limitations. Finally part II.5 presents the state space model obtained after some simplifications. The separation between the slow and fast dynamics is discussed introducing the guidance/control separation.

II.1 Aerodynamics Basis

This part presents some general knowledge on aerodynamics that will be useful for the modeling of the missile. For further details on aerodynamics and flight dynamics see [1]

II.1.a General reference systems

In this part the reference systems used to express the different vectors (speed, acceleration, forces, torques…) needed to compute a state space model for the missile.

Three main reference frames are used:

- The earth reference frame with origin G, the centre of mass of the missile and axis linked to the earth. The frame is used as a reference for the attitude of the missile.
- The body reference frame with origin G and axis linked to the missile. The angles that describe the rotation from the earth reference to the body reference frame are called Euler angles or attitude angles.
- The aerodynamics reference frame with origin G and axis linked to the velocity of the missile. This reference frame is convenient to compute the aerodynamic coefficient of the missile and describe the behaviour of the airflow on the body of the missile.

One can note that all this reference frames are not constants since they depend on the position of G so one can also use a last reference frame called local reference frame which is basically an earth reference system except that it has its origin constant in the initial position of G for example.

Earth Reference system : $R_0$

The $Gz_0$ axis corresponds to the local vertical line that passes through the aircraft centre of gravity, G. It is oriented positive downward and corresponds to the direction of the gravitation force $m \ddot{g}$ seen by the aircraft, see figure 6.

The $Gx_0y_0$ plane is the local horizontal plane that passes through the aircraft centre of gravity, G. It is usual to take, for $Gx_0$ direction, the geographic or magnetic Earth North.
Aircraft Reference system : $R_b$

Definition and conventions

The aircraft reference system $G_{xb}y_{xb}z_{xb}$ is linked to the aircraft with its centre of gravity $G$ as system origin.

- the $G_{xb}$ axis corresponds to the fuselage reference line and is oriented positive in the forward direction. It belongs to the aircraft plane of symmetry.

- the $G_{zb}$ axis is perpendicular with respect to $G_{xb}$ such that the plane $G_{xb}z_{xb}$ corresponds to the aircraft plane of symmetry. $G_{zb}$ axis is oriented positive downward.

- the $G_{yb}$ axis is perpendicular with respect to the aircraft plane of symmetry and is oriented positive towards the right of the pilot.

Rotations from $R_0$ to $R_b$

With respect to the Earth Reference system $G_{x0}y_{y0}z_{z0}$, the aircraft rotation vector $\dot{\Omega}_{b/0}$ is expressed within the Aircraft Reference system: $G_{xb}y_{xb}z_{xb}$ according to:

$$\dot{\Omega}_{b/0} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{R_b}$$

where $p$, $q$ and $r$ are respectively the roll, pitch and yaw angular velocity expressed in rd/s. $\dot{\Omega}_{b/0}$ represents also the rotation vector of the system $R_b$ with respect to $R_0$ because the system $R_b$ is linked to the aircraft.

Of course, the sign of each rotation component is coherent with the rotation around each associated axis.

Around $G_{xb}$ : positive rotation $p$ when the right wing goes down
p > 0 : aircraft roll angular velocity

Figure 7 Roll angle

Around $G_{y_b}$ : positive rotation $q$ when the fuselage nose goes up (pitch-up rotation)

q > 0 : aircraft pitch angular velocity (pitch up)

Figure 8 Pitch angle

Around $G_{z_b}$ : positive rotation $r$ when the aircraft nose goes right

$r > 0$ : aircraft yaw angular velocity

Figure 9 Yaw angle

Finally the angles describing the rotation from $R_0$ to $R_b$ are respectively:

- The roll angle $\varphi$
- The pitch angle $\theta$
- The yaw or azimuth angle $\psi$
The rotation vector $\vec{\Omega}_{b/0}$ can be expressed from these 3 rotations (see figure 7):

$$\vec{\Omega}_{b/0} = \dot{\psi} \cdot \vec{z}_0 + \dot{\theta} \cdot \vec{y}_\psi + \dot{\phi} \cdot \vec{x}_b$$

By expressing $z_0$ and $y_\psi$ with respect to $R_b$:

$$\vec{z}_0 = \begin{bmatrix} -\sin \theta \\ \cos \theta \sin \phi \\ \cos \theta \cos \phi \end{bmatrix}_{R_b} \quad \& \quad \vec{y}_\psi = \begin{bmatrix} \cos \phi \\ -\sin \phi \end{bmatrix}_{R_b}$$

It is possible to get a new expression for $p$, $q$, and $r$:

$$\vec{\Omega}_{b/0} = \begin{bmatrix} p = \dot{\phi} - \dot{\psi} \cdot \sin \theta \\ q = \dot{\psi} \cdot \cos \theta \sin \phi + \dot{\theta} \cdot \cos \phi \\ r = \dot{\psi} \cdot \cos \theta \cos \phi - \dot{\theta} \cdot \sin \phi \end{bmatrix}_{R_b}$$

That can be rewritten to get:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Note the singularity in $\theta = \pi / 2$ which will be treated later on by introducing the so called quaternions (see III.2.A)
Of course, the sign of each rotation component is coherent with the rotation around each associated axis.

By convention we assume:

\[ -\pi < \psi < \pi \]
\[ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \]
\[ -\pi < \phi < \pi \]  

From these three angles one can compute the rotation matrix from \( R_0 \) to \( R_b \), which is useful to change reference frame. Let \( X_0 \) and \( X_b \) be the expression of the vector \( X \) in \( R_0 \) and \( R_b \) respectively then:

\[ X_0 = \mathcal{R}(0 : b)X_b \]  

Actually \( \mathcal{R}(0 : b) \) is the product of three rotations, thus:

\[
\mathcal{R}(0 : b) = \begin{pmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{pmatrix}
\]  

Aerodynamic Reference system : \( R_a \)

**Definition and conventions**

The aircraft reference system \( G_{xa}y_aza \) is linked to the aircraft velocity vector \( \vec{V} \).

The \( G_{xa} \) axis corresponds to the aircraft velocity \( \vec{V} \) direction and is oriented positive in the same direction.

The \( G_{za} \) axis is perpendicular with respect to \( G_{xa} \) and located within the aircraft plane of symmetry \( G_{xb}z_b \); \( G_{za} \) axis is oriented positive downward.

The \( G_{ya} \) axis completes the Aerodynamic Reference system \( R_a \).

The expression of \( V \) with respect to \( R_a \) and \( R_b \) is:

\[
\vec{V}_{\text{Ra}} = V \cdot \cos \alpha \cos \beta \\
\vec{V}_{\text{Rb}} = V \cdot \sin \beta \\
\]  

Rotations from \( R_b \) to \( R_a \)
By convention we let:

\[ -\pi < \alpha < \pi \]
\[ -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2} \] (6)

And finally we found the rotation matrix from \( R_a \) to \( R_b \) as a product from the rotation of angle \( \alpha \) around \( Y_b \) and from the rotation of angle \( \beta \) around the axis \( Z_b \):

\[
\mathcal{R}(a:b) = \begin{pmatrix}
\cos \alpha \cdot \cos \beta & -\sin \beta \cdot \cos \alpha & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cdot \cos \beta & -\sin \beta \cdot \sin \alpha & \cos \alpha
\end{pmatrix}
\] (7)
Rotations from $R_0$ to $R_a$

In a similar way we can find the rotation matrix from $R_0$ to $R_a$:

$$
\mathcal{R}(0 : a) = \begin{pmatrix}
\cos \gamma \cos \chi & \cos \gamma \sin \chi & -\sin \gamma \\
\sin \gamma \sin \mu \cos \chi - \cos \mu \sin \chi & \sin \gamma \sin \mu \sin \chi + \cos \mu \cos \chi & \cos \gamma \sin \mu \\
\sin \gamma \cos \mu \cos \chi + \sin \mu \sin \chi & \sin \gamma \cos \mu \sin \chi - \sin \mu \cos \chi & \cos \gamma \cos \mu
\end{pmatrix}
$$

(8)
Definition and conventions

Usually aerodynamic force $F$ and torque $M_G$ are expressed according to: see [1 or 4]

\[
F = \frac{1}{2} \rho V^2 S \cdot C_F \quad M_G = \frac{1}{2} \rho V^2 S L \cdot C_M
\]  

where

- $\frac{1}{2} \rho V^2$ is the flow field dynamic pressure (N/m²) describing the influence of the physical parameters of the airflow (pressure, temperature…)
- $S$ is the reference area (m²), generally the wing area
- $L$ is the reference length (m), generally the wing mean aerodynamic chord M.A.C. (~mean length of the wings)
- $C_F$ & $C_M$ are dimensionless aerodynamic coefficients.

Of course, aerodynamic forces and torques can be projected, either within the aerodynamic reference system $R_a$, or within the aircraft reference system $R_b$; this defines two sets of aerodynamic coefficients:

According to $R_a$ : $G_{xa}y_{za}$

\[
\vec{R} = \frac{1}{2} \rho V^2 S \cdot C_x \quad \vec{M}_G = \frac{1}{2} \rho V^2 S L \cdot C_m
\]  

By definition, the projection of the aerodynamic forces along:

- $G_{xa}$ is called the drag $D$ and is associated to the drag coefficient $C_x$
- $G_{za}$ is called the lift $L$ and is associated to the drag coefficient $C_z$

$C_x$ & $C_z$ aerodynamic coefficients are widely used …

According to $R_b$ : $G_{xb}y_{zb}$

\[
\vec{R} = \frac{1}{2} \rho V^2 S \cdot C_y \quad \vec{M}_G = \frac{1}{2} \rho V^2 S L \cdot C_n
\]  

$C_y$ & $C_l$, $C_m$, $C_n$ aerodynamic coefficients are widely used …

Generally, aerodynamic coefficients depend on:

- Mach and Reynolds number
- aircraft angle of attack and sideslip : $\alpha$, $\beta$
- local angle of attack : $p.L/V$, $q.L/V$, $r.L/V$
- control surfaces deflections : $\delta l$, $\delta m$, $\delta n$
So far a linear aerodynamic model where the aerodynamic coefficients are linearly dependent of the parameters given above will be considered, but usually this coefficient are derived experimentally and then implemented as tabulated data in the models. One should note that the force and torque can then be expressed in any of the reference frame using the rotation matrix derived in II.1.a.

**II.2 Mass and inertia of the system missile+booster**

In this part the modelling and the calculation of the mass and inertia of the missile during the launch phase (i.e. the system MdCN with its booster called SAB) is described. We assume that the mass of the missile is a constant $M$ while the masse of the SAB $m_b$ depends linearly of the time as shown below:

Where $m_p$ is the mass at the beginning of the flight when the SAB is full, $m_f$ is the mass at the end of the booster phase when the SAB has burnt all its propellant and $t_0$ and $t_f$ are respectively the start and end instant of the booster phase.

Thus the total mass of the system is given by:
To compute the matrix of inertia of the system we will assume that the missile is a perfect cylinder of constant density and the SAB a punctual mass applied at the bottom of the cylinder in the point B.

Let $L$ be the total length of the missile and $R$ its radius. The inertia matrix of the system without SAB in $G_0$ the centre of gravity of the cylinder is given by:

$$I_{G_0} = \begin{pmatrix}
\frac{MR^2}{2} & 0 & 0 \\
0 & M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) & 0 \\
0 & 0 & M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)
\end{pmatrix} \hspace{1cm} (13)$$

Adding the SAB we have:

$$I_{G_0} = \begin{pmatrix}
\frac{MR^2}{2} & 0 & 0 \\
0 & M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) + m_p(t)\frac{L^2}{4} & 0 \\
0 & 0 & M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) + m_p(t)\frac{L^2}{4}
\end{pmatrix} \hspace{1cm} (14)$$
Finally according to the parallel axis theorem\(^1\) the inertia matrix in \(G\) the centre of gravity of the whole system is given by:

\[
I_G = I_G^0 + (M + m_b(t)) \begin{pmatrix}
G_0 Gx^2 & 0 & 0 \\
0 & G_0 Gy^2 & 0 \\
0 & 0 & G_0 Gz^2
\end{pmatrix}
\] (15)

\[
I_G^0 = \begin{pmatrix}
\frac{MR^2}{2} & 0 & 0 \\
0 & M(\frac{R^2}{4} + \frac{L^2}{12}) + m_b(t)\frac{L^2}{4}(1 + \frac{m_b(t)}{m_b(t) + M}) & 0 \\
0 & 0 & M(\frac{R^2}{4} + \frac{L^2}{12}) + m_b(t)\frac{L^2}{4}(1 + \frac{m_b(t)}{m_b(t) + M})
\end{pmatrix}
\] (16)

Note: The position of the centre of gravity of the whole system is given by:

\[
G_0 G = \frac{m_b(t)}{m_b(t) + M} G_0 B = \frac{m_b(t)}{m_b(t) + M} \begin{pmatrix}
-L/2 \\
0 \\
0
\end{pmatrix}
\] (17)

II.3 Expression of the booster’s Thrust

The SAB is a solid propellant rocket motor. The norm of the thrust produced by the SAB therefore can’t be controlled and follow the profile shown on the left.

In a first approximation we will consider that after the ignition the thrust follows a perfect step to \(F_p\).

\(^1\) According to the parallel axis if the axis of rotation is displaced by a vector \(R = (a \ b \ c)^t\) from the centre of mass, the new moment of inertia equals \(I_{new} = I_{old} + M \begin{pmatrix}
b^2 + c^2 & ab & ac \\
ab & a^2 + c^2 & bc \\
ac & bc & a^2 + b^2
\end{pmatrix}\) with the \(M\) the total mass of the system studied.
The missile can however be controlled since the nozzles can rotate around the axis Y and Z. Indeed a deflection of the same angle $\delta Y$ around the Y axis of the two nozzles will create a pitch; a deflection of the angle $\delta Z$ around the Z axis of the two nozzles will create a yaw and a different deflection of the nozzles around the Y axis will create a roll.

Thus it is possible to control the torque produced by the nozzles and to guide the missile in the good direction.

The expression of the force produced by the two swivelling nozzles is here expressed in the body reference system. The nozzles rotate around the axis Y and Z of the body frame so using two rotation one can express the thrust in the body frame. Hence the thrust is given by:

$$
\tilde{F}_{b} = \begin{bmatrix}
\cos \delta Y \cos \delta Z & \cos \delta Y \sin \delta Z & -\sin \delta Y \\
-\sin \delta Z & \cos \delta Z & 0 \\
\sin \delta Y \cos \delta Z & \sin \delta Y \sin \delta Z & \cos \delta Y
\end{bmatrix} F
$$

Thus

$$
\tilde{F}_{r} = P_{rb}(\delta \chi r, \delta \chi z) F
$$

and

$$
\tilde{F}_{l} = P_{lb}(\delta \chi l, \delta \chi z) F
$$

where $P_{rb}$ and $P_{lb}$ will be the rotation matrix from the “thrust axis” to the body axis system respectively for the right and left nozzle.

Assume that the force created by the nozzle is applied on the missile in the point $P_r$ for the right nozzle and in the point $P_l$ for the left one. Since these force are not applied directly on the centre of mass $G$ of the whole system they create the following torque around $G$ (see figure on the left) given by the moment arm formula:

$$
\dot{M}_{p} = \tilde{F}_{r} \times P_{r} G = P_{rb}(\delta \chi r, \delta \chi z) F \times \frac{L}{2(1 - \frac{m_b(t)}{m_b(t) + M})} e
$$
(19) for the right nozzle

\[ \vec{M}_i^e = \vec{F}_i^e \oplus \vec{P}_i^e = P_{m_i}^e (\delta X_l, \delta Z_l) \]

(20) for the left nozzle

\[ L / 2 (1 - \frac{m_i(t)}{m_i(t) + M_i}) - e \]

\[ \frac{\delta}{\delta t} \]

\[ 0 \]

\[ 0 \]

II.4 Nozzles actuators and limitations

The nozzles actuators are to be designed but they will have to answer to a step with a rise time, without overshoot, equal or smaller than the rise time obtained with the following model:

\[ taum = 0.006s \]

where \( Ksm = 63s^{-1} \)

Rate saturation = ±90°

Furthermore the thrusts produced by the nozzles have to stay within a cone of apex angle 16° around the missile X-axis:

Finally it is decided to set \( \delta Z_l = \delta Z_r = \delta Z \) to avoid collisions between the nozzles, thus we have three control variables \( \delta Z, \delta Yr \) and \( \delta Yl \) and we will see that this is enough to control the attitude vector of the missile which should be sufficient for the launch phase.
II.5 State space model

II.5.a Notations

Reference systems and rotations matrices

Let denote $R_n$ the local navigation reference system fixed with respect to the Earth. $R_0, R_b, R_a$ are respectively the Earth, body and aerodynamic reference systems linked to the missile.

Let $X$ be a vector, the expression of $X$ in the reference systems $R_n, R_0, R_b, R_a$ will be denoted $X_n, X_0, X_b, X_a$ respectively.

The rotation matrix from a reference system $x$ to a system $y$ will be denoted $P^y_x$, by this way $P^b_a$ will be the rotation matrix from the body system to the local navigation reference system.

Using this notation the rotation matrices computed in II.1.a will be denoted as follows:

$\mathbb{R}(0 : b) = P^b_0, \mathbb{R}(a : b) = P^b_a$ and $\mathbb{R}(0 : a) = P^a_0$

Note that all this rotation matrices are orthogonal so $P^y_x = P^{x^{-1}}_y = P^{x^T}_y$

Other notations

The following notations will be used below:

- $Q = \frac{1}{2} \rho V^2$ the dynamic pressure thus the aerodynamic force expressed in $R_a$ can be rewritten as :

$$F_a = \frac{1}{2} \rho V^2 S \begin{bmatrix} -C_x \\ C_y \\ -C_z \end{bmatrix} = \begin{bmatrix} -C_x \\ \mathcal{C}_y \\ -C_z \end{bmatrix} (Q)$$

which can be also expressed in the body reference frame using $P^b_a$ the rotation matrix from $R_a$ to $R_b$

$$F_b = P^b_a F_a = P^b_a \mathcal{Q} S \begin{bmatrix} -C_x \\ C_y \\ -C_z \end{bmatrix} = \begin{bmatrix} -C_x \\ C_y \\ -C_z \end{bmatrix} (22)$$

- $p_n = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ the position of the centre of mass in $R_n$, the local navigation reference frame.

- $V(G_0) = V_a = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$ the speed of the center of mass of the missile without SAB in $R_a$

Note that $\dot{p}_n = V_n (23)$
• $A_t = \theta$ the attitude or Euler angles of the missile.

• $\vec{\Omega}_{b/0} = \vec{p} \theta$ the angular velocity of the body towards the earth reference frame.

Recalling that

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = 
\begin{bmatrix}
p + (q \sin \phi + r \cos \phi) \tan \theta \\
q \cos \phi - r \sin \phi \\
q \sin \phi + r \cos \phi
\end{bmatrix} = 
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

Which yields to

\[
\dot{A_t} = 
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\vec{\Omega}_{b/0} \tag{25}
\]

II.5.b Force equation

The forces applied on the system \{MdCN+SAB\} are the gravity, the aerodynamic force and the two thrusts. Thus Newton’s second law of motion projected in $R_n$ gives:

\[
\frac{d}{dt}(m\vec{\ddot{V}}(G)_n) = m\vec{\ddot{V}}(G)_n + m\frac{d}{dt}(\vec{V}(G)_n + \vec{\dot{V}}(G)_0) = \sum \vec{F}_{ext,n} \tag{26}
\]

\[
m\frac{d}{dt}\vec{\dot{V}}(G)_n = P^n_a \sum \vec{F}_{ext,b} - m\vec{\ddot{V}}(G)_n - m\frac{d}{dt}\vec{\dot{V}}(G)_0 \tag{27}
\]

with \[
\sum \vec{F}_{ext,b} = mP^n_a \vec{g} + P^n_a QS \begin{bmatrix}
-C_x \\
C_y \\
C_z
\end{bmatrix} + \vec{F}_{p_r} + \vec{F}_{p_i}
\]

II.5.c Torque equation

Applied to the rotations Newton’s second law of motion on the same system projected in $R_n$ gives:

\[
\frac{d}{dt}\vec{\ddot{M}}_{b/0} = \vec{\ddot{I}}_b + \vec{\dot{I}}_b \times \vec{\dot{\Omega}}_{b/0} + \vec{\dot{\Omega}}_{b/0} \times \vec{\ddot{\Omega}}_{b/0} = \sum \vec{M}_{ext,b} \tag{28}
\]

\[
\vec{I}_\dot{b/0} = \sum \vec{M}_{ext,b} - \vec{I}_b - \vec{\ddot{\Omega}}_{b/0} \times \vec{\ddot{\Omega}}_{b/0} \tag{29}
\]
II.5.d Model simplification

In a first approximation the actuator dynamics will be neglected and the controllers will be designed assuming that the actuators are perfect. Since only small deviations are acceptable for the swivelling nozzles the classical first order approximation will be done and the thrust produced by a nozzle in the body axis system will be given by

\[
\tilde{F}_b = \begin{pmatrix}
1 \\ -\delta Z \\ \delta Y
\end{pmatrix} \delta F
\]

Let recall the force and torque equations:

\[
m \frac{d}{dt} \tilde{V}(G_0) = P^b_a \sum \tilde{F}_{ext_b} - \bar{m} \tilde{V}(G)_a - m \frac{d}{dt} \tilde{V}(G)_0 \tag{31}
\]

\[
\text{with } \sum \tilde{F}_{ext_b} = mP^a \hat{g} + P^b_a QS \begin{pmatrix}
-C_x \\
-C_y \\
-C_z
\end{pmatrix} - \tilde{F}_{pr_b} + \tilde{F}_{pi_b}
\]

\[
\dot{\Omega}_b/0 = \sum \tilde{M}_{ext_b} - \dot{\Omega}_b/0 - \tilde{\Omega}_b/0 \wedge \dot{\Omega}_b/0 \tag{32}
\]

\[
\text{with } \sum \tilde{M}_{ext_b} = QS \begin{pmatrix}
C_l \\
C_m + \bar{M}p_i^b + \bar{M}p_r^b \\
C_n
\end{pmatrix}
\]

Since \( m \frac{d}{dt} \tilde{V}(G)_0, \bar{m} \tilde{V}(G)_a \), and \( \dot{\Omega}_b/0 \) are very small compared to the forces (resp. the torques) applied on the system they will be neglected. Thus we have the following state space model:

\[
m \frac{d}{dt} \tilde{V}(G_0) = m \dot{\tilde{V}}_N = P^b_a \sum \tilde{F}_{ext_b} \tag{33}
\]

\[
\text{with } \sum \tilde{F}_{ext_b} = mP^a \hat{g} + P^b_a QS \begin{pmatrix}
-C_x \\
-C_y \\
-C_z
\end{pmatrix} - \tilde{F}_{pr_b} + \tilde{F}_{pi_b}
\]

\[
\dot{\Omega}_b/0 = \sum \tilde{M}_{ext_b} - \dot{\Omega}_b/0 \wedge \dot{\Omega}_b/0 \tag{34}
\]

\[
\text{with } \sum \tilde{M}_{ext_b} = QS \begin{pmatrix}
C_l \\
C_m + \bar{M}p_i^b + \bar{M}p_r^b \\
C_n
\end{pmatrix}
\]
II.5.e State space and guidance piloting decoupling

From equations (23),(25),(33) and (34) we have:

\[
\dot{\Omega} = \Omega - \dot{\Omega}_{/0} = W(\phi, \theta, \psi)\hat{\Omega}_{/0} \tag{35}
\]

If we define a state vector

\[
X = (x \ y \ z \ u \ v \ w \ \phi \ \theta \ \psi \ \ p \ \ q \ \ r)^T = (p_n \ V_n \ A_i \ \Omega)^T \tag{36}
\]

We do have a state space form \( \dot{X} = f(X,U) \) where the input \( U = \sum \vec{F}_{ext} \) represents the external forces and torques.

Furthermore one can show that this system has 6 fast dynamics and 6 slow dynamics, the 6 fast dynamics are the rotation dynamics and will define the control system while the 6 slow dynamics are the translation dynamics and will define the guidance system.

- Guidance: Slow dynamics \( \Rightarrow \) State vector \( \begin{bmatrix} p_n \\ V_n \end{bmatrix} \)

\[
\dot{X} = \begin{bmatrix} \dot{p}_n \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta \\ 0 & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \phi & \cos \theta \\ -\sin \phi & \cos \theta \end{bmatrix} \begin{bmatrix} \dot{\Omega} \\ \dot{\Omega}_{/0} \end{bmatrix} = W(\phi, \theta, \psi)\hat{\Omega}_{/0} \tag{37}
\]

- Control: Fast dynamics \( \Rightarrow \) State vector \( \begin{bmatrix} A_i \\ \Omega \end{bmatrix} \)

\[
\dot{At} = W(\phi, \theta, \psi)\hat{\Omega}_{/0} \tag{38}
\]

\[
\dot{\Omega}_{/0} = \sum \vec{M}_{ext} - \hat{\Omega}_{/0} \wedge \hat{\Omega}_{/0}
\]
In addition to the Guidance and Control we will call Navigation the process of estimating and filtering of the variables needed by the Guidance en the Control from the data collected by the different sensors of the aircraft. Navigation is not the core and will only be treated when interfacing Matlab/Simulink with the navigation software Oceani.

Figure 16 Guidance Control Navigation loop
III. Launch phase

III.1 The launch phase.

The main challenge during the launch phase is the lack of control inputs. The only thing that can be commanded during this phase is the orientation of the nozzles. There is no control on the magnitude of the thrust; the wings and fins are not available yet so the missile can’t be controlled using rudder deflections. It is decided to use the deviation of the thrust to control the attitude of the missile.

During a first phase ($\approx 1.5s$) the missile will accelerate in a vertical position and then turns over to a horizontal trajectory. During the swing the angle of attack (AoA) of the missile may grow a lot and this can cause stall phenomenon or make the ignition of the turbojet impossible. This leads us to the other requirements of the phase: the angle of attack should keep reasonable values and be as small as possible when the turbojet is ignited at the end of the launch phase, the speed of the missile should be high enough to ignite the turbojet and finally to avoid detection by potential enemies the missile should stay below 500m.

III.2 First controller design

In this part a first controller for the launch phase is designed. To accomplish this task it was decided to use a Lyapunov theory based technique called backstepping. Indeed the equations that describe the behaviour of the missile are highly non-linear so it is natural to use a nonlinear controller. Usually the industrials uses gain scheduling to deal with this issue, so gain-scheduled controller will not be investigated here. A Lyapunov theory based technique called back-stepping gave already promising results in previous studies of the DGA focusing on UAV guidance and control [11] and it was decided to investigate further whether or not this technique could give good results applied on the launch phase of the MdCN.

Feedback linearization, Sliding modes based controllers or optimal controller using Pontryagin’s Maximum Principle for example were other options but previous studies held at the DGA showed that feedback linearizing controller had poor behaviour in terms of robustness and external disturbances rejection. Furthermore sliding modes controller would have needed very fast switch of the control signal around a sliding surface which might be hard to achieve with the future nozzle actuators of the booster. This actuator has not been studied that much for now but the technical specification describes a low pass filter behaviour (see II.4). Concerning optimal controllers, it has to be taken in account that the system obtained are already quite complicated and has no simple explicit solution and the computational complexity required to solve the optimal control problem numerically might be to big to be implemented.

One challenge of the Back-stepping technique is that it needs a system under strict-feedback form to be applied easily. In first part the back-stepping technique will be presented more in detail, then the so called quaternions are introduced and the procedure used to rewrite the state space equation will be discussed finally the backstepping design and the first results of the obtained controller are detailed.
III.2. a The backstepping theory

Considering the system $\dot{x} = f(x, u)$ the idea of the backstepping design is to find a state feedback $u = u(x)$ that stabilises the system using Lyapunov control functions to design $u$ and prove the stability of the system.

Let take the following example to explain in detail how the procedure looks like:

Let $

\begin{align*}
\dot{x}_1 &= x_1^2 + x_2 \\
\dot{x}_2 &= u
\end{align*}

(39)

be a non linear system to control

This system is non linear and under strict feedback form (i.e. the state space variable $x_i$ do not depend on the next state space variable $x_{i+2}, x_{i+3}, \ldots$)

The following procedure called backstepping can be used to design a control signal $u$ that stabilises the system and an associated Lyapunov function.

Let $V_1(x_i) = \frac{x_i^2}{2}$ be a first storage function,

$$
\dot{V}_1 = x_i \dot{x}_1 = x_i (x_i^2 + x_2) = x_i (x_i^2 + x_2 - x_2^d + x_2^d)
$$

(40)

Letting $\bar{x}_2 = x_2 - x_2^d$ we have $\dot{V}_1 = x_1 (x_1^2 + \bar{x}_2 + x_2^d)$ thus if we set $x_2^d = -k_1 x_1$, $\dot{V}_1$ becomes: $\dot{V}_1 = -k_1 x_1^2 + x_1 (x_1^2 + \bar{x}_2)$ (41)

Let denote $\delta = x_i^2 + \bar{x}_2$ and let $V_2(x_1, x_2) = V_1(x_1) + \frac{\delta^2}{2}$ be a second storage function

$$
\dot{V}_2 = \dot{V}_1 + \delta \dot{\delta} = -k_1 x_1^2 + x_1 \delta + \delta (2 x_i \dot{x}_1 + \dot{x}_2)
$$

(41)

$$
\dot{V}_2 = -k_1 x_1^2 + \delta (x_1 + 2 x_i (x_1^2 + x_2) + u)
$$

(42)

$$
= -k_1 x_1^2 - k_2 \delta^2 + \delta (x_1 + 2 x_i (x_1^2 + x_2) + k_2 \delta + u)
$$

Thus with

$$
\begin{align*}
\dot{u} &= -(x_1 + 2 x_i (x_1^2 + x_2) + k_2 (x_1^2 + x_2 + k_1 x_1)) \\
\end{align*}

(43)

and $k_1, k_2 > 0$ we have $\dot{V}_2 = -k_1 x_1^2 - k_2 \delta^2$ (44)

- $V_2(0) = 0$
- $V_2(x_1, x_2) > 0 \ \forall x \neq 0$
- $V_2(x_1, x_2) = -k_1 x_1^2 - k_2 \delta^2 < 0 \ \forall x \neq 0$
- $V_2(x) \rightarrow \infty \text{ as } \|x\| \rightarrow \infty$

$V_2$ is a Lyapunov function and $u = -(x_1 + 2 x_i (x_1^2 + x_2) + k_2 (x_1^2 + x_2 + k_1 x_1))$, $k_1, k_2 > 0$ is a stabilizing state feedback.

III.2. b Quaternions
It was decided to use quaternions to control the missile. The quaternions are another way of describing the attitude of a missile. The quaternions are used because they are less sensitive to numerical issues and especially because of a singularity in $\theta = \pi/2$ when using the Euler angles.

Thus for any rotation defined by its Euler angles $(\varphi, \theta, \psi)$ we can find a quaternions $q = (q_1, q_2, q_3, q_4)$ given by:

\[
\begin{align*}
q_1 &= \sin(\varphi/2) \cos(\theta/2) \cos(\psi/2) - \cos(\varphi/2) \sin(\theta/2) \sin(\psi/2) \\
q_2 &= \cos(\varphi/2) \sin(\theta/2) \cos(\psi/2) + \sin(\varphi/2) \cos(\theta/2) \sin(\psi/2) \\
q_3 &= \cos(\varphi/2) \cos(\theta/2) \sin(\psi/2) - \sin(\varphi/2) \sin(\theta/2) \cos(\psi/2) \\
q_4 &= \cos(\varphi/2) \cos(\theta/2) \cos(\psi/2) + \sin(\varphi/2) \sin(\theta/2) \sin(\psi/2)
\end{align*}
\]

Thus using the quaternions we have the further model describing the attitude dynamics:

\[
\hat{\Omega} = \frac{1}{2} q^* \begin{bmatrix} 0 \\ \Omega \end{bmatrix}
\]

where $* \text{ denotes the multiplication in the quaternions space}.$

**III.2.c State space transformation**

Backstepping is Lyapunov function based control design that needs a special form of state space representation; the point of this part is to transform the state space representation into a backstepping friendly representation.

Let $q_d$ be the desired quaternion and $\Gamma = \sum M_{\text{ext}}$ the torque to apply on the system to get $q = q_d$. From a control point of view $q_d$ will be the reference signal, $\Gamma$ the control signal and $q$ the output signal.

Let $\Pi(p, q) = 2 \text{sgn}(q_1 p_1 + q_2 p_2 + q_3 p_3 + q_4 p_4) \begin{bmatrix} -q_1 p_2 + q_2 p_1 + q_3 p_4 - q_4 p_3 \\ -q_1 p_3 - q_2 p_4 + q_3 p_1 + q_4 p_2 \\ -q_1 p_4 + q_2 p_3 - q_3 p_2 + q_4 p_1 \end{bmatrix}$

$\Pi$ has the following properties (see [2] for further details on the derivation of $\Pi$)
\[
\begin{align*}
\Pi(q, q_d) &= 0 \iff q = q_d \\
\dot{\Pi}(q, q_d) &= \Omega
\end{align*}
\]

so letting \( \Pi \) be our new state space variable, the system becomes:

\[
\begin{bmatrix}
\dot{\Pi}(q, q_d) \\
\dot{\Omega}_{b/0}
\end{bmatrix} = \begin{bmatrix}
\Omega & \Pi \\
I^{-1}(\Gamma - \Omega_{b/0} \wedge \hat{\Omega}_{b/0})
\end{bmatrix}
\]

The system has now a strict feedback form and a Backstepping can be used to design a stabilizing state feedback and construct a Lyapunov function.

\section*{III.2.d Backstepping design}

Let \( S_1 = \frac{\Pi^2}{2} \) a first storage function.

\[
\dot{S}_1 = \Pi \dot{\Pi} = \Pi \Omega = \Pi(\Omega - \overline{\Omega} + \overline{\Omega}) = -K_1 \Pi^2 + \Pi \hat{\Omega}
\]

where \( \hat{\Omega} = \Omega - \overline{\Omega} = \Omega + K_1 \Pi \)

Let \( S_2 = S_1 + \frac{\overline{\Omega} \hat{\Omega}'}{2} \)

\[
\dot{S}_2 = -K_1 \Pi^2 + \Pi \hat{\Omega} + \overline{\Omega}(-\hat{\Omega} \wedge \hat{\Omega} + \Gamma) = -K_1 \Pi^2 + \hat{\Omega}(\Pi + \Gamma)
\]

\[
= -K_1 \Pi^2 - K_2 \hat{\Omega}^2 + \hat{\Omega}(\Pi + \Gamma + K_2 \hat{\Omega})
\]

Thus if we let \( \Gamma = -\Pi - K_2 \hat{\Omega} = -\Pi - K_2(\Omega - K_1 \Pi) \) (53) and \( K_1, K_2 > 0 \), \( S_2 \) is a Lyapunov function and \( \Gamma \) globally stabilizes the system. Furthermore given that \( \Pi(q, q_d) = 0 \iff q = q_d \)
we finally obtain the desired missile quaternion.
III.2.e Booster control

To achieve the torque \( \Gamma \) the nozzles have to be deviated so the control signal \( \Gamma \) has to be converted into SAB control signals \( \delta Z, \delta Y_r \) and \( \delta Y_l \).

\[
\Gamma = Q S C_m + \dot{M} p^b_r + \dot{M} p^b_l \Leftrightarrow \dot{M} p^b_r + \dot{M} p^b_l = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \Gamma - \Gamma_{aero} \tag{54}
\]

Letting \( l = L / 2(1 - \frac{m_b(t)}{m_b(t) + M}) \) the torques produced by the SAB are given by:

\[
\begin{align*}
\dot{M} p^b_r &= F^b_r \land P_r G = P^b_r (\delta Y_r, \delta Z) \quad &|F| &|l| \quad 0 &\land e \quad \tag{55} \\
\dot{M} p^b_l &= F^b_l \land P_l G = P^b_l (\delta Y_l, \delta Z) \quad &|F| &|l| \quad 0 &\land -e \quad \tag{56}
\end{align*}
\]

Furthermore the force produces by a nozzle is given by

\[
\tilde{F} p^b = \begin{pmatrix} 1 & \delta Z & -\delta Y \\ -\delta Z & 1 & 0 \\ \delta Y & \delta Y \delta Z & 1 \end{pmatrix} F
\]

Putting all the equations together yields to:

\[
\begin{align*}
\delta Y_r &= \frac{1}{F} \left( \frac{M_2}{l} - \frac{M_1}{e} \right) \\
\delta Y_l &= \frac{1}{F} \left( \frac{M_2}{l} + \frac{M_1}{e} \right) \tag{57} \\
\delta Z &= \frac{M_3}{Fl}
\end{align*}
\]
III.2. First results

The controller designed above is now implemented under a Simulink toolbox, called A3 (see Appendix A) and tested. Because of AoA problems the reference signal for the turn over of the missile has to be chosen carefully. It was decided to use a ramp from 90° to 0° as a reference pitch angle to perform the turn over and to tune its slope in order to keep a reasonable AoA angle and stay between 250 and 500m in altitude. Meanwhile the desired roll angle is set to 0° and the yaw angle heads to the first waypoint.

To tune the slope of the ramp several values are tested, Figure 11 shows the pitch angle of the MdCN for three different values of the reference slope. In all cases the turn over starts 2 seconds after the ignition of the booster but depending of the slope chosen the missile needs more or less time to reach the horizontal flight position with pitch angle of 0°.

Figure 12 shows the AoA of the missile during the turn over. The AoA increases a lot during the “transient phase” from the vertical to the horizontal position. High values of the slope gives high overshoot of the AoA but once the missile has reached a horizontal position the AoA decreases back to a constant value.

Figure 13 shows the altitude of the missile. When the slope chosen is too small the missile can reach too high altitude but on the other hand when it turns over too fast the missile may also crashes into the water.

Figure 14 shows the range of the MdCN, i.e. the distance covered by the missile during the booster phase. The missile has a better range when a high value of the slope is picked. Indeed when it turns over faster the booster push earlier horizontally.

Figure 15 shows the speed of the MdCN at the end of the booster phase, this parameter is of primary concern since the missile needs speed to ignite its turbojet propeller. Once again it appears that low values of the slope may give bad results, here low speed.

The higher the slope chosen for the reference signal, the quicker the missile turns over however this speed costs a lot in terms of AoA. Furthermore when the slope is too high the missile doesn’t have time to reach sufficient altitude. On the other hand when the slope is too low the missile keep reasonable AoA but it climbs too high and can’t reach the same speed and travel the same distance. Finally it comes out that a slope of 0.6°/s was a good compromise allowing the missile to reach an altitude of 300m, a range of 1300m for a maximum angle of AoA of −38° and a final speed of 290m/s.
Figure 17 Pitch angle of the MdCN for different values of the reference slope

Figure 18 Angle of Attack (AoA) of the MdCN for different values of the reference slope

Figure 19 Altitude of the MdCN for different values of the reference slope

Figure 20 Range of the MdCN for different values of the reference slope

Figure 21 Speed of the MdCN for different values of the reference slope
III.3 Controller improvement

The results of the previous controller were good but still it doesn’t guarantee that by the end of the booster phase the missile is in optimal conditions to ignite the turbojet, we have for example no control on the final values of the AoA of the missile at the end of the booster phase, so it was decided to use a special sequence in the reference signal of the previous controller. This choice was motivated by the simplicity of its implementation. This part describes the procedure used to improve the previous controller based on the new control sequence and a statistic analysis of the parameters of this sequence by Monte Carlo simulations.

III.3.a New control sequence

To make sure that the requirements in terms of turbojet ignition at the end of the booster phase are fulfilled, it was decided to use a more complex reference signal than the simple ramp used previously. Thus the desired pitch angle will have the following shape:

One can notice four different phases on the previous figure:

1. A vertical phase with a constant desired pitch angle of 90°
2. A first turn over phase with a high pitch rate
3. A second turn over speed with a much smaller pitch rate. This phase is called climb and acceleration phase to reach the desired speed and altitude.
4. A final phase to reduce the sideslip and AoA angles so that the airflow in the air entry be as high as possible and the conditions to ignite the turbojet are as good as possible.


III.3.b Angle of Attack and Sideslip reduction

The AoA and sideslip reduction phase is a bit special compared to the other phases since it needs some modifications of the autopilot. Indeed the controller was designed to control the attitudes of the missile from reference signal expressed in terms of Euler angles. But for the AoA and sideslip reduction it is necessary to control the AoA and the sideslip angles (cf fig.8 p.17) that describe the rotation from the body reference frame to the aerodynamic reference frame. The idea is to design a reductor that converts the reference in AoA and sideslip into reference in Euler angles.

Concerning the reduction of the AoA angle \( \alpha \) a classical approximation is used. For a pure longitudinal flight, i.e. for small values of the roll angle, which is the case here, the following relation is true:

\[
\theta = \alpha + \gamma
\]

with \( \theta \) the pitch angle and \( \gamma \) the flight path angle. This relation is illustrated by the figure below:

![Figure 23 Relation between the pitch and AoA angles for a longitudinal flight](image)

Thus if we want \( \alpha \to 0 \) then let \( \theta \to \gamma \) where \( \gamma = \arcsin(V_z/V) \)

Let recall the expression of the dynamics of the sideslip angle \( \beta \):

\[
\dot{\beta} = \frac{1}{mV}(QSC_\gamma (\alpha, \beta, \Omega) + (\bar{F}p_b^b + \bar{F}p_i^b)\cos \alpha \sin \beta + mg^\gamma) - r \cos \alpha + p \sin \alpha \quad (59)
\]

Assuming that the transverse aerodynamic force is proportional to the sideslip and that the AoA is small, we get the following expression of the dynamics of \( \beta \):

\[
\dot{\beta} = \frac{1}{mV}(QSC_\gamma \beta + (\bar{F}p_b^b + \bar{F}p_i^b)\beta + mg \sin \phi \cos \theta) - r \quad (60)
\]
Furthermore for small values of the roll we have \( \psi \approx r \) thus the dynamics of \( \beta \) become:

\[
\dot{\beta} = \frac{1}{mV}(QSC_y \beta + (\tilde{F}_{p_r}^b + \tilde{F}_{p_l}^b)\beta + mg\cos\theta) - \psi \quad (61)
\]

Letting \( u = \frac{g \cos\theta}{V} - \psi \) be a new control variable and \( a = \frac{1}{mV}(QSC_y + \tilde{F}_{p_r}^b + \tilde{F}_{p_l}^b) \) we have the following system \( \dot{\beta} = a\beta + u \) which will be controlled using the state feedback \( u = -(a + \tau)\beta \) with \( \tau \) the desired time constant of the sideslip angle.

Finally the commanded yaw angle will be \( \psi = \int (a + \tau)\beta + \frac{g \cos\theta}{V} \) \( dt + \psi_{wp} \) (62) where \( \psi_{wp} \) is the yaw angle to the next waypoint.

\[ III.3.c \text{ Statistic optimisation} \]

The last task in the design of the launch controller is to choose properly the timing of the phase and the amplitude of the turn over phases, i.e. the parameters \( T_1, T_2, T_3, \theta_1, \theta_2 \) introduced in fig.16. To perform this task a statistic approach is used where many set of parameters are tested and the results of the corresponding simulations analysed.

A set of conditions and criterions is defined to select the best set of parameters. The sets that didn’t match the following conditions were eliminated:

\begin{itemize}
  \item \( \forall t, \ z(t) \in [0,600m] \)
  \item \( \alpha(t_f) \) and \( \beta(t_f) \in [-5^{\circ},5^{\circ}] \)
  \item \( \text{mach}(t_f) > 0.4 \) and \( v(t_f) > 200m/s \)
  \item \( z(t_f) > 300m \)
  \item \( \dot{z}(t_f) > 0 \)
\end{itemize}

With \( t_f \) the time of the end of the phase when the booster is dropped.

Furthermore the following criterion for the set \( i \) was defined to select one of the remaining sets of parameter:

\[
J_i = \frac{v_i(t_f)}{v_{\text{mean}}(t_f)} + \frac{d_i(t_f)}{d_{\text{mean}}(t_f)} + \frac{1}{\delta_i(t_f)/\delta_{\text{mean}}(t_f)} \quad (63)
\]

Where \( d_i \) is the distance covered by the missile since its launch, \( \delta_i \) the distance to the plan heading to the first waypoint and
\[ v_{\text{mean}} = \frac{1}{n} \sum_{i \in [1,n]} v_i, \quad d_{\text{mean}} = \frac{1}{n} \sum_{i \in [1,n]} d_i \quad \text{and} \quad \delta_{\text{mean}} = \frac{1}{n} \sum_{i \in [1,n]} \delta_i \]

are the mean value of respectively the speed, distance covered and distance to the desired plan on the sets matching the conditions introduced above.

In the first series of simulation the parameters were picked in a determinist way around an initial guess, determined manually. The time parameters were picked every second and the angle parameters every 10 degrees.

This way a rough but acceptable set of parameters \( S_0 \) was obtained, to get a better set Monte Carlo simulations were launched. The parameters are picked following a normal statistic distribution centred on \( S_0 \). 1000 simulations were launch and the best set of parameters regarding the criterion \( J \) was selected.

Figure 18 shows all the set tested and Figure 19 shows the acceptable sets where the sets that do not match the conditions presented previously are eliminated. All the acceptable sets have roughly the same shape with a first turn over phase from \( \theta = 90^\circ \) to \( \theta = 15 - 25^\circ \) with high pitch rate from \( t = 2s \) to \( t = 3 - 5s \) and then second turn over phase with low pitch rate and the sideslip and AoA reduction phase that are hard to distinguish on figure 19.

Figure 20 and 21 shows the altitude of the missile corresponding to each set. Figure 20 shows that by tuning the five parameters \( T_1, T_2, T_3, \theta_1, \theta_2 \) a wide range of altitude at the end of the phase can be reached. Figure 21 shows the altitude evolution for the acceptable sets of parameters, as desired they are all bounded by 500 m and at the end of the phase the missile has an altitude between 250m and 500m.
Figure 25 Desired pitch angle for all the sets tested

Figure 26 Desired pitch angle for the acceptable sets
The best choice is shown in black

Figure 27 Altitude of the missile for all the sets tested

Figure 28 Altitude of the missile for the acceptable sets
Figure 29 3D trajectories of the acceptable set of parameters.

Figure 22 shows the 3D trajectories for all the acceptable sets of parameters, many different kinds of trajectories can still be covered by the MdCN. It will be the goal of the optimisation process to find the best set according to the criterion J defined previously.

After analysis of all the sets of parameters the best sets of parameters regarding the criterion J is:

\[
T_1 = 1s \quad T_2 = 1.7s \quad T_3 = 8.9s \quad \theta_1 = 31^\circ \quad \theta_2 = -4^\circ
\]
The following plots show the performances obtained with all the acceptable sets, on each plots the best choice according to the criterion J is shown in black. Figure 23 and 24 shows the evolution of the sideslip angle and AoA for all the acceptable sets. From these plots it comes out that the AoA and sideslip reduction algorithm presented in IV.3.b works good, the results are much better than those obtained in IV.2.f. with the first controller and even in some cases, including the one obtained by the optimisation process, the sideslip and AoA at the end of the booster phase are very close to 0° which means that the airflow at the entrance of the turbojet propeller will be optimal.

Figure 25 and 26 show the performance obtained in terms of speed and distance covered by the missile during the booster phase. Comparing the results obtained in IV.2.f and the one obtained with the best set of parameters one can note an increase of the range of the missile from 1250m to 1500m while the speed still ends around 300m/s but considering that this time the airflow flowing in the air entry of the turbojet is optimal the conditions are already much better to ignite the turbojet.

Figure 27 shows the pitch angle evolution during the booster phase for the acceptable sets. During the first seconds of the flight up to t = 2s the missile is in its launch container and follows the attitude of its carrier. Then the missile leaves the container and one can note the three phases of the turn over: the first phase with a high pitch rate, a second phase with a lower pitch rate and finally the AoA and sideslip angle reduction phase. Note that this time the missile ends the booster phase not exactly in a horizontal position and the pitch angle is just above 0°.

Finally figure 28 shows the trajectory of the missile in the horizontal plan. During these simulations the missile fired to the North but one can note that the missile slides a little to the west. Due to the lack of control inputs one has to choose to control exactly the heading of the missile or its sideslip angle. A high sideslip angle can highly reduce the airflow in the turbojet and thus threat the turbojet ignition. So it was chosen to accept this little deviation and guarantee the best ignition conditions. Furthermore this deviation, 100m for a distance covered of 1500m, can easily be corrected afterwards during the cruise phase.

![Figure 30 Sideslip angle for the acceptable sets. The best choice is shown in black](image)

![Figure 31 AoA for the acceptable sets. The best choice is shown in black](image)
Figure 32 Speed for the acceptable sets. The best set is shown in black.

Figure 33 Distance covered by the missile for the acceptable sets. The best set is shown in black.

Figure 34 Pitch angle for the acceptable sets. The best set is shown in black and the corresponding reference in blue.

Figure 35 Horizontal trajectories for the acceptable sets. The best set is shown in black.
III.4 Performance analysis

The performances of the selected set of parameters are now tested under several external conditions to evaluate the performance of the control law obtained and determined whether or not the missile can be fired under this conditions.

III.4.a Multi-azimuth launch

In this part the influence of the commanded heading of the missile will be investigated. The possibility to fire the missile to any direction is of primary interest for the operators since it makes the use of the missile much easier and much more convenient for the carrier. In the case studied here the missile will be fired to several directions from a frigate sailing at 5 m/s to the north.
Figure 29 and 30 show the trajectories of the missile in 3D and in the horizontal plan while Figure 31 shows the altitude of the missile. From these three plots it comes out that the missile follows in every case the same kind of trajectory, except for the heading of course, that seems to match the requirements of the launch phase. Now it has to be checked that the conditions to ignite the turbojet are good.

This is done by studying figure 32, 33 and 34 showing the speed, AoA and sideslip angle of the missile during the booster phase. In all cases the missile reaches good conditions to ignite the turbojet. With the controller obtained the altitude and speed are almost not sensitive to the heading commanded. It is also interesting to note that the AoA and sideslip angles reductors designed in III.3.b work well for various commanded headings.
III.4.b Carrier velocity influence

The carrier position and velocity has an impact on the performances of the missile since it determines the initial condition of the flight. Indeed if the sea is too bad the chance of success of the mission might be very low and a missile launch can even become dangerous for the carrier itself. According to the technical specifications of the MdCN it should be possible to launch the missile for different speed of the FREMM until 15m/s.

In the case studied here a missile is fired to the north from a FREMM sailing with different heading at 5, 10 and 15m/s. The altitude, speed and covered distance at the end of the booster phase are plotted on figure 35, 36 and 37. In all the cases the missile manage to reach good conditions of turbojet ignition, however the best performances are obtained if the FREMM is sailing at high speed in the direction of the first waypoint.

**Figure 42 Influence of the carrier velocity on the altitude at the end of the booster phase**

**Figure 43 Influence of the carrier velocity on the distance covered at the end of the booster phase**

**Figure 44 Influence of the carrier velocity on the speed of the missile at the end of the booster phase**
Wind is an uncontrollable parameter and it has often a big influence on aircraft systems. Off shore the carrier might encounter many different kinds of wind including strong wind that might affect a lot the performances of the missile. As in the previous case a missile is fired to the north from a FREMM sailing at 5m/s to the North under different wind.

Figure 45 Influence of the wind on the altitude

Figure 46 Influence of the wind on the speed

Figure 47 Influence of the wind on the Mach

Figure 38, 39 and 40 show the influence of the wind on the altitude, inertial speed and Mach of the missile. It comes out from these plots that the wind has not that much influence on the behaviour of the missile; the control law is robust to this perturbation. This can be explained because the wings of the missiles are not spread yet. During this phase thus the aerodynamic forces are really small compared to the high thrust produced by the booster. However if the wind is too high and blowing in the same direction as the missile the Mach might be very...
slow and it might be difficult to ignite the turbojet properly. Under a strong wind the missile should be fired against the wind to maximize the airflow in the air entry of the turbojet.

**III.4.d Conclusion**

The controller designed in this chapter gives satisfying results under various external conditions; it shows that the Backstepping can be used to design an autopilot of a complex system. Further works will be performed to test the influence of the attitude of the carrier at the ignition of the booster, what if the frigate rolls a lot due to the waves and can’t be stabilized properly? To achieve this task models of the sea are necessary are currently being derived at the LRBA. Now this controller should be compared with a classical linear controller to see whether or not it worth to use a non-linear technique to control the missile during the launch phase.

**IV. Cruise phase**

**IV.1 Control Strategy**

To simulate a complete flight a guidance-control loop dedicated to the cruise phase developed at the LRBA was used. The control strategy used for the cruise phase is the strategy commonly used by the aircraft manufacturers. The guidance law will try to follow a sequence of waypoints computed and implemented on the missile before the launch. The waypoints will be given in a horizontal plan tangent to the earth surface around a reference point while another loop will control the altitude of the missile. Concerning the pilot a linear controller called “three loops autopilot” is implemented. The guidance – control law designed will gives the possibility to test a non-linear technique called Anti-windup on the propulsion control loop of the missile during the cruise phase.

**IV.2 Guidance**

**IV.2.a Mid-flight guidance**

The guidance loop has to control the slow dynamics of the missile, i.e. the position of its centre of mass. During the preparation of the mission a trajectory is defined via an array of waypoints and then uploaded on the missile before the launch. The guidance control loop controlling the missile between the launch and terminal phases is usually called mid-flight guidance.

To follow the trajectory the guidance loop will send acceleration commands to the control loop to align the velocity of the missile and the vector missile-waypoint [3]. This requirement can be translated in terms of heading; the desired heading will be given by:

\[
\psi_s = \arctan\left(\frac{y_{WP} - y_M}{x_{WP} - x_M}\right)
\] (64)
This is then translated in terms of acceleration commands via a bank to turn strategy. To generate a desired acceleration:

- The missile is controlled to a specified roll angle
- The angle of attack of the missile is increased to generate the acceleration
- The sideslip angle is set to 0.

This technique is particularly adapted to aerodynamic configuration with a bigger lifting surface in pitch than in yaw like our MdCN.
IV.2.b Altitude Control

A simple linear PD controller controls the altitude of the missile:

\[ \Gamma_{z_{co}} = k_p (z - z_{co}) + k_d \dot{z} \]  \hspace{1cm} (65)

where \( k_p \) and \( k_d \) are tuned assuming a perfect autopilot: \( \Gamma_{z_{co}} = \Gamma_z = \dot{z} \)

\[ \frac{z}{z_{co}} = \frac{1}{s^2 - \frac{k_d}{k_p} s - s} = \frac{1}{s^2 + 2 \frac{\varepsilon}{\sigma} s + 1} \]  \hspace{1cm} (66)

The settings of \( \sigma \) and \( \varepsilon \) give then \( k_p \) and \( k_d \)

IV.3 Three-loop autopilot

The autopilot used in the control loop is the so-called three-loop autopilot as presented by Paul Zarchan in [4]. The calculation are presented in detail in appendix B

IV.4 Propulsion loop

IV.4.a Anti-windup

Saturations are a very common source of performance and/or stability losses in a control system. In the case of controllers involving integrators such as PID controllers, saturation often produces a phenomenon called windup. This occurs when the controller integrates an error that doesn’t converge to zero because the saturation of the system. Furthermore when the order of the system is bigger than three, windup can even induce a loss of stability as shown in the example below:

A PID controller controls a double integrator so that it converges to zero. A stabilizing tuning of the PID gives \( P = 10 \) \( I = 0.5 \) \( D = 2 \). However this system is subject to saturation between \(-1\) and \(1\) in input.

![Diagram](image)

Figure 42 compares the output the systems with and without saturation for an initial output of 50.
The previous example illustrates the effect of windup on the performance of a controller. In many cases this limitation should be taken into account when designing a controller. One way to deal with saturations is the so-called anti-windup compensators, among others things the biggest advantages of such compensators are:

- They have a limited impact on an existing controller, only additive blocks need to be added and it is not necessary to rebuild the all controller.
- They guarantee that the performance obtained on the system without saturation is still obtained when the signals do not saturate.
- They guarantee the stability of the system without extreme performances loss.

They are two big approaches when designing an anti-windup compensator: the conditional integration and the back-calculation or tracking. Two conditional integration techniques and one back-calculation will be designed in this thesis and tested on the propulsion control loop of the MdCN under cruise phase. Some other techniques are detailed in [5]

The idea behind the conditional integration technique is to reset the integrators when needed.
- The first compensator, called simple conditional integrating compensator designed will simply reset the integrators when the control signal saturates.
- The second one will be called complex conditional integrating compensator will reset its integrators when the control signal saturates and when the control efforts tend to increase the error to the reference.

The back-calculation based compensator works in slightly different ways. The idea is to recalculate the integral term of the controller when the control signal saturates. In practice another loop is added on the integral term and feeds it back with the difference between the saturated and unsaturated signals. The block diagram of a back-calculation based compensator applied on a PID controller is shown below:
Figure 51 Back-calculation anti-windup compensator applied on a PID controller

With this compensator the integral part of the control becomes:

\[
    u = K_p T_e s + \frac{1}{s} (u' - u) = K_p T_e s + \frac{1}{s} (sat(u) - u) \tag{67}
\]

Where \( T_e \) is called tracking time constant or anti-windup gain.

As mentioned previously if the control signal doesn’t saturate then \( sat(u) = u' = u \) and the system has the same behaviour as a system without saturation. Usually \( u' \) is not measurable directly, and one has to use a model of the actuator inducing the saturation that acts as an observer of the system. That’s why back-calculation compensators are also called observer based compensator.

The tuning of \( T_e \) defines the speed at which the compensator tracks the desired state; this parameter has an impact on the global performance of the system. Several tuning rules are proposed in the literature: \( T_e = \sqrt{T_i T_p} \) [6] for a PID controller or \( T_e = T_i \) [7] for a PI controller, which gives satisfying performances. Yet the tuning of this parameter is the main drawback of the technique and might be difficult for some more advanced controllers.

\textbf{IV.4.b Application on the propulsion control loop}

The three anti-windup techniques presented previously will be applied on the propulsion control loop of the MdCN during the cruise phase. The model used contains an advanced model for the turbojet TR 50 and the air entry [8]. The missile is controlled by the guidance law described previously and the three loops autopilot and follows a trajectory defined by some waypoints. The following diagram shows the propulsion control and guidance control loops in a global diagram:
The control signal of the turbojet is expressed as a percentage of the highest speed of the motor and is subject to saturation between 83.7 and 100%. Since this propeller is controlled by PI controller it might be subject to windup phenomena and the application of the anti-windup techniques presented previously is relevant.

**IV.4.c Case study**

The anti-windup compensator will be tested on a simulation. The MdCN performs a tight turn after a few seconds and then follows a line. The task of the propulsion controller is to maintain a constant speed of 266m/s for the missile despite of the turns.
Figure 44 shows the trajectory of the missile during the simulation, the turn performed after the second waypoint is too tight and the turbojet is not powerful enough to maintain the speed at 266 m/s during a certain time and the controller saturates. This simulation is repeated with the three anti-windup techniques and the results are shown below:

![Figure 54 Control signal of the MdCN for different Anti-Windup techniques](image1)

![Figure 55 Speed of the MdCN for different Anti-Windup techniques](image2)

The behaviour of the system without Anti-windup compensator is really poor. The saturation induced to maintain the speed of the missile “loads” the integral part of the controller and, when by $t \approx 60s$ the speed cross the reference of 266m/s, the integral term of the controller is still very high and induces a very high overshoot. When the controller “realises” that the missile is flying too fast the controller signal saturates to its lowest value which cause the undershoot observed by $t \approx 200s$. Either the PI controller should be retuned to avoid such long saturation or an anti-windup device should be added. Indeed anti-windup compensators give all good results, they all manage to avoid the overshoot observed in the first case without compensator, on the other hand the rise time is a bit longer for the back-calculation and the simple conditional integrator based compensators.

### IV.5 Results

The previous control laws are implemented and tested using the following specifications for the three loops autopilot:

- **Roll:** $\tau = 0.7s$, $\xi = 0.8$, $\omega_{CR} = 20rad / s$
- **Pitch:** $\tau = 0.4s$, $\xi = 0.7$, $\omega_{CR} = 25rad / s$
- **Yaw:** $\tau = 0.4s$, $\xi = 0.7$, $\omega_{CR} = 12rad / s$

A cruise phase trajectory is shown below:
Figure 45 shows the trajectory of the missile in a horizontal plan. The missile follows correctly the flight plan defined by the waypoints. However one should note that these results are obtained assuming a perfect navigation. In reality the missile has to estimate its position using different sensors like GPS, Inertial Measurement Units (IMU) or radio-altimeter that introduce errors and bias.

Figure 46 show the attitude of the missile during the cruise phase. The attitude angles are following the commands send by the guidance properly. One can note that the pitch and roll angles have a much faster dynamics than the yaw angle which is typical for a bank to turn strategy.
V. Navigation issues

V.1 Interface with Oceani

At the LRBA the navigation issues are studied on an other software called Oceani. Many sensors and IMU models were developed under Oceani with the associated filters. The idea behind this interface is to analyse the performances of the global guidance control navigation loop using the work done earlier on Matlab/Simulink and Oceani. Another objective was to validate the kinematics model used in A3 (see appendix A) by comparing it with the one developed under Oceani. Indeed the kinematics model developed in A3 was designed for tactical application and was not accurate enough for long range flight such as a cruise missile flight while the model used for navigation studies in Oceani was much more detailed and took all the inertial terms in account, especially the rotation of the Earth. The interface is a TCP/IP socket between Oceani and Simulink via a controller coded in C++ that manages the data exchanged between the two programs\(^2\). Two cases were studied during this thesis:

- In the first case Simulink is in charge of the guidance, control and model of the missile and computes the forces and torques applied on the missile. Oceani integrates the forces and torques and gives the kinematics of the missile that is used by the guidance and control. In parallel Simulink runs its own kinematics model to compare its performance with Oceani’s one.

\(^2\) The controller was designed during a previous thesis, only minor adjustments were added during the present thesis. A complete description of the way this Controller works in available in [9]
In the second case Simulink computes the data as previously but this time its kinematics model is in the loop. Oceani models an Inertial Measurement Unit and adds the navigation errors to the kinematics state to get an estimated state used by the guidance and control algorithms.

**V.2 Kinematics model and numerical integration**

For long range flight the influence of the Earth rotation has to be taken in account. Thus $R_n$, the local navigation reference frame fixed with respect to the Earth is not Galilean anymore and the fictitious forces should be added in Newton’s law. Thus the equation of the force becomes:

$$m \frac{d}{dt} \vec{V}(G_0)_n = P_b^o \sum \vec{F}_{extb} - 2 \vec{\Omega}_t \wedge \vec{V}(G_0) + \vec{\Omega}_t \wedge (\vec{\Omega}_t \wedge \vec{OG}_0)_n \quad (68)$$

With:

- $\vec{\Omega}_t = \begin{pmatrix} 0 \\ 0 \\ \omega_t \end{pmatrix}$ the rotation of the Earth.
- $2 \vec{\Omega}_t \wedge \vec{V}(G_0)_n$ the Coriolis force.
- $\vec{\Omega}_t \wedge (\vec{\Omega}_t \wedge \vec{OG}_0)_n$ the centrifugal force.
The integration of this new kinematics equation is not straightforward. First a block using the classical integrator “1/s” of Simulink was used (see [9]), launch in parallel with Oceani and compared with a reference trajectory.

The solver used in Simulink was a Runge-Kutta 4 at 100Hz and it comes out that this model was good yet it didn’t match the degree of precision required for a cruise flight especially on the altitude axis. The same simulation was launched at 1kHz and the results were much better. However, with such a sampling rate, simulations take too much time, yet it shows that the equations used are right but the integration is the origin of the problem. Thus it was decided to implement under Simulink the numerical integration method of Oceani which is a kinematics dedicated method called Edwards method [10].

![Diagram](image)

**Figure 58 Comparison of the latitude. 1/s classical integration method**
The method was implemented under Simulink and the simulation was launched again. The following table summarizes the errors to the reference trajectory of both methods in Simulink and the method of Oceani. The best improvement were obtained on the altitude axis, see figures 50 and 51, where the performances were the worst and Simulink is now much closed to the reference trajectory and the error is of the order as ones of Oceani which means that the model can be validated and used for simulations.

<table>
<thead>
<tr>
<th>Errors to the reference trajectory after 500s</th>
<th>Matlab Runge-Kutta 4 100Hz</th>
<th>Matlab Edwards method</th>
<th>Oceani Edwards method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>9.1e-7 rad</td>
<td>8.9e-7 rad</td>
<td>-2.4e-7 rad</td>
</tr>
<tr>
<td>Longitude</td>
<td>-3.4e-7 rad</td>
<td>2.8e-7 rad</td>
<td>2.7e-7 rad</td>
</tr>
<tr>
<td>Altitude</td>
<td>108 m</td>
<td>0.22 m</td>
<td>0.05 m</td>
</tr>
<tr>
<td>(V_{NS})</td>
<td>0.0325 m/s</td>
<td>0.0312 m/s</td>
<td>-0.0083 m/s</td>
</tr>
<tr>
<td>(V_{EO})</td>
<td>-0.0084 m/s</td>
<td>-0.0105 m/s</td>
<td>0.0106 m/s</td>
</tr>
<tr>
<td>(V_z)</td>
<td>-0.45 m/s</td>
<td>-0.0012 m/s</td>
<td>0.0021 m/s</td>
</tr>
</tbody>
</table>
V.3 Inertial Navigation System simulation

In this part the interface is used to let Oceani simulates an Inertial Measurement Unit (IMU). IMU are widely used in missiles systems but also in aircrafts, ships or satellites to estimate the position, speed and attitude of the system studied. An IMU is a combination of three accelerometers and three gyrometers to take measurements in the three axes, which are corrupted by noise bias etc… Today IMU are combined with other sensors like radioaltimeters or GPS to give a better estimation of the state of the system through Kalman filtering for example, that is not the point of this thesis but a complete navigation system with sensors and filters can easily be simulated under Oceani.

In the present case the navigation system is a IMU and its navigation filter. Two different class of IMU are tested on a complex trajectory to analyse the influence of navigation errors on the guidance and control laws. These IMUs are compared with the trajectory followed in V.5 (see fig 45) where the navigation was perfect.
Figure 52 shows the trajectory of the missile in the horizontal plan for different navigation systems. The results of the simulation show that the precision of the Navigation system has a global impact on the performances of the missile. Indeed for the second IMU that is a system with poor performances the position of the missile is so poorly estimated that the missile couldn’t finish the flight-plan and hit the ground after the sixth waypoint. While the results obtained with a better IMU are of course not perfect but acceptable to achieve a mission.

Conclusion

Summary

A presentation of the DGA and of the MdCN was first presented to the reader to introduce the context of the thesis and describes the general structure of the missile. The main components of the missiles were introduced and then the chronology of the launch has briefly described. Chapter 2 describes the Modelling of the missile. Some useful knowledge in aerodynamics are presented, including the different reference frames used to describe the state of a missile or an aircraft, and a general model for the aerodynamic forces applied on the missile. Then mass and inertia of the missile and their evolution during the booster phase are computed, the booster itself is modelled. Finally a state space model is derived and simplified.

Chapter 3 deals with the launch phase. The general problematic of this phase is first recalled and then a first controller is designed using the quaternion to describe the attitude of the missile and a backstepping design to control it. Then a more advanced control sequence is
introduced to match the specifications needed to ignite properly the booster. Based on the same controller an AoA and sideslip compensator is designed and finally the parameters of the control sequence are chosen using a statistic analysis. In a last part the performances of the controller designed are analysed for different conditions, it came out the backstepping based controller could give good results under many conditions.

Chapter 4 describes the cruise phase issue. To analyse the performances of different anti-windup compensators on the propulsion control loop a guidance control algorithm is briefly described and implemented, then the windup phenomenon is introduced and the two main anti-windup techniques are briefly presented and finally tested on a case study. The results of this analysis were that saturations have to be taken in account when designing the controllers and that the different anti-windup techniques gave comparable and satisfying results.

Chapter 5 presents the work done to take the navigation errors in account in the control guidance algorithms. Matlab/Simulink is interfaced with a navigation software called Oceani. A new kinematics model for Simulink is derived based on a comparison with Oceani’s one and a reference trajectory and finally a IMU simulated by Oceani is introduced in the loop to add some navigation errors to the state. Different types of IMU are compared with the results obtained with a perfect navigation. It came out that navigation errors can lead to the crash of the missile and that a simple IMU was not enough for a cruise flight.

**Future works**

There are several possible improvement of the work presented here, some of them are given here as future possible works:

- The robustness of the Backstepping controller to model errors should be analysed. This will be more instructive when the design of the MdCN will be a bit more advanced and when the DGA will have a better idea of the model of the missile.

- A study can be lead to analyse the influence of the waves and the sea on the attitude of the carrier and thus on the initial position of the MdCN to see whether or not the missile can be fired when the sea conditions are bad.

- The backstepping controller could be compared with a classical linear controller to see if the non-linear techniques improve the performance, availability and reliability of the missile.

- Concerning the anti-windup compensator applied on the propulsion loop, it could be interesting to study the fuel consumption induced by the different compensators and to study their impact on the global range of the missile.

- More advanced navigation system should be introduced in the loop using different sensors like GPS or radioaltimeter hybridised with the IMU and the corresponding filters to simulate a navigation system closer to the one that will be used in the MdCN.
Appendix A: A3

A.1 Presentation of A3

The model of the missile and its control laws are designed using A3 (Architecture d’aéronefs autopilotés *Autopiloted Aircrafts Architecture*) a special toolbox of Matlab Simulink developed at the LRBA.

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Aerodynamics</th>
<th>Atmosphere</th>
<th>Autopilote</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>Energy sources</td>
<td>Generic architectures</td>
<td>Interfaces – Visualisations</td>
</tr>
<tr>
<td>Kinematics</td>
<td>Masse &amp; Inertia</td>
<td>Payload</td>
<td>Propulsion</td>
</tr>
<tr>
<td>Sensors</td>
<td>Structure</td>
<td>Tools</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 64 A3 Toolbox](image)

A3 is meant to be a generic aircraft simulator: it should be as simple as possible to simulate various aircrafts under different flight conditions with different control laws… thus it has to use a specific architecture. The idea is to fill in the architecture using the blocs of the toolbox and to define its parameters in a mask. The standard architecture of the models is shown below:
The figure above shows the first level of the generic simulator, it consists of three blocs:

- The Environment bloc which describes the flight conditions: it is divided in two blocs an earth and an atmosphere blocs as shown below:

- The Interface/Visualisation bloc for all the visualisation tools such as graphs, charts, 3D rendering…
• The Aerial Vehicle bloc: divided in four main blocs: a vehicle bloc which will be described later on, a sensor and a payload blocs and the autopilot bloc where all the control laws will be implemented.

Figure 67 The aerial vehicle bloc level 1

The vehicle bloc is shown below:

Figure 68 Vehicle bloc – level 2

It consists of several blocs: propulsions, inertia, structure, kinematics... which give a systematic representation of the aircraft.
This system approach is very efficient and it allows the user to test very easily different models, different control laws. Furthermore it makes cooperation and team works easier since everybody has to use the same standard representation.

**A.2 Implementation of the model under A3**

**A.2.a Launch chronology**

For the implementation of the model under A3 a special attention is paid to the chronology of the launch. Indeed depending on the situation, the kinematics of the missile is not the same. The launch will be divided into different phases:

1. The carrier phase: the fire order hasn’t been given yet and the missile is still in its CTV (*Conteneur de Tir Vertical* Vertical Launch Container). During this phase the missile simply follows the dynamic of the carrier.
2. The CTV phase: this phase last from the fire order and the booster ignition to the torque when the missile leaves its CTV. During this phase the missile follows the attitude dynamic of the carrier, in the body axis \(x\) and \(z\) but \(y\) evolves according to the missile dynamics.
3. The missile has left the CTV: the missile behaves according to its full cinematic model.

Once the missile has left the CTV the same kinematics equations can be applied until the end of the simulation, however it was decided to introduce additional phases to describe structural changes of the MdCN like the SAB’s drop, the wings and fins spreading or the turbojet ignition.

**A.2.b A3 Model architecture**

**III.2.a A0 Level**

The architecture used to implement the model under A3 is slightly different from the generic architecture introduced previously. Level A0 is shown below:
Three blocks were added: a Mission Evolution block that deals with all the phases described previously the output of the block is a vector which components are Boolean variables that describe the evolution of the simulation. By now seven components are used, but one can easily add new phases if needed.

- Fire order (1 if given 0 else)
- SAB turned on
- CTV phase
- Turbojet turned on
- Cruise mode (i.e. wings and fins deployed)
- SAB dropped
- DCM dropped (launch from a submarine)

The block FPM deals with the mission data such as maps or enemy targets. The block is not used for the torque

The FREMM/BARR block simulates the behaviour of the carrier, either a frigate FREMM or a submarine SNA Barracuda, and computes the position of the missile on the carrier during the carrier and CTV phases.

II.2.b A1Level - Aerial Vehicle block

The Aerial Vehicle block is also a bit different from the classical architecture. Actually one level was added since the missile block has the same architecture as the former aerial vehicle block however the SAB and DCM blocks are new and will recreates the behaviour of the SAB and DCM. The DCM block is not used by now.
II.2.c A2 Level – SAB block

This block is really dedicated to the simulation of the launch phase and calculates the forces produced by the nozzles (top right), the inertia, the mass and the position of the centre of gravity of the SAB (bottom right) and finally the last block on the right will computes the kinematics of the SAB once it has been dropped from the missile. Indeed the SAB should fall in a given area due to safety issues.
Appendix B: S4

The work performed in the present thesis was hold in a so-called “Atelier Système” (System Workshop) named S4 for Scenarios, Simulation, Synthèse Système (Scenarios, Simulation, System Synthesis). The main goal of these workshops is to keep and share the knowledge between the different departments of the LRBA and the different centres of the DGA. Different tools are used in the workshops to create some advanced scenarios played in simulation and to display the results of the simulations. During this thesis two of these tools were used namely: FlightGear and Stage.

B.1. FlightGear interface

FlightGear is an open source flight simulator that has been interfaced at the S4 with Simulink to generate a 3D visualisation of the simulations. Basically a socket sends the position and attitudes data to the 3D rendering of the simulator.
B.2. Stage

Stage is a more advanced software designed to create some advanced scenarios, simulate and store them. Stage gives the possibility to simulate many objects in a complex environment. The scenario is defined by placing some entities such as aircrafts, ships, and missiles… on a tactical situation and defining their behaviour via waypoints on the map or via scenario editor.

During the thesis Stage has been interfaced with Matlab/Simulink to let Matlab/Simulink simulates the behaviour of the MdCN in Stage environment. A scenario was generated using Stage to simulate the whole mission of a MdCN. In this simulation a FREMM had to follow some waypoints until it gets the fire order, the Simulink model of the MdCN is launched and get its initial state, the flight plan and the target position. Simulink send to Stage through a socket the position, speed and attitude of the missile and Stage update the tactical situation. Thus a complete simulation could be visualized on Stage 3D rendering tool.
Appendix C: Three loops autopilot

The control loop of the pitch angle is the only one presented here but the procedure is the same on the roll and yaw angle.

C.1 Transfer functions calculation

The transfer functions from the equivalent fin deflection in pitch angle to the vertical acceleration and from the fin deflection the pitch rate are calculated here. Let denote:

\[ z_\alpha = \frac{QSCN_\alpha}{mV_{\text{ref}}} \, , \quad z_\delta = \frac{QSCN_\delta}{mV_{\text{ref}}} \, , \quad m_\alpha = \frac{IQSCm_\alpha}{I_{yy}} \, , \quad m_\delta = \frac{IQSCm_\delta}{I_{yy}} \, , \quad m_q = \frac{IQS}{V_{\text{ref}}} \frac{l}{I_{yy}} Cm_q \]

where \( V_{\text{ref}} \) is the desired speed of the missile during the cruise phase and \( Q \) the dynamic pressure. Then denote:
After some calculations we get the transfer functions:

\[
\Gamma_z = \frac{K_i (-s^2 + 1)}{\sigma_z^2 + 2 \epsilon_{af} \sigma_{af}} \quad \text{and} \quad q = \frac{K_3 (T_q s + 1)}{s^2 + 2 \epsilon_{af} \sigma_{af}} \quad s + 1
\]

\( T_o \) is called the incidence lag and denotes the lag of the aerodynamic speed with respect to the air frame, \( K_1 \) and \( K_3 \) are respectively the manoeuvrability in force and angle.

### C.2 Controller design

The flight control system with the three loops autopilot appears just below. A rate gyro feeds back the pitch rate while an accelerometer feeds acceleration information. Three autopilot gains are introduced \( K_d, \omega_i \) and \( K_r \) that are chosen to satisfy the design specifications while a fourth gain \( K_d \) will ensure the unity flight control gain.

![Flight Control System Diagram](image)

After some calculations the open loop transfer function can be written as:
Which can rewritten as:

\[
G(s) = -K_0 \left(1 + \frac{2\xi_0}{\omega_0} s + \frac{s^2}{\omega_0^2}\right)
\]

Where:

\[
K_0 = K_r \omega_1 K_d \left(\frac{K_3}{K_d} + K_1\right), \quad \frac{1}{\omega_0^2} = \frac{K_3 T_d - K_d \omega_1 K_1 / \omega_c^2}{\omega_1 K_3 - K_d \omega_1 K_1}, \quad \frac{2\xi_0}{\omega_0} = \frac{(K_3 + \omega_1 K_3 T_d)}{\omega_1 K_3 + K_d \omega_1 K_1}
\]

Assuming that the crossover frequency of the open loop is beyond the airframe dynamics, we have:

\[
G(\omega_{CR}) = 1 \approx -\frac{K_0 \omega_{CR}^2 / \omega_0^2}{\omega_{CR} \omega_c^2 / \omega_{af}^2}
\]

Solving for \( \omega_{CR} \):

\[
\omega_{CR} = \frac{-K_0 \omega_{af}^2}{\omega_0^2}
\]

After much algebra the transfer function of the closed system comes out:

\[
\frac{\Gamma_e}{\Gamma_c} = \frac{K_1 K_d K_4 K_3 \omega_1}{K_0} \left(1 - \frac{s^2}{\omega_{af}^2}\right)
\]

\[
\frac{1}{\omega_{af}^2 K_0} s^3 + \left(\frac{1}{\omega_0} - \frac{2\xi_0}{\omega_{af} K_0}\right)s^2 + \left(\frac{2\xi_0}{\omega_{af}} - \frac{1}{K_0}\right)s + 1
\]

The previous equation will now be rewritten under the following form:

\[
\frac{\Gamma_e}{\Gamma_c} = \frac{K_1 K_d K_4 K_3 \omega_1}{K_0} \left(1 - \frac{s^2}{\omega_{af}^2}\right)
\]

\[
\frac{1}{(1 + \xi_0)(\omega_0^2 + 2\xi_0 \omega / \omega_{af} + 1)}
\]

These transfer functions are equivalent if the denominators are the same, which yields to:
For a given airframe and given flight condition we know \( \sigma_{af} \) and \( \varepsilon_{af} \). Thus if we specify a desired time constant \( \tau \), a desired damping \( \xi \) for the closed loop system and a desired crossover frequency \( \omega_{CR} \) for the open loop system, we have four unknowns left \( \omega \), \( \xi_0 \), \( \omega_0 \) and \( K_0 \) which are directly linked to the autopilot gain. Recalling the equation for \( \omega_{CR} \) derived from the open loop analysis we have the following system of four equations and four unknowns:

\[
\begin{align*}
\frac{\tau}{\omega^2} &= -\frac{1}{\sigma_{af}^2 K_0}, \\
\frac{1}{\omega^2} + \frac{2\xi \tau}{\omega} &= \frac{1}{\sigma_0^2} - \frac{2\varepsilon_{af}}{\sigma_{af} K_0}, \\
\frac{2\xi}{\omega} + \tau &= \frac{2\xi_0}{\sigma_0} - \frac{1}{K_0}, \\
\omega_{CR} &= \frac{-K_0 \sigma_{af}^2}{\sigma_0^2}.
\end{align*}
\]

Which can be solved for the autopilot gains and we finally get the following expressions:

\[
K_c = \frac{-\omega_0^2}{\sigma_z^2} - 1 + 2\xi_0 \omega_0 T_a \\
\text{and}
\]

\[
K_D = 1 + K_c \\
K_A = \frac{K_2}{K_c K_1} \\
\omega_f = \frac{T_a K_c \omega_0^2}{1 + K_c + \frac{\omega_0^2}{\sigma_z^2}} \\
K_r = \frac{K_0}{K_1 K_D K_A \omega_f}.
\]
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