Parameter Study of Ferro-Resonance with Harmonic Balance Method

ALI ERBAY

Degree project in
Electric Power Systems
Second Level,
Stockholm, Sweden 2012

XR-EE-ES 2012:010
PARAMETER STUDY OF FERRO-RESONANCE WITH HARMONIC BALANCE METHOD

Ali ERBAY

Supervised by Mohamadreza BARADAR

Electric Power Systems Lab
Royal Institute of Technology
**ABSTRACT**

Ferro-resonance is an electrical phenomenon which can cause damage to electrical equipments of power systems by its characteristic steady state over voltages and over currents. Configurations where ferro-resonance is possible has more than one steady state operation. With time domain simulations, different dangerous steady state operations are hard to find due to the fact of dependancy of initial conditions and parameters of the system. Determination of risk of ferro-resonance needs special studies involving frequency domain and Fourier series based harmonic balance method. Two different types of harmonic balance method are used; namely analytical and numerical method. In order to draw two-parameter continuous curves, harmonic balance with hyper-sphere continuation method algorithm is created in MATHCAD environment. Work of two case studies in academic literature are extended by comparing different system parameter curves and calculating stability domain risk zones for fundamental ferro-resonance, subharmonic-1/2 and subharmonic-1/3 ferro-resonance. Alstom’s test system is also investigated with approximations. Application of numerical harmonic balance method is more superior than analytical method since it is ease of use with thevenin equivalents rather than deriving system equation by hand and possibility to study subharmonic ferro-resonance. Hyper-sphere continuation method worked well enough to turn limit points on parameter curves depending on considered Fourier components. Critical values for system parameters have been found for each type of ferro-resonance allowing to analyse normal operation and ferro-resonance operation regimes. Critical values of static damping resistor in the system can be calculated by harmonic balance method without using empirical formula. Damping resistor calculated by harmonic balance method showed difference than the one calculated by empirical formula. Fundamental and subharmonic ferro-resonance solutions existence zones are co-existant and sensitive to parameter changes therefore same attention should be given to subharmonic as in fundamental ferro-resonance. For future studies, three-phase models for harmonic balance method should be developed in order to study neutral isolated networks and a more customized method of solving non-linear harmonic balance equations for faster computation can also be developed in MATLAB environment.
I would like to thank Mr. Eusebio LOPEZ in ALSTOM Thermal Systems Department (Massy, France) for his support during my internship.
TABLE OF CONTENTS

1 INTRODUCTION ................................................................................................................. 10
2 FERRO-RESONANCE IN LITERATURE .................................................................................. 11
  2.1 TIME-DOMAIN ANALYSIS .............................................................................................. 11
  2.2 EFFECTS OF INITIAL CONDITIONS ............................................................................... 11
  2.3 NON-LINEAR TRANSFORMER CORE MODELS ............................................................... 12
  2.4 DAMPING AND MITIGATION OPTIONS ........................................................................... 12
  2.5 FREQUENCY DOMAIN ANALYSES .................................................................................. 13
3 LINEAR RESONANCE AND FERRO-RESONANCE ................................................................. 13
4 CAUSES AND EFFECTS OF FERRO-RESONANCE IN THE POWER SYSTEMS ................... 14
  4.1 SYSTEMS VULNERABLE TO FERRO-RESONANCE ......................................................... 15
    4.1.1 Voltage Transformer Energized Through Grading Capacitance .............................. 15
    4.1.2 Voltage Transformers Connected to an Isolated Neutral System ....................... 15
    4.1.3 Transformer Accidentally Energized in Only One or Two Phases ...................... 16
    4.1.4 Voltage Transformers and HV/MV Transformers with Isolated Neutral .......... 17
    4.1.5 Power system grounded through a reactor ....................................................... 18
    4.1.6 Transformer Supplied by a Highly Capacitive Power System with Low Short-Circuit Power .......... 19
5 PREVENTING FERRO-RESONANCE ................................................................................... 20
  5.1 DAMPING FERRO-RESONANCE IN VOLTAGE TRANSFORMERS .................................. 20
    5.1.1 Voltage Transformers with one Secondary Winding ........................................... 21
    5.1.2 Voltage Transformers with two Secondary Winding ........................................... 22
6 MODEL OF NON-LINEARITY ............................................................................................... 23
7 FERRO-RESONANCE IN TIME-DOMAIN ........................................................................... 25
  7.1 NORMAL OPERATION ...................................................................................................... 27
  7.2 FUNDAMENTAL FERRO-RESONANCE OPERATION ....................................................... 28
  7.3 SUBHARMONIC FERRO-RESONANCE OPERATION ....................................................... 30
  7.4 CHAOTIC FERRO-RESONANCE OPERATION ................................................................ 32
8 ANALYTICAL HARMONIC BALANCE METHOD ................................................................ 35
  8.1 APPLICATION OF HARMONIC BALANCE ON EXAMPLE SYSTEM .............................. 35
9 NUMERICAL HARMONIC BALANCE METHOD .................................................................. 43
  9.1 MATHEMATICAL FRAME ............................................................................................... 44
  9.2 CONTINUATION METHOD ............................................................................................. 45
  9.3 SELECTION OF HARMONIC COMPONENTS .................................................................. 49
  9.4 STABILITY DOMAINS BY NUMERICAL HARMONIC BALANCE METHOD ............... 50
10 FIRST APPLICATION OF NUMERICAL HARMONIC BALANCE ........................................ 52
  10.1 FUNDAMENTAL FERRO-RESONANCE ANALYSIS ....................................................... 53
    10.1.1 Flux – Source Voltage ......................................................................................... 53
    10.1.2 Flux – Capacitance ............................................................................................. 54
    10.1.3 Flux – Resistance ............................................................................................... 56
10.1.4 Stability Domain: Source Voltage against Capacitance ........................................... 56
10.1.5 Stability Domain: Source Voltage against Resistance ............................................ 57
10.2 SUBHARMONIC-1/2 FERRO-RESONANCE ANALYSIS .................................................. 57
10.2.1 Flux – Source Voltage.............................................................................................. 58
10.2.2 Flux – Capacitance .................................................................................................. 60
10.2.3 Flux – Resistance .................................................................................................... 60
10.2.4 Stability Domain: Source Voltage against Capacitance ........................................... 61
10.2.5 Stability Domain: Source Voltage against Resistance ............................................. 61
10.3 SUBHARMONIC-1/3 FERRO-RESONANCE ANALYSIS .................................................. 62
10.3.1 Flux – Source Voltage.............................................................................................. 62
10.3.2 Flux – Capacitance .................................................................................................. 63
10.3.3 Flux – Resistance .................................................................................................... 63
10.3.4 Stability Domain: Source Voltage against Capacitance ........................................... 64
10.3.5 Stability Domain: Source Voltage against Resistance ............................................. 64
10.3.6 Remarks .................................................................................................................. 65

11 SECOND APPLICATION OF NUMERICAL HARMONIC BALANCE .................................... 65
11.1 FUNDAMENTAL FERRO-RESONANCE ANALYSIS ......................................................... 67
11.1.1 Flux – Source Voltage.............................................................................................. 67
11.1.2 Flux – Capacitance .................................................................................................. 68
11.1.3 Flux – Series Resistance .......................................................................................... 68
11.1.4 Flux – Parallel Resistance ....................................................................................... 69
11.1.5 Stability Domain: Source Voltage against Capacitance ........................................... 70
11.1.6 Stability Domain: Source Voltage against Parallel Resistance ............................... 70
11.2 SUBHARMONIC-1/2 FERRO-RESONANCE ANALYSIS .................................................. 71
11.2.1 Flux – Source Voltage.............................................................................................. 71
11.2.2 Flux – Capacitance .................................................................................................. 72
11.2.3 Flux – Series Resistance .......................................................................................... 72
11.2.4 Flux – Parallel Resistance ....................................................................................... 73
11.2.5 Stability Domain: Source Voltage against Capacitance ........................................... 73
11.2.6 Stability Domain: Source Voltage against Parallel Resistance ............................... 74
11.3 SUBHARMONIC-1/3 FERRO-RESONANCE ANALYSIS .................................................. 74
11.3.1 Flux – Source Voltage.............................................................................................. 75
11.3.2 Flux – Capacitance .................................................................................................. 75
11.3.3 Flux – Series Resistance .......................................................................................... 76
11.3.4 Flux – Parallel Resistance ....................................................................................... 76
11.3.5 Stability Domain: Source Voltage against Capacitance ........................................... 77
11.3.6 Stability Domain: Source Voltage against Parallel Resistance ............................... 77
11.3.7 Remarks .................................................................................................................. 78

12 CASE STUDY ............................................................................................................... 78
12.1 SYSTEM DETAILS ...................................................................................................... 79
12.2 DAMPING RESISTOR CALCULATION BY EMPIRICAL METHOD ................................. 81
12.3 FUNDAMENTAL FERRO-RESONANCE ANALYSIS ......................................................... 81
12.3.1 Flux – Source Voltage.............................................................................................. 81
12.3.2 Flux – Capacitance .................................................................................................. 82
12.3.3 Flux – Damping Resistor ....................................................................................... 83
12.3.4 Stability Domain: Source Voltage against Capacitance ........................................... 83
12.3.5 Stability Domain: Source Voltage against Damping Resistor ................................... 84
12.4 SUBHARMONIC-1/2 FERRO-RESONANCE ANALYSIS .................................................. 84
<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.4.1</td>
</tr>
<tr>
<td>12.4.2</td>
</tr>
<tr>
<td>12.4.3</td>
</tr>
<tr>
<td>12.4.4</td>
</tr>
<tr>
<td>12.4.5</td>
</tr>
<tr>
<td>12.5</td>
</tr>
<tr>
<td>12.5.1</td>
</tr>
<tr>
<td>12.5.2</td>
</tr>
<tr>
<td>12.5.3</td>
</tr>
<tr>
<td>12.5.4</td>
</tr>
<tr>
<td>12.5.5</td>
</tr>
<tr>
<td>12.6</td>
</tr>
<tr>
<td>12.6.1</td>
</tr>
<tr>
<td>12.6.2</td>
</tr>
<tr>
<td>12.6.3</td>
</tr>
<tr>
<td>12.6.4</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>15</td>
</tr>
</tbody>
</table>
List of figures

FIGURE 4.1 Ferro - resonance of a voltage transformer connected in series with an open circuit breaker[46] ........ 15
FIGURE 4.2 Ferro-resonance of a voltage transformer between phase and ground in an isolated neutral system[46] ................................................................. 16
FIGURE 4.3 Examples of unbalanced systems[46] .................................................................................. 17
FIGURE 4.4 Faulty system[46] ........................................................................................................ 17
FIGURE 4.5 Ferro-resonance of voltage transformer between phase and ground with ungrounded/isolated neutral[46] ............................................................................. 18
FIGURE 4.6 PIM inductance between neutral and ground[46] ............................................................ 18
FIGURE 4.7 Resonant grounding system[46] ...................................................................................... 19
FIGURE 4.8 Power transformer supplied by capacitive system[46] ...................................................... 19
FIGURE 5.1 Damping for voltage transformer with one secondary[46] ................................................ 21
FIGURE 5.2 Damping system for voltage transformer with two secondary[46] .................................... 23
FIGURE 6.1 Example of Saturation Curve ....................................................................................... 24
FIGURE 7.1 System diagram ........................................................................................................ 25
FIGURE 7.2 Equivalent circuit ........................................................................................................ 26
FIGURE 7.3 Normal operation ....................................................................................................... 27
FIGURE 7.4 Normal operation ....................................................................................................... 27
FIGURE 7.5 Normal operation phase plane .................................................................................... 28
FIGURE 7.6 Normal operation frequency content .......................................................................... 28
FIGURE 7.7 Fundamental ferro-resonance operation .................................................................... 29
FIGURE 7.8 Fundamental ferro-resonance operation .................................................................... 29
FIGURE 7.9 Fundamental ferro-resonance phase plane ................................................................. 30
FIGURE 7.10 Fundamental ferro-resonance frequency content ...................................................... 30
FIGURE 7.11 Subharmonic ferro-resonance operation .................................................................... 31
FIGURE 7.12 Subharmonic ferro-resonance operation .................................................................... 31
FIGURE 7.13 Subharmonic ferro-resonance phase plane .................................................................. 32
FIGURE 7.14 Subharmonic ferro-resonance frequency content ...................................................... 32
FIGURE 7.15 Chaotic ferro-resonance operation ............................................................................ 33
FIGURE 7.16 Chaotic ferro-resonance operation ............................................................................ 33
FIGURE 7.17 Chaotic ferro-resonance phase plane ........................................................................... 34
FIGURE 7.18 Chaotic ferro-resonance frequency content ................................................................ 34
FIGURE 8.4 Source voltage against flux ....................................................................................... 40
FIGURE 8.5 Source voltage against flux with R/4 ........................................................................ 41
FIGURE 8.6 Limit points ............................................................................................................... 42
FIGURE 8.7 Stability domain ...................................................................................................... 42
FIGURE 8.8 Stability domain with R/4 ....................................................................................... 43
FIGURE 9.1 Thevenin model ...................................................................................................... 44
FIGURE 9.2 Simple continuation ................................................................................................. 46
FIGURE 9.3 Tangent at limit point ............................................................................................... 47
FIGURE 9.4 Hyper-sphere method[54] ...................................................................................... 48
FIGURE 9.5 Hyper-sphere continuation algorithm ...................................................................... 48
Figure 9.6 Stability domain algorithm ................................................................. 51
Figure 10.1 Equivalent circuit ............................................................................... 52
Figure 10.2 Flux against voltage source ............................................................... 53
Figure 10.3 Effect of capacitance change ............................................................. 54
Figure 10.4 Effect of resistance change ............................................................... 54
Figure 10.5 Flux against equivalent capacitance ................................................ 55
Figure 10.6 Flux against resistance .................................................................... 56
Figure 10.7 Stability domain: Source voltage against capacitance ....................... 56
Figure 10.8 Stability domain: Source voltage against resistance ......................... 57
Figure 10.9 Flux against source voltage ............................................................. 58
Figure 10.10 Effect of capacitance change ........................................................... 59
Figure 10.11 Effect of resistance change ............................................................. 59
Figure 10.12 Flux against capacitance ............................................................... 60
Figure 10.13 Flux against resistance .................................................................. 60
Figure 10.14 Stability domain: Source voltage against capacitance ..................... 61
Figure 10.15 Stability domain: Source voltage against resistance ....................... 61
Figure 10.16 Flux against source voltage ........................................................... 62
Figure 10.17 Flux against capacitance ............................................................... 63
Figure 10.18 Flux against resistance .................................................................. 63
Figure 10.19 Stability domain: Source voltage against capacitance ..................... 64
Figure 10.20 Stability domain: Source voltage against resistance ....................... 64
Figure 10.21 Comparison of ferro-resonance modes .......................................... 65
Figure 11.1 Ferro-resonant circuit [36] ................................................................ 66
Figure 11.2 Flux against source voltage ............................................................. 68
Figure 11.3 Flux against capacitance ............................................................... 68
Figure 11.4 Flux against series resistance .......................................................... 69
Figure 11.5 Flux against parallel resistance ....................................................... 69
Figure 11.6 Stability domain: Source voltage against capacitance ....................... 70
Figure 11.7 Stability domain: Source voltage against parallel resistance ............... 70
Figure 11.8 Flux against source voltage ............................................................ 71
Figure 11.9 Flux against capacitance ............................................................... 72
Figure 11.10 Flux against series resistance .......................................................... 72
Figure 11.11 Flux against parallel resistance ...................................................... 73
Figure 11.12 Stability domain: Source voltage against capacitance ....................... 73
Figure 11.13 Stability domain: Source voltage against parallel resistance ............. 74
Figure 11.14 Flux against source voltage ............................................................ 75
Figure 11.15 Flux against capacitance ............................................................... 75
Figure 11.16 Flux against series resistance .......................................................... 76
Figure 11.17 Flux against parallel resistance ...................................................... 76
Figure 11.18 Stability domain: Source voltage against capacitance ....................... 77
Figure 11.19 Stability domain: Source voltage against parallel resistance ............. 77
Figure 12.1 Single phase diagram of the test system ............................................ 78
Figure 12.2 Three-phase simplified circuit .......................................................... 79
Figure 12.3 Single phase equivalent ................................................................. 79
Figure 12.4 Flux against source voltage ............................................................. 82
Figure 12.5 Flux against capacitance ............................................................... 82
Figure 12.6 Flux against damping resistor ....................................................... 83
Figure 12.7 Stability domain: Source voltage against capacitance ....................... 83
Figure 12.8 Stability domain: Source voltage against damping resistor ................ 84
Figure 12.9 Flux against source voltage ............................................................ 85
FIGURE 12.10 FLUX AGAINST CAPACITANCE .................................................................................................................... 85
FIGURE 12.11 STABILITY DOMAIN: SOURCE VOLTAGE AGAINST CAPACITANCE ................................................................. 86
FIGURE 12.12 STABILITY DOMAIN: SOURCE VOLTAGE AGAINST DAMPING RESISTOR ......................................................... 87
FIGURE 12.13 FLUX AGAINST SOURCE VOLTAGE .................................................................................................................. 88
FIGURE 12.14 FLUX AGAINST CAPACITANCE .................................................................................................................... 88
FIGURE 12.15 FLUX AGAINST DAMPING RESISTOR .................................................................................................................. 89
FIGURE 12.16 STABILITY DOMAIN: SOURCE VOLTAGE AGAINST CAPACITANCE ................................................................. 89
FIGURE 12.17 STABILITY DOMAIN: SOURCE VOLTAGE AGAINST DAMPING RESISTOR ......................................................... 90
FIGURE 12.18 STABILITY DOMAIN: DAMPING RESISTOR COMPARISON ................................................................................. 90
FIGURE 12.19 STABILITY DOMAIN: DAMPING RESISTOR COMPARISON ................................................................................. 91
FIGURE 12.20 STABILITY DOMAIN: DAMPING RESISTOR COMPARISON ................................................................................. 91
1 Introduction

Ferro-resonance is an electrical phenomenon that has been a problem for power systems. The word “ferro-resonance” firstly used in 1920s to define complex oscillations between system components and ferro-magnetic material in RLC circuits where inductance is non-linear [1].

Ferro-resonant oscillations occur in systems which contain at least:

- A non-linear inductance
- A capacitor
- A voltage source
- Low losses

In the modern power networks, there are high amounts of saturable inductances (voltage measurement transformers, shunt reactors, power transformers) and also capacitances such as long line charging capacitor, series or parallel capacitor banks and grading capacitors. Voltage in the power system is provided by generators. These factors make ferro-resonance scenarios possible in the power systems.

Ferro-resonance is considered as a jump resonance. Jump resonance refers to a condition in a sinusoidally excited system: if an incremental change in amplitude or frequency of the input to the system or in the magnitude of one of the parameters of the system causes a sudden jump in signal amplitude somewhere in the system, jump resonance is said to have occurred [2]. Change in frequency is not very common but for some specific values of parameters (applied voltage, capacitor value, core losses etc...) there may exist two or more stable operation points where one of them is normal steady operation and other ones are ferro-resonant steady operation.

Ferro-resonant oscillations are very harmful to power system equipments. Large currents and over-voltages are characteristic of these oscillations. In the past, there are cases reported where transformer and other equipment insulation are damaged because of ferro-resonance.

Ferro-resonance depends on parameters of the system, initial conditions and transients such as transformer remnant flux residue, circuit breaker switching angles, faults and load shedding. Because of this wide dependency, special studies should be made to analyze ferro-resonance.

Due to dependency of initial conditions and transients, ferro-resonance occurrence seems to be randomly natured. A system can be in risk of ferro-resonance but never experience it in its life-time because “certain conditions” never happened. But when it ever happens it causes catastrophic failure. One would like to know if the system is in risk or not.
Setups, configurations and scenarios that may cause ferro-resonance are many. It is not easy to try every scenario because it will take so much computational time and some scenarios could be overlooked.

In this thesis, safety margin of system parameters is looked for the systems subject to ferro-resonance rather than finding out every possible “certain conditions” for ferro-resonance to occur. To be able to this study, a frequency domain analysis – a modified Harmonic Balance method is used with continuation techniques to draw continuous parameter curves. These parameter curves are used for assessing risk of ferro-resonance.

## 2 Ferro-resonance in Literature

First work on ferro-resonance field dates back to 1907, but in that time, the word of ferro-resonance has not been used for phenomenon. It is considered as a transformer resonance [3]. Up until 1960s graphical and experimental studies were popular then non-linear dynamics are applied by Hayashi and many other types of ferro-resonance are found [4]. In 1970s the work of Hayashi are improved in mathematical sense. In [2] Swift, analyzed ferro-resonance with describing function. In 1975, Galerkin’s Method is firstly applied to ferro-resonant circuits [6].

Publications before 1990 have weak connections between ferro-resonance and non-linear dynamics generally because of gap between experimental studies and theoretical studies. Bifurcation theory is used for ferro-resonance studies in 1990 [7]. After beginning of 1990s, lots of academic papers have been published mainly focused on non-linear models, damping of ferro-resonance, effect of initial conditions on ferro-resonance and frequency domain analyses. In 2002, Jacobson used separatrix calculation for the study of ferro-resonance [5].

### 2.1 Time-Domain Analysis

Vast majority of the academic studies on ferro-resonance is done in time-domain where the effects of parameters have been studied by using phase planes, poincare sections [8]-[30].

EMTP software and other non-linear dynamic methods have been used to study chaotic behavior of ferro-resonant circuits [23]-[30].

### 2.2 Effects of Initial Conditions

Ferro-resonance has a special behavior which is its different responses with same parameter values depending on initial conditions [8]-[17]. It means that time-domain solutions might
give different steady states depending on initial conditions. Reference [9] and [10] shows that exact fault clearing switch moments have effect on ferro-resonance. This makes it very hard to check all of scenarios on time-domain.

Small changes in initial flux values and voltage supply for voltage transformers lead to a large difference in long term behavior of the system [11], [12].

2.3 Non-linear transformer core models

Non-linearity of ferro-resonance is very important factor on its behavior. So representation of non-linearity of transformer core is crucial for ferro-resonance studies. Reference [13] shows that ferro-resonant behavior of the transformer under study, based on the piecewise linear and the polynomial saturation characteristics are significantly different.

Normally transformer core loss considered constant, it is shown that non-linear core loss models offers more accurate results [14]. Reference [15] provides information about how to determine magnetization characteristics of transformer by taking into account only the rms values and no-load losses. This model presents benefits over other models since magnetization characteristic can be directly obtained from only three measured rms values (voltage, current, losses).

Based on the Preisach theory, another transformer core model is represented and tested on voltage transformer and compared to others. It is seen that proposed model gives closer results to experimental results [16].

2.4 Damping and Mitigation Options

There are dynamic and static options to damp ferro-resonance oscillations. Common remedy is to use the damping resistors on the secondary windings or tertiary windings of voltage transformers which is the static damping [18]. Different types of connection of damping resistor are tested for damping different kinds of ferro-resonances [19].

A novel type of bidirectional thyristor based resonance eliminator is also mentioned which is in theory superior to static damping [20].

There is also a way to damp ferro-resonant oscillations by connecting shunt resistor to grading capacitances which causes system to have less sensitivity to initial conditions and variation in system parameters [21].
2.5 Frequency Domain Analyses

Main objective of the frequency domain analyses is to find periodic steady state of ferro-resonant non-linear circuits. Hayashi considers harmonic balance method is the best way to skip transients and directly calculate steady state solution to non-linear systems [4]. Analytical harmonic balance method has been used in some academic research and it is proven that this method is very advantageous on parameter study of ferro-resonance [31], [32], [33].

Galerkin’s Method and bifurcation theory is firstly used by Kieny [34], [35]. It is concluded that time-domain simulations are not providing better understanding of ferro-resonance phenomena. Author also concluded that adjustable accuracy and ease of use make proposed method better than analytical harmonic balance method. His work is extended by Ben Amar and Dhifaoui [36], [37].

Stability domains of different types of oscillations and determining damping resistor values with harmonic balance method are firstly studied late 1990s [38]-[45]. These studies are currently the latest development on ferro-resonance literature.

3 Linear Resonance and Ferro-Resonance

Linear resonance has one natural oscillation frequency which strictly depends on linear inductance and capacitance value of the system as in (3.1). Therefore, there is only one frequency \( \omega_n \) that causes over voltages and over currents in the system. The \( \omega_n \) is calculated as follows:

\[
\omega_n = \frac{1}{\sqrt{LC}}
\]  

(3.1)

When linear inductance is replaced by non-linear inductance as shown in (3.2) (Voltage transformer, shunt reactor etc...) oscillation frequencies may be network frequency or fractions of the network frequency.

\[
\omega_f = \frac{1}{\sqrt{f(i)L}}
\]  

(3.2)

When non-linear inductance is driven into saturation, it can exhibit many values of inductances therefore a wide range of capacitance values can cause ferro-resonance oscillations [46].

Moreover, change from one ferro-resonant state to another is also possible depending on initial conditions and transients.
4 Causes and Effects of Ferro-resonance in the Power Systems

Causes of ferro-resonance are many but it can be generalized as below;

- Transients
- Phase-to-ground, phase-to-phase faults
- Circuit breaker opening and closing
- Transformer energizing and de-energizing

The main cause of ferro-resonance cannot be known beforehand and it is generally found out by analyzing events in the power system prior to ferro-resonant oscillations.

Ferro-resonance can be identified by the following symptoms [46];

- High permanent over voltages of differential mode (phase-to-phase)
- High permanent over currents
- High permanent distortions of voltage and current waveforms
- Displacement of the neutral point voltage
- Transformer heating
- Loud noise in transformers and reactances
- Damage of electrical equipment (capacitor banks, voltage transformers etc...)
- Untimely tripping of protection devices

Some of the effects are not only special to ferro-resonance; an initial analysis can be done by looking at voltage waveforms. If it is not possible to obtain recordings or if there are possible interpretations for effects, not only system configuration should be checked but also events prior to ferro-resonance.

Following step is to determine if three conditions are met in order ferro-resonance to happen;

- Co-existence of capacitances and non-linear inductances
- Existence of a point whose potential is not fixed (isolated neutral, single phase switching)
- Lightly loaded system (unloaded power or voltage transformers)

If any of these conditions are not met, ferro-resonance is said to be very unlikely [46].

In reference [47], ferro-resonance occurred because of switching operations during commissioning new 400-kV substation where grading capacitance of a circuit breaker involved. It is reported that two voltage transformers are driven into sustained ferro-resonance state.
Ferro-resonance experienced in Station Service Transformer during switching operations by firstly opening the circuit breaker and then the disconnector switch located at the riser pole surge arrester [49]. Oscillations caused explosion of surge arrester.

In reference [48], explosion of a voltage transformer is reported. One of the buses was removed because of installing of new circuit breaker and current transformer, at the same time maintenance and line trip testing were conducted. Voltage transformers on the de-energized bus were energized by near on-operation bus bar through grading capacitors.

4.1 Systems Vulnerable to Ferro-resonance

In the modern power systems, there are many sources of capacitances, non linear inductances and wide range of operating setups. Configurations that may allow ferro-resonance to happen are endless. But there are some typical configurations that may lead to ferro-resonance [46].

4.1.1 Voltage Transformer Energized Through Grading Capacitance

Switching operations may cause ferro-resonance in voltage transformers which are connected between phases and ground. A sample case is illustrated in figure 4.1;

Opening of circuit breaker D started ferro-resonance by causing capacitance C (all the capacitances to ground) to discharge through voltage transformer. Through grading capacitance $C_{d}$, source delivers enough energy to maintain oscillation.

![Figure 4.1 Ferro-resonance of a voltage transformer connected in series with an open circuit breaker[46]](image)

4.1.2 Voltage Transformers Connected to an Isolated Neutral System

Transients due to switching operations or ground faults may start ferro-resonance by saturating iron core of voltage transformers shown in figure 4.2. This grounding system can be chosen on purpose or the system can become neutral isolated from a loss of system grounding due to different reasons.
A system operator may think there is a phase-to-ground fault in the system because of neutral point displacement and potential rise respect to ground on one or two phases.

![Figure 4.2 Ferro-resonance of a voltage transformer between phase and ground in an isolated neutral system][46]

### 4.1.3 Transformer Accidentally Energized in Only One or Two Phases

These setups can happen when one or two of the source phases are disconnected while the transformer is lightly loaded [46]. System capacitances in figure 4.3 may consist of underground cables or overhead lines. Primary of the transformers can be delta connected or wye connected with isolated or grounded neutral. Because of switching operations, ferro-resonant configurations are formed. Factors that are relevant is given below;

- Phase-to-phase and phase-to-ground capacitances
- Primary and secondary windings connections
- Voltage source grounding
4.1.4 Voltage Transformers and HV/MV Transformers with Isolated Neutral

There is possibility of ferro-resonance when HV and MV neutrals are ungrounded. When a ground fault happens in HV side, high potential is obtained at HV neutral point. With the help of capacitive effect between primary and secondary, over-voltages appears on MV side[46].

Conditions for ferro-resonance is formed with voltage source $E_0$, capacitances $C_e$ and $C_0$ and magnetizing inductance of a voltage transformer in figure 4.4 and figure 4.5.
4.1.5 Power system grounded through a reactor

In LV systems, Permanent Insulation Monitors (PIMs) are used to measure insulation impedance by injecting direct current between system and ground. Their impedance is inductive and it may contribute to ferro-resonance oscillations [46]. Any potential rise in neutral point may cause ferro-resonance between inductance of PIM and capacitances of the system.

In MV systems, a coil of inductance \( L \) is used between MV neutral of a HV/MV transformer and ground to limit ground fault currents. Excitation of ferro-resonance of the circuit consisting inductance \( L \) and zero-sequence capacitances may happen because of natural dissymmetry of transformer and capacitances shown in figure 4.7.
4.1.6 Transformer Supplied by a Highly Capacitive Power System with Low Short-Circuit Power

As shown in figure 4.8 when an unloaded power transformer is connected to a relatively low short-circuit power source through underground cable or long overhead line, ferro-resonance may happen.

With the experience from the past, it is concluded that system with features below are in danger of ferro-resonance [46];

- Voltage transformer connected between phase and ground on an isolated neutral system
- Transformer fed through capacitive lines
- Non-multi pole breaking
- Unloaded or lightly loaded voltage transformers
5 Preventing Ferro-resonance

Methods to prevent ferro-resonance and its harmful effects are listed as follows;

- Avoiding configurations vulnerable to ferro-resonance
- Ensuring system parameters are not causing risk of ferro-resonance
- Ensuring energy supplied by the source is not enough to sustain oscillations (introducing damping to the system)

International standards state that resonance over voltages should be prevented or limited, those voltage values cannot be taken basis for insulation design. So in theory, current design of insulations and surge arresters do not provide protection against ferro-resonance [56].

There are some research on dynamical damping of ferro-resonance, prototypes are introduced [19], [20] but the most common used practice is static damping with damping resistors.

For configurations in figure 4.3, following practical solutions are advised [46];

- Lowering capacitance between circuit breaker and transformer
- Avoiding use of transformers at 10% of its rated capacity
- Avoiding no-load energizing
- Prohibiting single-phase operations

In case of MV power systems grounded through a reactor figure 4.7, overcompensation of power frequency capacitance component of the ground fault current can be done or a resistive component to increase losses can also be added [46].

For power transformers whose are fed through capacitive lines, the best solution proposed is avoiding risky situations when active power delivery is less than 10% of the transformer rated power [46].

5.1 Damping Ferro-resonance in Voltage Transformers

As mentioned before, voltage transformers connected between phase and ground in neutral isolated systems is dangerous for ferro-resonance oscillations to happen.

It is advised that avoid wye-connections of voltage transformer primaries with grounded neutral by leaving neutral of primaries ungrounded or using delta connection instead [18],[40]. If wye-connection for primaries is used, only way left to damp a possible oscillation is to introduce load resistances.
5.1.1 Voltage Transformers with one Secondary Winding

Even though resistors will consume power during operation, damping resistors are used to damp possible ferro-resonant oscillations in figure 5.1.

Recommended minimum values of resistance $R$ and power rating of resistor $P_R$ are calculated with rated values of transformer in (5.1) and (5.2) [40], [46].

\[
R = \frac{U_s^2}{k.P_t - P_m} \tag{5.1}
\]
\[
P_R = \frac{U_s^2}{R} \tag{5.2}
\]

where;

$U_s$: rated secondary voltage (V)

$k$: factor between 0.25 and 1 regarding errors and service conditions

$P_t$: voltage transformer’s rated output (VA)

$P_m$: power required for measurement (VA)

Figure 5.1 Damping for voltage transformer with one secondary[46]
5.1.2 Voltage Transformers with two Secondary Winding

There is also an option to have two secondaries in voltage transformers. One is for measurement and second one is especially for damping (tertiary winding). The advantage to have damping resistors in the open delta connected secondary winding is that it is only active during unbalanced operation. During the balanced operation no current circulates in open delta.

Recommended minimum values of resistance $R$ and power rating of resistor $P_R$ are calculated with rated values of transformer in (5.3) and (5.4) [40], [46].

\[
R = \frac{3\sqrt{3}U_e^2}{P_e} \tag{5.3}
\]

\[
P_R = \frac{(3U_e)^2}{R} \tag{5.4}
\]

where;

$U_e$ : rated voltage of the tertiary winding (V)

$P_e$ : rated thermal burden of tertiary winding (VA)

Rated thermal burden is the apparent power than voltage transformer can supply without exceeding thermal constraints.
6 Model of Non-linearity

The complexity of the whole ferro-resonance problem is caused by non-linear inductances in the system. Relationship between flux and magnetizing current for voltage transformer should be formed in order to study ferro-resonance in time domain and also in frequency domain.

In many studies (6.1) is taken model for saturation curve characteristics for voltage transformers [12],[22],[27],[31],[33],[36],[37].

\[ i_m = k_1 \phi + k_2 \phi^n \]  

(6.1)

where;

\( i_m \) : Magnetizing current (p.u)

\( k_1, k_2 \) : Polynomial constants

\( \phi \) : Core magnetic flux (p.u)
Polynomial constants $k_1$, $k_2$ have impact on the linear and saturated regions of magnetization characteristics. $k_1$ is related to linear part of the saturation curve whereas $k_2$ is related to saturated zone when iron core is driven into saturation by high magnetizing current.

The behavior of a given system is extremely sensitive to non-linearity of the inductances so for sake of accurate results polynomial constants and index $n$ must be obtained with precision. Shape of magnetizing curve and saturation knee position are important characteristics of magnetizing curve of a voltage transformer. These curves can be created with help of records from real measurements of inrush currents during energization of a given voltage transformer.

Typical magnetization curve also called saturation curve is shown in figure 6.1. As the current is increased magnetic flux also increases. At saturation point, magnetic flux gets smaller and smaller increase compared to increase in magnetizing current. Slope of the magnetic flux and magnetizing current curve changes dramatically at saturation point and this slope is proportional with inductance value of the voltage transformer’s coil. It means that after knee point there is a big drop in inductance value of the coil as curve’s slope gets smaller in magnitude.

Since inductance value is not fixed, resonance frequency can change. Once coil is driven into saturation it will stay in there as long as magnetizing current is not decreased. When ferro-resonance happens in saturated zone, decreasing magnetizing current may not stop ferro-resonance oscillations [50].

In reference [8] and [14], it is discussed that non-linear core loss model gives different results against linear core loss models. It is more accurate as losses in saturated region are higher, it coincides with real life experiments also. In this thesis, non-linear core loss model is
not used for sake of simplicity for the analysis. But it will be kept in mind that in ferro-resonance operation core losses are getting relatively higher.

7 Ferro-resonance in Time-Domain

Time-domain solutions of ferro-resonant circuits do not give broad vision of these phenomena but can give examples of different kind of ferro-resonances. A sample configuration leading to ferro-resonance is examined.

Ferro-resonance in a voltage transformer fed through circuit breaker grading capacitance is studied in [28]. Three different kind of ferro-resonance is shown by changing parameters of the circuit.

![Figure 7.1 System diagram](image_url)

Figure 7.1 shows the ferro-resonant system components of a 275 kV substation. Voltage transformer is isolated from the bus bar by disconnector Ds2.

$C_{cb}$ is the grading capacitance for circuit breaker while $C_{bb}$ is the total capacitance to ground. In this system voltage transformer failure was experienced after opening of circuit breaker and Ds2 while Ds1 was opened [28].

In figure 7.2 Ferro-resonant circuit is shown after switching in the system is done.
Figure 7.2 Equivalent circuit

$E_s$ is the rms voltage applied and $C_{\text{series}}$ is circuit breaker grading capacitance whilst $C_{\text{shunt}}$ is the total phase to ground capacitance. Parallel resistor $R$ represents core loss of the voltage transformer.

Base values are given as;

Base voltage : $(275 / \sqrt{3})kV$

Base power : $100VA$

Base impedance : $252M\Omega$

Base angular frequency : $314rad/s$

Non-linear characteristic of the transformer is given in (7.1) with per unit values as follows;

$$i(\phi) = 3.42\phi + 0.41\phi^7 \quad (7.1)$$

System equation can be written as:

$$\frac{1}{\omega} \frac{dV}{dt} + \frac{1}{R\omega(C_{\text{series}} + C_{\text{shunt}})} V + \frac{1}{\omega(C_{\text{series}} + C_{\text{shunt}})} \left(3.42\phi + 0.41\phi^7\right) = \frac{C_{\text{series}}}{C_{\text{series}} + C_{\text{shunt}}} \sqrt{2}E \cos \omega t \quad (7.2)$$

In (7.2) $V$ is the first derivative of $\phi$ which is the voltage at the terminal of the voltage transformer.

Non-linear ODE of the system is solved by Runge-Kutta algorithm in MATHCAD. During simulation source voltage $E$ and frequency are fixed at 1 p.u. Grading capacitance $C_{\text{series}}$ is taken as 0.5 nF for each simulation [28]. $C_{\text{shunt}}$ was varied between realistic values of 0.1 nF and 5 nF.

Transformer core resistance is designated as $225M\Omega$ to cause 112 W losses which are thought to be reasonable [28].
Initial values for flux and flux first derivative are selected as \( V(t) = \sqrt{2} \) p.u and \( \phi(t) = 0 \) at \( t = 0 \). This means that circuit breaker operation is at maximum voltage.

### 7.1 Normal Operation

Graphs are drawn for the case where \( C_{\text{shunt}} \) is 3 nF. Other parameters such as \( C_{\text{series}} \) and parallel resistance \( R \) are kept at their original values.

In figure 7.6 it can be seen that frequency component is purely 50 Hz in steady state after initial transients have disappeared. Phase plane in figure 7.5 closes itself in steady state showing that response is purely fundamental.

Terminal voltage and flux in the voltage transformer in figure 7.4 and 7.3 seems to have normal sinusoidal wave response reaching up 0.2 p.u peak value.
In order to see steady-state fundamental ferro-resonance, $C_{\text{shunt}}$ is set to 1.25 nF. Relevant graphs are drawn again by changing shunt capacitance.

Waveforms of flux and voltage in figure 7.7 and 7.8 show differences from figure 7.3 and 7.4, it is noticeable that peak values of waveforms are much higher in fundamental ferro-resonance case.
Since it is a steady state operation phase-plane and frequency content are similar to normal operation only difference in magnitudes as can be seen in figure 7.9 and 7.10. Ferro-resonant behavior is experienced with a different value of $C_{shunt}$.

Figure 7.7 Fundamental ferro-resonance operation

Figure 7.8 Fundamental ferro-resonance operation
7.3 Subharmonic Ferro-resonance Operation

$C_{shunt}$ is set to 0.19 nF to find subharmonic ferro-resonant solution to the system.

The phase plane in figure 7.13 has two closing trajectories pointing out subharmonic content of the signal. It can be seen from figure 7.14 that there are frequency contents of 25 Hz, 50 Hz and 75 Hz. 25 Hz component is high enough to have an effect on phase-plane diagram.
Voltage waveform of the voltage transformer and flux in the voltage transformer diagrams in figure 7.11 and figure 7.12 show distorted waveforms including higher and lower harmonics. Peak-values are still much higher than normal operation.

**Figure 7.11 Subharmonic ferro-resonance operation**

**Figure 7.12 Subharmonic ferro-resonance operation**
7.4 Chaotic Ferro-resonance Operation

There is no reported chaotic ferro-resonance event until now mainly because it is possible with unrealistic values of circuit parameters [27]. This kind of ferro-resonance is still being investigated with chaos theory since its nature is much unknown.

To be able to observe chaotic operation, $C_{\text{series}}$ and $C_{\text{shunt}}$ are set to 3 nF and 0.1 nF respectively. Resistance value representing the core loss is set to $1900 \, \Omega$ in order to
reduce losses in the system. These unpractical parameters are able to give chaotic ferro-
resonance response showing that existence of this mode in theory.

Figure 7.15 Chaotic ferro-resonance operation

Figure 7.16 Chaotic ferro-resonance operation
In figure 7.15 and figure 7.16, waveforms are shown only for 800 ms, but phase plane and frequency content are shown for 10 seconds of time-domain response. Even though it does not seem so on flux and voltage waveforms, there are many higher and low order subharmonics present in the signal causing phase plane not to close down itself.

Normal operation, fundamental ferro-resonance operation and subharmonic ferro-resonance operation may happen under close range parameter values of the system. So it is better to study fundamental and subharmonic periodic steady state responses rather than chaotic ferro-resonance which happen for very extreme and unrealistic parameters of the circuit.
Time-domain analyses give narrow vision to understand the risk of ferro-resonance. In literature, it is found that differentiating the initial values to the ODE solver might give different results for same parameters of the system [8], [12], [27]. Since there are so many initial and parameter values that can be used, it is unrealistic to try out every single scenario. In reference [12], it is shown that a small increment on parameters can lead to different operation modes.

8 Analytical Harmonic Balance Method

Harmonic balance is a method for the study of non-linear oscillating systems which are defined by non-linear ordinary differential equations. The basis of the method is to substitute unknown in the system by assumed solution so that approximate periodic solutions of non-linear ordinary differential equations can be found. The assumed solution can be defined as a sum of steady state sinusoids (Fourier series) that includes the forcing frequency in addition to any significant harmonics.

Theory of harmonic balance method is explained in [51], [52], [53]. Practice of this theory on ferro-resonant circuits is done in [31], [32], [33].

An example of application of harmonic balance method will be given to show approach to fundamental ferro-resonant behavior.

8.1 Application of Harmonic Balance on Example System

In figure 4.3, there are cases shown where ferro-resonance configuration is formed when one or two of the source phases are lost while the transformer is lightly loaded. This may be caused by single phase switching operations such as clearing of single phase fusing and single phase reclosing.

In the following case, one phase of the system was open while other two are closed. This leads to an induced voltage in the open phase because of capacitances in the system [11].
The system under study in figure 8.1 consists of source feeding an unloaded transformer with one of the phase are open or in this case interrupted. Capacitive coupling with other two phases energizes the transformer. Ferro-resonance is due to interaction between the non-linear magnetizing inductance of the transformer’s open phase and capacitance of the distribution line.

In order to study harmonic balance on this circuit scheme, equivalent capacitance should be obtained. This can be done shorting first and second phase and omitting transformer windings and ground capacitance of first and second phase. From figure 8.2, one can see that there will be no current on mutual capacitance between node 1 and node 2 since they have the same potential therefore mutual capacitance between node 1 and node 2 can be omitted as well. Mutual and ground capacitances which are connected to node 3 will be taken for the equivalent circuit. Equivalent system capacitance is calculated in (8.1).

\[ C = C_g + 2C_m \]  

(8.1)

Equivalent voltage source should be;

\[ E = \frac{C_m}{C_g + 2C_m} V_i \]  

(8.2)
Core losses of the transformer can be neglected during loaded operation since flux induced in the primary is compensated by current flow in secondary of the transformer. If transformer is working in unloaded condition, induced flux flows through iron core of the transformer and causes non-negligible losses [11]. Figure 8.3 Shows additional resistance connected parallel to non-linear inductance to simulate transformer core losses.

![Figure 8.3 Equivalent circuit[11]](image)

Transmission lines are 100 km long with a ground capacitance 5.41 nF/km and mutual capacitance 1.18 nF/km and making equivalent capacitance equal to 777nF. Transformer has 25 MVA rated power and core loss given as %1 of rated transformer capacity defining parallel resistance as 48.4 kΩ.

Base voltage : 63.5kV
Base current : 131A
Base impedance : 484.73Ω

Transformer’s magnetization characteristic is approximated by two-term polynomial;

\[ i_m = a\phi + b\phi^n \]  \hspace{1cm} (8.3)

Polynomial power \((n = 11)\) mainly depends on capacity of the transformer, \(a = 0.28 \times 10^{-2}\) and \(b = 0.72 \times 10^{-2}\) are the fitting values of saturation curve. First term defines linear region and second one non-linear.

Mathematical equations of the circuit can be written as;

\[ E \sin \omega t = V_c + \frac{d\phi}{dt} \]  \hspace{1cm} (8.4)

Rewriting voltage across the capacitance and reforming (8.4);

\[ E \sin \omega t = \int \frac{i_c}{C} dt + \frac{d\phi}{dt} \]  \hspace{1cm} (8.5)

Currents in the system can be written in another way;
\[ i_c = i_m + i_r \]
\[ i_m = a\phi + b\phi' \]  
\[ i_r = \frac{1}{R} \frac{d\phi}{dt} \]  
(8.6)

By taking derivative of both sides of (8.5);

\[ E\omega\cos\omega t = \frac{d^2\phi}{dt^2} + \frac{1}{RC} \frac{d\phi}{dt} + \frac{1}{C} (a\phi + b\phi') \]  
(8.7)

To determine approximate periodic solutions, solution to magnetic flux linkage can be assumed as sinusoidal response written in fundamental components in Fourier series;

\[ \phi(t) = \lambda \sin(\omega t + \theta) \]
\[ \phi(t) = \lambda_x \sin \omega t \]
\[ \lambda_x^2 + \lambda_y^2 = \lambda^2 \]

(8.8)

Substitution of \( \phi(t) \) to circuit equation requires evaluation of \( \phi'' = \lambda'' \sin'' (\omega t + \theta) \);

Binomial odd power expansion of

\[ \sin'' (\omega t + \theta) = k_1 \sin(\omega t + \theta) + k_3 \sin(3\omega t + 3\theta) + \ldots + k_n \sin(n\omega t + n\theta) \]  
(8.9)

Only \( k_1 \) is used in (8.9);

\[ k_1 = \frac{(-1)^{n-1}}{2^{n-1}} \binom{n}{\frac{n-1}{2}} \]  
(8.10)

After substitution of (8.8) in (8.7);

\[ E\omega\cos\omega t = \sin\omega t \left( \left(\omega^2 - k_a\right)\lambda_x - \left(\frac{1}{RC}\right)\omega\lambda_y \right) + \cos\omega t \left( \left(\omega^2 - k_x\right)\lambda_y + \left(\frac{1}{RC}\right)\omega\lambda_x \right) \]  
+ \( k_1 k_b \lambda_x^{n-1} (\lambda_x \sin \omega t + \lambda_y \cos \omega t) \]

(8.11)

where;
\[ k_a = \frac{a}{C}, \quad k_b = \frac{b}{C} \]  

Equating sine and cosine terms in (8.11) makes the system dependent on the frequency and circuit parameters;

\[
\left( -\left( \omega^2 - k_a \right) + k_b \lambda^{n+1} \right) \lambda_x - \frac{\omega}{RC} \lambda_y = 0 \\
\left( -\left( \omega^2 - k_a \right) + k_b \lambda^{n+1} \right) \lambda_y + \frac{\omega}{RC} \lambda_x = \omega E
\]  

(8.13)

\[
\lambda_x^2 + \lambda_y^2 = \lambda^2
\]

Equation (8.13) can be seen as;

\[
a'x - b'y = 0 \\
a'y + a'x = H
\]  

(8.14)

After taking square and adding elements of (8.4);

\[
a^2(x^2 + y^2) + b^2(x^2 + y^2) = H^2 \\
x^2 + y^2 = \lambda_x^2 + \lambda_y^2 = \lambda^2
\]  

(8.15)

Finally a form of polynomial is found from (8.15);

\[
\xi^n - p_2 \xi^{(n+1)/2} + p_1 \xi - p_0 = 0
\]  

(8.16)

\[
\lambda = +\sqrt{\xi}
\]

where;

\[
p_0 = \frac{\omega^2}{(k_1k_b)^2} E^2
\]

\[
p_1 = \frac{\left( \omega^2 - k_a \right)^2 + \left( \frac{\omega}{RC} \right)^2}{(k_1k_b)^2}
\]

(8.17)

\[
p_2 = \frac{2\left( \omega^2 - k_a \right)}{k_1k_b}
\]

Logic is to find the roots of (8.16) for each given source voltage peak value; as expected for some values of source voltage there were more than one positive real solution. By increasing
source voltage 0.005 p.u at each step, real solutions of the polynomial have been tracked. Complex roots are disregarded.

![Graph showing source voltage against Flux](image)

**Figure 8.4 Source voltage against Flux**

Each point on figure 8.4 represents a steady state solution. For some values of source voltage three real solutions is found and marked on the graph. So it means that between 0.14 p.u and 0.98 p.u of source voltage there are more than one solution to the system. Upper solutions represent fundamental ferro-resonant solutions because assumed solution was in fundamental form of Fourier series and it exhibits high flux values in the transformer which also means over voltages. Totally risk free zone is between 0 p.u and 0.14 p.u. It can be said that system is in ferro-resonance risk since source voltage of the equivalent circuit is 0.15 p.u.

In order to see the effect of parameter on this S-shaped curve, parallel resistance is divided by 4 making it 12.1 kΩ. Same calculation and root finding of the polynomial is done in the same way and figure 8.5 is obtained.
With the change in the parallel resistance, losses in the circuit are now higher than previous setup. This made a change in shape of the curve making space for more normal operation between 0 p.u and 0.49 p.u. In this case since source voltage of the system is 0.15 p.u, there is only one operation point which is safe. Increasing losses in the system made system fundamental ferro-resonance risk free.

There are two points where bifurcation happens, these points correspond to local minimum and local maximum points of the S-curve. By freeing a second parameter in the system, in this case C, one can follow limit points for each value for C therefore stability domain for ferro-resonance can be drawn between parameters of source voltage and equivalent capacitance.

Stability domain term is firstly used by T. Van Craenenbroeck [44]. It is the point where stability of system is changes one to another. At each limit point, solution becomes unstable if it was stable before and vice versa. So mid branch of figure 8.4 are actually unstable solutions where physical system cannot work, it will evolve nearest stable point. Both ends of unstable solution mark the border of stable solutions. Since occurrence zones of ferro-resonance are searched for, stability of each solution is not very critical of this study.

In order to do this, \( \frac{dE}{d\xi} = 0 \) should be used to find critical values (limit points)

Equation (8.16) includes source voltage E term only in \( p_0 \) in this case, so it is easy to leave E at one side of the equation.

\[
E = \xi^n - P_2 \xi^{(n+1)/2} + P_1 \xi
\]

(8.18)
\[
\frac{dE}{d\zeta} = \frac{n\zeta^{n-1} - ((n + 1) / 2) \xi_p \zeta^{(n-1)/2} + \xi_n}{p_0} = 0
\]  

(8.19)

Roots of (8.19) will give peak flux linkages correspond to limit points, then E values can be found putting flux linkages in (8.16).

Limit points in figure 8.6 are followed by changing equivalent capacitance C 1 nF at each step in figure 8.7.

Area C in figure 8.7 represents fundamental ferro-resonance risk free area. Source voltage and corresponding capacitance value gives normal operation in all cases.
Area B in figure 8.7 represents the values where fundamental ferro-resonance may happen because in that area there is normal operation and as well as fundamental ferro-resonance which means there could be a jump between normal operation to ferro-resonant operation. Nature of this jump is not full known thus it is better to avoid working with source voltage and equivalent capacitance values inside this zone.

Area A in figure 8.7 represents saturated zone and this working conditions exhibits high flux values in the transformer therefore over voltages.

![Figure 8.8 Stability domain with R/4](image)

Change in system losses affects the stability domain graphs, by using a lower parallel resistance, Area C is increased in figure 8.8 giving more safe operation points.

The problem with this method is that deriving equations for every circuit to analyze is very cumbersome and it will get very complex if higher order harmonics are considered and because of presence of more circuit components.

9 Numerical Harmonic Balance Method

Numerical harmonic balance method has similar logic compared to analytical harmonic balance method which is computing directly steady response of non-linear differential equations. This method have been tried and showed its superiority over analytical method by its ease of use [35], [36], [37].
9.1 Mathematical Frame

This method studies the circuit in frequency domain. Assuming the solution of flux as a Fourier series as shown in (9.1), investigation on the specific harmonics can be done in a different way from analytical method.

\[ \phi(t) = \sum_{k=-n}^{n} \phi_{nk} \cos(k \omega t) + \sum_{k=-n}^{n} \phi_{ks} \sin(k \omega t) \]  \hspace{1cm} (9.1)

The great advantage of the method is that it is applicable to any circuit since circuit will be divided into linear thevenin part and non-linear part.

As long as circuit can be derived as in figure 9.1, numerical harmonic balance can be applied also. Frequency domain equation of the thevenin circuit will be used to find solution to unknown value which is flux in this case.

Linear part equations are as following;

\[ jk \omega \phi_k = E_k - Z_k I_k \]  \hspace{1cm} (9.2)

Equation (9.2) contains complex components of flux, current, voltage and impedance at pulsation \( k \omega \).

Real and imaginary part of \( I_k \) (here nonlinear current of the transformer) must be expressed in Fourier components in (9.3).

\[ I_{ck} = \frac{2}{T} \int_{0}^{T} i_m \cos(k \omega t) \, dt \]

\[ I_{ks} = \frac{2}{T} \int_{0}^{T} i_m \sin(k \omega t) \, dt \]  \hspace{1cm} (9.3)

Now these complex quantities can be introduced;
\[
\begin{align*}
\phi_k &= \phi_{k_c} - j\phi_{k_s} \\
I_k &= I_{k_c} - jI_{k_s} \\
E_k &= E_{k_c} - jE_{k_s} \\
Z_k &= R_{k_c} + jX_{k_s}
\end{align*}
\] (9.4)

So non-linear algebraic system of equations can be formed; (total number of equations depends on k)

\[
\begin{align*}
\omega \phi_{k_c} + R_{k_c} I_{k_c} + X_{k_s} I_{k_s} - E_{k_c} &= \xi_{k_c} \\
\omega \phi_{k_s} + R_{k_s} I_{k_s} + X_{k_c} I_{k_c} + E_{k_s} &= \xi_{k_s} \\
&\ldots
\end{align*}
\] (9.5)

The system of \( \xi(\phi, P) = 0 \) (9.5) can be solved in MATHCAD by built-in Levenberg-Marquardt algorithm by giving initial guess values to unknown Fourier coefficients.

Non-linear system \( \xi(\phi, P) = 0 \) consists of \( \phi \) unknown vector representing the \((2k)\) components of Fourier components of the flux. \( P \) is the free parameter, it can be source voltage, capacitance or resistance in the system.

Angular frequency \( \omega \) can start from forcing frequency of the system. For subharmonic analysis it may take lower values than forcing fundamental frequency.

This method provides solutions stable and unstable steady state where unstable solutions are not maintained by the system, they evolve to a stable solution [37]. But this does not have effect on finding stability zones of ferro-resonance.

It is considered that source voltage is purely fundamental, so contribution from any harmonic of the source voltage \( E_{k_c} \) and \( E_{k_s} \) will be taken zero. Only \( E_{k_c} \) and \( E_{k_s} \) will have non-zero values.

### 9.2 Continuation Method

Different than analytical method of the harmonic balance, numerical methods need a continuation method to draw continuous curves.

Let \( \phi_0 \) is a known solution for a given value of parameter \( P_0 \). The solution for \( P_0 + \Delta P \) is found from \( \phi_0 \) by a simple iteration where \( \phi_0 \) is used as the initial guess. So with small enough
steps of $\Delta P$, a branch of solutions can be calculated. Simple continuation flow chart is shown in figure 9.2 where $a$ and $b$ represent unknown Fourier coefficients couples.

Firstly, solver of the system should be initialized by a fixed parameter value and initial guess for unknowns Fourier coefficients. After initialization system enters red loop where difference between $P_{n+1}$ and $P_n$ gives $\Delta P$. Depending on the sign of $\Delta P$, solution branch follows a direction.

Simple continuation works normally until it encounters a limit point. Conditions of derivability of the system are given by:

$$\frac{\delta \xi}{\delta \phi} d\phi + \frac{\delta \xi}{\delta P} dP = 0 \quad (9.6)$$

If parameter $P$ is selected as source voltage $E$, then (9.6) becomes:

$$\frac{\delta \xi}{\delta \phi} d\phi = -\frac{\delta \xi}{\delta E} \quad (9.7)$$
Tangent in figure 9.3 corresponds to \( \frac{d\phi}{dE} = \infty \) since \( dE = 0 \) while \( d\phi \neq 0 \).

When the Jacobean matrix \( \frac{\partial \xi}{\partial \phi} \) is singular (its determinant is zero by definition), the slope of the tangent \( \frac{d\phi}{dE} \) to the curve is infinite [35],[36].

At these limit points, numerical harmonic balance method cannot converge since Jacobean of the equation system gets singular. In order to turn these limit points and then continue the solution branch pseudo-arc length continuation method is proposed by Kieny [35]. But in this thesis another more simpler method is used which is called hyper-sphere method [54].

By adding one more equation to the \( \xi(\phi, \lambda) = 0 \) system, singularity at the limit point can be overcome. Hence, complete curves can be drawn.

The new equation is a hyper-sphere equation; a sphere that encloses a part of solution curve always intersects the solution curve at least two times [54].

\[
g(\phi_i, \phi_c, P) = (\phi_i - \phi_{ic})^2 + (\phi_c - \phi_{ic})^2 + \ldots + (\phi_m - \phi_{ic})^2 + (\phi_n - \phi_{ic})^2 + (P - P_n)^2 - r^2 = 0
\]

m indices represents the known coefficient or parameter from last iteration.
By using previous solution as a center of hyper-sphere in figure 9.4, radius of the sphere determines the distances between two adjacent solutions, for convergence problems radius should be selected according to scales of both axis of the graph in study.

By adding one more equation, $P$ is also considered unknown and will be found with Fourier coefficients of the flux. Sign of $\Delta P$ will decide on the direction of solution branch.
In figure 9.5 MATHCAD algorithm of the hyper-sphere method is shown. Again, initialization is needed and done by setting radius of the hyper-sphere to zero for only one iteration. After finding the first unknown values, solution branch direction is set by $\Delta P$ and radius is given to the solver.

Number of iterations of the red loop is decided beforehand, it might be increased if the graph is believed to not complete.

It uses a basic algorithm to check if the solver reached a limit point. If the solver calculates the same point more than once, it is programmed to change sign of $\Delta P$. Calculation of a point more than once indicates that solver is stuck and cannot move further with the same sign of $\Delta P$, exploiting this feature make the solver turn around limit points most of the time. For the times that it does not turn, radius, convergence tolerance (TOL) and constraint tolerance (CTOL) should be adjusted manually.

### 9.3 Selection of Harmonic Components

At the very beginning of the calculation Fourier series in (9.1) should be decided. But the number of elements mainly depends on what kind of accuracy is demanded. $n$ is selected in a fashion that adding more high order components of the flux will not affect the final result anymore [37]. But it will also increase the unknowns needed to be calculated. There is a tradeoff between computation time and accuracy of the solutions. MATHCAD slows down exponentially if number of unknowns is increased.

Higher order harmonics show their effects for non-realistic values (very high) of the circuit as seen in [37]. So it is not totally necessary to add so many components to solve. It is a general thought that FFT of the system response in time-domain could give hints harmonics to include in order to increase accuracy of this method. For, example, if there is no component of 7th harmonic during a fundamental-resonance in time-domain. It is not needed to add 7th harmonic for flux because the components will be zero or very close to zero. So there is no need to increase computing and waste resources to find coefficients which are zero. Same goes for even harmonics.

When fundamental solution is searched for estimated flux solution is given in form of;

$$
\phi(t) = \phi_s \sin(\omega t) + \phi_c \cos(\omega t)
$$

(9.9)

Resulting flux is given by;

$$
|\phi| = \sqrt{\phi_s^2 + \phi_c^2}
$$

(9.10)

In (9.10), unknowns to be found are $\phi_s$ and $\phi_c$.

When a subharmonic solution is tried to be found estimated flux solution is given in (9.11);
\[ \phi(t) = \phi_{1s} \sin(\omega t) + \phi_{1c} \cos(\omega t) + \phi_{2s} \sin\left(\frac{\omega}{m} t\right) + \phi_{2c} \cos\left(\frac{\omega}{m} t\right) \]  \hspace{1cm} (9.11)

Resulting flux is given by;

\[ |\phi| = \sqrt{\phi_{1s}^2 + \phi_{1c}^2 + \phi_{2s}^2 + \phi_{2c}^2} \]  \hspace{1cm} (9.12)

where \( m \) is the fraction of fundamental frequency which is appointed for subharmonic in analysis.

Fundamental solution branches do not contain subharmonic components. \( \phi_{2s} \) and \( \phi_{2c} \) are non-zero only for sub-harmonic solution branches. Therefore it is easy to identify which branch is being worked on.

### 9.4 Stability Domains by Numerical Harmonic Balance Method

It is shown that if the tangent \( \frac{d\phi}{dE} \) of the curve at limit point is infinite, \( \frac{\delta \xi}{\delta \phi} \) matrix is singular.

\( \frac{\delta \xi}{\delta \phi} \) matrix is also the Jacobean matrix of the non-linear algebraic equation system, and if it is singular, its determinant is zero \( \det(J[\xi]) = 0 \).

This mathematical feature at limit point is used by T.Van Craenenbroeck to track limit points in two parameter space [38]-[44].

By adding one more equation \( \det(J[\xi]) = 0 \) to the system, one more parameter can be freed and treated as unknown \( P_1 \). Stability domain graphs will be the coordinates of the limit points on \( P_1 \) and \( P_2 \) parameter space.
In figure 9.6, algorithm is shown for stability domain calculation. First initialization is very important because first guess values should be very close to limit point so that solver will converge. After finding the first limit point data, algorithm can continue tracking in red loop by hyper-sphere method in $P_1$ and $P_2$ parameter space. The logic of the algorithm is pretty much the same with the one in figure 9.5.
10 First Application of Numerical Harmonic Balance

The circuit given in analytical harmonic balance method will be resolved with numerical harmonic balance method with some additional analyses.

Equivalent circuit in figure 10.1 was found and analytical harmonic balance method has been applied. Now, in order to study this circuit in numerical harmonic balance method, circuit should be divided into linear thevenin part and non-linear part.

Per unit values of the system is given as;

Base voltage: 63.5kV

Base current: 131A

Base impedance: 484.73Ω

Base angular speed: 2π50rad/s

Parameters of the system;

E = 0.15 p.u (63.5 kV)

C = 0.118 p.u (777 nF)

R = 99.85 p.u (48.4 kΩ)

Magnetizing curve equation is given in polynomial;

\[ i_m = a\phi + b\phi^n \]  \hspace{1cm} (10.1)

where \( n = 11 \), \( a = 0.28 \times 10^{-2} \) and \( b = 0.72 \times 10^{-2} \) in p.u

Thevenin impedance and thevenin voltage have been calculated at non-linear inductance terminals;

\[ E_{th} = \frac{E}{1+i\omega RC} \]  \hspace{1cm} (10.2)
\[ Z_{ik} = \frac{R}{1 + ik\omega RC} \quad (10.3) \]

k value in (10.3) depends on which harmonic component is taken for the calculation, it could be positive integers or positive fractions.

### 10.1 Fundamental Ferro-Resonance Analysis

Assumed flux solution has only two components to be found;

\[ \phi(t) = \phi_{1c} \sin(\omega t) + \phi_{1c} \cos(\omega t) \quad (10.4) \]

Equation system consists of two non-linear algebraic equations and hyper-sphere equation in this case;

\[ \begin{align*}
\omega \phi_{1c} + R_{1c} I_{1c} + X_{1c} I_{1s} - E_{1c} &= \xi_{1c} = 0 \\
\omega \phi_{1c} + R_{1c} I_{1s} + X_{1c} I_{1c} + E_{1s} &= \xi_{1s} = 0 \\
(\phi_{1s} - \phi_{1m})^2 + (\phi_{1c} - \phi_{1m})^2 + (P - P_m)^2 - r^2 &= 0
\end{align*} \quad (10.5) \]

Equation (10.4) and (10.5) are used in algorithms in figure 9.5 and figure 9.6.

#### 10.1.1 Flux – Source Voltage

![Figure 10.2 Flux against voltage source](image)

Numerical method gave similar results in figure 10.2 which is found by analytical method. System is in fundamental ferro-resonance risk with having 0.15 p.u value as source voltage. Curve is started from (0,0) coordinate and advanced further by hyper-sphere.
In figure 10.3 and 10.4, effect of parameter change on S-curve can be observed. S-shaped curve can be influenced by capacitance and resistance values. Lowering both of the values has the same effect making the system safe at 0.15 p.u source voltage.

10.1.2 Flux–Capacitance

One of the advantages of numerical method is that ease of drawing graphs with other parameters by doing some little adjustments on MATHCAD.

In this study, voltage source and other parameters except equivalent capacitance are kept constant.
In figure 10.5, it can be seen that there is a normal operation point for every capacitance value but after 611 nF of capacitance value, ferro-resonant solutions begin to exist. Hence, capacitance value below 611 nF provides safe normal operation. It should be remembered that system’s actual equivalent capacitance is 777 nF which implies the system with 0.15 p.u source voltage is in danger zone for fundamental ferro-resonance.
10.1.3 Flux - Resistance

Same character is observed in figure 10.6 Flux – resistance graph as well as in Flux – capacitance. When source voltage is 0.15 p.u, 48.4 kΩ of parallel resistance value puts the system in fundamental ferro-resonance risk. On the other hand, if there were less than 38 kΩ valued parallel resistance, system would be in safe operation condition.

![Figure 10.6 Flux against resistance](image)

10.1.4 Stability Domain: Source Voltage against Capacitance

By using algorithm shown in figure 9.6, limit points in figure 10.7 have been tracked on source voltage and capacitance parameter space.

![Figure 10.7 Stability domain: Source voltage against Capacitance](image)
Red line on the figure 10.7 shows system voltage 0.15 p.u. 611 nF is the intersection point of limit curve and source voltage of the system. These values at intersection points will be named as “critical values”. Here the critical value shows the maximum value of capacitance which would not put the system in risk zone. From the graph it can be seen that 777 nF of equivalent capacitance makes the system risky.

10.1.5 Stability Domain: Source Voltage against Resistance

In figure 10.8, it can be seen that critical resistance value is 38 kΩ, system resistance value is 48.4 kΩ so again it is confirmed that the system is in risk.

![Figure 10.8 Stability domain: Source voltage against resistance](image)

10.2 Subharmonic-1/2 Ferro-Resonance Analysis

Assumed flux solution has only four components to be found;

\[ \phi(t) = \phi_{1s} \sin(\omega t) + \phi_{ic} \cos(\omega t) + \phi_{2s} \sin\left(\frac{\omega}{2} t\right) + \phi_{2c} \cos\left(\frac{\omega}{2} t\right) \]  \hspace{1cm} (10.6)

Equation system consists of four non-linear algebraic equations and hyper-sphere equation in this case;

\[
\begin{align*}
\omega \phi_{1s} + R_{1c} I_{1c} + X_{1s} I_{1s} - E_{1c} &= \xi_{1c} = 0 \\
\omega \phi_{ic} + R_{1c} I_{1s} + X_{1s} I_{1c} + E_{1s} &= \xi_{1s} = 0 \\
\frac{\omega}{2} \phi_{2s} + R_{2c} I_{2c} + X_{2s} I_{2s} - E_{2c} &= \xi_{2c} = 0 \\
\frac{\omega}{2} \phi_{2c} + R_{2c} I_{2s} + X_{2s} I_{2c} + E_{2s} &= \xi_{2s} = 0
\end{align*}
\]  \hspace{1cm} (10.7)
Since subharmonic solutions have no trivial solutions as in fundamental ferro-resonance, it is important to find at least one solution which belongs to subharmonic solution branch in order to draw solution curves. Unfortunately, there is no easy and reliable way to find a subharmonic solution point. Trial and error method is done by changing initial guess values for unknown coefficients. There is a small clue that will tell about the origin of the solution. If it has non-zero subharmonic components it means that the solution in focus belongs to subharmonic branch. If it has zero contribution from subharmonic coefficients, it belongs to fundamental solution branch [37]. After finding a solution on subharmonic branch, algorithm can draw the whole branch.

### 10.2.1 Flux – Source Voltage

Subharmonic solutions form a closed curve as seen in figure 10.9 which is also called “isolate”. At each limit point solution changes its stability, so upper side or lower side of this curve has unstable solutions. Since existence zone of subharmonic solutions is looked for, knowing which part of the isolate is stable or not is not important.

\[
\begin{align*}
(\phi_{1s} - \phi_{1m})^2 + (\phi_{1c} - \phi_{1cm})^2 + (\phi_{2s} - \phi_{2m})^2 + (\phi_{2c} - \phi_{2cm})^2 + (P - P_m)^2 - r^2 &= 0
\end{align*}
\]

Equation (10.6) and (10.7) are used in algorithms in figure 9.5 and figure 9.6.
Figure 10.10 Effect of capacitance change

Figure 10.11 Effect of resistance change

Lowering the resistance and capacitance has the same damping effect on the system like in fundamental ferro-resonance case as shown in figure 10.10 and figure 10.11. As long as subharmonic solution branch moves away from source voltage 0.15 p.u, it is favorable for the safety from subharmonic ferro-resonance.
10.2.2 Flux – Capacitance

Subharmonic solutions start at 780 nF in figure 10.12. It is almost the same value as equivalent capacitance 777 nF. It should be noted that normal operation curve has only fundamental components.

![Figure 10.12 Flux against capacitance](image)

10.2.3 Flux – Resistance

Subharmonic solutions in figure 10.13 start at 48.8 kΩ which is again very close to system resistance value.

![Figure 10.13 Flux against resistance](image)
10.2.4 Stability Domain: Source Voltage against Capacitance

Critical value of capacitance for subharmonic ferro-resonance is 780 nF as shown in figure 10.14.

![Figure 10.14 Stability domain: Source voltage against capacitance](image)

10.2.5 Stability Domain: Source Voltage against Resistance

In figure 10.15, it can be seen that critical value of resistance for subharmonic ferro-resonance is 48.8 kΩ.

![Figure 10.15 Stability domain: Source voltage against resistance](image)
10.3 Subharmonic-1/3 Ferro-Resonance Analysis

Assumed flux solution has four components to be found;

\[ \phi(t) = \phi_{1s} \sin(\omega t) + \phi_{1c} \cos(\omega t) + \phi_{3} \sin\left(\frac{\omega}{3} t\right) + \phi_{3c} \cos\left(\frac{\omega}{3} t\right) \]  \hspace{1cm} (10.8)

Equation system consists of four non-linear algebraic equations and hyper-sphere equation in this case;

\[
\begin{align*}
\omega \phi_{1s} + R_{ic} I_{ic} + X_{ic} I_{ic} - E_{ic} &= \xi_{ic} = 0 \\
\omega \phi_{1c} + R_{ic} I_{ic} + X_{ic} I_{ic} + E_{ic} &= \xi_{ic} = 0 \\
\frac{\omega}{3} \phi_{3} + R_{3c} I_{3c} + X_{3c} I_{3c} - E_{3c} &= \xi_{3c} = 0 \\
\frac{\omega}{3} \phi_{3c} + R_{3c} I_{3c} + X_{3c} I_{3c} + E_{3c} &= \xi_{3c} = 0
\end{align*} \hspace{1cm} (10.9)
\]

\[ (\phi_{1s} - \phi_{1cm})^2 + (\phi_{1c} - \phi_{1cm})^2 + (\phi_{3s} - \phi_{3cm})^2 + (\phi_{3c} - \phi_{3cm})^2 + (P - P_m)^2 - r^2 = 0 \]

Equation (10.8) and (10.9) are used in algorithms in figure 9.5 and figure 9.6.

Checking other type of subharmonics could be interesting in order to see their existence zones compared to 25 Hz subharmonic.

10.3.1 Flux – Source Voltage

Subharmonic-1/3 solutions are very close to subharmonic-1/2 solutions. Even though there is no information about which kind of subharmonic ferro-resonance is likely to happen, it is important to see that they have close existence zones.
10.3.2 Flux – Capacitance

After the value of 810 nF, subharmonic-1/3 solutions begin to exist showing that system is not in subharmonic-1/3 ferro-resonance risk as can be seen in figure 10.17.

![Figure 10.17 Flux against capacitance](image)

10.3.3 Flux – Resistance

In figure 10.18, subharmonic-1/3 solutions start after the resistance value 51 kΩ.

![Figure 10.18 Flux against resistance](image)
10.3.4 Stability Domain: Source Voltage against Capacitance

Critical value of the capacitance for subharmonic-1/3 ferro-resonance is 810 nF as shown in figure 10.19.

![Figure 10.19 Stability domain: Source voltage against capacitance](image)

10.3.5 Stability Domain: Source Voltage against Resistance

In figure 10.20, it can be seen that critical value of the resistance for subharmonic-1/3 ferro-resonance is $51 \, k\Omega$.

![Figure 10.20 Stability domain: Source voltage against resistance](image)
10.3.6 Remarks

Flexibility of the numerical harmonic balance method helped to draw parameter curves with different kinds of parameters. By adopting per unit values, it is much easier to adjust guess values and find subharmonic solutions since component values ranges from -2 to 2.

Existence zone of different ferro-resonance modes seems to have a common parameter ranges as shown in figure 10.21.

![Figure 10.21 Comparison of ferro-resonance modes](image)

Which kind of ferro-resonance may happen is not fully known and understood, system may never experience subharmonic ferro-resonance but in order to be sure existence range should also be analyzed.

11 Second Application of Numerical Harmonic Balance

As a second example, the circuit which F. Ben Amar investigated in 2006 will be analyzed further [36]. The author has analyzed a ferro-resonance risked system with harmonic balance and pseudo-arc length continuation method. The following study will show that hyper-sphere method is working as intended.
In figure 11.1, an equivalent circuit of a system is given. This system can be a result of a voltage transformer (400/20 kV) connected to a bus bar separated by circuit breaker grading capacitance. It is also explained by Ferraci in figure 4.1 [46].

Per unit values of the system is given as;

Base voltage: 327 kV
Base current: 100 A
Base impedance: 3270 Ω
Base angular speed: \(2\pi 50 \, \text{rad/s}\)

Parameters of the system;

\(E = 1 \, \text{p.u} \) (327 kV phase to ground peak value)

\(C = 4.1 \times 10^{-4} \, \text{p.u} \) (0.4 nF equivalent capacitance)

\(R_1 = 9.8 \, \text{p.u} \) (32 kΩ series losses)

\(R_2 = 2.186 \times 10^5 \, \text{p.u} \) (714 MΩ parallel losses)

Magnetizing curve equation is given in polynomial;

\[ i_m = a\phi + b\phi^n \] (11.1)

where \(n = 9\), \(a = 10^{-8}\) and \(b = 2.34 \times 10^{-34}\) in p.u

Thevenin impedance and thevenin voltage have been calculated at non-linear inductance terminals;

\[ E_{th} = \frac{E}{\frac{1}{i\omega C} + R_1 + R_2} R_2 \] (11.2)
\[
Z_{sh} = \frac{1}{\frac{1}{ik\omega C} + \frac{1}{R^2}}
\]  

(11.3)

11.1 Fundamental Ferro-Resonance Analysis

Assumed flux solution has only two components to be found:

\[
\phi(t) = \phi_{ts} \sin(\omega t) + \phi_{tc} \cos(\omega t)
\]  

(11.4)

Equation system consists of two non-linear algebraic equations and hyper-sphere equation in this case:

\[
\begin{align*}
\omega\phi_{ts} + R_{tc} I_{tc} + X_{ts} I_{ts} - E_{tc} &= \xi_{tc} = 0 \\
\omega\phi_{tc} + R_{ts} I_{ts} + X_{tc} I_{tc} + E_{ts} &= \xi_{ts} = 0 \\
(\phi_{ts} - \phi_{tsm})^2 + (\phi_{tc} - \phi_{tcn})^2 + (P - P_m)^2 - r^2 &= 0
\end{align*}
\]  

(11.5)

Equation (11.4) and (11.5) are used in algorithms in figure 9.5 and figure 9.6.

11.1.1 Flux – Source Voltage

In figure 11.2, peak value of the source voltage corresponds to 1 p.u peak value of flux in the transformer. The system is in danger of fundamental ferro-resonance since it has more than one solution at 1 p.u of source voltage.
11.1.2 Flux – Capacitance

In figure 11.3, it can be seen that very large range of capacitance value system has more than one solution. After 20 pF, system enters fundamental ferro-resonance risk zone.

11.1.3 Flux – Series Resistance

Series losses of the system shows different behavior compared to parallel losses on the occurrence of the fundamental ferro-resonance solutions in figure 11.4. Around the value of 1620 \( k\Omega \), system gets risk free of fundamental ferro-resonance. Since the normal value of
the series resistance in the system is 32 kΩ, system is in definite risk. 50 times larger resistance is needed to make the system fundamental ferro-resonance risk free.

**Figure 11.4 Flux against series resistance**

### 11.1.4 Flux – Parallel Resistance

In figure 11.5, Fundamental ferro-resonant solutions start at 45 MΩ.

**Figure 11.5 Flux against parallel resistance**
11.1.5 Stability Domain: Source Voltage against Capacitance

It is practical to check the stability domain of a limit point which is closer to source voltage value on flux – source voltage graph. So from now on only the first limit point will be tracked. In figure 11.6, critical value of the capacitance for the fundamental ferro-resonance is 20 pF.

![Figure 11.6 Stability domain: Source voltage against capacitance](image)

11.1.6 Stability Domain: Source Voltage against Parallel Resistance

In figure 11.7, critical value of the parallel resistance is approximately around 45 MΩ.

![Figure 11.7 Stability domain: Source voltage against parallel resistance](image)
11.2 Subharmonic-1/2 Ferro-Resonance Analysis

Assumed flux solution has only four components to be found;

\[ \phi(t) = \phi_{1s} \sin(\omega t) + \phi_{1c} \cos(\omega t) + \phi_{2s} \sin(\frac{\omega}{2}t) + \phi_{2c} \cos(\frac{\omega}{2}t) \]  \hspace{1cm} (11.6)

Equation system consists of four non-linear algebraic equations and hyper-sphere equation in this case;

\[ \begin{align*}
\omega \phi_{1s} + R_{1c} I_{1c} + X_{1s} I_{1s} - E_{1c} &= \xi_{1c} = 0 \\
\omega \phi_{1c} + R_{1s} I_{1s} + X_{1c} I_{1c} + E_{1s} &= \xi_{1s} = 0 \\
\frac{\omega}{2} \phi_{2s} + R_{2c} I_{2c} + X_{2s} I_{2s} - E_{2c} &= \xi_{2c} = 0 \\
\frac{\omega}{2} \phi_{2c} + R_{2s} I_{2s} + X_{2c} I_{2c} + E_{2s} &= \xi_{2s} = 0
\end{align*} \hspace{1cm} (11.7)

\[ (\phi_{1s} - \phi_{1im})^2 + (\phi_{1c} - \phi_{1im})^2 + (\phi_{2s} - \phi_{2im})^2 + (\phi_{2c} - \phi_{2im})^2 + (P - P_m)^2 - r^2 = 0 \]

Equation (11.6) and (11.7) are used in algorithms in figure 9.5 and figure 9.6.

11.2.1 Flux – Source Voltage

As seen again, subharmonic-1/2 ferro-resonance happens in vicinity of fundamental ferro-resonance shown in figure 11.8.

![Figure 11.8 Flux against source voltage](image-url)
11.2.2 Flux – Capacitance

In figure 11.9, subharmonic-1/2 solutions begin to happen around 35 pF.

![Figure 11.9 Flux against capacitance](image)

11.2.3 Flux – Series Resistance

Up to value of $3100 \times \Omega$, system is in risk of subharmonic-1/2 ferro-resonance in figure 11.10.

![Figure 11.10 Flux against series resistance](image)
11.2.4 Flux – Parallel Resistance

In figure 11.11, subharmonic-1/2 solutions start around 80 $M\Omega$. System value is 714 $M\Omega$ so system is in danger of subharmonic-1/2 ferro-resonance.

![Figure 11.11 Flux against parallel resistance](image)

11.2.5 Stability Domain: Source Voltage against Capacitance

In figure 11.12, critical capacitance value for subharmonic-1/2 ferro-resonance is around 35 pF.

![Figure 11.12 Stability domain: Source voltage against capacitance](image)
11.2.6 Stability Domain: Source Voltage against Parallel Resistance
In figure 11.13, critical parallel resistance value for subharmonic-1/2 ferro-resonance is around 50 MΩ.

![Figure 11.13 Stability domain: Source voltage against parallel resistance](image)

11.3 Subharmonic-1/3 Ferro-Resonance Analysis

Assumed flux solution has four components to be found;

$$\phi(t) = \phi_{1s} \sin(\omega t) + \phi_{1c} \cos(\omega t) + \phi_{3s} \sin\left(\frac{\omega t}{3}\right) + \phi_{3c} \cos\left(\frac{\omega t}{3}\right)$$  \hspace{1cm} (11.8)

Equation system consists of four non-linear algebraic equations and hyper-sphere equation in this case;

$$\omega \phi_{1s} + R_{1s} I_{1s} + X_{1s} I_{1s} - E_{1s} = \xi_{1s} = 0$$
$$\omega \phi_{1c} + R_{1c} I_{1c} + X_{1c} I_{1c} + E_{1s} = \xi_{1s} = 0$$
$$\frac{\omega}{3} \phi_{3s} + R_{3s} I_{3s} + X_{3s} I_{3s} - E_{3s} = \xi_{3s} = 0$$  \hspace{1cm} (11.9)
$$\frac{\omega}{3} \phi_{3c} + R_{3c} I_{3c} + X_{3c} I_{3c} + E_{3s} = \xi_{3s} = 0$$

$$(\phi_{1s} - \phi_{1cm})^2 + (\phi_{1c} - \phi_{1cm})^2 + (\phi_{3s} - \phi_{3sm})^2 + (\phi_{3c} - \phi_{3cm})^2 + (P - P_m)^2 - r^2 = 0$$

Equation (11.8) and (11.9) are used in algorithms in figure 9.5 and figure 9.6.
11.3.1 Flux – Source Voltage

In figure 11.14, subharmonic-1/3 ferro-resonant solutions are within the region of subharmonic-1/2 ferro-resonant solutions.

![Figure 11.14 Flux against source voltage](image)

11.3.2 Flux – Capacitance

In figure 11.15, subharmonic-1/3 ferro-resonant solutions start at 19 pF meanwhile system’s capacitance is 400 pF. This value is off the graph but subharmonic-1/3 solutions continue to exist even at larger values.

![Figure 11.15 Flux against capacitance](image)
11.3.3 Flux – Series Resistance

In figure 11.16, subharmonic-1/3 solutions exist until around 1370 kΩ value of series resistance.

![Figure 11.16 Flux against series resistance](image1)

11.3.4 Flux – Parallel Resistance

In figure 11.17, critical parallel resistance value for subharmonic-1/3 ferro-resonance is around 52 MΩ.

![Figure 11.17 Flux against parallel resistance](image2)
11.3.5 Stability Domain: Source Voltage against Capacitance

In figure 11.18, critical capacitance value for subharmonic-1/3 resonance is 19 pF. 400 pF is well above the critical value leaving system in danger of subharmonic-1/3 ferro-resonance.

![Figure 11.18 Stability domain: Source voltage against capacitance](image)

11.3.6 Stability Domain: Source Voltage against Parallel Resistance

Critical parallel resistance for subharmonic-1/3 ferro-resonance is 50 $M\Omega$ showing that system is in subharmonic-1/3 ferro-resonance risk in figure 11.19.

![Figure 11.19 Stability domain: Source voltage against parallel resistance](image)
11.3.7 Remarks

The system studied by F. Ben Amar is analyzed further and it is shown that for given parameters the system is in danger of fundamental ferro-resonance, subharmonic-1/2 and subharmonic-1/3 ferro-resonance. By stability domain graphs, safe range of parameters has been shown for all kinds of ferro-resonances in study. Possibility of use of these parameters is not discussed.

12 Case Study

Alstom’s test system is gone under study in this section. Setup is shown in figure 12.1.

![Figure 12.1 Single phase diagram of the test system](image)

After a routine circuit breaker opening, the side of VT became neutral-isolated although VT is connected between phase and ground.

In this setup, voltage source is high voltage grid. Non-linear inductor is VT and capacitance is grading capacitance of the circuit breaker.

To damp a possible oscillation, VT’s are in open-delta with a resistor connected to tertiary winding as in shown in figure 5.2. The logic in this is to introduce additional system zero-sequence losses in case of phase-ground faults therefore damp ferro-resonant oscillations. Ferro-resonance state can sustain itself provided that transferred energy from source for this state is higher than total system losses.

Simplified equivalent circuit in figure 12.2 shows one example way how ferro-resonant currents can flow.
Damping resistor will be present in any equivalent circuit when there is an unbalanced operation. Currently, the harmonic balance method used is only applicable to one phase system, basically where a single phase system can be simplified from a three phase system. Neutral isolated systems cause problem when deriving a single phase equivalent therefore possible ferro-resonant current flow ways used to derive a single phase equivalent for this case. This also simulates the case when one of the phases is open while other two is still connected; unbalanced situation allows adding damping resistor to the equivalent single phase circuit in figure 12.3.

### 12.1 System Details

Per unit values of the system is given as;

- Base voltage: 17.1 kV
- Base current: 0.019 A
- Base impedance: 882 kΩ
Base angular speed: \(2\pi 50 \text{rad} / \text{s}\)

Parameters of the system:

\(E = 0.66 \text{ p.u} \) (12 kV phase to ground)

\(C = 27.7 \text{ p.u} \) (100 nF grading capacitance)

\(R_1 = 6.87 \times 10^{-3} \text{ p.u} \) (6059 Ω Transformer Primary Resistance)

\(L_1 = 0.01 \text{ p.u} \) (29.2 H Transformer Primary Inductance)

\(N = \frac{21000/\sqrt{3}}{100/3} \) (Transformer turn ratio to tertiary winding)

Magnetizing curve equation is approximated in polynomial form in (12.1);

\[ i_m = a\phi + b\phi^n \]  \hspace{1cm} (12.1)

where \(n = 19\), \(a = 0.001\) and \(b = 5.9 \times 10^{-39}\) in p.u.

Values are taken from AREVA’s ferro-resonance report of voltage transformer in study [55]. Saturation curve is approximated from this report also since real saturation curve could not been obtained from manufacturer. In addition, core loss is neglected.

Thevenin impedance and thevenin voltage have been calculated at non-linear inductance terminals;

\[ Z_x = \frac{1}{i\omega C} \left( \frac{1}{i\omega C} + R_p + R_1 + i\omega L \right) \]  \hspace{1cm} (12.2)

\[ E_{th} = E \cdot \frac{Z_x}{Z_x + \frac{1}{i\omega C} \left( R_p + R_1 + i\omega L \right)} \]  \hspace{1cm} (12.3)

\[ Z_{th} = \frac{\left( \frac{1}{i\omega L} + R_1 + i\omega L_1 \right) R_p}{\frac{1}{i\omega C} + R_1 + i\omega L_1 + R_D} \]  \hspace{1cm} (12.4)
12.2 Damping Resistor Calculation by Empirical Method

There are some suggestions how to select damping resistors for voltage transformers based on their rated values [40], [46]. These suggestions intend to link a known transformer parameter to the ohmic value of the resistor.

Tertiary winding voltage: 33.33 V

Thermal burden of voltage transformer: 200 VA

From these values by using (1.5), damping resistance $R_D$ is found 28.86 $\Omega$.

$R_D$ is reflected in the equivalent circuit as $R_D \cdot N^2$ where $N$ is the transformer ratio.

12.3 Fundamental Ferro-Resonance Analysis

Assumed flux solution has only two components to be found;

$$\phi(t) = \phi_{i_s} \sin(\omega t) + \phi_{i_c} \cos(\omega t) \quad (12.5)$$

Equation system consists of two non-linear algebraic equations and hyper-sphere equation in this case;

$$\begin{align*}
\omega \phi_{i_s} + R_{i_s} I_{i_s} + X_{i_s} I_{i_s} - E_{i_s} = \xi_{i_s} = 0 \\
\omega \phi_{i_c} + R_{i_c} I_{i_c} + X_{i_c} I_{i_c} + E_{i_c} = \xi_{i_c} = 0 \\
(\phi_{i_s} - \phi_{i_s_m})^2 + (\phi_{i_c} - \phi_{i_c_m})^2 + (P - P_m)^2 - r^2 = 0
\end{align*} \quad (12.6)$$

Equation (12.5) and (12.6) are used in algorithms in figure 9.5 and figure 9.6.

12.3.1 Flux – Source Voltage

Test system is not in danger of fundamental ferro-resonance because there is no possible jump to another operation point around 0.66 p.u of rated source voltage in figure 12.4.
12.3.2 Flux – Capacitance

In figure 12.5, this time flux-capacitance graph shows irregular behavior. Between 0 and 10 nF, transformer gets into saturated zone. Less than values of 32 nF capacitance, system has fundamental ferro-resonance solutions as well as normal operation.
12.3.3 Flux – Damping Resistor

In figure 12.6, there are no fundamental ferro-resonance solutions for any value of the damping resistor.

![Figure 12.6 Flux against damping resistor](image)

12.3.4 Stability Domain: Source Voltage against Capacitance

At 100 nF, stability curve is well above rated voltage level making the system safe for fundamental ferro-resonance. But it can be seen that in figure 12.7 between 5 nF and 35 nF system is in fundamental ferro-resonance risk.

![Figure 12.7 Stability domain: Source voltage against capacitance](image)
12.3.5 Stability Domain: Source Voltage against Damping Resistor

In figure 12.8, at any value of damping resistor there is no danger of fundamental ferro-resonance since stability curve never goes under source voltage mark.

![Figure 12.8 Stability Domain: Source voltage against damping resistor](image)

12.4 Subharmonic-1/2 Ferro-Resonance Analysis

Assumed flux solution has only four components to be found;

$$\phi(t) = \phi_{1c} \sin(\omega t) + \phi_{1s} \cos(\omega t) + \phi_{2c} \sin\left(\frac{\omega t}{2}\right) + \phi_{2s} \cos\left(\frac{\omega t}{2}\right)$$  \hspace{1cm} (12.7)

Equation system consists of four non-linear algebraic equations and hyper-sphere equation in this case;

$$\omega \phi_{1c} + R_{lc} I_{lc} + X_{lc} I_{ls} - E_{lc} = \xi_{lc} = 0$$
$$\omega \phi_{1s} + R_{ls} I_{ls} + X_{ls} I_{lc} + E_{ls} = \xi_{ls} = 0$$
$$\frac{\omega}{2} \phi_{2c} + R_{2c} I_{2c} + X_{2c} I_{2s} - E_{2c} = \xi_{2c} = 0$$ \hspace{1cm} (12.8)
$$\frac{\omega}{2} \phi_{2s} + R_{2s} I_{2s} + X_{2s} I_{2c} + E_{2s} = \xi_{2s} = 0$$

$$(\phi_{1c} - \phi_{1cm})^2 + (\phi_{1s} - \phi_{1sm})^2 + (\phi_{2c} - \phi_{2cm})^2 + (\phi_{2s} - \phi_{2sm})^2 + (P - P_m)^2 - r^2 = 0$$

Equation (12.7) and (12.8) are used in algorithms in figure 9.5 and figure 9.6.
12.4.1 Flux – Source Voltage

Subharmonic-1/2 ferro-resonance is away from 0.66 p.u of source voltage therefore system with given parameter is safe for subharmonic-1/2 ferro-resonance in figure 12.9.

![Figure 12.9 Flux against source voltage](image)

12.4.2 Flux – Capacitance

In figure 12.10, subharmonic-1/2 solutions appear almost exactly after fundamental solutions between 34 nF and 69 nF which well far from 100 nF grading capacitance value.

![Figure 12.10 Flux against capacitance](image)
12.4.3 Flux – Damping Resistor

Since there is no possible subharmonic-1/2 ferro-resonance. The graph is same with fundamental ferro-resonance case seen in figure 12.6.

12.4.4 Stability Domain: Source Voltage against Capacitance

It can be seen that the range where subharmonic-1/2 ferro-resonant solution exists in figure 12.11, can also be seen on stability domain graph. The limit points lie under source voltage mark shows risky capacitance values.

![Figure 12.11 Stability domain: Source voltage against capacitance](image)

12.4.5 Stability Domain: Source Voltage against Damping Resistor

Similar to fundamental ferro-resonance, at any value of damping resistor in figure 12.12 system does not exhibit ferro-resonance risk. It might be interesting to note that even at 25 kV of source voltage there are relatively small values of damping resistance that could make the system safe.
**12.5 Subharmonic-1/3 Ferro-Resonance Analysis**

Assumed flux solution has four components to be found;

\[
\phi(t) = \phi_{1s} \sin(\omega t) + \phi_{1c} \cos(\omega t) + \phi_{3s} \sin(\frac{\omega}{3} t) + \phi_{3c} \cos(\frac{\omega}{3} t) \tag{12.9}
\]

Equation system consists of four non-linear algebraic equations and hyper-sphere equation in this case;

\[
\begin{align*}
\omega \phi_{1s} + R_{1c} I_{1c} + X_{1s} I_{1s} - E_{1c} &= \xi_{1c} = 0 \\
\omega \phi_{1c} + R_{1c} I_{1c} + X_{1s} I_{1s} + E_{1c} &= \xi_{1c} = 0 \\
\frac{\omega}{3} \phi_{3s} + R_{3c} I_{3c} + X_{3s} I_{3s} - E_{3c} &= \xi_{3c} = 0 \\
\frac{\omega}{3} \phi_{3c} + R_{3c} I_{3c} + X_{3s} I_{3s} + E_{3c} &= \xi_{3c} = 0
\end{align*}
\tag{12.10}
\]

\[
(\phi_{1s} - \phi_{1cm})^2 + (\phi_{1c} - \phi_{1cm})^2 + (\phi_{3s} - \phi_{3sm})^2 + (\phi_{3c} - \phi_{3cm})^2 + (P - P_m)^2 - r^2 = 0
\]

Equation (12.9) and (12.10) are used in algorithms in figure 9.5 and figure 9.6.

**12.5.1 Flux – Source Voltage**

In figure 12.13, subharmonic-1/3 solutions pass at 0.66 p.u of source voltage, so system is in risk of subharmonic-1/3 ferro-resonance.
12.5.2 Flux – Capacitance

Subharmonic-1/3 solutions in figure 12.14 appear between 98 nF and 140 nF including 100 nF grading capacitance value.

12.5.3 Flux – Damping Resistor

At 30 $\Omega$ system has more than one solution in figure 12.15, a lower value of resistance around 25 $\Omega$ will damp oscillations and system will return back to normal operation.
12.5.4 Stability Domain: Source Voltage against Capacitance

Limit point which corresponds to 100 nF lie under source voltage 12 kV in figure 12.16. It is not safe to use 30 Ω damping resistance.

12.5.5 Stability Domain: Source Voltage against Damping Resistor

In figure 12.17, critical value of damping resistor is 26 Ω. Value higher than this leaves the system in risk of subharmonic-1/3 ferro-resonance.
12.6 Different Damping Resistor Comparison

Numerical harmonic balance method can be used to determine damping resistance required to make the system safe from periodic ferro-resonances without using an empirical formula.

12.6.1 Fundamental Ferro-resonance
In figure 12.18, it can be seen that system is always in safe condition at 100 nF grading capacitance. But it is noticeable that decreased damping resistance also decreases the risk interval values of capacitance. At no damping it is at its maximum interval and system is still not in risk.
12.6.2 Subharmonic-1/2 Ferro-resonance

Test system is in risk free operation condition as in fundamental ferro-resonance case in figure 12.19.

![Figure 12.19 Stability domain: Damping resistor comparison](image)

12.6.3 Subharmonic-1/3 Ferro-resonance

In figure 12.20, for subharmonic-1/3 ferro-resonance, 30 Ω damping resistor does not provide safety for this kind of ferro-resonance but 20 Ω damping resistor makes system safe at 100 nF grading capacitance.

![Figure 12.20 Stability domain: Damping resistor comparison](image)
12.6.4 Remarks

Empirical calculation of the resistance did not provide safety from subharmonic-1/3 if it is considered 30 Ω, but numerical harmonic balance method showed that 20 Ω damping resistor will be needed. It does seem that empirical formula suggested might not be the best way to determine damping resistor, and it will be hard to support the way of its calculation. Harmonic balance method can show that if there is a danger of ferro-resonance or not. It can be shown that selection of resistor really provides a protection.

13 Current Issues with Harmonic Balance Study of Ferro-resonance

1 – Unfortunately the algorithm for numerical harmonic balance method is not fully automated. Hyper-sphere radius, ΔP and tolerance for MATHCAD solver needs to be adjusted manually each time when there is a convergence problem or if the solver is having hard time to turn limit points. It may take some to find correct parameters. Pseudo-arc length continuation method has been tried but due to mathematical problem of defining tangent vector at limit point, turning around limit points could not been achieved.

2 – MATHCAD seems to be very slow when calculating more Fourier components. The advantage of MATHCAD is that equation system solvers are already built-in functions. A basic Newton-Rhapson method coded manually may not provide enough convergence capabilities therefore create some other mathematical problems to solve equations.

3 – Magnetization curve as known as real saturation curve is needed from the manufacturer of transformer. During the time of internship, it could not be obtained and an approximation had to be used. By keeping in mind that parameters are very important on solution regimes, equivalent circuit and its parameters should be known correctly.

4 - Finding subharmonic solutions is not easy because they do not have a trivial solution like fundamental does (0,0) point. So by adjusting initial guess values to the solver, a single point should be found on the subharmonic isolate to start continuation method. That single point is found by trial and error. Since p.u values are used, initial guess value range is around -2 and 2. But still it might take time to find them.

5 – Deciding on components of the assumed flux solution is not clear in literature. Effect of components on each other should be analyzed in another study. There is also no comparison between occurrence of different type of ferro-resonances, this study included fundamental and subharmonics but information about which ferro-resonance would likely to happen could not be given.

6 - Determining the equivalent circuit for neutral-isolated systems is controversial. To use this method explained here, it is needed to work with a single phase equivalent. But it is said
that neutral-isolated systems cannot be simplified into one-phase equivalent [44]. The nonlinearity between flux and magnetizing current does not allow using linear transformation to use zero-sequence circuit to analyze damping effect of damping resistance. Studying “forced” unbalanced system may not be correct. To apply harmonic balance to three-phase circuits, mutual impedances should be found between phases which gets very complicated since numbers of unknown get tripled.

7 – Finding real cases to test numerical harmonic balance method on ferro-resonance is hard to find. Especially saturation curves are not given most of the time causing use of harmonic balance meaningless in that case.

14 Summary and Conclusion

Ferro-resonance even though it is not very common is a problem to power systems. It has dangerous consequences like stable over voltages and over currents. Risky configurations are mentioned and prevention of ferro-resonance is discussed. Time-domain simulations are not enough to study for these kinds of phenomena since ferro-resonance may happen or may not happen depending of circuit parameters and initial conditions prior to any event in the system. Due to this problem of time-domain analysis, direct calculation of steady states is done with harmonic balance method. Two methods of harmonic balance have been discussed namely analytical and numerical harmonic balance method. Numerical harmonic balance method showed advantages on application and study on different kinds of ferro-resonances. In order to deal with continuation problem while drawing two parameter graphs, a hyper-sphere method is applied. Two examples from literature have been extensively analyzed with harmonic balance and also Alstom’s test system went under the same study with other cases. Stability domain of different kind of ferro-resonances provides large insight to the ferro-resonance phenomena. By changing any kind of parameter in the system it can be observed if that change put the system in or out of ferro-resonance danger. This feature is used to determine damping resistance to damp ferro-resonant oscillations and do not allow them to become steady-state. Without using empirical method to calculate resistance, decision on damping resistor by harmonic balance method is more scientifically backed up. There are still problems with the method has been applied and those were discussed in chapter 13.

This study considers that static damping is used for prevention of ferro-resonance in voltage transformers. Due to problems in this method, dynamical damping may be an option for power system designers, it would automatically control itself for any kind of oscillation and adjust itself to damp oscillations. Then it will not be necessary to think about which kind of ferro-resonance should be studied and for which kind of ferro-resonance the damping resistor should be calculated. Harmonic balance study showed that different ferro-resonance types needs different damping resistance values. As the parallel losses increased by lowering
the damping resistance, it is possible to provide protection from other types of ferro-resonance if it is not provided before. Lowering resistance value might have some thermal restrictions for voltage transformers but that study is not done here.

Existence zone for fundamental and subharmonic ferro-resonances are very close so even if the real possibility of occurrence is not known, it is better not to leave it to chance and provide a damping resistance that would make the system risk free of ferro-resonance. In Alstom’s test system, empirical method to calculate damping resistance provided safety for fundamental and subharmonic-1/2 ferro-resonance but not for subharmonic-1/3 after it has been checked with harmonic balance method. Not only for damping resistance but values of the grading capacitance value can be calculated with this way also if it is needed. The power of harmonic balance method comes from its flexibility of use.

Correct selection of damping resistor will prevent ferro-resonance therefore it can avoid unpredictable accidents that may happen in power systems and power plant networks. It contributes to general safety of power systems and even safety for human life.

For future studies, algorithm proposed can be tried with another continuation method such as pseudo-arc length method and solve computation problems at limit points. MATHCAD seems to be slow on non-linear equation solving when it comes to calculate many Fourier components. An algorithm can be coded from scratch in FORTRAN or MATLAB but it is not sure it would drastically reduce computation time. Because of this problem studies done in this project do not contain many Fourier components that would make solutions more accurate.

It is known that voltage transformers have non-linear core loss, and during saturation losses are increased. This could have effect on calculations of ferro-resonance. By adopting a proper model for core loss, calculations will present more accurate real life cases. Increased losses help damping of the system so calculations done without including the extra losses by non-linear core loss would be in safe design side.

Three-phase analysis can be done for isolated neutral systems. In this project one-phase equivalents are used because three-phase analysis turned out to be has not been done therefore no information could have been found on this matter for guidance.
15 References
3- J. Bethenod, "Sur le Transformateur à Résonance", L'Éclairage Électrique, vol. 53, 1907


34- C. Kieny, “Application of the bifurcation theory in studying and understanding the global behavior of a ferroresonant electric power circuit”, *IEEE Transactions on Power Delivery*, vol. 6, no. 2, April 1991.


