HELPING COGNITIVE RADIO IN THE SEARCH FOR FREE SPACE

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Abstract

Spectrum sensing is an essential pre-processing step of cognitive radio technology for dynamic radio spectrum management. One of the main functions of Cognitive radios is to detect the unused spectrum and share it without harmful interference with other users. The detection of signal components present within a determined frequency band is an important requirement of any sensing technique. Most methods are restricted to the detection of the spectral lines. However, these methods may not comply with the needs imposed by practical applications.

This master thesis work presents a novel method to detect significant spectral components in measured non-flat spectra by classifying them in two groups: signal and noise frequency lines. The algorithm based on Fisher's discriminant analysis, aside from the detection of spectral lines, estimates the magnitude of the spectral lines and provides a measure of the quality of classification to determine if a spectral line was incorrectly classified. Furthermore, the frequency lines with higher probability of misclassification are regrouped and the validation process recomputed, which results in lower probabilities of misclassification.

The proposed automatic detection algorithm requires no user interaction since any prior knowledge about the measured signal and the noise power is needed. The presence or absence of a signal regardless of the shape of the spectrum can be detected. Hence, this method becomes a strong basis for high-quality operation mode of cognitive radios.

Simulation and measurement results prove the advantages of the presented technique. The performance of the technique is evaluated for different signal-to-noise ratios (SNR) ranging from 0 to -21dB as required by the IEEE standard for smart radios. The method is compared with previous signal detection methods.
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Gävle, January 2012
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List of Abbreviations

AIC  Akaike’s Information Criterion
CR   Cognitive Radio
DSA  Dynamic Spectrum Access
ED   Energy detection
FCC  Federal Communications Commission
FFT  Fast Fourier Transform
ML   Maximum-Likelihood Estimator
MME  Maximum -Minimum Eigenvalue
MoM  Method of Moment
PU   Primary User
SDR  Software-Defined Radio
SNR  Signal-to-noise ratio
SSA  Static Spectrum Access
SU   Secondary User
Chapter 1

Introduction

Emerging wireless services indicate an increasing demand of consumers for spectrum-based communication links. Governmental agencies regulate the spectrum usage assigning frequencies to license holders for exclusive use. The limited available spectrum and inefficient usage of the allocated spectrum under the current fixed spectrum allocation policy make of the radio electromagnetic spectrum a scarce natural resource [1]. The underutilization of the spectrum stimulated the development of new technologies to exploit the existing resource efficiently.

Efficient and intensive spectrum use by licensees within their own bands as well as opportunistic spectrum access to the licensed bands by other users without interfering with the existing users require flexible spectrum allocation systems. Flexibility implies dynamic spectrum sensing, access and sharing techniques [2] working on heterogeneous architectures. Hence, an intelligent system characterized by its flexibility on changing its functionality by software is needed. Application of digital signal processing (DSP) software to radio communications resulted in the advent of a technology that can perform most radio functions without the need to replace hardware. Cognitive radio is the technology which complies with these characteristics.

The term “cognitive radio” was introduced by Joseph Mitola in 1999 as a spectrum-sharing technology that emerged from the application of advanced software techniques to radio processing for improvement of the spectral use [3]. By 2002 the Federal Communications Commission (FCC) assisted by its multi-disciplinary team Spectrum Policy Task Force concluded that radio technologies enable better access to spectrum and that changes on the spectrum policy should be considered [4]. In 2003, the FCC issued a Notice of Proposed Rule Making that identifies cognitive radio as the right candidate for spectrum sharing under negotiable and opportunistic conditions between users [4]. In response to this, in 2004 the IEEE
formed the 802.22 Working Group to develop a standard for wireless regional area networks based on cognitive radio technology [4].

Cognitive radio (CR) is defined as the intelligent wireless technology that is “cognizant” of its surrounding, “learns” from it and adapts its transmission parameters to maximize the spectrum use and the access of unused spectrum by exploiting it or transferring it when requested by owners such that risk of interference between users is prevented and good quality of service is insured [5].

The platform for the realization of a cognitive radio is known as Software-defined Radio (SDR), where the radio system is easily defined or reconfigured by software to operate on different frequencies and formats than the ones supported by its hardware design making the implementation of the mentioned cognitive capabilities feasible. Therefore, the available spectrum can be obtained through the cognitive capability and reconfigurability features of cognitive radio.

Since the challenge is to share the licensed spectrum without interfering with the transmission of other licensed users also known as primary users (PU), when a primary user is inactive, a cognitive radio should be able to sense the unused licensed spectrum and relinquish its usage to a secondary user (SU). Hence, the “sensing” feature of the cognitive radio is fundamental for the efficient management of the spectrum. Several methods are suggested for this purpose, the most important are: geo-location database and spectrum sensing.

*Spectrum sensing* function enables the cognitive radio to adapt to its environment by identifying the unused portions of spectrum and transmit in such “spectrum holes”. Cognitive radio should therefore determine if a signal from a user is locally present in a determined band. *Signal detection* approach is based on the detection of the weak primary user signal. The spectrum sensing problem can be therefore formulated as a binary hypothesis test, where $H_0$ is a null hypothesis in which no primary user signal is present in a certain band, and $H_1$ is an alternative hypothesis which indicates the presence of a licensed user signal [1].

The simplest approach for spectrum sensing is visual analysis of the power spectrum of the signal. However, some factors can turn visual approach unfeasible: low SNR, fading and multipath for wireless communication, and noise power uncertainty. Diverse techniques have been developed in order to overcome these shortcomings with variable success. However, most of the methods require some prior knowledge on either noise power information or signal characteristics. Some relevant characteristics of these techniques are introduced in the next section.
1.2 Previous Research

Some spectrum sensing techniques are already available. The Energy Detection (ED) method does not need any information of the signal to be detected, but requires a good estimate of the noise power, becoming very vulnerable to noise uncertainty and prone to false detection [6]. Since, it is easy to implement it became a generally adopted technique. Matched-filter detection is an alternative method that maximizes the received SNR and presents short execution time, where the need of prior knowledge on the licensed user signal as well as implementation complexity and large power consumption makes it impractical [1]. Cyclostationary feature detection is based on the observed statistical properties varying over time demanding long sensing time and complex computations [1].

To overcome the shortcomings of these methods, test statistics-based methods: Covariance method and Maximum-Minimum Eigenvalue (MME) are blind algorithms insensitive to noise for a limited number of primary users [7], [8]. High probability of detection and low probability of false alarms with little information about the primary user signals and noise spectrum can be obtained. However, they require additional user interaction and present high complexity. An investigation of the performance of these methods done in [9] proves the superiority of the latest developed methods over the initial methods. A latest method based on random Vandermonde matrices presents a better performance than the previous methods even for small numbers of measurement samples [10].

Most of these methods depend on different signal features, and hence are classified as parametric methods. No information beyond the detection of the signal line is given.

Recently a novel method [11] based on a statistical test has been developed to overcome most of these problems. This technique modified Fisher’s discriminant analysis to automatically discriminate the noise lines from the signal lines, having three major advantages over the existing methods:

1) The developed method is simple and user friendly since it requires no user interaction and minimal postulated noise assumptions.
2) The probability of false classification is obtained.
3) The estimate of the magnitude of the spectral component is computed for every detected frequency line.

However, the method has also a major disadvantage: since the disturbing noise is assumed to be ‘white’ i.e. it has constant spectrum, and hence the signal spectrum is assumed to be flat, which cannot be assumed for normal operation conditions of cognitive radios.
1.2 Problem statement

An automatic signal detection algorithm based on a statistical test to detect the spectral components in any kind of spectrum is needed.

1.3 Goal

The main goal is to develop an automatic signal detection method based on Fisher’s discriminant analysis. One approach is to segment the spectrum and apply the method over each segment. Hence, this method should be able to work over non flat measured spectra without any knowledge a priori, becoming useful for cognitive radios. Minimal postulated assumptions, system identification theory and knowledge about polynomial regression models shall be used.

1.4 Thesis outline

Chapter 1: Introduction - presents an overview of the content of this thesis by describing the background, stating the problem and the goals to achieve.

Chapter 2: Cognitive Radio – contains the insights of Cognitive radio theory and operation, describing in particular, the spectrum sensing function.

Chapter 3: Signal Detection- explains the principles of the basic and proposed methods used to detect the spectral components. The proposed method represents the main contribution of this thesis work.

Chapter 4: Probabilistic Validation – contains the explanation of the method used to determine the quality of classification of the technique detailed in Chapter 3.

Chapter 5: Simulation Results – presents the evaluation of the technique performance over computer-generated signals at different SNR ranging from 0 to -20dB.

Chapter 6: Measurement Results – shows some measurement examples where realistic telecommunication signals and conditions are considered.

Chapter 7: Conclusions – presents conclusions from analysis of the results shown in Chapter 5 and 6.

Chapter 8: Future Prospects – suggests some challenges and opportunities for the continuation of this work.
Chapter 2

Cognitive Radio

In this chapter, an overview to cognitive radio technology and some definitions used throughout this thesis are given. Software-defined radio, cognitive radio and dynamic spectrum access are explained, with a particular emphasis on spectrum sensing function and techniques.

2.1 Introduction

Over the past years, several research challenges emerged due to the spectral needs imposed by the existing and upcoming wireless technologies. Since some spectrum bands are already licensed to services for exclusive use, crowded frequency allocation becomes notorious. Spectrum shortage results from the static spectrum management policies rather than a physical scarcity of usable frequencies. In contrast to this static spectrum access, dynamic spectrum access (DSA) of radio systems to the idle frequency bands by license-exempt users is proposed.

For this purpose, radio systems have been evolving from hardware-model based radios to a traditional combination of hardware and software radios. Later on, they evolved to software-defined radios (SDR) [12] which incorporate computer processing capabilities into radio systems in order to increase the abilities of their internal models [13]. One step further are cognitive radios (CR), which upon the platform of SDR are intelligent radios that can be autonomous reconfigurable by learning and adapting to the communication environment. These different radio systems are illustrated in Fig. 1 [14].
2.2 Cognitive radio

Cognitive radio is a radio system that has the ability to sense its radio frequency environment and modify its communication parameters based on what it detects [15]. From this definition, the two main characteristics of cognitive radios can be synthesized as cognitive capability and reconfigurability.

2.2.1 Cognitive capability

The tasks required for the adaptive operation of cognitive radios are shown in an enhanced cognitive cycle as illustrated in Fig. 2 [1], [5]. The cognitive cycle starts with a passive sensing of radio frequency (RF) stimuli and culminates with action. Three main steps of this cycle stand out [1]:

1. Spectrum sensing, monitors the available spectrum channel, learns their information and determines the unused channel.
2. Spectrum analysis estimates the characteristics of the detected channel.
3. Spectrum decision determines the operating parameters. According to the user requirements and spectrum characteristics, a spectrum channel is chosen.
2.2.2 Reconfigurability

The capability of adjusting the transmission operating parameters without modifying the hardware components such that the radio system can adapt to the radio environment is called reconfigurability. The communication parameters can be reconfigured not only at the beginning of the transmission but also during the transmission, and at the reception for an appropriate communication. The CR can determine the most suitable operating frequency, reconfigure the modulation scheme to the user requirements and channel conditions for spectral efficiency, adapt the transmission power within the power constraints by decreasing the power and interference with other users, and provide interoperability among different communication systems [1], [5].

2.2.3 Cognitive radio system

A cognitive radio system can be generally viewed in Fig. 3. A reconfigurable radio based on radio parameters (operating frequency, power, bandwidth, etc.) is present. A sensing engine may accept inputs from the radio environment. A policy database determines a behavior according to the observations. These two input to a reasoning block, so an appropriate configuration for the radio system can be determined, being capable of learning from this experience. Finally, a configuration database stores the current configuration of the system as a behavior stereotype [15].
2.4 Dynamic Spectrum Access (DSA)

The specific behavior of a cognitive radio is termed Dynamic Spectrum Access (DSA), which is the process of increasing spectrum efficiency by real-time adjustment of radio resources [2]. In order to increase the number of radio access points, the secondary use of underutilized spectrum which originally was allocated to another primary purpose is encouraged. The users of the spectrum at certain time can be categorized as in [4], [16], and [17]:

- **Primary users (PU)** also known as licensed users or licensees are defined as the owners of certain frequency channel and therefore, have legal rights on the usage of that specific part of the spectrum.
- **Secondary users (SU)** also known as unlicensed users or lessees transmit or receive over the licensed spectra when primary users are inactive to avoid interference.

*Spectrum hole* also known as unused spectrum, white space or spectrum opportunity, defines the inactivity of a primary user when the allocated spectrum is not fully utilized or remains idle.

DSA techniques allow the cognitive radio to select the best available portion of the spectrum. Hence, cognitive radio enables the secondary users to (1) determine the available channel and detect a licensed user presence, *spectrum sensing*; (2) select the best available channel, *spectrum management*; (3) negotiate access to the channel with other users, *spectrum sharing* and (4) liberate the channel when a licensed user requests it, *spectrum mobility* [1].

Therefore, a SU needs to have cognitive radio capabilities, such as sensing spectrum to reliably detect weak primary signals over a targeted frequency band to exploit the spectrum opportunities.
2.5 Spectrum sensing

Sensing emerges in the early stages of the cognitive cycle as a fundamental step that enables spectrum use agility by providing to CR systems of awareness and sensitivity to the environment changes. Efficient spectrum use is attained when minimum time in sensing the degrees of freedom (time, frequency, and space) is spent.

One approach to detect spectrum holes is to detect the primary users that receive data. Generally, spectrum sensing techniques focus on primary transmitter signal detection. Hence, the spectrum sensing problem can be reduced to *signal detection*. Based on the detection of a weak primary signal, hence a binary hypothesis can be formulated as:

\[
\mathcal{H}_0: \ r(n) = v(n) \\
\mathcal{H}_1: \ r(n) = x(n) + v(n)
\]  

(1)

where \(\mathcal{H}_0\) is a null hypothesis that represents the absence of a primary user signal, i.e. contains noise only \(\mathcal{H}_1\) is an alternative hypothesis that indicates the presence of a primary user signal whereas \(r(n)\) denotes the measured signal at a sampling instant \(n\), \(x(n)\) the primary user signal and \(v(n)\) the noise.

2.5.1 Limitations of Spectrum Sensing

Most of CR applications impose certain requirements to the spectrum sensing techniques. A sensing algorithm should comply with the needs imposed by practical applications such as large operating bandwidth, short sensing time, low implementation complexity as well as low power consumption and hardware cost.

In cognitive radio, terminals are required to process wide frequency bands, so spectrum opportunities can be easily identified. However, this requires additional hardware components and long sensing duration. To overcome this, high speed processing units for short delay are required. Trade-offs between speed and sensing reliability arise, since spectrum sensing demands some time to identify PUs while on the other hand requires vacating the band as fast as possible if solicited.

According to IEEE standards for cognitive radios, the sensitivity of these techniques is evaluated when the method performs under SNR ranging from 0 to -21dB [18]. The complexity of the method increases as the detector sensitivity approaches certain critical value called SNR walls. Below the SNR walls it is almost impossible to distinguish the two above defined hypothesis [19].
Some parameters that evaluate the performance of sensing methods should be calculated as part of the proposed technique. Probability of detection ($P_d$) is the probability of detecting correctly a signal on the evaluated frequency band, while probability of false alarm ($P_f$) is the probability that the method incorrectly detects a signal in the evaluated frequency band when actually it is not. A large detection probability and a low false alarm probability are required to prevent underutilization of the spectrum. Therefore, these probabilities determine the sensitivity ($P_d$) and specificity ($P_f$) of the method respectively [1], [16].

### 2.5.2 Signal detection methods for Spectrum Sensing

This section gives an overview of the existing spectrum sensing techniques. These techniques can be classified in two groups: parametric and non-parametric methods. Having prior knowledge about how the process was generated allows to parametrically estimate the spectral content, generally they are slow but accurate. Some parametric methods are: matched-filter detection, cyclostationary feature detection, energy detection, and maximum to minimum eigenvalue detection. In the other hand, the non-parametric methods are considered to be fast but a rough estimation. One non-parametric method is the discriminant analysis detection. These mentioned methods will be described here.

#### 1. Matched filter detection

*Matched-Filtering* also referred as a *coherent detector*, is known as an optimum detector that maximizes the SNR in noisy environments if the transmitted signal properties (modulation type and order, the pulse shape, bandwidth and the packet format) are known a priori [1].

Cognitive radio needs receivers for the different signal types and to demodulate the received signals. Therefore, accurate knowledge of signal properties and different detection algorithms make the implementation complex, impractical and require large power consumption [16].

The main advantage is the short time to achieve high performance due to synchronization [20] while its major disadvantage is that it performs poorly when the prior knowledge of every primary user is not accurate and at very low SNR.
2. Energy detection

The non coherent energy detector, also known as radiometer is a simple approach that does not need any knowledge on the primary user signal but requires information on the noise power. Signals can be detected by comparison of the output of the detector with a noise-dependant threshold that decides whether a primary user signal is present or not. Initially, the decision threshold required knowledge of noise and signal power. Estimation of the noise power is feasible but that is not the case for signal power, since the signal strength varies according to transmission parameters and the distance between radio and the primary user. Hence, the selection of the threshold is sufficed with noise variance knowledge.

The performance of the detection algorithm can be evaluated observing two probabilities: high $P_D$ and low $P_F$. Under low SNR, low $P_D$ would represent missing potential primary user signals which would increase interference to the primary users. A high $P_F$ would represent underutilized spectrum. Clearly, sensitivity is conditioned on the SNR. An optimal threshold can be determined when a balanced relation between $P_D$ and $P_F$ is achieved [16].

Low computation and implementation complexities make it an optimal detector when no sufficient information about the signal is provided. However, the method is susceptible to noise uncertainty having poor performance at low SNR. Moreover, it cannot differentiate in signal types since only the presence of signal can be determined, which makes it prone to false detection.

3. Cyclostationary feature detection

Cyclostationary–based detector or feature detector is an alternative method based on the statistical properties of a signal varying periodically over time [21]. The inherent periodicity of the signal features can be detected and analyzed by spectral correlation functions for accurate detection [20].

The main advantage is that it differentiates the noise power from the signal power even at low SNR. Furthermore, it can distinguish among different types of transmissions and primary users. The method is better than energy detector when detecting noise due to the robustness to the uncertainty noise [1]. It becomes a coherent detector if the noise power is known [20].

The number of known features in the signal determines the robustness of the method in multipath or fading environments. However, this demands long sensing time and complex computation. The performance of the cyclostationary detector in terms of $P_D$ and $P_F$ is mathematically difficult to determine and it requires higher computationally complex algorithms i.e. Monte Carlo method [1].

Energy detection becomes suitable in cases where limited information about the primary signals is present. Instead, coherent detection and feature detection can be used for sensing refinement or signal classification when more information of the primary signal is available [20].
4. Maximum minimum eigenvalue detection

*Maximum–minimum eigenvalue detection* is a test statistic method, where the ratio of the maximum to the minimum eigenvalue derived from the covariance matrix of the received signal is compared to a computed threshold. Based on random matrix theories (RMT), the ratio can be quantified and the threshold based on the probability of false alarm [8].

The method outperforms most of the previous methods, and it can be used for different applications since no knowledge on the signal, channel conditions and noise power is needed. Therefore, it is a blind algorithm with certain computational complexity.

However, the calculation of the threshold and probabilities of detection and false alarm is based on asymptotical distributions of eigenvalues, while eigenvalues are approximated by deterministic values. In other words, it is assumed that the number of samples tends to infinity. In practical situations, parameters are finite to achieve optimal performances with the thresholds estimated with this method. Therefore, the estimation of thresholds for a finite number of samples becomes a challenge [22]. Some other disadvantages are the need of user interaction and information on the number of primary users.

A latest research on the asymptotic behavior of random *Vandermonde* matrices and Gaussian matrices is presented in [10]. The natural connection to fast Fourier transform (FFT) and Hadamard transforms allows to sense spectrum opportunities reliably with a small number of received samples.

5. Discriminant analysis detection

Automatic detection [11] based on a statistical test known as Fisher’s quadratic discriminant, automatically detects primary user signals. The only information a priori is that within a frequency band, the method discriminates two groups: a signal and a noise group.

The major advantages over existing methods are: (i) low complexity, (ii) no prior knowledge is required i.e. signal features, noise power, number of primary transmitter users, as others, (iii) every detected signal or noise line receives an estimate of the magnitude of the signal and noise power, and (iv) the probability of being falsely classified as signal or noise is calculated for every line.

However, the method works but under assumptions that are generally not met in practice: (i) high SNR such that Rice distribution can be approximated by Gaussian distribution, (ii) the disturbing noise is assumed to be white, and (iii) the signal spectrum is assumed to be flat.
2.7 Comparison of sensing techniques

The limitations present in most sensing techniques leads to further techniques to overcome the shortcomings while keeping the advantages of the previous methods.

Some aspects of these methods are mainly observed in order to determine the effectiveness of the method. The most important properties of a sensing technique are:

- *Prior knowledge* defines the quantity of signal information needed by the method.
- *Noise rejection* describes the immunity of the method against noise variation.
- *Interference rejection* shows the capacity of the method to be immune to other disturbances.
- *Sensing time* gives an idea of the performance of the method in real-time applications.
- *Computational complexity* depicts the quality of difficulty required to execute the method.

These properties are evaluated for the methods previously explained, and compared to each in other in Table I [21], where the red colored block represents a bad property, a yellow colored block represent a medium property, a green colored block represents a good property and a light green colored block represents an optimal property.

<table>
<thead>
<tr>
<th>Method</th>
<th>Prior knowledge</th>
<th>Noise rejection</th>
<th>Interference rejection</th>
<th>Sensing time</th>
<th>Computational complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matched filter</td>
<td>HIGH</td>
<td>LOW</td>
<td>HIGH</td>
<td>LOW</td>
<td>HIGH</td>
</tr>
<tr>
<td>Energy detection</td>
<td>NONE</td>
<td>LOW</td>
<td>LOW</td>
<td>MEDIUM</td>
<td>LOW</td>
</tr>
<tr>
<td>Feature detection</td>
<td>HIGH</td>
<td>HIGH</td>
<td>HIGH</td>
<td>HIGH</td>
<td>HIGH</td>
</tr>
<tr>
<td>Eigenvalue detection</td>
<td>NONE</td>
<td>HIGH</td>
<td>LOW</td>
<td>MEDIUM</td>
<td>MEDIUM</td>
</tr>
<tr>
<td>Discriminant Analysis</td>
<td>NONE</td>
<td>HIGH</td>
<td>HIGH</td>
<td>LOW</td>
<td>LOW</td>
</tr>
</tbody>
</table>

In conclusion, the method that appears to be most flexible and simple is the automatic detection. Therefore, this thesis work extends the methodology of the discriminant analysis in such a way that the assumptions may be relaxed to meet normal operating conditions for cognitive radios.
Chapter 3

Signal Detection

This chapter introduces the signal detection theory and discriminant analysis philosophy for spectrum sensing. The discriminant analysis detection technique is concisely summarized since it forms the basis of the extended methodology proposed as main contribution of this thesis work. The automatic detection algorithm can be divided in three major parts: the detection of spectral components, the estimation of the magnitudes of the signal and noise power and the probabilistic validation of the detected spectral components. The detection of spectral components will be discussed throughout this chapter.

3.1 Introduction

The general theory of signal detection is based on statistical decision theory that treats detection as a decision process to test statistical hypotheses [23]. It is particularly used in the selection of the criterion for signal presence, where ambiguous stimuli can be sensed and classified. It is assumed that the “sensory” response of a detection process can be perturbed by the presence of random interference or noise. Thus, the fundamental detection process is limited to the observation of two classes of stimulus events: events containing noise alone and events containing signal plus additive noise. The criterion is therefore, based on what the detector “detects” and some other relevant parameters. The expected values of the decision made by a detector can be summarized as the four possible outcomes shown in Table II [24]. A ‘Yes’ response given by the detection of a significant signal presence is a correct response, but a ‘Yes’ to a falsely detected signal presence is considered as a mistake and denoted as a false alarm. A ‘No’ response when a signal is absent is a correct response and it is called correct rejection while a ‘No’ to an actual signal presence is an error called misdetection.
The detection and false alarm responses are equiprobable. These responses depend on two determiners. One is the difficulty of the detection where a simple detection is given when signal and noise are well separated or the distance between signal and noise is large. The second determiner is the intensity of the stimulus events where a signal is present when the intensity exceeds certain criterion, and noise alone is determined whenever the intensity is lower than this criterion. The criterion that elicits a response is called threshold [24].

Some assumptions are made: (i) the noise follows a Normal or Gaussian distribution which is described by two parameters the mean \( \mu \) and the variance \( \sigma^2 \). Within the signal theory detection framework, these values are 0 and 1 respectively. (ii) Since the signal is added to the noise and the signal is treated deterministically, the distribution of the signal has the same shape (and the same variance) as the noise distribution [24].

Hence, the signal detection theory provides a measure of sensitivity [23] that should be practically exempt of all the possible variables expected to affect the detection decision. Consequently, signal detection for cognitive radios simplifies the spectrum sensing problem to the detection of weak primary transmitter signals, where the presence or absence of a primary user should be determined.

As seen in the previous chapter, the signal detection technique based on discriminant analysis complies with both previously mentioned determiners. The discriminant analysis partition the data in two groups: frequency lines containing noise alone and frequency lines containing signal, such that there is a maximum separation between them. The best separator of these two groups is the threshold, known as discrimination height. Considering that the amplitudes in the frequency-domain of a disturbed signal follow a Rice distribution, some assumptions were made: (i) at high SNR, Rice distributed signals are approximated by a Gaussian distribution, ii) the time-domain noise is assumed to be white, (iii) the spectrum of the signal is assumed to be flat. However, these assumptions affect the detection decision for non-flat spectra, and therefore the method become impractical.
An extension of the discriminant analysis technique to meet practical requirements is proposed here. In order to do so, the spectra can be portioned into small segments where the flatness of the power spectrum can be assessed. This methodology will be fully described in this chapter.

### 3.2 Foundation of discriminant analysis

Discriminant analysis is a statistical technique that allows the study of differences between two or more groups, introducing among others: discriminant and classification functions. Data cases are the basic units of analysis and are classified into two or more well defined groups. One should be able to “discriminate” between the groups based on the characteristics that differentiate one group from the other. These characteristics are called discriminating variables. How closely these variables and the function are related, enable to discriminate between groups. It is also important how the function is related to the variables within the groups [25]. The determination of a linear discriminant function as a linear combination of the discriminating variables is used for discrimination analysis. This function should maximize the distance between the groups with an equation that minimizes the possibility of misclassifying cases into their respective groups. The number of discriminant functions is equal to the number of groups minus one [26].

Fisher posed this problem: Let \( x \) and \( y \) be two 2-D Gaussian random vectors. Assume that there are \( N \) measurements \( x(n) \) and \( y(n) \) where \( n = 0, \ldots, N - 1 \) for \( x \) and \( y \). The vector \( z \) consists of \( x \) and \( y \), and therefore there are \( 2N \) measurements of \( z \). Under the assumption that \( z \) is Gaussian distributed, \( x \) and \( y \) are also Gaussian distributed. The problem is to differentiate which measurements belong to \( x \) and to \( y \). Fisher approached the problem by finding a linear separator that discriminates the measurements of \( x \) and \( y \) such that the probability of misclassification is minimized. Hence, this reasoning can be applied to discriminate spectral components into noise and signal line groups without any user interaction [11].

Visual inspection of the signal power spectrum is a simple way to detect spectral components. However, some signal assumptions are formulated before getting into details of the method itself.

### 3.3 Signal assumptions

Let \( x(t) \) be a continuous time signal i.e.

\[
x(t) = g(t) + n(t) \quad (2)
\]

where \( g(t) \) is a multisine with \( K \) arbitrary tones, and \( n(t) \) is a noise process such that its power spectral density \( S_n(j\omega) \) and variance \( \sigma_n^2 \) exist.
For simplicity, two assumptions are made:

(i) The number of periods of the signal \( g(t) \) is an integer such that its power spectral density \( G(j\omega) \) is discrete, and (ii) the noise spectrum is white such that the process has zero mean and finite variance, having a flat spectrum.

The signal \( x(t) \) is digitized and the resulting signal is \( x_d(n) \), where \( n = 0, \ldots, N - 1 \). In the frequency domain, the amplitude of the signal \( A_x(k) = |X_d(k)| \), where \( X_d(k) \) is the discrete Fourier coefficient of the signal \( x_d(n) \) at frequency bin \( k \).

### 3.4 Discriminant Analysis detection of spectral lines

The automatic detection algorithm based on statistical test needs knowledge on the distribution of the measurements \( A_x(k) \). The user interaction is eliminated with use of the discriminant analysis where Gaussian disturbances are assumed. Furthermore, the advantages and the disadvantages of this technique are elaborated on.

#### 3.4.1 Distribution of spectral components

The probability distribution of the amplitude measurements \( A_x(k) \) can be obtained from analysis of \( X_d(k) \). Hence, the amplitude measurements \( A_x(k) \) can be represented by:

\[
A_x(k) = |G_d(k) + N_d(k)|
\]

where \( G_d(k) \) and \( N_d(k) \) are the discrete Fourier coefficients at frequency bin \( k \) of the signal \( g(t) \) and noise \( n(t) \), respectively. \( N_d(k) \) is complex circular Gaussian distributed with zero mean and variance \( S_n(j\omega_k) \). Thus, the distribution of the amplitude \( A_x(k) \) is equal to

\[
A_x(k) \approx Rice\left(|G_d(k)|, \frac{1}{2}S_n(j\omega_k)\right)
\]

More details on (4) are found in [11].

#### 3.4.2 Signal detection

The main philosophy of discriminant analysis is to partition the data in two groups such that the groups are maximally separated under the constraint that the variance within every group is as small as possible.

The data cases for a measured spectrum consist of two groups: frequency lines containing noise alone and frequency lines containing signal. Spectrum analyzer measurements follow Rice distribution which can be approximated by Gaussian distribution under high signal-to-noise ratio conditions.
Let $I$ and $J$ be the groups of frequencies containing signal lines and noise lines respectively. No prior knowledge on these groups is required. The mean amplitudes of the spectral lines are defined as

$$\hat{A}_x^I = \frac{1}{|I|} \sum_{k \in I} A_x(k)$$

$$\hat{A}_x^J = \frac{1}{|J|} \sum_{k \in J} A_x(k)$$

where $\hat{A}_x^I$ represents the mean amplitude of the spectral lines classified as signal while $\hat{A}_x^J$ represents the mean amplitude of the classified noise lines. The variables $|I|$ and $|J|$ represent the respective number of classified signal and noise lines.

The main philosophy of discriminant analysis is to partition the data in two groups such that the groups are maximally separated under the constraint that the variance within every group is as small as possible.

Expressing this objective in a statistical testing framework results under Gaussian noise assumptions in Fisher’s quadratic discriminant [11], such that

$$T^2 = \frac{(\hat{A}_x^I - \hat{A}_x^J)^2}{\sigma_I^2(|I|-1) + \sigma_J^2(|J|-1)} \left(|I| + |J| - 2\right)$$  \hspace{1cm} (5)

The objective of the discriminant analysis is to maximize (5). Therefore the set of frequency bins of the signal lines $I$ and of the noise lines $J$ should be chosen in such a way that the numerator or distance between the group means is maximized, and the denominator or distance within the group variances is minimized. A binary grid search is used to come to the correct discrimination height.

The philosophy of this method is illustrated in Fig. 4, where the gray curve is the amplitude of a disturbed signal, the bold red line is the discrimination height. The frequencies with amplitudes below the discrimination height are classified as the group containing only noise lines, while the ones with amplitudes above are the group of frequencies with signal components. The dashed lines are the averages of the amplitudes in each group. The solid arrows denote the distance within one group, while the dotted arrow denotes the distance between the group averages. The objective of the method is to find the bold red line such that, it complies the requirements of the discriminant analysis.

Aside from the spectral line classification, the automatic method can also provide with an estimate of the signal and noise amplitudes as well as the probability of misclassifying the data cases into signal or noise groups by studying the probability distribution given in (4). These two other tasks of the method are explained in the next chapter.
The discriminant analysis method clearly has the following advantages:

1. It is fully automatic, with no user interaction and no prior knowledge.
2. It provides an estimate of the amplitude spectrum of signal and noise.
3. It provides a user-friendly and simple validation.

However, the presented technique only works optimally under the assumption that the considered power spectrum of both signal and noise is flat, which cannot be assumed in practical applications e.g. normal operation conditions of cognitive radios.
3.5 Segmentation Algorithm

In this section, we propose an extension of the previously described discriminant analysis method to non-flat spectra. This can be done by partitioning the spectra into small segments in which the power spectrum of the noise can be approximated as being flat. To assess this, we need to detect the frequency lines that are purely noise contributions. Doing so, the width of the region where the flatness condition holds can be determined.

3.5.1 Detection of signal and noise spectral lines

By a simple visual inspection, one can already have a rough idea of which parts of the spectrum contain signal, and which contain noise. A signal line typically has larger amplitude than its neighboring noise frequency lines. One issue that remains to be solved, is how much larger the amplitude of the analyzed frequency line needs to be, compared to the neighboring lines. In the proposed detection algorithm, a frequency line is a potential signal line if the amplitude difference between this frequency line and its neighboring frequency lines is larger than a user-defined value $\delta_G$. To implement the above idea, we use the function provided in [27] to obtain maximum and minimum amplitude values:

$$A_x(l) < A_x(k) - \delta_G \quad (6)$$

$$A_x(l) > A_x(k) + \delta_G \quad (7)$$

where $k < l$. Every $A_x(k)$ satisfying (6) is a local maximum, denoted as $A_x^{\max}(k)$, and $A_x(k)$ satisfying (7) is a local minimum, denoted as $A_x^{\min}(k)$.

The goal is to find the frequency lines $k$ with $A_x^{\max}(k)$ which contain signal contributions and frequency lines $k$ with $A_x^{\min}(k)$ which contain noise contributions. This method does not detect all signal lines but gives a rough idea.

The objective of this algorithm is illustrated in Fig. 1. The crosses represent the frequency lines $k$ with amplitude $A_x^{\max}(k)$, these lines can contain signal contributions. The circles represent the frequency lines $k$ with amplitude $A_x^{\min}(k)$ and these lines are definitely noise contributions. The remaining frequency lines can contain either noise or signal contribution.
3.5.2 Segmentation boundaries

Based on the noise lines with amplitude $A_x^{\text{min}}(k)$ detected in the previous section, we can now divide the spectrum into segments with local flat spectrum. Having $k < l$, let $k_{\text{upper}}$ and $k_{\text{lower}}$ be the frequency lines where maximal amplitude and minimal amplitude are found such that they satisfy (8) and (9).

$$A_x^{\text{min}}(l) < A_x^{\text{min}}(k_{\text{upper}}) - \delta_{SG} \quad (8)$$

$$A_x^{\text{min}}(l) > A_x^{\text{min}}(k_{\text{lower}}) + \delta_{SG} \quad (9)$$

The order in which these frequency lines appear define the right bound of a segment, while the left bound of the segment is the same bound as the right bound of the previous segment. An exception occurs for the frequency lines located at the beginning and end of the spectrum, for which the left and right bounds are respectively 1 and $N$. Hence, having $a$ maximal values and $b$ minimal values, the bounds of the different segments are defined as:

1. $k_{\text{upper}}(1)$, $k_{\text{upper}}(1)$ - $k_{\text{lower}}(2)$, $k_{\text{lower}}(2)$ - $k_{\text{upper}}(3)$, ..., $k_{\text{upper}}(a)$ - $k_{\text{lower}}(b)$, $k_{\text{lower}}(b)$ - $N$.

An illustrative example is presented in Fig. 2. The diamonds represent the amplitudes of the upper bounds of the segments and the squares represent the amplitude of the lower bounds. Clearly, the main difficulty is to select the proper values for $\delta_G, \delta_{SG}$. Although, these values can be selected arbitrarily by the user, the Chapter 5 provides some interesting rules-of-thumb.
3.5.3 Discrimination Curve

In this section, the segmentation technique is used for signal detection. The idea is to apply the method described in Section 3.4, as detailed in [11], to each of the detected segments. Hence, each segment receives a discrimination height which discriminates between signal and noise within the segment (Fig. 7). To obtain a smooth discrimination curve over the full frequency band of interest, a polynomial is fitted on the centers of every discrimination height over the different segments (Fig. 8). For this purpose, data fitting using polynomial regression models will be used.

Given $N$ data points, let $f_k$ with $k = 0, \ldots, N - 1$ be the frequency lines, and $h$ be the discrimination height function, this response variable $h$ is modeled as a combination of the predictor variables $f_k$. Given data on $f_k$ and $h$, regression estimates the model parameters $\alpha$. The model proposed takes the following form

$$h(f_k) = \alpha_0 + \alpha_1 f_k + \cdots + \alpha_{p-2} f_k^{p-2} + \alpha_{p-1} f_k^{p-1} + \varepsilon(f_k)$$ (10)
The discrimination height variable is modeled as a linear combination of the variables $f_k$. Some uncontrolled errors are present and modeled by $\epsilon$. The polynomial model can be synthesized as

$$h(f_k) = \sum_{i=0}^{p-1} \alpha_i (f_k)^i + \epsilon(f_k) \quad (11)$$

For this, the linear regression model can be also expressed as an $N$-by-$p$ system of equations:

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & f_0 & f_1^2 & \ldots & f_1^{p-1} \\ 1 & f_1 & f_2^2 & \ldots & f_2^{p-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & f_{n-1} & f_{n-1}^2 & \ldots & f_{n-1}^{p-1} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{p-1} \end{bmatrix} + \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ \vdots \\ \epsilon_{n-1} \end{bmatrix} \quad (12)$$

$$h = X \alpha + \epsilon \quad (13)$$

To fit the model to the data, the system must be solved for $p$ coefficient values in $\alpha = (\alpha_0, ..., \alpha_{p-1})^T$. The $p$-vector $\alpha$ of parameters that gives the “best fit” to the data points can be found when the unknown error is minimized, this can be seen when (13) is rewritten as

$$\epsilon = h - X \alpha \quad (14)$$
\( h - X \propto \) is named residual vector and can be also expressed as \( h(f_k) - \sum_{i=0}^{p-1} \propto_i k^i \). The norm of the residual should be minimized. For computational convenience, Euclidean or 2-norm is used to minimize the difference between the actual data trend and proposed polynomial model. The solution will be given by the vector that minimizes the sum of squares of differences between the data points and the model, which is known as least square method. The best fit expressed in least squares sense is given by

\[
\arg\min_{\propto_i} \| h - X \propto \|_2^2 \quad (16)
\]

The estimation of the polynomial coefficients can be computed as follows

\[
X^T h = X^T X \propto \quad (17)
\]

\[
\propto = (X^T X)^{-1} X^T h \quad (18)
\]

Once the coefficients are found, the polynomial is evaluated over the full range of frequency lines. The obtained polynomial curve is able to separate the signal lines from the noise lines without any user interaction.

Polynomial regression is one linear model that has the advantage of being simple and flexible for following data trends. However, polynomial models oscillate between data points, particularly at higher
degrees and eventually the prediction of new values becomes poor due to the inaccuracy of the model. Clearly, there is a trade-off between data fitting and complexity.

The complexity of the polynomial model is given by some model parameters, thus the selection of an appropriate model and the estimation of its parameters should be determined. Some confidence boundaries for $\varepsilon$ as well as an optimal degree selection can be facilitated by means of model selection criteria. Among the different model criteria, Akaike’s information criterion (AIC) and $C_p$ or Mallows statistic can be used to find the best model.

The AIC model criteria attempts to find the model that fits the data with minimum degrees of freedom based on the residual sum of squares that minimizes the AIC value. It does not need any prior information on the model parameters [29]. Mallow’s $C_p$ assesses the regression model using least squares, which is found by selecting a subset of the variable predictors [30]. Both improve their performances as more model variables are considered which avoids overfitting.
Chapter 4

Probabilistic Validation

The discriminant analysis method automatically classifies the spectral lines but also provides information on the magnitude of the spectral components and quality of classification. For this purposes, an estimate of the amplitudes of noise and signal lines and probabilistic validation of the method are computed.

4.1 Introduction

The quality of the discrimination method gives some confidence on the reliability of the performed detection. Since no information on the groups of frequencies is given in advance, a databased validation is not possible. However, a probabilistic validation can be computed to know if the measured amplitude is wrongly classified either as corresponding to a signal or to a noise line. To compute this probability of false classification, the probability distribution of the amplitude measurement $A_x(k)$ should be determined.

In Chapter 3, it was assumed that the signal $g(t)$ is periodic and corrupted by zero-mean Gaussian noise (white or colored) $n(t)$. In the frequency domain, the amplitude of this disturbed signal $A_x(k)$ is distributed according to the Rice probability density function. This distribution is fully described by two parameters: the amplitude of the signal $A_x(k)$ and the standard deviation $\sigma$ of the noise [31]. For instance, the Rice distribution of a variable that depends on complex circular Gaussian distributed variables as in Section 3.4.1 can be derived as follows. Let $Z$ be Rice distributed $Z \sim Rice(\nu, \sigma^2)$ if $Z = \sqrt{X^2 + Y^2}$ [32], where $X \sim N(\nu \cos \theta, \sigma^2)$ and $Y \sim N(\nu \sin \theta, \sigma^2)$ are two independent Gaussian distributed variables with means $\nu \cos \theta$ and $\nu \sin \theta$ and variance $\sigma^2$ while $\theta$ is a real number [11]. Hence, one obtains
\[ Z \triangleq Rice(v, \sigma^2) \triangleq \sqrt{(N(v \cos \theta, \sigma^2))^2 + (N(v \sin \theta, \sigma^2))^2} \quad (19) \]

This reasoning can be applied to (3) since \( N_d(k) \) is complex circular Gaussian distributed with zero mean and variance \( S_n(j\omega_k) \), the amplitude of the disturbed signal can be also expressed in a complex way: \( A_x(k) = A_{d,r}(k) + jA_{d,i}(k) \) having a real and an imaginary part so as the signal \( G_d(k) = G_{d,r}(k) + jG_{d,i}(k) \) and the noise \( N_d(k) = N_{d,r}(k) + jN(k) \) [11]. The Rice distribution of the amplitude is then given by

\[
A_x(k) \triangleq Rice\left(|G_d(k)|, \frac{1}{2}S_n(j\omega_k)\right) \triangleq \sqrt{\left(G_{d,r}(k), \frac{1}{2}S_n(j\omega_k)\right)^2 + \left(G_{d,i}(k), \frac{1}{2}S_n(j\omega_k)\right)^2} \quad (20)
\]

clearly (20) is equal to (4). It becomes obvious that to determine the two parameters of a Rice distribution, the amplitude of the signal and the noise should be known. Since no knowledge on the true signal and noise amplitudes is available, the goal is to estimate these two values.

The Maximum Likelihood Estimator (ML) and the method of moments (MoM) are two estimation procedures. The MoM provides consistent estimators but not as efficient as ML, they are used for their simple computation which contrasts with the ML. However, MoM estimators can give poor solutions for low SNR, while ML requires good assumptions for initial values [31], [33]. Other estimation procedures like Bayesian approach could be used in order to overcome the shortcomings of the other estimators.

### 3.4.3 Estimation of the magnitudes of the signal and noise power

The discriminant analysis detection gives an estimate of the magnitudes of the signal and noise power using ML and MoM procedures as detailed in [11]. Assuming that the noise spectrum is white, the ML estimator of the noise power for the set of noise frequency lines \( k \in J \) is

\[
\hat{S}_n(j\omega_k) = \frac{1}{|J|} \sum_{k \in J} (A_x(k))^2 \quad (21)
\]

The ML for the signal component \( G_d(k) \) is not available. The MoM estimator [34] gives the expected value of the amplitude measurements per frequency for Rice distributed amplitude [11], [32]

\[
\mathbb{E}[A_x^2(k)] = |G_d(k)|^2 + S_n(j\omega_k) \quad (22)
\]

For simplicity, the expected value in (22) can be reduced to \( \mathbb{E}[A_x^2(k)] = A_x^2(k) \). Using (21) and (22) results in an estimate of the signal amplitude for the set of signal frequency lines \( k \in I \)

\[
|G_d(k)| = \sqrt{A_x^2(k) - \hat{S}_n(j\omega_k)} \quad (23)
\]
4.2. Probabilistic validation of the detected spectral lines

The discriminant analysis method should correctly classify the signal presence as the detection of a signal line, and the signal absence as a correct rejection of a noise line. In order to assess the quality of the classification, we compute the probability of false classification which embraces the misdetection of actual signal frequency lines and the false alarm of signal presence when noise frequency lines are actually present. This wrong classification can be denoted as misclassification.

To formally introduce the probability of misclassification, we denote $A_s(k)$ to be the random variable describing a new amplitude measurement at frequency $k$. The probability that values $A_s(k)$ larger than the amplitude measurement $A_x(k)$ are observed while $k$ is a noise line such that the line $k$ is incorrectly classified as a signal line can be denoted as $\mathbb{P}(A_s(k) > A_x(k)|k \notin I)$. The probability that values $A_s(k)$ smaller than the amplitude measurement $A_x(k)$ are observed while $k$ is a signal line such that the line $k$ is incorrectly classified as a noise line can be denoted as $\mathbb{P}(A_s(k) < A_x(k)|k \in I)$. Hence, the probability of misclassification is given by

$$\pi(k) = \begin{cases} \mathbb{P}(A_s(k) > A_x(k)|k \notin I) \\ \mathbb{P}(A_s(k) < A_x(k)|k \in I) \end{cases}$$  \hspace{1cm} (24)

Based on the nature of $k$, the values in (21) and (23) are used to compute the probabilities in (24). Since $A_s(k)$ is a Rice distributed amplitude value. For $k$ noise lines, the Rice parameters are $\left(0, \frac{1}{2}S_n(j\omega_k)\right)$. Thus, an estimate of the probability of misclassifying noise lines is given by

$$\mathbb{P}(A_s(k) > A_x(k)|k \notin I) = 1 - F_{\text{Rice}}\left(0, \frac{1}{2}S_n(j\omega_k)\right)(A_x(k))$$  \hspace{1cm} (25)

where $F_{\text{Rice}}$ denotes the cumulative distribution function of a Rice distribution.

For $k$ signal lines, the Rice parameters are $\left(|G_d(k)|, (\frac{1}{2})S_n(j\omega_k)\right)$. As a result, an estimate of the probability of misclassifying signal lines is given by

$$\mathbb{P}(A_s(k) < A_x(k)|k \in I) = F_{\text{Rice}}\left(|G_d(k)|, \frac{1}{2}S_n(j\omega_k)\right)(A_x(k))$$  \hspace{1cm} (26)

Therefore, the probabilities of misclassification can be summarized as

$$\hat{\pi}(k) = \begin{cases} 1 - F_{\text{Rice}}\left(0, \frac{1}{2}S_n(j\omega_k)\right)(A_x(k)) \\ F_{\text{Rice}}\left(|G_d(k)|, \frac{1}{2}S_n(j\omega_k)\right)(A_x(k)) \end{cases}$$  \hspace{1cm} (27)

It is desired that these probabilities remain below the 50% which assures a good performance of the method. Simulation and measurement examples are presented in the next chapter, where for higher probabilities of misclassification the method allows regrouping the frequency lines to the correct group.
Chapter 5

Simulation Analysis

In this section, the segmentation technique is used for signal detection. The idea is to apply the method described in Sections 3 and 4, as detailed in [11], to each of the detected segments. Hence, each segment receives a discrimination height which discriminates between signal and noise within the segment. To obtain a smooth discrimination curve over the full frequency band of interest, a polynomial is fitted on the centers of every discrimination height over the different segments. Furthermore, the validation process is computed to assess the quality of the method in different shapes of spectrum.

5.1. Numerical Examples

The proposed method is illustrated with different simulation examples, a time series $y(t)$ is generated with $t = 0,...,2048$. The signal $y(t)$ is represented:

$$y(t) = g(t) + n(t)$$

where $n(t)$ is a zero mean white or colored Gaussian noise sequence. The signal $g(t)$ is a multisine with equal or different amplitude spectrum and eight tones arbitrarily chosen.

Therefore, the simulation results can be divided in 3 cases:

1. Case I: Different signal amplitude under white noise
2. Case II: Equal signal amplitude under colored noise
3. Case III: Different signal amplitude under colored noise

The performance of the technique will be evaluated in these different cases for different SNR ratios ranging from 0 to -20dB as required by the IEEE standard for cognitive radios [18]. Robustness of the method will be discussed and some interesting rules-of-thumb will be elaborated on.
The technique clearly presents 5 steps to be performed in order to assess the method for all the cases.

1. Detection of signal and noise spectral lines
2. Detection of segment boundaries
3. Signal detection
4. Probability of misclassification
5. Regrouping of frequency lines and the validation process is recomputed.

5.1 Case I: Different signal amplitude under white noise

A. SNR of 0dB

An initial detection of signal and noise lines is performed in Fig. 9a (left), where local maximum and minimum spectral lines are detected: the frequencies lines $k$ with amplitude $A^\text{max}_x(k)$ can contain signal contributions while the frequency lines $k$ with amplitude $A^\text{min}_x(k)$ contain definitely noise contributions. These local maximum points at the frequency $k$ exist if there is some $\delta_G > 0$ such that $A_x(k) \geq A_x(l)$ when $|A_x(k) - A_x(l)| > \delta_G$. Similarly, the local minimum points at frequency $k$ exist if $A_x(k) \leq A_x(l)$ when $|A_x(k) - A_x(l)| > \delta_G$. At high SNR, $\delta_G$ can be defined as:

$$\delta_G = |\mu + 2\sigma|$$  \hfill (28)

where $\mu$ is the mean of the amplitude of the signal, and $\sigma$ is the standard deviation of the amplitude of the signal. If the SNR is sufficiently large, amplitudes that follow Rice distribution can approximate Gaussian distribution, and $\delta_G$ reflects the 95% confidence interval for Gaussian random variables. Amplitude values that exceed at least this $\delta_G$, assures that maximum and minimum values are found. Applying formula 28, the value is $\delta_G = 20.2046$.

Next, the segment boundaries also need a new $\delta_{SG}$ to be defined. Based on the noise lines with amplitude $A^\text{min}_x(k)$ previously detected, we can define $\delta_{SG}$ as the standard deviation of these amplitudes, which represents the minimum difference to determine maximal and minimal amplitudes. $\delta_{SG} = 1.7663$. The upper and lower bound detection of segments is illustrated in Fig. 9b (center).

The signal detection is performed in Fig. 9c (right). Red dark lines represent the discrimination heights of each segment, a smooth curve is obtained with a polynomial fitting curve of degree $p=2$. Below the curve, all noise lines are found while over the curve only signal lines are found.

The method is able to separate the signal and noise lines perfectly without any observed errors. This is supported by the lower graph in Fig. 10a (left) which indicates that the probability of falsely detected signals is almost zero. In total, with the validation process 8 lines were classified as signals and received a
misclassification probability of $1.5e-08$, while 2040 were classified as noise lines having a misclassification probability ranging from $3.8875e-4$ to 0.62.

According to the main philosophy of the discriminant analysis, misclassification probabilities should be reduced. The fact that some noise lines present a higher misclassification probabilities indicate that there is a high risk they might be signal lines, therefore these frequency lines should be regrouped to the other group and the misclassification probabilities recomputed. As seen in Fig. 10b (right), other 23 lines are regrouped as signal lines and received a considerable misclassification probability of 0.44, while a reduction of misclassification probability for the noise lines is seen, ranging from $3.8875e-4$ to 0.45.

**B. SNR of -10dB**

Since the formulas (28) and (29) seem not to hold at low SNR, a proper choice of $\delta_G$ and $\delta_{SG}$ are required from the user. For the initial detection of signal and noise spectral lines is shown in Fig. 9b (left) using a user-defined $\delta_G = 8$. Next, the Fig. 9b (center) shows the bound segments which are found using a user-defined $\delta_{SG} = 3$. The signal detection is performed in Fig. 9b (right), where a polynomial curve of degree $p=7$, visually this curve is able to separate signal from noise lines. Initially, the validation process in Fig. 10b (left) recognizes 10 signal lines and they received a misclassification probability of 0.0085, while for the noise lines the misclassification probability ranges from $5.3464e-11$ to 0.92. Obviously, 2 extra lines were classified as signals, but after regrouping 443 extra lines are classified as signals since they previously presented high misclassification probability but still a probability of 0.4847, see Fig. 10b (right). While the noise lines have lower misclassification probability (<0.5) since they are correctly classified as noise lines.

**C. SNR of -20dB**

An initial detection of signal and noise lines and bound segment detection are performed in Fig. 9c (left) and Fig. 9c (center), respectively using user-defined values for $\delta_G = 6$ and $\delta_{SG} = 3$. These lower values are required since at low SNR, many signal lines might be buried in the noise and therefore, the signal detection algorithm will require smaller segments in order to detect these lines. The signal detection is applied to each segment as shown in Fig. 9c (right) presenting a polynomial fitting curve of degree $p=7$.

By simple visual inspection, it is clear that the detected signal lines present a low misclassification probability of 0.3595 in the lower graph of Fig. 10c (left) and many classified noise lines with a misclassification probability of 0.9645 in the middle graph in Fig. 10c (left). After regrouping the frequency lines, 440 extra signal lines are found with a probability of 0.4922 in the lower graph of Fig. 10c (right). This demonstrates that they present high risk of being confused as noise lines due to the poor SNR, whereas lower misclassification probability for noise lines in the middle graph of Fig. 10c (right) indicates that they are correctly classified as noise lines.
Fig. 9 Segmentation Algorithm for different SNR: (a) 0dB, (b) -10dB and (c) -20dB. Left: Local maximum and minimum spectral line detection: local maximum (cross marked lines) and local minimum (circled lines). Center: Upper and lower bound detection of segments: Amplitude of the lower bounds (diamond) and amplitude of the upper bounds (square). Right: Polynomial fitting curve for an SNR of 0dB: (gray) amplitude of frequency lines, (bold dark curve) polynomial fitting curve, (black circles) center of discriminant heights, (solid red line) discrimination heights.
Fig. 10 Probabilistic validation for different SNR: (a) 0dB, (b) -10dB and (c) -20dB. Left: Validation process without regrouping of frequency lines. Right: Validation process with regrouping of frequency lines with high misclassification probabilities. Segmented spectra (upper graph), probability of falsely classified noise (middle graph) and probability of falsely classified signals (lower graph).
Case II: Uniform amplitude under colored noise

This case will be evaluated for a multisine signal \( g(t) \) having uniform amplitude spectrum and eight tones arbitrarily chosen, whereas \( n(t) \) is a zero-mean colored Gaussian noise sequence. A colored noise is generated by sending white noise through a dynamic system like a low pass filter, which is denoted as \( \text{shaping filter} \). Adjusting the parameters of the shaping filter the colored noise can be varied. The low pass filter to be used will be a Butterworth filter of order 2, and normalized cutoff frequency of 0.5 and 0.75 for this case. For the following simulations an SNR of -10dB is used.

A. SNR of -10dB and colored noise at a normalized cutoff frequency of 0.5

An initial detection is performed in Fig. 11a (left) using (28), one obtains \( \delta_G = 14.4368 \). The boundaries of the segments are determined in Fig. 11a (center) use a user-defined \( \delta_{SG} = 1 \). The signal detection is performed in Fig. 11a (right), and 8 tones are correctly classified as signal lines and this coincides with the discrimination curve of degree \( p=2 \) which correctly separates the signal from the noise lines.

The validation process is performed, and some signals present a misclassification probability ranging from 0 to 6.3408e-11 while some noise lines present a misclassification probability ranging from 2.6882e-40 to 0.9627. As expected, the misclassification probabilities for signal lines are zero in the lower graph of Fig. 12a (left), while for noise lines is also zero in the middle graph in Fig. 12a (left). After regrouping, see. Fig. 12a (right), the misclassification probabilities for noise lines decreases values are lower than 0.49931, while for the 311 extra signal lines probabilities lower than 0.4540 are seen.

Case III- Different amplitude under colored noise

A. SNR of -10dB and colored noise at a normalized cutoff frequency of 0.75.

An initial detection is performed in Fig. 11b (left) using (28), one obtains \( \delta_G = 15.1399 \). The boundaries of the segments are determined in Fig. 11b (center) using a user-defined value of \( \delta_{SG} = 1 \). The signal detection performed is shown in Fig. 11b (right), and 14 tones are correctly classified as signal lines. The discrimination curve of degree \( p=5 \) adapts well to the shape of the spectra but at higher frequencies than the normalized cutoff frequency, it classifies many noise lines as signal lines.

The validation process for the technique is computed, and some signals present a misclassification probability ranging from 0 to 0.2779 while some noise lines present a misclassification probability ranging from 3.7825e-41 to 0.98247 as in Fig. 12b (left). After regrouping, the misclassification probability for noise lines decays to 0.4946, while for the 212 extra signal lines present a probability of 0.4697 as can be seen in Fig. 12b (right).
Fig. 11 Segmentation Algorithm for signals with an SNR of -10dB with (a) uniform amplitude disturbed by colored noise at a normalized cutoff frequency of 0.5 and (b) different amplitude disturbed by colored noise at a normalized cutoff frequency of 0.75. Left: Local maximum and minimum spectral line detection: local maximum (cross marked lines) and local minimum (circled lines). Center: Upper and lower bound detection of segments: Amplitude of the lower bounds (diamond) and amplitude of the upper bounds (square). Right: Polynomial fitting curve: (gray) amplitude of frequency lines, (bold dark curve) polynomial fitting curve, (black circles) center of discriminant heights, (solid red line) discrimination heights.

An interesting observation is that, for the disturbed signals by filtered noise, the method is able to suggest a polynomial curve that adapts to the shape of the spectrum. At higher frequencies than the normalized cutoff frequency, it is easier to differentiate the signal lines from the noise lines since the signal lines are sticking out from the noise floor which contrasts when observing at low frequencies where the signal lines can be buried in noise which makes more difficult to determine the existence of signal lines.
Fig. 12 Probabilistic validation for signals with an SNR of -10dB with (a) uniform amplitude disturbed by colored noise at a normalized cutoff frequency of 0.5 and (b) different amplitude disturbed by colored noise at a normalized cutoff frequency of 0.75. Left: Validation process without regrouping of frequency lines. Right: Validation process with regrouping of frequency lines with high misclassification probabilities. Segmented spectra (upper graph), probability of falsely classified noise (middle graph) and probability of falsely classified signals (lower graph).
Chapter 6

Measurement Results

In the measurement example, some cases proposed in the previous section will be measured. The test setup consists of a Rhode & Schwarz SMU 200A Vector Signal Generator (SG), a Rode & Schwarz FSQ 26 Signal Analyzer (SA), and a personal Computer (PC) interconnected as shown in Fig. 13. For this purpose, the time-domain disturbed signal which is computer-generated will be sent to a SG and then measured by a SA. The measured data is sent to a PC for further application of the method.

Fig. 13 Measurement setup for assessment of the signal detection technique: A PC sends data to a Vector Signal Generator (SG), the signal coming from this SG is measured using a Spectrum Analyzer, and the measured data is stored in a PC for application of the method
7.1 Measurement results

In the measurement examples, the generated time-domain disturbed signals presenting an SNR of -10dB for the different cases presented the previous chapter are measured. These multisine signals with eight tones distorted by zero-mean white and colored noise were sent to the SG using a carrier frequency $f_c = 1.5GHz$. The amplitude of the signals was measured by an SA using a bandwidth of $B_W = 100MHz$. The method was applied to the measured signals, presenting the following results.

Case I: Different signal amplitude under white noise

Using a $\delta_G = 10$, an initial detection of signal and noise spectral components as shown in Fig. 14a (left). The bounds detection for $\delta_{SG} = 1$ can be seen in Fig. 14a (center). The signal detection is performed in Fig. 14a (right), where a discrimination curve of degree $p=3$ separates the signal from noise detected lines.

- **Signal detection**

In total 168 lines were classified as signal lines, 7 of them correspond to the tones of the generated multisine while the rest of extra lines correspond to falsely detected signal lines. The noise amplitudes estimate values range from -50 to -53.62dBm, while the signal amplitudes estimate values varying from -48 to -36.03dBm which corresponds with visual analysis of the 6 segments.

- **Probabilistic Validation**

The misclassification probabilities for signal lines had a maximum value of $0.0901\ (= 1 - F_{Rice}(a_{\frac{3}{2}n(j\omega_k)})(A_s(k)))$ for all the detected lines while the misclassification probability for the noise lines is about from 0.0181 to 0.63022 as seen in Fig. 15a (left).

- **After regrouping:**

The misclassification probabilities for the detected signal lines had a maximum value of $0.4391\ (= 1 - F_{Rice}(a_{\frac{3}{2}n(j\omega_k)})(A_s(k)))$ due to the increment of frequency lines in the signal group, however they present significant misclassification probably values. For the actual signal lines these probabilities are ranging from 0.0002 to 0.1057($= 1 - F_{Rice}(a_{\frac{3}{2}n(j\omega_k)})(A_s(k)))$. The misclassification probabilities for noise lines are lower than 0.5. Both probabilities can be seen in Fig. 15a (right).
Case II: Uniform signal amplitude under colored noise

A signal having uniform amplitude and disturbed with colored noise (using Butterworth lowpass filter at the normalized frequency of 0.5) was generated. The method is applied to the measured amplitude, which results in: an initial detection of signal and noise spectral components for $\delta_G = 6$ as shown in Fig. 14b (left). The bound detection is performed with $\delta_G = 3$ as illustrated in Fig. 14b (center). Next, the signal detection and a discrimination curve of degree $p=2$ as can be seen in Fig. 14b (right).

- The method initially obtains 94 detected signal lines.
- The noise amplitudes are ranging from -70 to -50dBm, while the signal amplitudes are ranging from -39 to -33dBm which coincides with the visual analysis of the spectrum.
- The misclassification probability values for signal lines were from 0 to 0.066 $(= 1 - F_{Rice}(\frac{1}{2}S_n(j\omega_k))(A_x(k)))$ for all the detected lines while the misclassification probabilities for the noise lines were from 0.041 to 0.6274 as in Fig. 15b (left).
- After regrouping of frequency lines, the misclassification probability values were around 0.427 for the regrouped lines, while $0(= 1 - F_{Rice}(\frac{1}{2}S_n(j\omega_k))(A_x(k)))$ for the actual frequency lines. The misclassification probabilities for the noise lines lower than 0.4938 as seen in Fig. 15b (right).

Case III: Different signal amplitude under colored noise

In this case, a multitone signal presenting different amplitude and disturbed with colored noise (Butterworth lowpass filter at the normalized frequency of 0.75) was generated. The method is applied to the measured amplitude, which results in: an initial detection of signal and noise spectral components for $\delta_G = 8$ is shown in Fig. 14c (left). The bound detection is performed with $\delta_G = 2$ as illustrated in Fig. 14c (center). Next, the signal detection is performed presenting a discrimination curve of degree $p=3$ as can be seen in Fig. 14c (right).

- The method initially obtains 42 detected signal lines.
- The noise amplitudes are ranging from -75 to -50dBm, while the signal amplitudes are ranging from -47 to -33dBm which coincides with the visual analysis of the spectrum.
- The misclassification probability values for signal lines were from 0 to 0.0123 $(= 1 - F_{Rice}(\frac{1}{2}S_n(j\omega_k))(A_x(k)))$ for all the detected lines while the misclassification probabilities for the noise lines were from 0 to 0.6321 as shown in Fig. 15c (left).
After regrouping of frequency lines, the misclassification probabilities were around 0.4348 for the regrouped lines, while between 0 and 0.0243 ($= 1 - F_{Rice}(\frac{1}{2}, 1)A_x(k)$) for the actual frequency lines. The misclassification probabilities for the noise lines lower than 0.4946 as illustrated in Fig. 15c (right).

Fig. 14 Segmentation algorithm for measured amplitude of signals with an SNR of -10dB having (a) different amplitude under white noise, (b) uniform amplitude under colored noise at a normalized cutoff frequency of 0.5 and (c) different amplitude under colored noise at a normalized cutoff frequency of 0.75. Left: Local maximum and minimum spectral line detection: local maximum (cross marked lines) and local minimum (circled lines). Center: Upper and lower bound detection of segments: Amplitude of the lower bounds (diamond) and amplitude of the upper bounds (square). Right: Polynomial fitting curve: (gray) amplitude of frequency lines, (bold dark curve) polynomial fitting curve, (black circles) center of discriminant heights, (solid red line) discrimination heights.
Fig. 15 Probabilistic validation for signals with an SNR of -10dB having (a) different amplitude under white noise, (b) uniform amplitude under colored noise at a normalized cutoff frequency of 0.5, and (c) different amplitude under colored noise at a normalized cutoff frequency of 0.75. Left: Validation process without regrouping of frequency lines. Right: Validation process after regrouping of frequency lines. Segmented spectra (upper graph), probability of falsely classified noise (middle graph) and probability of falsely classified signals (lower graph)
7.2. Measurement Analysis

- The method applied to the different cases of the measured amplitude signal proves that the method is able to detect spectral components by discriminating the noise from the signal lines.

- The segmentation requires of a trade-off between the $\delta_G$ and $\delta_{SG}$, increments on $\delta_G$ requires of decrements on $\delta_{SG}$, while for lower SNR lower values of $\delta_G$ and $\delta_{SG}$ are required.

- The signal lines are more the difficult lines to classify specially at very low SNR (-20dB) where some noise lines can be detected as signal lines, the lower the SNR the more the falsely detected signals.

- The polynomial model gives a smooth discrimination curve that adapts to the different discrimination heights, presenting low polynomial order. Lower polynomial orders adapt smoothly to the discrimination heights and follow the shape of the spectrum, which is clearly seen when having signals disturbed by filtered noise. Higher polynomial orders can pass through each center of the different discrimination heights but the discrimination curve does not adapt properly and therefore, it is not able to correctly separate noise from signal lines. This is known as overfitting, where the polynomial is not setting a proper relationship for the data.

- The probabilistic validation results suggests that the segmentation and hence, the discrimination was correctly performed.

- The regrouping of frequency lines results in lower misclassification probabilities, the fact that these probabilities can be altered indicates that the segmentation can be further updated to improve the performance and the method and hence, reduce the misclassification probabilities.

- The method is applied with no mathematical or computational complexities in a short execution time.

The method can be analyzed in terms of risk of misclassification, where the quality of segmentation and signal detection can be evaluated for the spectrum in the full frequency range. The risk of misclassification for the different measured amplitude of the three different cases of disturbed signals is shown in Fig. 16(a-c). Gray colored frequency lines present very low risk of being misclassified ($\hat{\pi}(k) < 10\%$), green colored frequency lines present low risk ($10\% < \hat{\pi}(k) < 30\%$), blue colored frequency lines present a considerable risk ($30\% < \hat{\pi}(k) < 50\%$), and red colored frequency lines ($\hat{\pi}(k) > 50\%$) present high risk of being misclassified whether as noise lines or signal lines.
Fig. 16 Risk of misclassification probabilities for signals with an SNR of -10dB having a) different amplitude under white noise, (b) uniform amplitude under colored noise at a normalized cutoff frequency of 0.5, and (c) different amplitude under colored noise at a normalized cutoff frequency of 0.75. Left: Risk of noise lines misclassification. Right: Risk of signal lines misclassification, in both cases (inverted gray triangle) frequency lines with very low risk, (green circles) frequency lines with low risk, (blue triangle) frequency lines with significant risk, and (red triangle) frequency lines with high risk of misclassification.
For the measurement examples, the discriminant analysis finds signal lines with very low probability of being misclassified as seen in the Fig. 16a-c (right) while the noise lines with amplitude values close to the one of the discrimination height for each segment will have high risk of being misclassified, while the frequency lines below enough present the lower risk as well as the ones above as seen in Fig 16a-c (left). Clearly, the signals are the more difficult lines to classify statistically, since their amplitude are the sum of a deterministic and a stochastic part (random noise). The suspicion that some signal lines buried in noise are misclassified suggests that the validation step, besides giving the quality of the method, can be also used to properly select the boundaries of the segments such that the detection is improved.
Chapter 7

Conclusions

The main goal in this thesis work was to extend the signal detection methodology for cognitive radios, based on the discriminant analysis philosophy in such a way that the technique is able to detect the presence or absence of signal components in non-flat measured spectra without any knowledge a priori.

Cognitive radios are a new paradigm for wireless communication that intends to solve the underutilization of the radio electromagnetic spectrum through efficient and intensive use. One of the main functions of cognitive radios is spectrum sensing: detection of the unused spectrum, efficient use of licensed spectrum and spectrum sharing without harmful interference with other users. Since it is required the sensing of spectrum holes, one approach is signal detection by visual analysis of the signal power spectrum. In the past many algorithms have been developed. However, all these methods require some prior knowledge on the transmission parameters or signal properties from primary users (noise power, signal patterns, type of modulation, number of primary users, etc.) since they determine the robustness of the method. Having such information is no possible for real-time applications, therefore these methods become impractical. Moreover, no further information than the detection of spectral lines is obtained from these methods.

This thesis work extends an automatic detection methodology based on Fisher’s discriminant analysis which allows the detection of spectral components in measured amplitude spectra. The advantage of the method over previous methods is the minimal user interaction. Minimal postulated noise assumptions, system identification theory and polynomial regression models are used to detect signal components regardless of the shape of the spectra and under practical measurement conditions.

For this purpose, a segmentation algorithm of the spectrum proposes to partition the spectrum into small segments, in which the flatness condition required by the initial method holds such that the discriminant
analysis method can be applied to each segment. The algorithm detects local maxima and minima spectral lines which determine the boundaries of each segment. Hence, each segment receives a discrimination height that separates the signal and noise within the segment. A polynomial curve is fitted to the center of the discrimination heights of each segment resulting in a smooth discrimination curve that separates signal from noise components.

The segmentation algorithm requires two user-defined values $\delta_G$ and $\delta_{SG}$ which are the minimum distances required to determine local maximum and local minimum spectral lines. At high SNR, it is possible to determine these values as explained in Section 5.1. At lower SNR this requires a proper choice from the user, the lower the SNR the lower values for $\delta_G$ and $\delta_{SG}$ are required. Since lower SNR require a higher number of segments (smaller portions of spectra) in order to be able to detect signal lines that might buried in noise.

At lower SNR, the degree of the polynomial discrimination curve also increases to fit all the discrimination heights of the larger number of segments. However, lower polynomial degrees for the discrimination curves ($p \leq 7$) were required even at the extreme SNR case i.e SNR= -20dB.

Obviously, the performance of the method is optimal at high SNR: the method is able to detect correctly the signal and noise components in shorter time and the probabilities of misclassification are lower, which comply with the requirements of the desired spectrum sensing technique. At very lower SNR (around -20dB), the method detects the signal lines plus some noise lines that are classified as signal lines. The lower the SNR, the greater the number of noise lines classified as signal lines.

Since the probability of misclassification gives an estimate of the quality of classification process, frequency lines with misclassification probabilities above 50% might denote a high risk that the segmentation was incorrect. These frequency lines can be regrouped to the other group such that the probabilities of misclassification are reduced. Doing so, a connection between the probability of misclassification and the segment boundaries becomes obvious and lead us to think that these probability values can help us to define better the values of $\delta_G$ and $\delta_{SG}$ yielding to the proper bounds of the segments and hence, to an optimal discrimination of the signal and noise lines.

The advantages of the extended method over existing methods are given here.

- The developed method is simple and fast, presenting low computational complexity and no user interaction which makes it user friendly.
- The only prior knowledge needed is that within a frequency band, the algorithm separates two groups: a signal and noise group as well as some minimal postulated noise assumptions. Hence, it requires short observation and execution time.
• Beyond the signal detection, estimate of signal magnitude and noise power as well as a measure of the quality of classification is given by the probability of misclassification for every frequency line.

• Frequency lines with high probability of misclassification (>50%) can be regrouped and the validation process recomputed, which results in lower probabilities of misclassification.

• No information on the disturbing noise power for the proper selection of a threshold is required w.r.t ED method.
  No information on the number of primary users transmitting in the band of interest is required w.r.t. MME method.
  Since no prior knowledge on the signal properties or transmission parameters, the proposed technique is fully blind.

• The extended method is able to discriminate between signal and noise components under real conditions for cognitive radios such as low SNR (0 to -20dB), under white and colored noise and regardless of the shape of the spectra.

The advantages of the proposed technique make this signal detection method very useful for spectrum sensing function of cognitive radios.
Chapter 8

Future prospects

The extended method certainly implies simplicity and shorter time execution. Nevertheless, some suggestions might improve the robustness of the method:

- The probabilistic validation reveals that some lines were wrongly classified as being signal, the validation part detect these when the misclassification probability is above 50% for those lines. This information can be used to update the $\delta_G$ and the $\delta_{SG}$ values and improve the choice of segment boundaries as well as the polynomial curve. The best choice for these values depends on the SNR, an iterative procedure is needed such that these values reflect better the SNR and reduce the falsely detected signal lines and so the misclassification probabilities.

- The probabilities of misclassification can be also improved by improving the expected value of the amplitude measurement (22) which is an approximation for the further computation of the estimate of the signal amplitude.

- For smoother discrimination curves, not only polynomial fitting regression models can be used. Some other models can be also considered:
  - Weighted regression models can be also used since they consider not only some fixed points (center of discrimination heights) but also the width of the segment, this might take full advantage of a particular discrimination height.
Orthogonal polynomials can be used for better fitting. A proper choice of the polynomial is required. One should consider the computational complexity that this model implies which contrasts with the real-time processing required by sensing techniques.

The optimal polynomial choice depends on the complexity of the chosen model which can be defined by the degree of the polynomial. Model selection criteria i.e. Akaike’s information criterion (AIC) or Cp Mallows statistic can be used to find the best model to fit the data.

Although, the extended technique works under more practical assumptions it still assumes that the disturbing noise is white-noise since it assumes that the flatness condition holds within a spectrum segment. Corrections for possible violations of this assumption i.e. colored noise are beyond the scope of the work but can be considered for future method improvement.
References


