Wave Energy of an Antenna in Matlab

Master’s Thesis in Computational Science and Engineering

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Abstract

In the modern world, because of increasing oil prices and the need to control greenhouse gas emission, a new interest in the production of electric cars is coming about.

One of the products is a charging point for electric cars, at which electric cars can be recharged by a plug in cable. Usually people are required to pay for the electricity after recharging the electric cars. Today, the payment is handled by using SMS or through the parking system. There is now an opportunity, in cooperation with AES (the company with which we are working), to equip the pole with GPRS, and this requires development and maintenance of the antenna. The project will include data analysis of the problem, measurements and calculations.

In this work, we are computing energy flow of the wave due to the location of the antenna inside the box. We need to do four steps. First, we take a set of points (determined by the computational mesh) that have the same distance from the antenna in the domain. Second, we calculate the angles between the ground and the points in the set. Third, we do an angle-energy plot, to analyse which angle can give the maximum energy. And last, we need to compare the maximum energy value of different position of the antenna.

We are going to solve the problem in Matlab, based on the Maxwell equation and the Helmholtz equation, which is not time-dependent.
Keywords

Antenna, charging point, electric cars, Matlab, Maxwell equation, Helmholtz equation, Energy Flow
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1 Thesis work in cooperation with AES

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2 Introduction

An electric car is a motor vehicle which uses electric motors, using electrical energy stored in batteries or another energy storage device. Electric cars were popular in the late 19th century and early 20th century, until vast production of cheaper gasoline vehicles reduced the use of electric car. The energy crises of the 1970s and 80s brought back an interest in electric cars, but in the mid 2000s a new interest in the production of electric cars came about, because of fast increasing oil prices and the need to control greenhouse gas emissions.

Widespread adoption of electric cars faces several obstacles and limitations. As of 2011 electric cars are more expensive than conventional internal combustion engine vehicles and hybrid electric vehicles because of the cost of their lithium-ion battery pack. The lack of public and private recharging points and the driver's fear of the batteries running out of energy before getting to their destination because of the limited range of existing electric cars are some obstacles for spreading out using of electric cars. Several governments have set up policies and economic incentives to get the better of existing barriers, to increase the sales of electric cars, and to reach further development of electric vehicles, more cost-effective battery technology and their components.

Electric vehicles have been used for years now and a number of new technical innovations and tax advantages mean they are increasing in popularity. They are powered by an electric motor (DC or AC) which draws its power from an on-board battery pack which play the role of energy store. Batteries, usually Nickel metal-hydride or Lithium-ion are charged by simply connecting the vehicle to a main power supply, and is usually charged overnight for a full charge. A regenerative braking system which allows the battery to be topped up when the brakes are used is a feature of most modern electric cars.

In new electric cars, a range of 40 to 100 miles is achievable, with top speeds from 25 to 45 mph. Because of that they are popular for city drivers, especially in cities such as London where massive savings can be made and traffic is generally low speed with shorter distances travelled. Because of being zero-emission vehicles, electric cars generally have many tax advantages besides the fact you never need to fill up with petrol. Emissions are however produced during the
generation of electricity to charge the car, but this can be better controlled by use of a renewable energy supply.

2.1 Why Electric Cars?

There are some advantages for electric cars that can be listed as:

1. Electricity is cheaper than gas, and can come from renewable resources such as solar and wind power.
2. They pollute less than gas-powered cars (especially when renewable energy sources are used to generate the electricity).
3. They are much more reliable and require less maintenance than gas-powered cars.
4. In case of using domestically-generated electricity instead of oil, there will be less cause for conflict in the Middle East.

2.2 Charging an Electric Car

Any electric car that uses batteries requires a charging system to recharge the batteries. The charging system has two aims:

• To charge batteries as fast as the batteries will allow
• To observe the batteries and avoid damaging them during the charging process

Batteries can be charged from a standard electric socket, so businesses or individuals can recharge them at their own premises and homes. However, there is also an increasing network of public and private electric car charging points. Nowadays, many public charging points are free. There are also charging networks accompanied by electric car charging schemes, where for a small subscription to the charging scheme, you get access to a network of charging points, a range of perks (charging cable, free parking in certain areas, points etc.) but you have to pay for the electricity you use. At this point, these networks only exist in London but this is going to spread. In near future electricity points will be as common as petrol stations.

2.3 Charging Point

The electric car charging points market is developing with new companies and products coming to the market almost every few months. The basic technology has become established - a secure post with one socket, accessed by an electronic tag.

Charging posts are usually high posts with a plug socket mounted near the top. They are proper for installation indoors or out in the street.
The question for the engineers will be if the technology that they use is vandal-proof, has a secure way of collecting credit card payments, and allow for low cost to establish a vast network of charging points or not!

![Charging post made by GARO](image)

*The charging post which is made by GARO*

### 2.4 How the Antenna Works

The key design challenges for a network of charging/parking meters would be security, reliability and cost. To make the charging point vandal-proof, a metal enclosure was the most obvious choice. This, however, presents the challenge of finding an antenna that can operate effectively despite the metal enclosure. Putting the antenna onto the PCB would mean that the metal enclosure would block the RF signal, whereas a conventional wire antenna, outside the enclosure, could easily be bent and damaged by vandals.

GARO engineers overcame these problems with a GSM antenna | 2J664B, operating in the AMPS 824-894 /900/1800/1900/2100/2400 MHz frequency band. Encased in rubber, this antenna is virtually vandal-proof, measures 70mm x 15mm and operates across of temperature range of -40˚ to +85˚ Centigrade, and impedance is 50 Ohms, with horizontal polarization.
This antenna is a kind of Micro strip Patch Antenna that is used to send on-board parameters to the ground under operating conditions.

In its most fundamental form, a Micro strip Patch antenna consists of a radiating patch on one side of a dielectric substrate which has a ground plane on the other side. The patch is generally made of conducting material such as copper or gold and can take any possible shape. The radiating patch and the feed lines are usually photo etched on the dielectric substrate.

This antenna is used for paying system with GPRS. In our thesis we are trying to find out the best place for the antenna inside the box, and to find out the maximum energy value of different positions of the antenna due to the angle-energy plot.

2.5 Problem Description
Our topic comes from AES Company, which has this charging point product with the antenna inside. The charging point can be divided into two parts, the head, which we can also call the box, the other part being the bottom. The material of the box is aluminium casting approximately 3mm thick. There is a small semitransparent plastic window which is located in front of the head. There are two expected results from the company:

- How is the energy flow of the antenna and in which direction from the ground can the antenna send out the maximum energy?
Wave Energy of an Antenna in Matlab

- Which is the best location for antenna inside the head/box? ‘Best’ here meaning that the antenna can send out the maximum energy from that location.

The main idea of our topic is trying to solve these two questions. We need to do a project including data analysis of the problem, measurements and calculations. And we will solve the problem in Matlab. The antenna is supposed to be set in space, which gives us infinity boundary condition except at the ground. In our project, the ground is assumed to give perfect reflection of the wave.

We need to:

- Measure the size of charging point, the head, and the window;

- Model the problem in Matlab;

- Calculate the wave energy in Matlab;

- Analyse the ‘Energy Flow and Angle plot’ of different position of antenna.

2.6 FEM and Matlab

From reference [1], we collect the information from Wikipedia, we know: The finite element method (FEM) (its practical application often known as finite element analysis (FEA)) is a numerical technique to find approximate solutions of partial differential equations (PDE). It is a systematic way of approaching the subject. When the form of the equations has been established, a more efficient implementation can be made using the matrix form as a starting point.

MATLAB (matrix laboratory) is a numerical computing environment and fourth-generation programming language. Developed by Math Works, MATLAB allows for matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, Java, and FORTRAN.
3 Theory

To solve the problem, we need to use the Maxwell’s Equations, Electromagnetic Wave Equation, Helmholtz Equation and the Wavelength Formula. In this Chapter, we will introduce all the theories we used in this thesis work.

3.1 Maxwell’s Equations

In reference [2], the group of partial differential equations that compose Maxwell’s equations can be found. Maxwell’s equations have two major variants. From the 'microscopic' view, it is total charge and total current that are used. Here ‘total charge and total current’ means they are the difficult-to-calculate atomic level charges and currents in materials. From the 'macroscopic' view, there are two new auxiliary fields that are defined in Maxwell’s equations. And we don’t need to know the 'atomic' size of charges and currents.

Here are the famous Maxwell’s Equations in differential form:

\[
\nabla \cdot \mathbf{E} = 0 \\
\n\nabla \cdot \mathbf{B} = 0 \\
\n\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\n\n\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]

(3.1-1) (3.1-2) (3.1-3) (3.1-4)

Here B is the magnetic field, E is electric field, \( \mu_0 \) is the permeability of free space, also called the magnetic constant, and \( \varepsilon_0 \) is the permittivity of free space, also called the electric constant. Gauss's law is the integral form of Eq. (3.1-1). Ampere's law is the integral form of Eq. (3.1-4) for the case where the electric field does not vary with time. The Ampere-Laplace-Biot-Savart law is derived from a combination of Eq. (3.1-2) and (3.1-4), also for the case where the electric field does not vary with time. The Faraday-Henry law of magnetic induction is the integral form of Eq. (3.1-3).

3.1.1 Gauss's Law for Magnetism

Reference [3] states that in physics, we know one of Maxwell’s equations is Gauss's law for magnetism. The set of four equations of Maxwell’s equations underlie classical electrodynamics. Gauss's law for magnetism states that the magnetic field B is a solenoidal vector field. It
equivalently states that there does not exist magnetic monopoles. Rather than "magnetic charges", the basic entity for magnetism is the magnetic dipole.

Gauss's law for magnetism in differential form is

\[ \nabla \cdot \mathbf{B} = 0 \quad (3.1-2) \]

The applied magnetic field is denoted by \( \mathbf{B} \). The applied magnetic field is related to the magnetization and the induced magnetic field by the magnetic permeability:

\[ \mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H}) = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_{\text{mag}} \mathbf{H} \quad (3.1.1-2) \]

Where \( \mathbf{H} \) is the induced magnetic field and \( \mathbf{M} \) is the resulting magnetization of the material. The magnetic permeability of free space \( \mu_0 \) is \( 4\pi \times 10^{-7} \) H/m. The unit of inductance \( \mathbf{H} \) is termed a Henry and is equal to 1 Vs/A. The magnetic permabilities [Vs/A] of other materials are given by \( \mu_{\text{mag}} = \mu_0 (1 + \chi_m) \), where \( \chi_m \) is the magnetic susceptibility of the material. Gauss's law for magnetism takes the same form as Gauss's law for electricity, but the right-hand side is zero because magnetic monopoles are absent.

### 3.1.2 Faraday's Law of Induction

Reference [4] states that Faraday's law of induction is a basic law of electromagnetism. It is related to the operating principles of transformers, inductors, and many types of electrical motors and generators. It says that in any closed circuit, the induced electromotive force (EMF) is equal to the time rate of change of the magnetic flux through the circuit.

The integral form of the Faraday's law of induction can be expressed by:

\[ \mathcal{E}^{\text{ind}} = -\frac{\partial}{\partial t} \int_{\mathbf{S}} \mathbf{B} \, d\mathbf{S} = \oint_{\mathbf{L}} \mathbf{E} \, dl \quad (3.1.2-1) \]

\( \mathcal{E}^{\text{ind}} \) is the induced electromotive force. Since \( \mathcal{E}^{\text{ind}} \) is measured in volts, it is also called the induced voltage. If the closed path \( \mathbf{L} \) is a metallic loop, an electric current is produced in the loop by an induced electromotive force. If the flux through a circuit in a certain direction is increasing, the induced current sets up a flux in the opposite direction, and if the flux is decreasing, this current sets up a flux in the same direction. The induced currents always seek to maintain the initial status of the magnetic field.

By applying Stokes' theorem to the line integral on the right-hand side of Eq. (3.1.2-1), we may derive the local differential form of Faraday's law.

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3.1-3) \]
Eq. (3.1.2-1) states that the line integral of a static electric field \( E \) about any closed path is always zero. This means that all static electric fields are conservative.

### 3.1.3 Ampère's Circuital Law

From reference [5], we know, using Ampere's law, how to determine the magnetic field associated with a given current or current associated with a given magnetic field, provided that there is no time changing electric field present. In its historically original form, Ampère's Circuital Law relates the magnetic field to its electric current source. The law can be written in two forms, the ‘integral form’ and the ‘differential form’.

Ampère's Circuital Law in integral form:

\[
\oint_C B \cdot dl = \mu_0 I_{enc} \tag{3.1.3-1}
\]

\[
\oint_C H \cdot dl = I_{f,enc} \tag{3.1.3-2}
\]

Where \( \oint_C \) is the closed line integral around the closed curve \( C \); \( B \) is the magnetic B-field in Teslas; \( H \) is the magnetic H-field in ampere per metre; \( \cdot \) is the vector dot product; \( dl \) is an infinitesimal element (a differential) of the curve \( C \).

Eq. (3.1.3-1) and Eq. (3.1.3-2) can also be written in differential form:

\[
\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \tag{3.1-4}
\]

### 3.2 Electromagnetic Wave Equation

The electromagnetic wave equation is derived from Maxwell's equations. The electromagnetic wave equation is a second-order partial differential equation that describes the propagation of electromagnetic waves through a medium or in a vacuum.

It can be written as below:

\[
(\nabla^2 - \varepsilon \mu \frac{\partial^2}{\partial t^2}) E = 0 \tag{3.2-1}
\]

\[
(\nabla^2 - \varepsilon \mu \frac{\partial^2}{\partial t^2}) B = 0 \tag{3.2-2}
\]

Where \( \varepsilon = \frac{1}{\sqrt{\mu \varepsilon}} \) is the speed of light in the medium, and \( \nabla^2 \) is the Laplace operator, \( B \) is called the magnetic flux density or magnetic induction.

The general solution to the electromagnetic wave equation is a linear superposition of waves of the form:

\[
E(r, t) = g(\phi(r, t)) = g(\omega t - k \cdot r) \tag{3.2-3}
\]
\[ B(r, t) = g(\phi(r, t)) = g(\omega t - k \cdot r) \quad (3.2-4) \]

Eq. (3.2-3) and Eq. (3.2-4) are for virtually any well-behaved function \( g \) of dimensionless argument \( \Phi \), where \( \omega \) is the angular frequency (in radians per second), and \( k = (k_x, k_y, k_z) \) is the wave vector (in radians per meter). We rewrite those two equations, and then we get:
\[
E(r, t) = \text{Re}\{E(r)e^{i\omega t}\} \quad (3.2-5)
\]

Where \( i \) is the imaginary unit, \( \omega \) is the angular frequency in radians per second, \( f \) is the frequency in hertz, and \( e^{i\omega t} = \cos(\omega t) + i \sin(\omega t) \) is Euler’s formula.

### 3.3 Helmholtz Equation

The Helmholtz equation often arises in the study of physical problems involving partial differential equations (PDEs) in both space and time. The Helmholtz equation, which represents the time-independent form of the original equation, results from applying the technique of separation of variables to reduce the complexity of the analysis. It can be written as:
\[
\nabla^2 A + k^2 A = 0 \quad (3.3-1)
\]

Where \( \nabla^2 \) is the Laplacian, \( k \) is the wave number, and \( A \) is the amplitude.

### 3.4 Derivation of Maxwell’s Equations and Electromagnetic Wave Equation

From Maxwell’s equations, Eq. (3.1-1), Eq. (3.1-2), Eq. (3.1-3), and Eq. (3.1-4), and Electromagnetic wave equation, (3.2-5), we can derive them as below:

From Eq. (3.1-1) and Eq. (3.1-2), we get:
\[
\nabla \times \left( \frac{\partial B}{\partial t} + \nabla \times E = 0 \right) \quad (3.4-1)
\]

From Eq. (3.1-4), we get:
\[
\frac{\partial}{\partial t} \left( \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} - \nabla \times B = -\mu_0 I \right) \quad (3.4-2)
\]

Above Eq. (3.4-1) and Eq. (3.4-2) can be rewritten as below:
\[
\frac{\partial \nabla \times B}{\partial t} + \nabla \times \nabla \times E = 0 \quad (3.4-3)
\]
\[
\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} - \frac{\partial}{\partial t} \nabla \times B = -\mu_0 \frac{\partial I}{\partial t} \quad (3.4-4)
\]
\[
\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} + \nabla \times \nabla \times E = -\mu_0 \frac{\partial I}{\partial t} \quad (3.4-5)
\]

In our work, we only consider the problem in 2D, that gives us \( E = [0,0,E_z] \), which can gives us the following equation:

---

10
\[ \nabla \times E = \left( \frac{\partial E_z}{\partial y}, -\frac{\partial E_z}{\partial x}, 0 \right) \]
\[ \nabla \times \nabla \times E = \left( 0, 0, -\frac{\partial^2 E_z}{\partial x^2} - \frac{\partial^2 E_z}{\partial y^2} \right) \]
\[ = (0, 0, -\nabla^2 E_z) \quad (3.4-6) \]

Finally, we get:
\[ \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2} - \nabla^2 E_z = -\mu_0 \frac{\partial I_z}{\partial t} \quad (3.4-7) \]

Furthermore, in our work we supposed the problem is not time-dependent. We know that \( E_z = e^{i\omega t} \hat{E}_z \) and \( I_z = \alpha e^{i\omega t} \). By using this, we can rewrite the Eq. (3.4-7) as below:
\[ -\mu_0 \varepsilon_0 \omega^2 \hat{E}_z - \nabla^2 \hat{E}_z = -\mu_0 \alpha i \omega \]

Remove the \(-\mu_0\) in both two sides, we get:
\[ \varepsilon_0 \omega^2 \hat{E}_z + \frac{\nabla^2 \hat{E}_z}{\mu_0} = \alpha i \omega \quad (3.4-8) \]

When the time dependence part is separated out, the space part of both wave equation and heat equation is given by the Helmholtz equation. Thus Eq. (3.4-8) can be seen as the form of Helmholtz equation, Eq. (3.3-1), which we mentioned before.

### 3.5 Wavelength Formula

In physics, the wavelength of a sinusoidal wave is the spatial period of the wave – the distance over which the wave's shape repeats. The wavelength \( \lambda \) of a sinusoidal waveform travelling at constant speed \( v \) is given by:
\[ \lambda = \frac{v}{f} \quad (3.5-1) \]

Where \( v \) is called the phase speed (magnitude of the phase velocity) of the wave and \( f \) is the wave's frequency.

### 3.6 Boundary Conditions

Our boundary conditions can be viewed as a version of the Sommerfeld radiation condition. \( du/dn = (+-) \omega \hat{u} \). By integration by parts in the Laplace operator, we get out the normal derivative and replace it by the factor \( \omega \hat{u} \), which when integrated gives an imaginary mass matrix on the boundary. This only works perfectly on a circle (sphere) when the wave hits the circle (sphere) as a plane wave (i.e., not at an angle), but it still gives a low order approximation of a non-reflecting surface [6].
4 Matlab Modelling

After analyzing the problem and knowing which formula to use, now we need to model the problem in Matlab. What we do in this Chapter is following the list in Chapter 2, problem description.

4.1 Measurement of the Problem

We need to measure the height of charging point, the head, and the antenna.

Since we only solve the problem in 2D, we can measure the data from the x-y plane, front view.

4.1.1 Charging Point

The height from ground to the top of charge is 1.2m.

4.1.2 Head of Charging Point

The height from bottom to the top is 0.272m.
Fig. 4.1.2-1 Measurement of charging point-head height

The width from right to left is 0.238.8m (front view).

Fig. 4.1.2-2 Measurement of charging point-head width
We also know that the height of the plastic window is 0.03m and the thickness of the box is 0.003m.

4.1.3 Frequency of Antenna

From the company’s data, we know the frequency of antenna is 2.4GHz. From Chapter 3.5, we know the wavelength formula is:

$$\lambda = \frac{v}{f}$$  \hspace{1cm} (3.5-1)

So we can calculate the wavelength:

$$\lambda = \frac{3 \times 10^8 \text{m/s}}{2.4 \times 10^9 \text{Hz}} = 0.125 \text{m}$$ \hspace{1cm} (4.1.3-1)

That means in our project frequency is 24Hz and the wavelength should be 0.125m.

4.1.4 Distance/Curve from the Point Source

In order to do the analysis, we need to take out all the points in domain that have the same distance from the point source (also the location of antenna). And we use the symbol ‘r’ for this distance. This means that we can draw a circle centred in the point source with radius equal to ‘r’. We won’t be strict with this ‘r’. It can be any certain length inside the domain. But the best way to choose this distance is that, when we draw a circle, it should has two intersectional points with the top and the bottom boundaries of domain. Then, we draw two lines from point source to those two intersectional points. Meanwhile we get an angle between these two lines. We assumed this angle should be less than 180°.
So finally, we decided this ‘r’ should be 1.4m.

4.2 Choosing Domain and Units

In Matlab code, we choose a domain which is $2 \times 2$ units large and the unit is meter. In order to have same unit we use meter in this thesis. And correspondingly, we use unit Hz for frequency. To show all these together, we got a sketch as below:

![Sketch of problem](image)

Now we can model the problem in Matlab according to this sketch.

4.3 Modelling

After measurement, we can model the problem in Matlab. We used the $2 \times 2$ domain stands for the space. Since the main idea of FEM is to create mesh of the objects, we can’t get the quite precise data as the entity. So we have to take the approximate values to assemble the problem.

4.3.1 Assemble the Box/Head

To assemble the box (head of charging point), we take three lines represent the top, the right and the bottom of the box, they are: $y = 1.4688, x = 2, y = 1.2031$. And for the left of the box, we use two vertical lines with a gap around 0.03m between those two lines. The gap stands for the plastic window. And the x-axis coordinate of those two lines are the same, they are
\( x = 1.765 \), and \( y \)-axis coordinate is from \( y = 1.2031 \) to \( y = 1.2188 \) stands for the lower left boundary and from \( y = 1.2500 \) to \( y = 1.4688 \) stands for the upper left boundary. The distance between 1.2500 and 1.2188 is approximately 0.0312, which stands for the height of plastic window.

Since the thickness of the box is only 3mm, which is too small to model in this Matlab domain, so we will neglect this distance. Also we will not consider the material properties of the object.

### 4.3.2 Assemble the Antenna

To assemble the antenna, we just take a point inside the box. And in Matlab, the way to do that is by multiplying frequency with an imaginary number. In our case, we do it like: \( j \times \omega \) where \( j \) is an imaginary number and \( \omega \) stands for the frequency valued by 24Hz.

The location of antenna should be inside the box, and we should compare the results of the different positions, thus we choose three vertical positions and three horizontal positions where antenna can be located. The vertical ones are \((1.7813,1.3906),(1.7813,1.3281),(1.7813,1.2344)\) and the horizontal ones are \((1.7813,1.3906),(1.8594,1.3906),(1.9357,1.3906)\).

The point \((1.7813,1.3906)\) is the same point. So in total we choose five points.

### 4.3.3 Assemble the Curve

To assemble the curve, we take the location of antenna as the point source, which is also the centre of the circle. Then we use the \( r = 1.4 \)m to draw the circle. When the curve line reach the intersection points with the top and bottom boundaries of domain, then we will stop drawing. Therefore, we will have a curve. All the points along this curve have the same distance to the antenna. In our code, we use `elementlist.m` function file to calculate all those points along the curve and then, we plot the result of the `elementlist.m`.

### 4.3.4 Assemble the Angle

To assemble the angle from the ground and the points along the curve, we choose the point on the ground, which is also vertical to the position of antenna, as the vertex of the angle. In our case, the coordinates of this point is \((1.7813,0)\). Then we will assemble the angle between the ground and curve. We solve it in the `Angle.m` file in Matlab.
4.4 Boundary Conditions

In this Chapter, we will talk about the boundary conditions. In our problem, the mainly boundary conditions come from two areas. One is the boundary for the space, and the other is for the head of charging point, which we also call the box.

4.4.1 Space Boundary

Our charging point is in space, this gives us the infinite boundaries except the ground. And thus, when the antenna sends out the wave, the only reflection comes from the ground. We need to consider this in our project and express it in the Matlab code.

4.4.2 Head /Box Boundary

The charging point itself also gives us four boundary conditions. The antenna is located in the head of charging point, and we assumed that the wave only goes out from the left side of the box. That means the top, the bottom, and the right boundary will not allow the wave to go through and we have reflection there.

For the left boundary of the box, it is only the plastic window that can send out the wave, and the length of this window along the y-axis is assumed to be 0.03m. There is no reflection on the left either.

4.5 Expected Output

We expect Matlab can give us four plots as output. The first is picture of the antenna and the wave, the second is the circle with ‘r’ radius centred in the point source, the third is the energy flow plot of the circle and the fourth is the angle-energy plot, in which we can see the relationship between the angle and the energy flow.

All those plots we mentioned above are from the same location of the antenna. What we need to do next is to change the different positions of antenna and compare each set of plots, to analyse which position is the best for the antenna inside the box.
5 Result

Our results include plot pictures and data from Matlab, and analysis by ourselves.

5.1 Overview of the Output

Here are the overview pictures of the output. The antenna is located at $[1.7813, 1.39006]$, and we looked at the four plot pictures and the data from one calculation.

Four Plot Pictures

![Wave Plot](Fig.5.1.1 Wave Plot)
Fig.5.1-2 Curve Plot

Fig.5.1-3 Energy Flow Plot
Fig. 5.1-4 Angle-Energy Flow Plot

Data of Calculation

\[
\begin{align*}
  v &= 342.0011 \\
  \text{MaxEnergy} &= 2.9770 \\
  \text{IND} &= 112 \\
  \text{MaxAngle} &= 45.6211
\end{align*}
\]

Fig. 5.1-5 MaxEnergy and MaxAngle
5.2 Analysis of Wave Energy at One Position

5.2.1 Analysis of Wave Plot

5.2.1.1 Boundary Reflection of Wave

From Fig.5.1-1, we can see that the location of the antenna has the red colour and the wave comes from left of the box and goes towards the space. The waveform is affected by reflection from the ground.

5.2.1.2 Wavelength

By using the formula of Chapter 3.5, we can calculate that wavelength is equal to 0.125m. And left of domain where the wave flows towards space is 1.765m. Then we get \( \frac{1.765}{0.125} \approx 14 \), which means that we should have approximately 14 waves in domain. In Fig.5.1-1, the blue colour and the cyan-blue colour indicate the waves.

5.2.2 Analysis of Energy Flow Plot

From Fig.5.1-3, we can see the maximum energy is almost 3J. This energy flow plot is supposed to show how the energy flows out when the particles go through the curve. The energy distributes curvy.

5.2.3 Analysis of Angle- Energy Flow Plot

From Fig.5.1-4 and Fig.5.1-5, we can see clearly the relationship between the energy and the angle. This figure represent that along the curve, the angle 46.6° from the ground towards to space, has the maximum energy, which is 2.977J.

5.3 Analysing Different Locations of Antenna

To compare the different results of antenna, we need to choose different locations, which we also call point sources. As mentioned before in Chapter 4.3.2, we will take six points inside the box as point sources, which three of them are vertical and others are horizontal. First, we will compare the maximum energy in vertical pointes, and second, we will take the y-axis coordinate of the maximum one and compare the energy in horizontal pointes. In fact, two of those six points will be the same, so we have actually five points in total.

5.3.1 Compare Vertically

For these Four points \((1.7813,1.3906),(1.7813,1.3281),(1.7813,1.2344)\), we find the nodes index for those three points in Matlab, they are 11787, 11675 and 11631, then we just change the point source to those three index and we get the results as below:
Wave Energy of an Antenna in Matlab

<table>
<thead>
<tr>
<th>Point Source</th>
<th>Coordinate</th>
<th>Maximum Energy</th>
<th>Corresponding Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>11787</td>
<td>(1.7813, 1.3906)</td>
<td>2.9770J</td>
<td>46.6211°</td>
</tr>
<tr>
<td>11675</td>
<td>(1.7813, 1.3281)</td>
<td>1.9859J</td>
<td>62.1027°</td>
</tr>
<tr>
<td>11631</td>
<td>(1.7813, 1.2344)</td>
<td>1.4510J</td>
<td>82.8750°</td>
</tr>
</tbody>
</table>

Table.5.3.1-1 Data of Comparison Vertically

Here we can see the maximum energy among those three is 2.977, then next, we take the y-axis coordinate of this point, which is 1.3906 and to compare horizontally.

We also compare the other point in this vertical line, the final result shows that the point 11787 has the maximum energy in this line $x = 1.7813$.

<table>
<thead>
<tr>
<th>Point Source</th>
<th>Coordinate</th>
<th>Maximum Energy</th>
<th>Corresponding Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>11787</td>
<td>(1.7813, 1.3906)</td>
<td>2.9770J</td>
<td>46.6211°</td>
</tr>
<tr>
<td>3010</td>
<td>(1.8594, 1.3906)</td>
<td>0.8533J</td>
<td>85.6013°</td>
</tr>
<tr>
<td>11796</td>
<td>(1.7813, 1.4375)</td>
<td>2.1606J</td>
<td>45.00°</td>
</tr>
<tr>
<td>3012</td>
<td>(1.7813, 1.4531)</td>
<td>1.2152J</td>
<td>45.00°</td>
</tr>
</tbody>
</table>

Table.5.3.1-2 Data of Comparison Vertically-Other Points

5.3.2 Compare Horizontally

For points (1.7813,1.3906), (1.8594,1.3906), (1.9357,1.3906), we find the nodes index are 11787, 11803 and 11739, then we just change the point source to those three index and we get the results as below:

<table>
<thead>
<tr>
<th>Point Source</th>
<th>Coordinate</th>
<th>Maximum Energy</th>
<th>Corresponding Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>11787</td>
<td>(1.7813, 1.3906)</td>
<td>2.9770J</td>
<td>46.6211°</td>
</tr>
<tr>
<td>11803</td>
<td>(1.8594, 1.3906)</td>
<td>0.8533J</td>
<td>85.6013°</td>
</tr>
<tr>
<td>11739</td>
<td>(1.9357, 1.3906)</td>
<td>1.0929J</td>
<td>76.9708°</td>
</tr>
</tbody>
</table>

Table.5.3.2-1 Data of Comparison Horizontally

Here we can see, if we compare horizontally, maximum energy still comes out from point source 11787, maximum energy is 2.977J and corresponding angle is 46.6211°.


6 Conclusion

In this thesis work, we measured the charging point and the antenna; calculated the wave, the energy flow and the corresponding angle. Finally we did the simulation from all the data and plot pictures we got. And we came to the conclusion that inside the head of charging point, if the antenna is located at the point \((1.7813, 1.3906)\), we have the maximum energy. The corresponding angle from the ground towards the space is approximately \(46.6^\circ\). assuming the radius 1.4\(m\) and the centre point the same as the point source, we drew a curve in domain, and then we got the maximum energy value \(2.977\text{J}\).

We solved the problem finally in Matlab and we gave two solutions to AES, and we hope this thesis work will aid the company when they are going to make the decision about the location of the antenna.
7 References

8 Appendix

Code of Wave Energy of Antenna in Matlab

Main code in ‘mainbil.m’

clc
clear all
close all

 tic
xnod=[0 2 2 0]';ynod=[0,0,2 2]';
nodes=[1 2 3 4];
frequency=24;
mu0=1;eps0=frequency;
icase=1;

 for i=1:7
 [nodes,xnod,ynod]=refine(nodes,xnod,ynod);
 end
nei = neighbours(nodes);

 u=solve(frequency,nodes,nei,xnod,ynod);
u=imag(u);
figure(1),clf,w=patch(xnod(nodes'),ynod(nodes'),u(nodes'),u(nodes'));axis normal

 xmid=mean(xnod(nodes),2);
ymid=mean(ynod(nodes),2);

 R=1.4;
ellist=elementlist(R,xnod,ynod,nodes);
Angle=angle(R,eellist,xnod,ynod);

 figure(2),clf,w=patch(xnod(nodes'),ynod(nodes'),'w'),hold on
 title('R=1.4 Cycle')
patch(xnod(nodes(eellist,:)',ynod(nodes(eellist,:)','r')
vec=zeros(size(eellist));
val=zeros(size(eellist));

 for ii=1:length(eellist)
 iel=eellist(ii);
 iv=nodes{iel,:}';xc=xnod(iv);yc=ynod(iv);
 [fi,fix,fiy,detj]=basefn(0,0,xc,yc);
bmat=[fix;fiy];
E=[0,0,dot(fi,u(iv))];
curlE=[dot(fiy,u(iv)),dot(fix,u(iv)),0];
S=(-1/(2*mu0))*cross(E,conj(curlE/(j*frequency)));
vec(ii)=ymid(iel);
val(ii)=norm(S);
end
 [vec,ind]=sort(vec);
 [ang,ind2]=sort(Angle);
val=val(ind);
[MaxEnergy,IND]=max(val)
MaxAngle=ang(IND)
figure(3),clf,plot(vec,val,'-o')
title('energy plot')
figure(4),clf,plot(ang,val,'-o')
title('angle-energy plot')
Mainly solve solution in function ‘solve.m’

```matlab
function u=solve(omega,nodes,nei,xnod,ynod)
global pointnr
[nele,s]=size(nodes);
nno=length(xnod);
f=spalloc(nno,1,nno);

```
dx=diff(x);
for i=1:(length(in)-1)
  mele=[dx(i)/3, dx(i)/6;dx(i)/6, dx(i)/3];
  iv=[in(i),in(i+1)];
  M(iv,iv)=M(iv,iv)+j*omega*mele;
end
in=find(xnod==max(xnod));
[y,ind]=sort(ynod(in));in=in(ind);
dx=diff(y);
for i=1:(length(in)-1)
  mele=[dx(i)/3, dx(i)/6;dx(i)/6, dx(i)/3];
  iv=[in(i),in(i+1)];
  M(iv,iv)=M(iv,iv)+j*omega*mele;
end
in=find(xnod==min(xnod));
[y,ind]=sort(ynod(in));in=in(ind);
dx=diff(y);
for i=1:(length(in)-1)
  mele=[dx(i)/3, dx(i)/6;dx(i)/6, dx(i)/3];
  iv=[in(i),in(i+1)];
  M(iv,iv)=M(iv,iv)+j*omega*mele;
end

Mainly curve drawing solution in function ‘elementlist.m’

function ellist=elementlist(R,xnod,ynod,nodes)
global pointnr
xp=xnod(pointnr);yp=ynod(pointnr);
xx=xnod(nodes)-xp;yy=ynod(nodes)-yp;
size(xx)
size(radius)
rmin=min(radius,[],2);
rmax=max(radius,[],2);size(rmax)
Mainly angle calculate solution in function ‘angle.m’

```matlab
function Angle=angle(R,ellist,xnod,ynod)
    ff=length(ellist);
    AngVal=zeros(ff,1);
    Angle=zeros(ff,1);
    xp=xnod(2143);
    for g=1:ff
        xang=xnod(ellist(g,:));
        yang=ynod(ellist(g,:));
        AngVal(g,:)=atan(abs(yang./(xp-xang)));
        Angle(g,:)=AngVal(g,:)*(360/(2*pi));
    end
end
```