Product Costing for Sawmill Business Management
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Abstract


Several Swedish sawmill groups have recently developed product-costing systems to possibly compensate for diminishing production knowledge in recently centralized market organizations. The concept of product costs is challenging in sawmilling, since production is of a joint type – each log typically yields many products. The newly developed costing systems rely on traditional accounting-based methods and are of little use in decision-making because the resulting cost figures do not normally estimate actual cost changes.

This thesis develops an alternative, theoretically defendable method based on linear programming, and tests it on a pine sawmill. Computer simulations are compared with traditional methods, and to analyze the effects of managing the salesmen by the product costs of the suggested method. The thesis relies on the joint-cost accounting discourse from the 1980s, which was abandoned before any essential application was found. This application has now been found through changes in the sawmill industry, and the discourse is here revived practically and theoretically.

The sawmills are modeled with relative capacity restrictions and with constraints on the flexibility of their timber supply. Sales decisions based on product costs from the suggested method seem to successively improve company profit. To be successful, the product costs have to be recalculated regularly. Analyses indicate that with flexibility in purchasing timber and a low cost difference between buying scarce products and selling surplus products externally, the necessary length of the recalculation period and the usefulness of the suggested method both increase markedly.

Keywords: joint cost, lumber costing, linear programming, sawmilling, cost allocation, sales management, opportunity cost.
Appended Papers

This thesis is based on results presented in the following papers:

**Paper #1:**

**Paper #2:**

**Paper #3:**

**Paper #4:**
Preface

This thesis is the result of work carried out at the Department of Technology and Design at Växjö University. Several people and organizations have helped me. First, my work was financed and made possible by the project “Wood Design and Technology” and by my employer, Sveaskog. I am deeply grateful for this support.

Second, I would like to thank my supervisor, Professor Kaj Rosling, who has guided me wonderfully through the entire process. Our discussions have sometimes been confusing, but always interesting and helpful in driving my work ahead. I am also grateful for his co-operation in paper #1 where he clarified the mathematical theories, while I was responsible for the rest of the paper.

I would also like to thank those from the former AssiDomän and its successor sawmill company, The Setra Group, for all their help and advice. Their support and positive attitude have been invaluable to me.

Last, but not least, I would like to send a grateful thought to my family, for their support and patience with all my personal shortcomings during this writing.
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1. Introduction

According to recent discussions in the branch, several sawmills have changed from acting as relatively small solitary producers of standardized commodities, sold further by middlemen, to being parts of large sawmill groups that produce value-added products sold by the companies’ marketing units directly to industrial users and retailers. The new structure, possibly caused by changes in the lumber distribution system, creates new possibilities and new challenges for sawmills.

Conversations with practitioners indicate that typical marketing organizations are considerably larger and the salesmen in these new sawmill groups are more specialized in their markets compared to more traditionally organized companies. In fact, the salesmen are now often separated from the sawmills and thus, their knowledge of production diminishes. Presumably to improve coordination of sales and production, several Swedish sawmill companies have recently developed product-costing systems.

The product cost of lumber is a complicated concept due to the joint character of production, where each log typically yields many products. Several authors have analyzed joint cost allocation where products are made in fixed proportions. Most conclude that several popular cost allocation methods are arbitrary and the resulting costs may typically be of little use for decision-making.

In contrast to true joint production, an output mix that may be changed by choice of sawing patterns characterizes the production of lumber. This case is sometimes referred to as joint production in variable proportions, which has been analyzed very little in the literature.

Joint cost allocation was active discussed within academia from the 1960s to the beginning of 1990s, with several articles in major accounting journals. However, more recent articles dealing with the question are rare, possibly because its practical usefulness in true joint production was small, and joint production in variable proportion may be empirically unusual.

The sawmill industry seems to be an interesting exception – an industry that produces their output jointly, but in variable proportions, as well as needing product costs to manage their new organizations. However, further research is necessary to answer how the costs should be used to manage the sawmill
most effectively. Hopefully, some important insights are presented in this dissertation.

The two main objectives of this thesis are to develop a theoretically defendable method for allocation of timber and sawing costs to products that can be used by a sawmill group with several production units, and to discuss the feasibility of managing the sales force by product costs in market-oriented organizations, where informed choices of offerings to customers can be made based on such costs as well as opportunity information.

Chapter 2 describes some general conditions and recent changes in the Swedish sawmill industry. A theoretical background is presented in chapter 3 and the studied mill is presented in chapter 4. The suggested allocation method for sawmills is described in chapter 5 and a discussion of how the costs may be used is presented in chapter 6. The scientific contribution of this thesis is discussed in chapter 7, and further research questions are outlined in chapter 8. Chapter 9 summarizes the appended papers.
2. The Swedish sawmill industry

2.1. Recent changes

The Swedish sawmill industry has changed rapidly in recent years. However, many of these changes have not yet been fully captured by scientific literature; hence, the changes presented in this section are largely based on empirical information and general discussions within the branch.

Several customers of lumber have grown and increased their requirements, both on physical products and logistics (Falk 2004; Nord 2005). Typically, the products have to be further processed after the sawing process, which is increasingly done by the sawmills. The Sawmilling Inventory from the year 2000 (Staland, Navrén et al. 2002) shows that value adding operations at the sawmills, such as cutting into exact lengths, strength grading, and making parts for building, furniture and joinery industries, increased markedly between 1990 and 2000.

Also, improved logistical solutions may be important for several customers, e.g. deliveries with time constraints, with small packages and bar coded products (Gustavsson 2003). Not only have changes encouraged closer cooperation between the sawmill and its customers, but also possibly the growth of companies, since a larger company may be better prepared to work with innovations (Hovgaard and Hansen 2004).

The use of middlemen has decreased in recent years, particularly for large sawmill companies that have built up sales units in their main markets, as indicated by Norberg (2000). The change to more direct sales facilitates the exchange of information between user and producer, an advantage when the sawmill makes increasingly customized products (Lönner 1985).

Furthermore, for years the number of sawmills has decreased, whereas the capacities of the remaining mills have increased. About 160 sawmills closed down in Sweden from 1990 to 2000, corresponding to nearly 34% of all mills with an annual output above 1000 m³ (Staland, Navrén et al. 2002). The increased need for product development and marketing may partly explain the fast change (Falk 2004), along with the trend of improved productivity in large production units (Staland, Navrén et al. 2002).
2.2. Organizational changes

Presumably to meet changes in the market and utilize the advantage of becoming a larger company, several sawmill companies have changed their marketing organizations from relatively small ones, where independent middlemen did much of the sales work, to large marketing units with several salesmen. This change would be difficult to realize in small companies, because a specialized salesman needs a relatively large volume to be cost-effective.

The new marketing organizations, with salesmen responsible for a single geographical market and numerous products from several mills in the group, should offer several advantages compared to the traditional set-up, where each sawmill has its own marketing people who sell to several markets through mainly middlemen. This is further discussed in section 3.1.3.

However, the need to coordinate sales efforts and production will probably increase with a centralized marketing organization, since the salesmen work with products from several sawmills, making it very difficult for them to estimate the costs of the products, which of course differ between the mills. The dialogue between salesmen and production managers, which is presumably used to lead to good production and sales planning, becomes more difficult because there are more people involved and the salesmen are not located at the sawmills in these new organizational set-ups.

By supporting the market organization with costs that facilitate profitable measurements and thus support the salesmen in directing their sales efforts effectively, the product costs from recently developed costing systems are most likely intended to mitigate the effects of the salesmen’s diminishing knowledge about production costs.
**Figure 1.** The use of product cost. The sawmill communicates the cost of production through calculated product costs and the marketing organization estimates sales changes and returns the result or prognosis to the production units.

Figure 1 illustrates how product costs and sales plans are interchanged between the sawmills and marketing unit during the budgeting process. The cause-effect model is a simplification of a more sophisticated real planning process, including an overall business planning process and separate production and marketing planning processes (c.f. Oskarsson, Aronsson et al. (2006)).

The main theoretical costing problem in lumber production arises from the saw, where several products simultaneously emanate from each single log. Typically, the process steps are scaling, sorting into saw classes, sawing, sorting, drying and, finally, a further sorting/trimming step before the products are packaged and distributed.
In Sweden, an independent timber measurement association measures and scales the logs, since this usually forms the basis of the payment to the timber supplier. The logs are sorted into saw classes based on log diameter, length, and sometimes also quality. This typically increases the speed of the sawing process and facilitates production planning because the classes are relatively homogeneous.

In the next step, the logs are sawed according to a sawing pattern, each yielding a specific mix of products with different dimensions and qualities, referred to as battens, boards, studs and spars depending on the dimension and sawing method (Juslin and Hansen 2002). The boards are typically length trimmed and sorted into dimensions and quality classes after the saw. Besides boards, the sawing process also yields chips that are normally sold to the pulp industry and sawdust that is sold to the board industry.

Since moisture content influences the strength, form stability and the risk of fungi growth, the lumber is typically dried after the sawing process. Lumber for construction purposes is generally dried to a moisture content of about 18%, whereas other uses such as furniture and floor making require a lower moisture content, typically 8-10%. The time of the drying process depends
on wood species, dimension and moisture of the sawed pieces, and normally varies from a couple days to a week when kilns are used.

The dried lumber is then trimmed and sorted again, either according to a standardized grading system, for example, as described in “Nordic Wood” (Borg, Föreningen Svenska Sågverksmän et al. 1999) or according to rules specified by the customer, and then packed for shipment. In many cases the lumber is further processed at the mill, e.g. planed, cut into an exact length, or finger joined. These operations are usually carried out after the grading stage.

The prevalent sawing method in Nordic countries is square sawing (Grönlund 1992), where a primary saw breaks down the log into a thick block and side boards, and a secondary saw turns the block 90 degrees and divides it into boards.

![Figure 3](image.png)

**Figure 3.** A schematic view of a sawing pattern. The number of boards may vary in different sawing patterns.

The sawmill process can be described as a joint process with variable output, because the amount of different sawing patterns may be varied to change the mix of products in the output. Normally, it is possible to also use several sawing patterns for each saw class, but the actual variability will increase if the timber procurement is also flexible, i.e. the sawmill may change the amount of different log types available to the mill. In fact, this
variability may influence the product costs significantly in the sawmill case (c.f. the appended papers #2 and #3).

2.3. Production planning

From discussions within the branch, one can conclude that the product cost has not traditionally been a critical concept in the sawmilling industry and that sales decisions have largely been based on previous experiences. Alternative sales opportunities may have been analyzed by calculating total mill profitability or estimating volume yields of different sawing patterns. These analyses might have been carried out partially as described by Persson and Norbäck (1990) or through some commercial saw optimization program as a total analysis. Prasal, Karlsson et al. (1991), who analyze the profitability of side boards, present a specific decision situation.

Typically, production has been managed through a combination of sales orders and forecasts about future orders (Brodin, Lundberg et al. 1995). The diverging flow in the sawing process, the heterogeneity in the raw material, fast changes in the markets of lumber and a great number of products contribute to making management complicated (Brodin, Lundberg et al. 1995; Liljeblad, Lycken et al. 1999). The complexity of the planning situation possibly increases further when the sawmill grows or is incorporated into a group with several mills.

In several sawmills, data about the production process, e.g. saw yields, have been unreliable and have contributed to making production planning uncertain and difficult, as indicated by Brodin et. al. (1995). The lack of basic production data has possibly also obstructed systematic performance analyses in many companies.

A complex management situation has possibly been the reason for several mills to adopt rather careful strategies that aim to sell products in proportions normally used to be produced in the sawmill (Brodin, Lundberg et al. 1995).

2.4. Production costs

The cost of timber represents most of the price of sold lumber, normally between 65-70% and practitioners therefore usually assume that a high volume yield in the saw corresponds to good profitability (Alkbring 2003). Other major costs are typically the sawing cost, about 33% of the total
manufacturing costs (excluding timber costs), trimming (17%), drying (16%) and timber handling (about 13%) (Bergkvist, Karlsson et al. 1988).

Assuming a proportional expansion of production, a majority of the product cost is variable also in the short-run. In the analyzed mill, about 80% of the total cost is variable for one year. The cost allocation discussed in this study mainly concerns the costs of timber, log handling and sawing, which summarize the joint costs of sawing.

2.5. Timber procurement

The ability to manage the mix of logs arriving at the sawmill is a critical prerequisite in varying the use of different sawing patterns in production, which thus strongly influences the variability of output. To obtain the demanded log types, important aspects are how the bucking operations in the forest are managed and what alternative purchasing methods are available to the mill.

Harvesters, equipped with computers that optimize the bucking operation, log most softwood timber in Sweden. The optimizing algorithms use price lists and measurements of the stem to maximize the profit of the bucking operation. Each sawmill typically makes their own timber price list that is intended to give a mix of logs with a desired diameter, length and quality. (Uusitalo and Kivinen 2001).

The majority of the timber is usually bought by contracts that compensate the seller according to the sawmill’s official log price list, where diameter, length, quality and wood species determine the price of each log. The sawmill, a company associated to the sawmill, or the forest owner may carry out the felling operations. One important drawback with this method is that the sawmill has a limited ability to decide on which type of stands the price list will be applied, considered an important determinant in the log distribution of the supply (Uusitalo and Kivinen 2001). Complementing the price list in the contract by the desired log distribution might remedy this, and pay a bonus to the forest owner as compensation for the suboptimal outcome (as measured by the price list).

Buying forest stands that are typically restricted to felling within two years is presumably the most flexible way of controlling supply, since the sawmill may then use temporarily evaluated price lists when optimizing the bucking operation. The sawmill also has great freedom to choose when to harvest the stand. The drawbacks with this kind of procurement are that the sawmill has
to deal with undesired products from the stand, e.g. pulpwood, undesired species and logs with undesired diameters, which have to be sold to other users. Generally, the cost of buying stands is also relatively high compared to other types of contracts.

Even if most timber arriving at a sawmill usually comes from forests in the same region as the mill, imports from other countries may be an important temporary source of timber supply for some mills. The possibilities to influence the log mix in these cases vary greatly depending on how the specific sales contract is designed.

A prerequisite of a long-term flexible timber supply should be that the sawmill does not have to utilize all timber from the stands, but only the desired part of the harvest. Traditionally, timber from a cutting has been divided into pulpwood, logs with small diameters (typically 10-18 cm in the smaller end) and logs with large diameters (18 cm in the small end and above), and the sawmill was restricted to buying one of these assortments. Today, the specialization has advanced in several mills that would like to buy more complicated or more restricted interval assortments that fit the mill setup and product mix better.
3. The theoretical framework

The theoretical framework is divided into three major parts. The first part, presented in sections 3.1 to 3.3, outlines some theoretical issues that may be helpful to explain the occurrence of specific management systems and how they correspond to the organizational context of the firms. This background may be relevant to understand the recently raised interests of product costing in the sawmilling industry.

The second part presented in section 3.4 describes the characteristic of joint production and how it influences product costing. This is essential for the whole study, since it delimits it from other studies that discuss the more common assembly case.

The third part, theories of cost allocations and different cost allocation techniques for joint production processes, is discussed in sections 3.5 to 3.8. Among other things, this part should clarify some problems related to decision-making based on arbitrary cost figures.

Hopefully, the theoretical framework in section 3 may be relevant background for the main results of this study presented in sections 5 and 6.

3.1. Management systems design in its organizational context

3.1.1. Structural contingency theory

The structural contingency theory aims to explain how different conditions influence the organization of firms and how their management tools are adapted. Three points summarize the basic principles of the approach:

1. The work may be organized in several ways.
2. The best management system and organization alternative depends on the environment that the organization works in.
3. The efficiency may vary for different types of management systems and in different types of organizations.
From the above points, it may be concluded that there is a relation between the usefulness of a system and its influences on the behaviour of the organization, which in turn is important for organizational outcomes. If the system is well adapted, the personnel make correct decisions and their work is more efficient. The connection between the environment, system adoption and decisions is usually denoted “fit”. Contingency variables are sometimes assumed to influence the organizational structures, and thus the design of the management systems. In this case, the “fit” concept describes the connection between organizational structures and management systems. In other cases, the contingency variables are thought to influence the managing systems directly, i.e. the “fit” concept describes the relation between the contingency variables and the design of the management systems. Figure 4 summarizes the two models.

Figure 4. The general models in contingency based MCS research. The figure is based on Ask and Ax (1997).

How much well adapted management systems actually improve the companies’ results in practice has been investigated very little. According Ask and Ax (1997), an important problem with such studies is the difficulty to appoint and measure the efficiency of an organization, which of course complicates comparisons between investigated cases and studies. The fact that the same “fit” may be attained through several systems and in different ways may be a second problem (Gerdin 2005). Further, the investigated
Contingency variables are often indistinctly described (Kimberly 1976; Chenhall 2002).

Chenhall (2002) states that investigations of how contingency variables influence management systems may be motivated, but conclusions of how these adoptions influence organizational outcomes should be avoided. A considerable problem is that all investigations, for practical reasons, have only studied part of the available information. The information that may be used by decision-makers is not usually controlled. A related problem is when contingency variables co-vary, e.g. organizational size and complexity, as commented by Khandwalla (1974) and Child (1973).

### 3.1.2. Management systems

Management accounting (MA), management accounting systems (MAS), management control systems (MCS) and organizational control (OC) are sometime used without distinctions. However, Chenhall (2002) describes MA as a collection of methods, such as budgeting processes and product costing, whereas MAS refers to the systematical use of the methods to achieve some specific goals. MCS is a broader concept that includes MAS along with other, more informal methods, such as personal control and clan control. OC is sometime used to describe controls that are merged to activities and processes, e.g. just-in time management (JIT) and Total Quality Management (TQM).

The management systems may have different designs, but the usual aim is to improve and, in the best case, secure (Khandwalla 1972):

1. desired quality and quantity of output to the right cost,
2. coordination between departments,
3. reasonably correct information about the activities,
4. management by exceptions.

Some examples of different management systems are:

1. Standard costs and variance analysis,
2. marginal and incremental costing to make and buy decisions and pricing,
3. activity level budgeting,
4. internal auditing,
5. internal rate of return or present value for investment analyses,
6. statistical quality control,
7. inventory control and production scheduling by operational research techniques,
8. systematic evaluation of managers and senior staff personnel.

The control system concept initially concerned formal systems and financial measurements, but gradually broadened to today also include informal systems such as external non-financial information (Chenhall 2002). As an example, Chapman (1998) states that several studies conceptualize accounting as a collection of techniques, which may be problematic. He instead suggests that accounting should be studied as a collection of ongoing processes. Also, Gerdin (2005) points out that information from several alternative sources can give equal utility for management and control.

### 3.1.3. Contextual variables

The literature describes several types of contextual variables that are assumed to influence the design of organizations and managing systems. The variables are thought to not directly influence the designs, but indicate some conditions that have logical connections to the designs. Therefore, the variables are often denoted proxy-variables and their logical influences are denoted theoretical dimensions, c.f. (Scott 1975; Fry 1982).

**Contextual variables in general**

Contextual variables are general if they influence all current objects, and specific if they influence just some of the objects. Thus, the classification depends on the current situation and the delimitation of the group of objects. The most important contingency variables are probably general for the kind of companies, i.e. large softwood companies in Sweden, which are of interest for this study. However, each variable may of course influence the specific company at different levels.
The contextual variables are often divided in three groups:

1. technology,
2. environment,
3. organizational characteristics.

Some examples of concepts that belong to technological variables are operations technology, manageability of raw material, routine vs. non-routine, operations variability, interdependencies and technological complexity.

Heterogeneity, dynamism, competition, hostility, complexity and heterogeneity are examples of environmental variables.

Examples of organizational characteristics include organizational structure, number of employees, turnover and strategy.

However, it may be difficult to assign a concept to a specific group of variable because they may be related to each other. A common example is production technology, which may influence organizational size and in turn possibly influence the degree of decentralization.

Two contextual variables of particular interest for this study are organizational size and vertical integration, which have changed recently in the Swedish sawmilling industry as indicated by Staland, Navrén et al. (2002) and discussions in the branch. The following discussions may explain much of the recent changes in the organisations’ structures and management methods.

Organizational size
Organizational size is one of the most investigated and discussed contextual variables. Many authors believe it to be one of the most important variables, though the concept is not always distinctly defined. Kimberly (1976) writes that the indistinct definition of the concept makes it difficult to generalize the results from different studies because different studies actually test different variables, even if all denote them size. A second problem is that the size variable is often studied together with other variables, typically in medium or large size organizations (Chenhall 2002). This makes interpreting the results more complicated due to the difficulty in isolating the effect of size from other variables. A further factor that may complicate
the definition is that many firms have close co-operation with their suppliers and customers, making it difficult to delimit the organization.

Child (1973) studies organizational structures and how they relate to size. He suggests that size, technology, location and environmental influence complexity. The complexity, not size, in turn influences the degree of formalization. However, size seems to explain directly the degree of decentralization. These assumptions are described in Figure 5.

Larger size may lead to specialization, decentralization, and increased efficiency. However, much more information has to be processed and more decisions have to be coordinated in a large organization. Bruns and Waterhouse (1975) found that large administrative systems were used more frequently in large organizations, whereas smaller organizations were more often managed by informal methods, like personal controls. Also Khanwolla (1972), Bruns and Waterhouse (1975), and Merchant (1981, 1984) concluded that large organizations are more often decentralized and use formal systems more frequently.

Ask and Ax (1997) state that knowledge, financial strength, administrative intensity and organizational complexity increase when the organizational size increases. Chenhall (2002) summarizes the effects of the organizational size, based on twenty years of research:

- Large organizations are more diversified, have more formalized procedures and more specialist functions than small ones.
- Large organizations are more often divided.
- The usage of sophisticated controls is more frequent in large organizations.
Organizational size is perhaps the contextual variable that has changed most obviously in the sawmill industry during recent years. Company sizes have changed quickly through investments, acquisitions and mergers.

**Vertical integration**

Khandwalla (1974) evaluates a model that describes the relationship between production technology and different organizational parameters, like vertical integration, decentralization and the occurrence of control mechanism.

One conclusion is that companies try to decrease production uncertainty, in particular when they utilize some kind of mass output technology that is usually quite sensitive to unexpected situations. Vertical integration, i.e. incorporation of processes before the main process, after the main process, or both, may be a way to achieve this because it gives the company more control over the processes. Generally, vertical integration implies growing organizational size, and such diversification increases when new types of activities are incorporated into the company. Diversified activities thus usually imply staff with a broader range of qualifications. Personnel are
usually located close to their specific process, which often implies that the company divides the processes into specialized sub divisions. A consequence of this is that the company would need sophisticated management methods, like costing systems and budgeting processes, to be efficiently managed. These causes and effects relations are summarized in Figure 6.

![Diagram of relations of mass output technology and the use of sophisticated controls according Khandwalla. From Khandwalla (1974).](image)

**Figure 6. A model of relations of mass output technology and the use of sophisticated controls according Khandwalla. From Khandwalla (1974).**

An important example of vertical integration in the sawmill industry is the decreased use of middlemen in recent years as indicated by discussions with practitioners. This is particularly evident for large sawmill companies that have built up sales units in their main markets (Norberg 2000). The change to more direct sales facilitates the exchange of information between user and producer, which should be an advantage when the sawmill makes increasingly customized products (Lönner 1985; Norberg 2000). Some authors have argued that the use of middlemen, which may have blocked direct communications between the sawmill and user, has been an obstacle and contributed to unsatisfactory profits in several companies (Lönner 1985) and (Brege and Överberg 2001).
To probably meet the changes in the market and utilize the advantage of becoming a larger company, several sawmill companies have changed their marketing organizations from relatively small ones where independent middlemen did much of the sales work to large marketing units with several salesmen that worked more directly with the final customers. This change could most likely not be realized as long as the sawmill companies were small, since a sales force needs relatively large sales volumes to be cost-effective.

These new marketing organizations, where the salesman might be responsible for one geographical market and several products from several mills in the group, should increase control of the sales process and offer several advantages compared to a traditional set-up, where each sawmill has its own marketing people who sell mainly through middlemen to several markets.

First, a large marketing organization that allows the salesmen to be specialized in their markets increases the company’s knowledge of important markets and customers. This becomes increasingly important when the share of non-standardized products increases because specific customers’ needs have to be met to a greater extent (Norberg 2000).

Second, a centralized marketing organization facilitates the coordination of marketing activities and negotiations related to business agreements, which are probably growing issues when several mills are incorporated in a group and the customers increase in size. If the company has a marketing unit at each mill, a large buyer will negotiate with several mills in the group and perhaps obtain different prices or services from each mill, most likely unfavourable for the sawmill company. In contrast, with a centralized marketing unit, the sawmill has only one channel to the market, thus simplifying the sales and buying processes both for the sawmill company and the customer.

A third advantage of a centralized marketing unit might be the facilitated coordination of production between sawmills in the group, since each mill does not have to supply all products. This may lead to a specialization of each mill and result in simplified and more effective production.

Developed from Child (1973), Figure 7 suggests the cause and effect relationships in the sawmilling industry.
3.2. Managing sales and production – two approaches

This section describes two principally different approaches for managing sales and production in sawmill companies. The first approach, described in section 3.2.1, optimizes the processes directly in one step. Much of the earlier research work follows this approach, possibly denoted the “traditional” approach. In contrast, this thesis develops a managing method that is divided into two sub-problems, one production-planning problem and one marketing-planning problem. Repeatedly solving the sub-problems, the overall problem is solved indirectly. The approach is described in section 3.2.2.

Let us assume a company setup with two departments: a) a production department and b) a sales department. The company acquires and transports the logs to the mill yard. The log price lists, which valuate each log type based on species, dimension and quality, influence the relative amount of different log types. The production department is responsible for the sawing, drying and grading operations. Besides the lumber, i.e. the main product, there are of course considerable volumes of by-products such as chips,
sawdust and bark. These products are typically used by the mill as fuel or sold at fixed prices.

The production department chooses a mix of sawing patterns and grading programs to coordinate production with the market demand and available logs. The sales department is responsible for all market activities. The sales volumes can be modified by purposeful sales efforts, both through changes to regular customers and by marketing efforts at new markets.

The company sells $N$ sawed products that are indexed $j = 1, 2, \ldots, N$. The sales quantity of product $j$ is denoted $d_j$, which is a decision variable, though restricted so that $d \in D$, the set of feasible sales plans. The total revenue of the company is a general function, $TR(d)$, where $d = (d_1, d_2, \ldots, d_N)$. The function describes the profit after the sawing, drying, grading, marketing and transportation costs. $TR(d)$ may typically be quite a complicated function that in practice would be hard to describe explicitly in advance.

The sawmill can buy $K$ different log types, $k = 1, 2, \ldots, K$. The amount of log type $k$ is denoted $l_k = (l_1, l_2, \ldots, l_K)$. The set $K$ describes the feasible plans. The total cost of plan $l$ is described by the function $IC(l)$, which may also be a complicated function depending on, for example, the relative amount of each log type. Further, $\varepsilon_{ik}$ indicates if the sawing pattern $i$ uses log type $k$ ($\varepsilon_{ik}=1$ or 0).

The set $M$ denotes the sawing patterns that are indexed $i = 1, 2, \ldots, M$. Each sawing pattern is applied to a log type with specific dimensions and quality. The quantity of sawing pattern $i$ is denoted $x_i$ and $x = (x_1, x_2, \ldots, x_M)$. The cost of sawing pattern $i$ is $c_i$, i.e. the accumulated cost up to and including the sawing minus net incomes from by products, and $c = (c_1, c_2, \ldots, c_M)$.

Feasible production plans must also satisfy other production restrictions that may be summarized as $x \in S$. The saw yield of product $j$ from sawing pattern $i$ denotes $a_{ij}$. Note that normally only a few $a_{ij} \neq 0$ for each $j$. 

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The objective of the company is expressed in problem P:

\[ v_P = \text{Max}_{d,x,y} \ TR(d) - \left[ \sum_i c_i x_i \right] - IC(l) \]  

such that

for each product j:
\[ \sum_i a_{ij} x_i \leq d_j, \] 

for each log type k:
\[ \sum_i \varepsilon_{ik} x_i \leq l, \]

and \( d \in D, x \in S, l \in K \)

In the model the company strives to maximize total incomes minus costs such that the sales plan (2) is satisfied and the amount of different log types (3) is available. Note that the inequality from (2) and (3) assumes free disposals of surplus products (2) and logs (3).

Several variants of problem P, e.g. relative log mix restrictions, that change (3) to express the possible log mix in relation to the total log volume are possible. This in turn implies that the log costs may be expressed as linear functions of volume, in particular if the log mix variability is set to be rather moderate and thus, \( IC(l) \) may be eliminated in (1). Both these \( P \) changes are implemented in the models suggested in the thesis and may be summarized as problem \( P' \):

\[ v_P = \text{Max}_{d,x,y} \ TR(d) - \left[ \sum_i c_i x_i \right] \]  

such that

for each product j:
\[ \sum_i a_{ij} x_i \leq d_j, \] 

for each log type k:
\[ r_{k \text{min}} \sum_i x_i \leq \left[ \sum_i \varepsilon_{ik} x_i \right] \leq r_{k \text{max}} \sum_i x_i, \]

and \( d \in D, x \in S \).
Where \( r_k^{\max} \) denotes the maximal possible share of log type \( k \) and \( r_k^{\min} \) the minimum share.

### 3.2.1. Direct optimization

Several models directly solve the total planning problem \( P \), or variants of it, e.g. (Jackson and Smith 1961; Mendoza and Bare 1986; Carino and Willis 2001 a). Some have also been implemented in commercial computer programs and used for practical planning and control. The common assumption is the linearity of \( TR(d) \) and fixed log costs, so that problem \( P \) is specified as problem \( P2 \)

**Problem P2:**

\[
v_{SPP1} = \text{Max} \sum_j p_j d_j - \sum_i c_i x_i \tag{7}
\]

subject to (2) and (3),

where \( p = (p_1, p_2, ..., p_N) \) denotes the given, fixed net prices.

Jackson and Smith (1961) present an LP model that maximizes the profit of some given sawing patterns, assuming fixed prices, costs, available amounts of logs and maximal demand. The optimal solution of the model suggests what amount of each sawing pattern should be used at given log availability and product demands. Several subsequent research papers referring to the model coincide with \( P2 \).

Having solved for the optimal production and sales plan, sales quotas may most naturally manage the salesmen. Opportunities for new business may be managed through resolving \( P2 \) after new data are added into (7), (2) and (3).

Carino and Willis (2001 a) present an evaluated model that optimizes the profit in a company over several periods with both sawing and further processing, and considering stocking costs. The model may answer questions about optimizing: a) production and storage volumes, b) sales volumes, c) capacity usage and d) how input/output changes influence the company’s profit. Carino and Willis (2001 b) test the model on a real case. Johannisson (2006) evaluates a one period optimization model for a company with both sawing and planning processes, and the model is also tested on a real sawmill case.
Mendoza and Bare (1986) present a model that considers both the operation of bucking stems into logs and the allocation of logs to different industries. The problem is divided into two sub-problems that are solved with a column generation technique. Also, Mercado, Carino et al. (Mercado, Carino et al. 1990) discuss the allocation of raw material to the industry. They use an LP model to analyze the profit of a sawmill and an improved timber price list is evaluated from the optimal solution of the LP model that in turn improves the total profitability of the mill.

A drawback with the above approach may be that much information about market demand and prices needs to be known in advance. Maness and Adams (1991) state that the approach does not help to find optimal sales volumes, which would be one of the most important questions for decision makers in sawmills. Maness and Adams suggest that formulating the price/demand functions of the products in the model might solve the problem. However, this may require the practical application of rather easy price/demand relations.

### 3.2.2. Indirect optimization through decomposition

In large sawmill companies, with many persons involved in decision-making, customized products and high sales costs, dividing the total planning problem into one production problem, SPP1’, and one marketing problem, SPM1’, would be perhaps natural. The sub-problems are connected with product quantities and product costs, as presented in Figure 1.

*The production problem, SPP1’:

Assuming linear production costs and given product quantities, a long-run production problem SPP1’ that is evaluated from problem P’ can be written as:
\[ v_{SPP1'} = \min_{x \in S} \sum_i c_i x_i, \quad (8) \]

subject to (5) to (6)

and \( x \in S \)

Let \( \Pi_j \) denote the dual variables to (5). The dual of SPP1’ may then be expressed as:

\[ v_{DSPP1'} = \max_{\Pi \geq 0} \min_{x \in S} \{ \sum_j d_j \Pi_j + \sum_i c_i x_i \}, \quad (9) \]

which is then reduced to a regular linear programming dual when \( S \) is described by linear inequalities.

Note that \( \Pi \) is a vector of which element \( j \) may be interpreted as the marginal cost of product \( j \).

The marketing problem SPM1’:
For fixed \( \Pi \), the marketing department solves the problem SPM1’:

\[ v_{SPM1'} = \max_{d \in D} TR(d) - \sum_j \Pi_j d_j, \quad (10) \]

which may be solved by each individual salesman as decentralized sales decisions in a budget process. Iterations between SPP1’ and SPM1’ may lead to successively improved plans that increase the company’s profit.

Paper #3 tests a coordination scheme based on restricted sales changes per iteration, whereas paper #4 tests a scheme based on averaged product costs. Both methods are heuristic and lead to asymptotical converges in the tested cases, but for one exception in each paper #3 and #4, where the changes were too large between the iterative steps.

3.3. Sales management systems in sawmill companies

The use of formal management systems to manage sales has probably increased in recent years, mainly because many companies have grown in size. Large companies may be divided more often and rely on decentralized decisions, e.g. salesmen who are detached from the production units make sales decisions. Figure 8 summarizes some hypotheses that may be drawn from the sections above.
For companies that sell standardized products at fixed prices, basing their decisions on profit measurement of sawing patterns as described by **problem P2** would be natural. This approach is denoted as “costing of sawing patterns” in the figure above. These analyses may be calculated manually in small and medium sized companies, where the objective (7) is possibly calculated for only certain sawing patterns at a time, and the restrictions (2) and (3) are controlled manually. Individual contact and verbal communications between production and marketing staff would be important for efficient decision-making. Large companies would need formal systems, e.g. databases containing market and production data combined with optimization systems that optimize the whole problem simultaneously, to compensate for the large coordination needed because several persons and units are involved. When production and sales are appointed, sales quotas would mainly direct the salesmen.

Companies with large marketing departments that sell customized products at flexible prices may need product costs to support sales efforts. Small companies may possibly still make their decisions based on the cost of sawing patterns. If a single salesman is at least responsible for the sales of
all products in a sawing pattern, the salesman can then calculate and compare the profit of possible alternatives. However, in larger companies, it would be more common if the salesmen are organized into geographical markets or product groups, and are thus not responsible for all outputs from each single sawing pattern. A frequent dialog between production staff and salesmen in small and medium size companies may possibly solve this problem, where the outcomes from possible decisions are compared. If many persons were involved in the decision-making, it would be difficult to get efficient decisions because of the large amount of data that needs to be processed. Therefore, large companies would more often need formal costing and budgeting systems to be effective. Because of the complicated cost estimates due to the joint production in the saw, formal costing systems would presumably be required for correct cost estimates and profitable decisions. Of note, the joint cost problems may be less in spruce production, where just one product typically dominates in a sawing pattern, compared to pine production, where several equally important products emanate in most patterns because each dimension is sorted into several products depending on the quality grades. This may indicate that the usefulness of sophisticated allocation models for product costing is more common in pine than spruce dominated companies.

3.4. Characteristics of joint production

Manes and Cheng (1988) use the term joint production to describe a production process that necessarily and simultaneously results into two or more outputs, either fixed or semi-fixed proportions. Some authors, e.g. Balachandran and Ramakrishnan (1981), also include costs from facilities shared by two or more products in the term joint costs. However, this paper follows the definition in Manes and Cheng (1988), who refer to these costs as common costs, e.g. costs of capacities and depreciation, and are further discussed in section 3.4.

Baumol, Panzar et al. (1988) suggest that joint production arises because some production factors are public inputs in the sense that once they are acquired for use in producing one good, they are available free of cost for use in the production of other goods. They also consider that joint production is a marked case of economy of scope, since it should be cheaper to produce pairs of goods together instead of producing them separately. They formally depict the joint process as:
where $y_i$ denotes the output of product $i$, $t_i$ a vector of inputs directly traceable to the product $i$ and $M$ is the public input. Manes and Cheng (1988) make a distinction between fixed and non-fixed joint costs and extend (11) to:

$$y_i = f^i (t_i, A, J_j)$$

where $A$ denotes the common fixed costs in the short run, e.g. supervisory salaries, costs of physical plants, etc. $J_j$ denotes the joint costs that are proportional to the volume of joint product $y_i$, e.g. raw materials and units of conversion inputs used up to the split off point.

Several papers have analyzed joint production when the output is assumed to be a truly fixed proportion of the input, sometimes referred to as “joint production in fixed proportions” or “true joint production”. However, this case differs from the sawmill case, since the sawmill may change the output mix through conscious changes of sawing patterns. Amey and Goffin (1988) cover this case using the broader term of “joint production in variable proportions”. These authors distinguish between cases where the variability is a either result of substitutions between products from a single process, or a result of combining processes, each yielding constant proportions of the products, though the interpretation of the product costs should be identical in both cases.

In contrast to an assembling process where the marginal cost are often assumed constant over a considerable interval, the marginal costs typically change in a joint production process depending on the current sales of different products. Figure 9 shows the marginal cost of a product produced in a joint process with two products in fixed proportions. Let $X$ denote the amount of input, $c$ the cost of per unit of input, $a_i$ the output of product $i$ per unit of input, and $S_i$ the sales volume of product $i$. 
The MC changes depending on the relative sales of product 1 and 2, and is undefined if the sale equals the production.

Cost function: Total cost = \( c X \)

\[ MC, \]
\[ \text{Product 1} \]
\[ c_0/a_1 \]
\[ 0 \]
\[ (a_1/a_2) S_2 \]
\[ \text{Sales,} \]
\[ \text{Product 1} \]

Figure 9. The marginal cost of product 1 (true joint production). The arrows indicate the point where sales correspond to production. Assuming no additional costs when disposing of excess volumes.

Note that the marginal cost of product 1 is zero as long as \( S_1 < (a_1/a_2) S_2 \), since the product is then available in abundance. The marginal cost increases to \( c/a_1 \) when the sales of product 1 is greater than \( (a_1/a_2) S_2 \), i.e. the whole cost should then be allocated to product 1, since only it is the cause of the cost change. The arrows indicate that the most likely sales ratio in the long run is possibly when the sales correspond to the production rate, i.e. \( S_1/a_1 = S_2/a_2 \), a point where the marginal costs of product 1 and 2 are undefined. Thus, the marginal cost of specific products may be of limited use for decision-making, though the two products should instead be managed together. However, Cheng and Manes (1992) illustrate how to manage sales with three products in a true joint production when the sales do not correspond to the production ratio.

Figure 10 shows the marginal cost of product 1 when two products are processed in a joint process and where the production is the mixed output of two joint production inputs. \( c_j \) denotes the cost per unit of input \( X_j \), and \( a_{ij} \) the output of product \( i \) from input \( j \).
Cost function: Total cost = $c_1 X_1 + c_2 X_2$

$a_{11} > a_{12}, a_{22} > a_{21}$ and $c_2 a_{22} > c_1 a_{11}, c_1 a_{21} > c_2 a_{22}$.

The MC changes depending on the relative sales of product 1 and 2, is undefined only if the sale equals the production rate of a specific production process.

The marginal cost of each product also changes in the variable case, though contrary to the fixed case, the sales rate may end up between the maximal and minimal production rates of each product, as indicated by the arrows in Fig. 10. The marginal cost of each product is defined in this interval and they may thus be useful for supporting sales decisions, in particular if the variability in production is significant.

### 3.5. Cost allocation

Several definitions of cost allocations are in the literature. For example, Manes and Cheng (1988) use the definition: the appointment of general expense of a business to the account of its particular departments according to some arbitrary rule. This could mean 1) to charge an item or group of items of revenue or cost to one or more objects, activities, processes, operations and products in accordance with cost responsibilities or benefits received, 2) to distribute the total cost of a lump-sum purchase over the items purchased or department affected or 3) to spread a cost systematically over two or more periods.
Very critical to all one-to-many cost allocations, Thomas (1974) directs his criticism to allocation-free financial accounting, though this cannot be achieved unless allocations are eliminated in management accounting as well. In contrast to Thomas, Manes and Cheng (1988) believe that joint cost allocations are purposeful and can lead to enhanced decision-making. Also, Zimmerman (1979) argues that cost allocations may be useful to control and motivate managers in decentralized organizations, because the costs may appear to proxy for certain hard-to-observe costs within the firms. This exemplifies a division between authors who say that allocations are arbitrary and can lead to dysfunctional behaviors and suboptimal decisions and those who think that allocations are needed to promote certain behaviors and make better decisions (Hirsch 1994).

The main reasons for cost allocations should usually be to meet internal or external profit measurement and inventory valuation requirements (Drury 2000). Even though allocations of joint costs typically are of little use for decision-making, it has been frequently applied because of legal requirements on external reporting (Slater and Wootton 1988).

Thomas (1974) suggests three general conditions that should be considered when an allocation method is developed:

**Additive.** The whole should equal the parts; the allocation should exhaust the total, dividing up whatever is there, no more and no less.

**Unambiguous.** Once the allocation method has been specified, it should be impossible to divide the total into more than one way, i.e. only one specific cost per product should correspond to the allocation.

**Defensible.** Any choice among allocation methods is a choice among different ways to divide the total into parts. Once the person who makes the allocation has chosen the method, he or she should be able to provide a conclusive argument for choosing it, defending the method against all possible alternatives.

He distinguishes between unarbitrary, or theoretically justified methods that fulfill the above criteria and those that are arbitrary and do not. The third criteria might be considered as an overall criterion, whereas criteria 1 and 2 are necessary conditions to fulfill the third one.
Baumol, Panzar et al. (1988) define the incremental cost associated to good \( i \) as “the firm’s total cost with the given vector of outputs, minus what that cost would be if production of good \( i \) were abandoned, all other output quantities remaining unchanged”. Since the cost reflects how the costs change when the output of the product changes, the cost would typically be a useful input for decision-making. Perhaps assuming a somewhat broader interpretation of the concept of incremental cost, including changes of output of a good from one positive level to another, Noreen (1991) lists three conditions to be fulfilled for an accounting system to generate cost figures that may be interpreted as incremental costs:

The underlying real cost function can be partitioned into cost pools, each dependant upon only a single activity.

The cost in each cost pool is strictly proportional to its activity.

Each activity can be divided among products so that the portion attributed to each product depends only upon that product.

The above conditions are formulated with regards to Activity Based Costing (ABC) systems, but they are also applicable to more traditional systems that may be considered as variants of ABC systems (Noreen 1991).

However, regular accounting methods that typically estimate average costs may have a problem in fulfilling the third condition in a joint production environment, which is characterized by the fact that an activity does not depend on only one product.

### 3.6. Accounting based allocation methods

Even though theoreticians agree that the average cost is not a very meaningful concept for a firm facing joint costs, several allocation methods are frequently described in the accounting literature. The methods are typically additive and unambiguous according to Thomas’s (1974) definition, and because the resulting costs are difficult to interpret, the methods should not fulfill Thomas’s third condition: defensibility.

Demski (1994) illustrates the problematic average cost in a joint production firm through the following example: Suppose the firm produces two
products whose quantities are denoted $q_1$ and $q_2$. The economic cost function of the firm is stated as:

$$C(q_1, q_2) = G(q_1) + H(q_2) + J(q_1, q_2). \quad (13)$$

Where $C(q_1, q_2)$ is the total cost of the firm, $G(q_1)$ the portion of the cost function that depends on product 1 only, $H(q_2)$ the portion of the cost function that depends on product 2 only and $J(q_1, q_2)$ is the portion that depends on both products, i.e. the joint cost.

To compute the average costs, the total cost must be divided between the two products and then averaged over each output quantity. Dividing the separable costs, $G(q_1)$ and $H(q_2)$, is a straightforward task, but to divide the joint cost, $J(q_1, q_2)$ of a respective product, the company has to use some arbitrary fraction between 0 and 1, here denoted $\alpha$.

The arbitrary unit costs could then be expressed as:

$$\text{unit cost product 1} = [G(q_1) + \alpha J(q_1, q_2)]/q_1 \quad (14)$$

and

$$\text{unit cost product 2} = [H(q_2) + (1-\alpha) J(q_1, q_2)]/q_2 \quad (15)$$

The unit cost of each product obviously depends on the arbitrarily selected $\alpha$, with the resulting costs being of little use for decision-making.

Most firms have cost functions that partly correspond to (13) in the short run, e.g. a head office or a service department that serves several manufacturing units. The costs are usually denoted “common costs” and include “capacity costs”, i.e. the case when $J$ in (13) is a constant cost of some limited capacity, of which $a_1$ units are required for each unit produced of product 1 and $a_2$ units for each unit produced of product 2. These costs are typically “common” only in the short run. In the long run they appear as parts of $G$ and $H$.

A special case of joint production that may be easier to handle through traditional accounting methods is when only one product accounts for most of the revenues so that the revenues from other products are insignificant. A defendable allocation method may then be to allocate the total cost minus revenues from the by-products to the main product. Thus, only the main
product is priced. However, this is not the case in sawmills, since no single product is very dominant.

To handle joint processes with more than one main product, several allocation methods that estimate average costs are described in the literature, of which three common methods are described below. Each has been used as the basis for recently developed costing systems in three large Swedish sawmilling groups.

In the sections below, a small example is used to illustrate the methods. The separable costs of products 1 and 2 are assumed to be linearly proportional to the quantity, thus \( G(q_1) = c_i^{\text{separable}} \cdot q_1 \) and \( H(q_2) = c_i^{\text{separable}} \cdot q_2 \), respectively. \( c_i^{\text{separable}} \) denotes the separable cost of each product. The joint input is transformed to the outputs in linear proportions to the input volume, i.e. \( J(q_1, q_2) = c \cdot \max(q_1/a_1, q_2/a_2) \). \( c \) denotes the cost per unit of input, and \( a_i \) the amount of output of product \( i \) per unit of input.

### 3.6.1. The physical measurement method

The joint costs of the process are allocated to products in proportion to certain physical measurements, usually relative weight or volume.

This method assumes that all products receive similar benefits from the joint cost. Its relative simplicity may be the main advantage (Drury 2000) and possibly explain the frequent use in practice (Kaplan and Atkinson 1989). However, because the resulting costs have no relationship to the revenue-producing power of the individual products in the process, some products may appear very profitable, whereas others may seem rather unprofitable. This may result in disadvantageous actions if the costs are used for sales decision.

Equations (16) and (17) illustrate the principle of the allocation method:

\[
\Pi_1 = [G(q_1) + (a_1/(a_1+a_2)) J(q_1, q_2)]/q_1 \quad (16)
\]

\[
\Pi_2 = [H(q_2) + (a_2/(a_1+a_2)) J(q_1, q_2)]/q_2. \quad (17)
\]

where \( \Pi_i \) denotes the product cost.

Kjesbu, Eikenes et al. (2001) present a recent sawmill application and evaluate an ABC based costing system for sawn lumber. The suggested
allocation method assigns costs in proportion to physical volume. It has also been used in a recently developed costing system currently in use. For products appearing in several sawing patterns, the product costs are calculated as averages of the different cost estimates.

### 3.6.2. The value based method

The joint costs of the process are allocated to products in proportion to their net market values.

This method may be varied somewhat. If the market prices for all products are known at the split off point, the joint cost may be allocated in proportion to the market price of each product. Otherwise, the market prices have to be adjusted for product specific costs incurred after the split off point. The adjusted market prices, typically denoted net realizable values, are used as weights to allocate the joint cost.

If \( P_i \) denotes the net realizable value of product \( i \) per unit of output, the allocated costs are obtained as:

\[
\Pi_1 = \frac{[G(q_1) + (q_1P_1/(q_1P_1 + q_2P_2)) J(q_1, q_2)]/q_1}{q_1} \quad (18)
\]

\[
\Pi_2 = \frac{[H(q_2) + (q_2P_2/(q_1P_1 + q_2P_2)) J(q_1, q_2)]/q_2}{q_2} \quad (19)
\]

Several authors seem to prefer some variation of the value method to the physical measurement method (e.g. Drury, 2000, Kaplan and Atkinson, 1989, and Demski, 1994). If the joint process is profitable, the value method does not allocate higher costs than the net realizable value to products, contrary to the physical measurement method. Thus, it is consistent with the doctrine of conservatism in financial accounting, which states that the assigned cost to a product should not exceed its value (Kaplan and Atkinson 1989). However, if average prices are used as weights and the market prices decrease as the output increases, the resulting allocation may overstate the actual marginal value of a product (Bierman 1967).

A large Swedish sawmill company uses the method to allocate timber costs to sawed products. The product costs are calculated as weighted averages of the cost estimates if the products appear in several sawing patterns.
3.6.3. The main product/by-product method

Here, one product is considered the main product, while the others are by-products. The market price is estimated for the by-products and the revenues from them are subtracted from the joint cost, which is then entirely allocated to the main product (Drury 2000).

In effect this means that a cost figure is estimated for the main product only. If $P_2$ denotes the net realizable value of the by-product, the allocated cost per unit of the main product is obtained as:

$$\Pi_1 = \frac{[G(q_1) + \{J(q_1, q_2) - q_2P_2\}]}{q_1} \quad (20)$$

This method has been applied in a Swedish sawmilling group to each sawing pattern, whose joint cost is mainly the cost of the log and its breakdown. When the same main product appears in several sawing patterns, the average cost of the product is computed as a weighted average of the different estimates. All main products were selected among the centre products of the sawing patterns so that most by-products were sideboards.

3.7. The marginal cost approach to common costs

In contrast to the average cost of products, the marginal cost is also typically well-defined for a firm that faces joint production. “Marginal cost is well defined, it just happens to depend (generally) on the quantities of other products” (Demski 1994). Some allocation methods utilize this fact by determining how the total cost, or profit, changes if the amount of a product is marginally changed. These costs are allocated to the processes or the products. The methods are based on some type of optimized mathematical model.

A typical model that allocates common costs is found, e.g., in Kaplan and Velam (1974). The model assumes that the common cost represents a limited capacity, of which some unit is required to produce each product. Thus, if the overhead cost is proportional to the capacity, it is naturally understood that as a cost it is proportionally variable in the long run to each of the product volumes. Consequently, it is not a joint cost.
The approach may be illustrated by the following LP-problem:

\[ \text{Max } W_{X \geq 0} = P_1 S_1 + P_2 S_2 \]  \hspace{1cm} (21)

\[ \text{s.t. } \begin{align*}
    a_{11} S_1 + a_{12} S_2 & \leq E_1 \text{ (dual variable } \lambda_1) \hspace{1cm} (22) \\
    a_{21} S_1 + a_{22} S_2 & \leq E_2 \text{ (dual variable } \lambda_2) \hspace{1cm} (23)
\end{align*} \]

\( S_1, S_2 \) denote the sales volume and \( P_1, P_2 \) the net revenue per unit of product 1 and 2. The corresponding dual formulation is stated as:

\[ \text{Min } Z_{\lambda \geq 0} = E_1 \lambda_1 + E_2 \lambda_2 \]  \hspace{1cm} (24)

\[ \text{s.t. } \begin{align*}
    a_{11} \lambda_1 + a_{21} \lambda_2 & \geq P_1 \text{ (primal variable } S_1) \hspace{1cm} (25) \\
    a_{12} \lambda_1 + a_{22} \lambda_2 & \geq P_2 \text{ (primal variable } S_2) \hspace{1cm} (26)
\end{align*} \]

Let \( \Delta E_j \) denote a small change of the available amount of (overhead) resource \( j \). \( (S_1^*, S_2^*) \) denote the optimal values of \( (S_1, S_2) \) when the right hand side (available resources) is \( (E_1, E_2) \) and \( (S_1^* + \Delta S_1, S_2^* + \Delta S_2) \) is optimal when the right hand side is \( (E_1 + \Delta E_1, E_2 + \Delta E_2) \). If \( \Delta E_i \) is sufficiently small, the corresponding dual variable may be unchanged in the two solutions and, according to the dual theorem, the following equalities must hold (ref. to Winston, 1994, pp. 282-284, for a proof of the dual theorem):

\[ P_1 S_1^* + P_2 S_2^* = E_1 \lambda_1^* + E_2 \lambda_2^*, \] \hspace{1cm} (27)

\[ P_1 (S_1^* + \Delta S_1) + P_2 (S_2^* + \Delta S_2) = (E_1 + \Delta E_1) \lambda_1^* + (E_2 + \Delta E_2) \lambda_2^*. \] \hspace{1cm} (28)

Subtracting (27) from (28) gives:

\[ \Delta W = P_1 \Delta S_1 + P_2 \Delta S_2 = \Delta E_1 \lambda_1^* + \Delta E_2 \lambda_2^*. \] \hspace{1cm} (29)

The left hand side is the change of the objective function, \( W \), when the amount of the recourses are changed by \( \Delta E_1 \) and \( \Delta E_2 \). Consequently, \( \Pi_i^* \) may be interpreted as the marginal value of resource \( i \).
Moreover, if the available resources are fully used by both solutions, then

\[ \Delta W = a_{11}\lambda_1^* \Delta S_1 + a_{21}\lambda_2^* \Delta S_1 + a_{12}\lambda_1^* \Delta S_2 + a_{22}\lambda_2^* \Delta S_2 \quad (30) \]

In fact this equality also holds if a resource is not fully used, since the corresponding dual variable is then zero. Thus, the overhead cost has in effect been transformed into costs proportional to the output of each product, i.e. \( \Pi_1 = a_{11}\lambda_1^* + a_{21}\lambda_2^* \) and \( \Pi_2 = a_{12}\lambda_1^* + a_{22}\lambda_2^* \).

An allocation scheme, according (21) to (30), does not necessarily fulfill all of Thomas’s three conditions:

**Additive:** the allocated cost will be lower than the actual cost unless the actual production is optimized (according the model), thus breaking the additive condition. A remedy for this is suggested in papers #1 and #2, where the unallocated costs are assigned to the products in a second step by a proportional mark-up.

**Unambiguous:** if the optimal solution is degenerated, there are several possible values of the optimal \( \Pi_i \), and the solution should therefore be considered ambiguous. Typically, the optimization algorithm decides which of the solutions is selected. Whether the solutions were degenerated or not, or implications of degenerated solutions, the solutions were not directly analyzed in this work, but implicitly by the simulations in paper #3.

**Defendable:** if the solution is not degenerated, the allocated costs may be interpreted as marginal costs for moderate changes; hence, this should make the method defendable according Thomas’s third condition.

In practice, capacity costs are usually assigned to products through a simpler approach. The cost per unit of capacity, denoted \( c_j \), is typically calculated as the total overhead cost of capacity \( j \) over the available capacity of \( j \). In the next step, the capacity costs are assigned to the products in proportion to their capacity consumption:

\[ \Pi_1 = a_{11}c_1 + a_{12}c_2 \quad (31) \]
\[ \Pi_2 = a_{21}c_1 + a_{22}c_2 \quad (32) \]
The incremental cost of the firm may be stated as:

\[
\Delta \text{Capacity cost} = a_{11} c_1 \Delta S_1 + a_{21} c_2 \Delta S_1 + a_{12} c_1 \Delta S_2 + a_{22} c_2 \Delta S_2, \tag{33}
\]

which may be compared to (30).

When the overhead cost is proportional to the capacity in the long run, the average costs per unit of capacity, \(c_i\), are constant and should be reasonable estimates of the shadow prices, \(\lambda_i\), as long as the capacities are normally utilized. The method is additive, unambiguous and presumably also defendable and should therefore be consistent with Thomas’ (1974) requirements.

In contrast to capacity costs, the marginal product costs of a joint cost process are typically dependent on relative sales volumes and thus not proportional to the sales volume.

3.8. The marginal approach to joint costs

Manes and Smith (1965) were perhaps the first in accounting literature to suggest the marginal approach to allocate joint costs (Manes and Cheng 1988). Assuming known independent demand functions, they identify at least one situation for which a unique cost allocation exists – when at least one of the joint products may be disposed of without additional costs immediately after the split-off point. No cost should then be appointed to the product subject to disposal.

In the literature, some principally different models have been analyzed and suggested for cost allocation. Several authors suggest models that maximize total profit of the company, typically assuming diminishing prices or restricted maximum demands on products or capacity, e.g. (Bierman 1967; Weil 1968; Hartley 1971; Jensen 1973; Cheng and Manes 1992). Several papers also analyze problems related to joint production, but which are not primarily directed to solve the specific cost allocation problem. Examples are Hartley (1971), who analyses the optimal production level for joint products in fixed and variable proportions, Fujimoto and Ranade (1998), who analyze profit increases in joint production as a result of cost reducing technical changes, and Lager (2001), who studies disposals of products with negative prices, e.g. waste and pollutants.

The analyses of joint cost allocations may be divided into cases that assume joint production in both fixed proportions and variable proportions.
3.8.1. **True joint costs**

To facilitate decisions, especially marginal decisions like make-or-buy or special order decisions, Manes and Cheng (1988) present a linear one-period, profit maximizing model that does not consider storage between periods and restrict the maximum sales volumes. The case corresponds to Figure 4, joint production in fixed proportions.

$D_i$ and $S_i$ denote the maximal and actual sales volume of product $i$. The activity of the joint process is $X$. When two outputs and one input are assumed, the model reads:

$$Max \ W_{S \geq 0, X} = P_1 S_1 + P_2 S_2 - c \cdot X$$  \hspace{1cm} (34)

s.t.

$$S_i - a_i X \leq 0 \hspace{0.5cm} for \ i = 1,2,$$  \hspace{1cm} (35)

$$S_i \leq D_i \hspace{0.5cm} for \ i = 1,2.$$  \hspace{1cm} (36)

The objective function, $W$, maximizes the net revenues minus the costs of input. The first constraint restricts sales to be less or equal to production, while the last constraint restricts sales to be less or equal to the demand. Let $\Pi_i$ be the dual variable of the constraints (35), and $U_i$ of (36). The dual problem then reads:

$$Min \ Z_{\Pi U \geq 0} = D_1 U_1 + D_2 U_2$$  \hspace{1cm} (37)

s.t.

$$\Pi_i + U_i \geq P_i \hspace{0.5cm} for \ i = 1,2.$$  \hspace{1cm} (38)

$$a_1 \Pi_1 + a_2 \Pi_2 = c.$$  \hspace{1cm} (39)

Manes and Cheng (1988) suggest that the dual variables of the first constraint set, $\Pi_i^*$ (the product costs), be used for cost allocation, purchasing and selling decisions. The company should purchase a product from an outside company only if the price is lower than the marginal cost ($\Pi_i^*$) and sell an additional unit only if the selling price is higher than $\Pi_i^*$. The dual variable $U_i^*$ (the net profit per unit) may be used to allocate marketing efforts.
The allocated cost \( (\Pi_1^*) \) depends on the relative sales quantities of products 1 and 2, illustrated in Figure 11. If the sales correspond exactly to production, i.e. on the solid line, several allocations are possible and thus the allocation is arbitrary.

![Figure 11. The relations between production and sales of products 1 and 2. The diagonal line from the lower left corner denotes the production possibilities. The dotted lines denote ISO-cost curves.](diagram)

### 3.8.2. Variable joint costs

The variable case, where production can be selected as a variable mixture of several joint production processes, is not described very much in accounting literature. Manes and Smith (1965) describe a few cases mainly based on examples from the refinery industry and the variable case was then probably considered a relatively unusual. Yet, it is the most convenient way to describe the sawmill process.

Hartley (1971) describes a model for decision-making. The analysis is based on a primary process that yields two products and two subsequent processing steps. The variability is modelled as a linear trade-off between the two products. When the rate of one product is increased compared to a standard share, the rate of the other product is assumed to decrease. Incremental costs are assumed when the ratio changes from the standard distribution, perhaps understandable if the material has to be processed several times, e.g. as in the refinery industry. Amey and Goffin (1988)
extend Hartley’s case to three products and three subsequent processes and make some general conclusions. In contrast to Hartley, they do not assume any incremental costs when the standard distribution is changed, though this makes no essential logical difference. Manes and Cheng (1988) also present and analyse a case with joint production in variable proportions. Their formulation is logically equivalent to Amey and Goffin (1988) and Hartley (1971). Let $b_i$ denotes the coefficients of the relation between $M_1$ and $M_2$, and $X$ the amount of input according fig. 12.

![Joint production in variable proportions](image)

**Figure 12. Joint production in variable proportions. The diagonal lines from the lower left corner denote the limits of the production possibilities. The dashed lines show an ISO-cost curve.**

Manes and Cheng (1988) then state their model as:

$$\text{Max } W_{S,X,M \geq 0} = P_1 S_1 + P_2 S_2 - c \cdot X$$  \hspace{1cm} (40)$$

s.t.

\begin{align*}
S_1 - M_1 & \leq 0 \quad (41) \\
S_2 - M_2 & \leq 0 \quad (42) \\
a_1 M_1 + a_2 M_2 - X & = 0 \quad (43) \\
b_1 M_1 - M_2 & \leq 0 \quad (44) \\
b_1 M_1 - M_2 & \leq 0 \quad (45) \\
S_1 & \leq D_1 \quad (46) \\
S_2 & \leq D_2 \quad (47)
\end{align*}
The model is depicted in Fig. 12, where the feasible production quantities for given X appear within the triangle formed by origo, M1 and M2.

In the sawmill case, however, it would be more natural to reformulate the model above as a linear combination of the two joint processes of size \(X_1\) and \(X_2\), corresponding to each of the solid lines in Fig.12, each representing a sawing pattern, cf. Fig.13.

![Figure 13. Joint production in variable proportions when two inputs (sawing patterns) are assumed. The two diagonal lines from the lower left corner denote the output mix in sawing patterns 1 and 2. The dashed lines show an ISO-cost curve.](image)

The model is formulated as:

\[
\text{Max } W_{S,M \geq 0,X} = P_1 S_1 + P_2 S_2 - c_1 X_1 - c_2 X_2 \quad (48)
\]

s.t.

\[
\begin{align*}
S_1 - a_{11} X_1 - a_{12} X_2 & \leq 0 \quad (49) \\
S_2 - a_{21} X_1 - a_{22} X_2 & \leq 0 \quad (50) \\
S_1 & \leq D_1 \quad (51) \\
S_2 & \leq D_2 \quad (52)
\end{align*}
\]

The interpretation of the objective function and the restrictions correspond to equations (40) to (47) above. Now, \(c_1\) and \(c_2\) correspond to the cost per m\(^3\) sawed timber according to two different sawing patterns; \(a_{21}/a_{11}\).
correspond to $b_1$ and $a_{22}/a_{12}$ to $b_2$. Let $\Pi_i$ denote the dual variables of (49) and (50), and $U_i$ of (51) and (52).

The dual then reads:

$$\text{Min } Z_{\Pi, U \geq 0} = D_1 U_1 + D_2 U_2$$  \hspace{1cm} (53)

s.t.

$$\Pi_1 + U_1 \geq P_1$$  \hspace{1cm} (54)
$$\Pi_2 + U_2 \geq P_2$$  \hspace{1cm} (55)
$$a_{11} \Pi_1 + a_{21} \Pi_2 = c_1$$  \hspace{1cm} (56)
$$a_{12} \Pi_1 + a_{22} \Pi_2 = c_2$$  \hspace{1cm} (57)

$\Pi_i$ may be used as product cost and $U_i$ may be interpreted as the incremental value of increasing the demand of product $i$, and should therefore be useful for allocating sales efforts.

3.8.3. The practical usefulness of the marginal approach for joint cost allocation

The practical usage of the marginal approach as a tool for joint cost allocation has probably been quite limited, possibly because product costs like decision support have not been a critical concept in most industries facing joint costs. Many of these industries have perhaps sold standardized products at fixed prices and with rather low sales management costs. Thus, it would be unnecessary to use product costs to guide business efforts and sales. A better approach would probably be to use the models in sections 3.2.1., 3.8.1. or 3.8.2. to direct activities directly towards optimal sales and production and managing the sales force by quotas, instead of the more indirect method of managing by product costs.

The limited range of dual variables may also explain the limited use of the marginal costs for decision-making in joint cost firms, possibly encouraging the market organization to change the sales of a product too much. However, this should probably be less of a problem the more the firm can change the output mix.
4. Material and methods

The model and method developments presented in sections 5 and 6 are tested on a pine sawmill belonging to a Swedish sawmill and forest group that run several mills. An important reason to base the model tests on the investigated company was that the author was employed by the company during the work of this thesis and therefore had access to company’s databases and other information considered critical for the model developments.

4.1. The sawmill

The mill, with a yearly sales of about 160,000 m$^3$, produces about 150 centreboard products that are specified by width, thickness and standard quality. The customers are mainly from the furniture and joinery industries, and buy commercial qualities that are typically mixtures of the standard ones.

The sawmill might be considered as a relatively typical sawmill in several respects, but with a capacity that ranks it among the larger Swedish mills (c.f. Starland and Navrén, 2002).

Timber procurement is carried out via a separate business unit that purchases timber from both the company’s forest department and private forest owners. Payments are based on a timber price list, with varying log prices depending on diameter, length and quality.

Several reorganizations have recently been carried out. A considerable change has been the separation of salesmen from the sawmills to a centralized marketing organization consisting of sales units responsible for specific geographical markets. Salesmen employed by the company conduct most sales, though some independent middlemen are still used in less important markets. The main reason for the organizational changes was due to an increased need of coordinated sales work and specialization of the salesmen. The company also believed that the new set-up would encourage a successive specialization of the different mills and thus increase production efficiency.
Due to a perceived need for management tools to coordinate production and sales in the new organization, the company developed a product costing system that was introduced into the organization in 2002. The system can be used to measure product, market and customer profitability and was intended to support sales and production decision.

However, the system allocates the timber costs with an arbitrary method (described in paper #1, denoted “recent method”) and should therefore be expected to give less useful products costs. Even if the actual use of the cost figures are not investigated in this study, conversations with personnel indicate that they have little confidence in the system’s cost figures, perhaps indicating that these costs have little actual influence on the organization. Traditional concepts, e.g. profitability of sawing patterns and sawing yield measurements, are therefore still essential key measurements for actual decision-making. The lack of support to the salesmen, who compensate for their diminishing knowledge of production costs, may be an obstacle to efficient allocation of their sales efforts.

4.1.1. Data collection

In paper #1, the analyses are mainly based on data obtained from interviews with the sawmill personnel. Data about saw yields, i.e. in what ratio products emanate from different sawing patterns, are typically difficult to obtain in practice because the measurements are usually made on mixes of log types. Thus, the sawmill personnel had to estimate this data based partly on their experiences from test sawing and more general assumptions.

The analyses in papers #2, #3 and #4 are based on data from the company’s present costing systems. To obtain the necessary data about sawing yields, the company has developed an algorithm that splits the aggregated measurements into sawing patterns. The algorithm has not been validated, but because experienced sawmill personnel were involved in the development and consider the outcome as reasonable, the sawing yield data may be regarded as reliable.

Net market prices of each product were calculated as average prices of a time-period and based on data from a system used to follow up business deals. In papers #3 and #4, the product prices were reduced with corresponding sales costs, which were assumed to increase proportionally with the deviation from the original sales budget. To avoid extreme cost allocations, it was necessary to use maximum and minimum limits of the prices (c.f. section 5.3). The price estimates were partly based on the
company’s experience of buying scarce volumes from external mills and selling surplus volumes to non-regular customers.

4.2. Model development

The model and method development have followed the outline in appended papers #1 to #4. The developments have been continuously checked with key individuals in the company as well as information in the company’s costing and management systems. The development of the allocation model and the coordinating mechanism started quite simply in paper #1 and were successively refined and adapted to the company’s situation in subsequent papers.

5. The cost allocation model for sawmills

5.1. Cost minimization with restricted variable output

In contrast to the models in sections 3.2.1, 3.8.1, and 3.8.2., the allocation model developed in the appended papers minimizes the total costs when a specified sales budget is assumed to be satisfied. This is because the sawmill has to fulfill the current sales contract of the period and the price/demand functions for the main products are unknown, even though the salesmen successively estimate them. One may describe it as the maximizing models in the above sections finding the best profit with given known prices, whereas the minimizing model in section 3.2.2 and below finds the lowest cost with a given sales budget.

Thus, an essential difference when comparing both models is that maximizing models will find the optimal plan in the first optimization, whereas the costs have to be recalculated several times with new sales plans before the best possible solution is reached with the minimizing approach.
Since the sawmill strives to minimize the total cost of production, given that a sufficient amount of each product is produced, the basic model may be stated as:

\[
\text{Min } Z_{X\geq0} = c_1 X_1 + c_2 X_2 \tag{58}
\]

\[
s.t.
\]

\[
a_{11} X_1 + a_{12} X_2 \geq D_1 \tag{59}
\]

\[
a_{21} X_1 + a_{22} X_2 \geq D_2 \tag{60}
\]

c_i denotes the cost of sawing pattern i, X_j the amount of sawing pattern i and a_ij the yield of product j from pattern i.

The optimal values of the dual variable s in (59) and (60) show how much the objective value changes when the sales (D_i) change, and may therefore be interpreted as the marginal costs of products 1 and 2. Let \(\Pi_i\) denote the dual variables of (59) and (60), the dual is then formulated as:

\[
\text{Max } W_{\Pi\geq0} = D_1 \Pi_1 + D_2 \Pi_2 \tag{61}
\]

\[
s.t.
\]

\[
a_{11} \Pi_1 + a_{21} \Pi_2 \leq c_1 \tag{62}
\]

\[
a_{12} \Pi_1 + a_{22} \Pi_2 \leq c_2 \tag{63}
\]

The allocated costs depends on the current production and demand. The cost of product 1 varies between 0 and \(c_1/a_{11}\) (assuming the same relations as in fig. 13), and the cost of product 2 between 0 and \(c_2/a_{22}\).

5.2. Restricted timber supply

In practice, it is unrealistic to assume that the sawmill can freely buy different log types. The forest in the region typically restricts the maximum share of different log types. This should probably be the case even if the sawmill buys some of the timber from other regions.
Let $r_1^{\text{min}}$ denote the minimum share of sawing pattern 1, which uses log type 1, and $r_1^{\text{max}}$ the maximum share, and the model is then extended with the timber restrictions:

$$\text{Min } Z_{X \geq 0} = c_1 X_1 + c_2 X_2 \quad (64)$$

s.t.

$$a_{11} X_1 + a_{12} X_2 \geq D_1 \quad (65)$$
$$a_{21} X_1 + a_{22} X_2 \geq D_2 \quad (66)$$
$$X_1 \geq r_1^{\text{min}} (X_1 + X_2) \quad (67)$$
$$X_1 \leq r_1^{\text{max}} (X_1 + X_2) \quad (68)$$

$T_1^{\text{min}}$ is the dual variable of (67) and $T_1^{\text{max}}$ the dual variable of (68). The dual is formulated as:

$$\text{Max } W_{\Pi, \Pi_2 \geq 0} = D_1 \Pi_1 + D_2 \Pi_2 \quad (69)$$

s.t.

$$a_{11} \Pi_1 + a_{21} \Pi_2 - (1-r_1^{\text{max}}) T_1^{\text{max}} + (1-r_1^{\text{min}}) T_1^{\text{min}} \leq c_1 \quad (70)$$
$$a_{12} \Pi_1 + a_{22} \Pi_2 + r_1^{\text{max}} T_1^{\text{max}} - r_1^{\text{min}} T_1^{\text{min}} \leq c_2 \quad (71)$$

When the sawmill has to buy different log types in semi-fixed proportions, the flexibility in production decreases because each production possibility is now a combination of sawing patterns one and two. Figure 14 shows the production possibilities and a corresponding iso-cost line.
Figure 14. Production possibilities with restricted timber supply. The oval dots indicate the range of production possibilities that are a combination of the two sawing patterns, $X_1$ and $X_2$. The dashed line denotes an ISO-cost curve.

**Case A**

If sales correspond to point A in Fig. 14, the allocated costs are:

$$
\Pi_1^* = \frac{a_{22}}{a_{11}a_{22} - a_{12}a_{21}} c_1 - \frac{a_{21}}{a_{11}a_{22} - a_{12}a_{21}} c_2 \quad (72)
$$

$$
\Pi_2^* = - \frac{a_{12}}{a_{11}a_{22} - a_{12}a_{21}} c_1 + \frac{a_{11}}{a_{11}a_{22} - a_{12}a_{21}} c_2. \quad (73)
$$

Note that the cost increases in one sawing pattern and decreases in the other. This is because the amount of one sawing pattern will increase, while the amount of the other pattern decreases when the sale of one product is marginally changed.

**Case B:**

If the sales correspond to point B in Fig. 14, product 2 is available in excess and hence, $\Pi_2^* = 0$. The company would like to use only sawing pattern $X_1$, but because of the timber restriction, they have to use some amount of $X_2$ as well.

50
The amount of sawing pattern $X_1$ depends on $D_1$ as:

$$X_1^* = D_1 / [a_{11} + a_{12} \{ (1-r_1^{max})/r_1^{max} \}] \quad (74)$$

The first part of the right hand side, $D_1/a_{11}$, should be the changed amount if only sawing pattern $X_1$ is used and $X_2$ changes. Thus, the second term in the denominator expresses the simultaneous change of the amount of $X_2$ times the saw yield of $D_1$ in $X_2$.

Analogously, the amount of sawing pattern $X_2$ depends on $D_1$ as:

$$X_2^* = D_1 / [a_{12} + a_{11} \{ r_1^{max}/(1-r_1^{max}) \}] \quad (75)$$

Consequently, the cost of product 1 may be stated as:

$$\Pi_1^* = c_1/ [a_{11} + a_{12} \{ (1-r_1^{max})/r_1^{max} \}] + c_2/ [a_{12} + a_{11} \{ r_1^{max}/(1-r_1^{max}) \}], \quad (76)$$

i.e. the changes of the amount of each sawing pattern times their costs.

### 5.3. Max and min prices

Since the sawmill can usually buy scarce products externally if they pay a sufficiently high price and sell surplus quantities to non-regular customers, normally at a lower price, it seems reasonable to restrict the product costs with maximum and minimum limits. $P_j^{max}$ denotes the cost of product $j$ when it is bought externally and $P_j^{min}$ the price on product $j$ when its sold to non-regular customers, typically with less service commitments. The cost of product $j$ is then restricted by following inequalities in the dual problem:

$$P_j^{max} \leq \Pi_j \leq P_j^{min} \quad (77)$$

meaning that the allocated cost to product $j$ is between the maximum and minimum price.
5.4. Quality hierarchy

Traditionally, the lumber has been sorted and sold in a falling quality scale and customers have typically accepted a product of better qualities, as long they not had to pay a higher price. This is still prevalent for several customers and products, even if more and more customers desire a product of “right” quality – neither better nor inferior. Let \( \text{sup}(j) \) denote the product with closest superior quality to product \( j \). The allocated costs are then restricted by (78) in the dual formulation.

\[
\Pi_j \leq \Pi_{\text{sup}(j)} \quad (78)
\]

Thus, a product of the superior quality has the same (if it is available in abundance) or a higher (if the sales equals the output) price than the product of the inferior quality.

5.5. Relative production capacities among mills

To be practically applicable in companies with several mills, the model should be extended with restricted relative capacity at each mill. These restrictions are also justified in a long run, e.g. because the cost of timber transports increases with mill size. The restrictions are implemented by (x) in the primal, which limit the relative production capacity at each mill.

\[
\gamma_u^{\text{max}} = \text{sawmill } u \text{'s largest possible share of total company production } ([\text{log volume, m}^3, \text{mill } u]/[\text{total log volume, m}^3]),
\]

\[
\gamma_u^{\text{min}} = \text{sawmill } u \text{'s smallest possible share of total company production } ([\text{log volume, m}^3, \text{mill } u]/[\text{total log volume, m}^3]),
\]

for each mill \( u = 1, 2, ..., S \):

\[
\gamma_u^{\text{min}} \sum_i X_i \leq \sum_{i \in SAW(u)} X_i \leq \gamma_u^{\text{max}} \sum_i X_i, \quad (79)
\]

Now, the terms of the opportunity cost of (79), the production share of sawmill \( u \), are added to the dual. The term \( \{(1 - \gamma_u^{\text{max}}) \rho_u^{\text{max}} \rho_v^{\text{max}} \gamma_v^{\text{max}} \} \)
increases the costs of sawing patterns at sawmills that use up their production share and \((1 - \gamma_u^{min}) \rho_u^{min} - \Sigma_{v \neq u} \rho_v^{min} \gamma_v^{min}\) decreases the costs for patterns at sawmills that produce as little as possible.

5.6. Relative capacity restrictions within mills

The inequality (80) below restricts the production capacities at sawmill \(u\), but the capacity limits are expressed in relation to the total procurement volume at the sawmill. Thus, they are not absolute, but relative and represent capacity imbalances that can be expected to prevail in the long run. These restrictions may concern available capacities after the saw, e.g. in edgers and drying ovens, in addition to the regular sawing capacity. The following notations are used:

\{1, 2, \ldots , K\} is the set of capacity types available at the mills.

- \(b_{jc}\) = product \(j\)’s consumption of capacity type \(c \in \{1,2,\ldots,K\}\) (e.g. \([\text{m}^3 \text{ usage}]/[\text{m}^3 \text{ product}]\) or \([\text{hour usage}]/[\text{m}^3 \text{ product}]\)),

- \(t_{uc}\) = the amount of scarce capacity \(c\) at sawmill \(u\) in relation to its total volume of timber (e.g. \([\text{m}^3 \text{ available recourse}]/[\text{m}^3 \text{ total timber volume}]\) or \([\text{available resource, hour}]/[\text{m}^3 \text{ total timber volume}]\)).

For each capacity \(c = 1,2,\ldots, K\), and each sawmill \(u = 1, 2, \ldots, S\)

\[
\Sigma_{i \in SAW(u)} (\Sigma_j b_{jc} \alpha_{ij}) X_i \leq t_{uc} \Sigma_{i \in SAW(u)} X_i. \tag{80}
\]

In the case of insufficient capacity, the effect of these relative restrictions is that the common timber costs are reallocated from products that use little of the scarce resource to products that use much. As demonstrated in paper #4 the relative restrictions assure, contrary to absolute capacity restrictions, that the full cost is allocated to products, which the case company desired – it may be rationalized by the belief that present capacity imbalances will persist.

Let \(\omega_u\) denote the dual variable to (80). \(\Sigma_c \{ \Sigma_f(\alpha_{ij} b_{jc}) - t_{uc} \} \omega_{uc}\) is then added to the dual. This term mirrors the opportunity costs of scarce capacity (80) and adds the costs to sawing patterns in relation to the capacity usage of their products, \(\Sigma_c \Sigma_f(\alpha_{ij} b_{jc})\) and accounts for opportunity incomes of the
capacity made available by the sawing volume of $X_i$ in the relative capacity constraint, (80).
6. Managing the sawmill by product costs

In contrast to traditional sawmill management, where the salesmen are guided by sales quotas derived through a centralized planning process based on profit measurements of each sawmill or sawing pattern, a shift towards using product costs may make profit measurements of products, customers or markets possible. Hence, to decentralize sales decision to the salesmen, the company may change from being production-focused to being more market driven. This is probably a significant change for most sawmills and would give the salesmen a more active role than they normally would have in the traditional set-up.

How sawmill companies actually use product costs, when they exist, and what influences the cost figures may have on sales decisions and organizational set-ups are not investigated in this study. However, a simplified approach is studied in papers #3 and #4, simulating the economical outcome of the sawmill when it is managed with product costs. The production unit communicates with the sales units in terms of product costs. Based on the costs, the salesmen allocate their sales efforts so that the share of profitable products increases and the share of unprofitable products decreases. When sales have changed, the market unit communicates the new sales to the production unit, which use it for a new production plan. The efficiency of the approach assumes that the changes are moderate between each updated sales/production plan. The process corresponds to an iterative process where the plans are repeatedly updated and the product costs recalculated.

Regular data updates and cost recalculation may defend assumptions about linear costs and fixed prices for products sold to regular customers. If the updates and recalculations are carried out often, the changes in prices and capacities between each calculation would be moderate and linearity assumptions should thus be probably acceptable approximations of the actual changes.

An important question is how often the plans have to be updated and the costs recalculated. This may indicate how useful the costs would be in practice, as one may imagine that the difficulty of salesmen planning would significantly increase with the updating frequency and the consequential erratic cost information. Cost information may also be somewhat irrelevant if updated too often, like many sales decisions in the long run.
The sales response models in the third and fourth papers are speculative and a simplification of actual behavior. Even if the logical outline is similar to Darmond and Rouziés (2002), the model parameters are not tested as comprehensively as theirs. The model parameters in paper #3 are estimates partly based on interviews of salesmen and personnel at the sawmill. The assumption that the salesmen allocate their sales efforts strictly to the price-cost relationship of each product is most likely an exaggeration. In practice, the company would probably add further information to the decisions as well as use compensation plans (commissions) to adjust the behavior of the salesmen.

The analyzed sales decisions deal with the sales of main products sold to regular customers, i.e. typical business deals of a long-term nature. This may imply that it is practically difficult to decrease a sales volume during a period that follows after a sales increase, a fact that is not included in the model. This may eventually imply that, in practice, sales decisions are more long-term considerations than as indicated in paper #3 and thus, the use of long-term costs may be justified.

The resulting sales plans will probably form queues of some products that need more capacities than available at the moment, probably partly mitigated by the capacity balance discussion in paper #4. This would be an incentive to acquire more of the insufficient capacity for the following periods. If the sales decisions are based on full costs, the cost of extra capacity is considered, since the cost of capacity is included in the costs. This may be a main motive to use the long-term cost in the sales decisions.

However, an extension of the allocation model in section 5 may be to add absolute capacity restrictions, e.g. available sawing capacity. It would then be economically justified to also use the resulting cost when short-term decisions are assumed. Let $K$ denote the sawing capacity and $h_j$ the sawing pattern $j$'s capacity usage. The capacity restrictions may then be modeled by adding:

$$h_1 X_1 + h_2 X_2 \leq K, \quad (81)$$

to the primal formulation in section 5.1. To avoid unfeasible solutions, the model should be extended with externally bought amounts of product $i$ (c.f. (77)). Further, let $\lambda$ denote the dual value to (79). The dual formulation may then be stated as:
Max $W_{\Pi, \lambda \geq 0} = D_1 \Pi_1 + D_2 \Pi_2 - K \lambda$ \hfill (82)

s.t.

\begin{align*}
  a_{11} \Pi_1 + a_{21} \Pi_2 - d_1 \lambda & \leq c_i^s \hfill (83) \\
  a_{12} \Pi_1 + a_{22} \Pi_2 - d_2 \lambda & \leq c_2^s \hfill (84) \\
  \Pi_1 & \leq P_1^{\text{Max}} \hfill (85) \\
  \Pi_2 & \leq P_2^{\text{Max}} \hfill (86)
\end{align*}

$c_i^s$ denotes the short run variable costs. The variable $\lambda$ may be interpreted as the marginal value of extra capacity, i.e. how much the firm would spend on increased capacity. As long as there is idle capacity, $\lambda = 0$, and the capacity constraint therefore does not influence the cost allocation – only short-term variable costs are allocated, except when the saw is fully utilized, $\lambda > 0$, and the costs are shifted upwards to $P_1^{\text{Max}}$ and $P_2^{\text{Max}}$. This indicates that the capacity constraint probably strengthens the “jumps” in the cost figures, which possibly insists on more frequent cost recalculation.
7. The scientific contribution of this thesis

Discussions about joint cost allocations seem to have grown quiet in accounting journals during the last 15 years, possibly because its practical usefulness was considered very limited to a certain types of production. However, recent extensive changes in the sawmill industry and its markets have led to a rising interest of costing methods by branch practitioners. Several questionable allocation methods have been employed in companies. Together with the fact that sawmills are characterized of joint production with variable output, a sparsely analyzed case in the literature makes this further research interesting and meaningful. Hopefully, this thesis contributes to a revival of the discussion about using product costs as a managerial tool in joint production companies.

Analyses based on a Swedish sawmill show that a detached marketing unit could use product costs to make economically justified sales decisions, which systematically may improve the profitability of the sawmill. The cost figures need to be recalculated regularly, and at a shorter recalculation interval if the timber flexibility is small.

To calculate economically defendable cost figures, a linear programming method to allocate jointly caused sawing and timber costs to products is developed. Tests of the modeled sawmill showed the necessity to include restrictions on the flexibility in timber supply and thus avoid unrealistic purchasing plans. Assumptions about realistic timber flexibility make capacity restrictions superfluous, which simplify the model formulation.

Restricted flexibility in the timber supply also amplifies the changes in cost figures because a small change of a single product may lead to the changed needs of numerous log types. Tests based on the investigated company showed that the costs have to be limited with maximum and minimum prices. In addition, the relative capacity among mills has to be restricted when the production is carried out in several mills. Relative capacity balances that describe the capacities within each mill may be necessary to prevent unrealistic production plans.
8. Future research

It would be worthwhile to study how different sawmill companies accomplish practical sales management, especially companies that have changed from a traditional organization to centralized sales units detached from the mills. This would possibly generate knowledge and ideas about what information is important to support sales decisions and how the salesmen should be managed and organized to focus sales efforts on profitable business opportunities.

The suggested allocation method concerns mainly products that the company already sells. An interesting question is how the company should estimate the profitability of new products. The suggested allocating model in paper #2 handles this partly through the use of standardized qualities that may be combined with new commercial products. However, the question remains for products with new dimensions. Since new products may imply radical changes in the timber price list and production, the problem should perhaps be handled through separate analyzes and not be incorporated in regular product costing systems.

The results in papers #2 and #3 indicate that flexibility in the timber supply influences the usefulness of the product costs: the higher the flexibility, the more constant the costs. Timber flexibility is modeled through restricted proportions of different saw classes, though it would perhaps be better modeled in other ways. An interesting question would be to study the actual timber supply flexibility of a sawmill. The flexibility may perhaps be large in some cases, but the cost of each log type perhaps changes greatly when the “standard” mix of logs is changed. Thus, it would possibly to imply that the flexibility should better be modeled with nonlinear cost functions.

Paper #4 tests three averaging methods for the cost figures. They show a quite rapid convergence in the opening iterations, which would be necessary to be practically applicable. The methods are similar to Mean Value Cross Decomposition (MVCD) (Holmberg 1992). However, MVCD assumes a procedure that weighs both cost figure and sales volumes, i.e. the figures that coordinate each sub-problem, to give asymptotical convergence. The simulations in paper #4 present convergent solutions, even if just the cost figures were weighed. Since both convergence patterns and rate largely depend on sales changes, modeled rather simply in this study, it would be
interesting to also investigate this for other price/demand relations than those assumed in the paper.
9. Summary of appended papers

9.1. Paper #1

*The Timber Cost of a Board, Mats Johansson and Kaj Rosling, 2002.*

The long run planning problem of a sawmill is depicted and decomposed into a marketing planning problem and a production-planning problem. Based on a small stylistic example, two arbitrary costing methods used in industry are compared with a theoretically defendable allocation procedure that is based on mathematical programming. Summarized in two propositions, the analysis points out some fundamental differences between the two arbitrary methods and the suggested method.

The products’ costs calculated by the arbitrary methods are all non-decreasing in the costs of the sawing patterns and the total cost equals the total cost of the sawing pattern, i.e. full cost allocation.

The product costs of the suggested procedure typically increase in the costs’ coefficients of some sawing patterns and decrease in others. The total product cost underestimates the actual cost of the sawing patterns, unless the current production happens to be optimized.

Based on an arbitrary excerpt from a pine sawmill, product costs were calculated with both arbitrary methods and the suggested procedure. Considerable differences in cost estimates between the methods were observed, which may lead to suboptimal sales decisions if the arbitrary methods were used. The study also shows that the sawmill, thanks to flexibility in the production process, may differ from true joint production and thus render the product costs useful. The small case also indicates that the actual total cost may differ considerably from the calculated optimal cost. The unallocated cost was suggested to be allocated through a proportional adjustment upwards of each product cost.
9.2. Paper #2

*Allocating the Timber Cost to Sawn Products – A Case Study, Mats Johansson, 2003.*

A theoretically defendable method to allocate the costs of timber and sawing, denoted timber cost, is evaluated and tested. Timber costs are derived from the dual formulation of an LP model describing the production process and restricted with the budgeted sales volumes. Data about costs, capacities and sales budgets were collected from the present costing system of the company. Also, saw yield parameters, i.e. yields of specific products from a sawing pattern, were collected from the database of the present costing system. These data are normally difficult to collect and confirm because only the yields regarding the mixture of log qualities and sawing patterns are measured in the production. However, the present costing system utilizes an algorithm evaluated by the company to estimate the sawing yield from each combination of log type and sawing pattern group, denoted setting. Several findings were made.

Assuming realistic flexibility in timber procurement, the deviation of log mixture and sales volumes compared to the current budget were moderate. Since the current budget should reflect production restrictions, it would imply an unnecessary extension of the allocation model with detailed production capacity restrictions.

It was necessary to use ex-ante boundaries reflecting maximum and minimum product costs to avoid extreme cost allocations. Maximum prices, i.e. the marginal cost of buying the product externally, and minimum prices, i.e. the marginal price when products are sold to irregular customers, are quite simple to derive in principle, but probably difficult to measure in practice. The need for cost boundaries seemed to be more marked when little flexibility in timber procurement was assumed, probably because a small change in the sale volume of a product may lead to a change in the volumes of many log types.

Due to differences of actual total cost and the (theoretical) total cost in the optimal solution, the allocated timber cost had to be adjusted upwards with a “mismanagement charge” to obtain full cost allocation. However, this charge was relatively small, about 7% of the total cost, when the timber flexibility was assumed to be 10%, which is probably quite a realistic assumption for many Swedish sawmills. To obtain full product costs, the
paper suggests a model based on traditional allocating methods to designate costs after the sawing, e.g. drying, grading and shipping costs.

The study indicates that the estimated marginal costs of products are sensitive to changes in the market plan, which, of course, may make the use of product cost as a decision-making tool more difficult in the practice. The sensitivity seems to be more marked when assuming low or moderate timber flexibility.

9.3. Paper #3


The paper analyzes the effects of using product costs to direct sales efforts. A sales model mimics the effects of sales changes due to sales time. The model assumes that purposeful sales work is needed both for sales increases and decreases. Different recalculation intervals of product costs, varied timber flexibility and different assumptions about maximum and minimum product costs are analyzed.

The analysis shows that the lesser the timber flexibility, the shorter the intervals the costs have to be recalculated. Assuming 10% timber flexibility, the total average profit of the sawmill successively increases if the sales changes of each product are restricted to a maximum 10% of the current sales volume, whereas the total profit successively decreases when the sales changes were restricted to 30%. Assuming 100% timber flexibility, which is perhaps too high a flexibility to be realistic, also resulted in successively increased total profits of the sawmill when the product costs were recalculated in intervals given by a maximum 30% sales change, i.e. believed to approximately correspond to a recalculation of the cost between once and twice a year.

If the gap between maximum and minimum prices was assumed to be low, the recalculation intervals could be prolonged, and still obtain increased profit. This is because the error in the costs’ estimates is probably smaller, and the cost figures will consequently lead to smaller sales changes and support the sales efforts more effectively.

The study also indicates that the recalculation interval may be longer if the sawmill has a great share of standard products sold in arbitrary amounts at
fixed market prices, probably because the cost of a specific product is influenced less by changes of other products.

Even moderate sales changes may cause large changes of the cost figures, which may, of course, confuse the users of the costs. Therefore, to obtain organizational confidence in the cost figures, it is important that the users understand the specific costing situation and its consequences on the costs.

9.4. Paper #4

*Product costing for sales management in sawmill groups, Mats Johansson, 2007.*

The paper develops the model of paper #3 and analyzes the effects of managing the sales efforts with averaged cost figures.

The simulations show that the LP model should restrict the relative production capacities among mills to avoid unrealistic solutions when several mills share parallel production in a company. The model is also extended with restrictions of production capacities within the mills to express capacity unbalances that prevail also in the long run. These restrictions are expressed as relations and the long run characteristics of the model may thus be maintained. Finally, the paper briefly discusses the implications of absolute capacity restrictions and cost allocations in the short-term.

The analysis is based on a sales model that mimics the effects of sales changes due to sales time. The model assumes that purposeful sales work is needed for both sales increases and decreases. Three different variants of cost-averaging methods, influenced by Mean Value Cross Decomposition (Holmberg 1992), are evaluated and tested.

The analysis shows that the non-averaged cost figures lead to decreasing profits and solutions that cycle between suboptimal sales quantities because the sales changes are too large at each iteration. In contrast, averaged cost figures manage sales effectively and result in asymptotical converges. Near optimal solutions are achieved in just 3 to 5 sales adjustments.

The cost averaging procedures result in smaller and smaller cost changes as the iterations proceeds. This may be advantageous in practical situations because the users may feel more confident about the rather stable cost figures.
10. Errata appended papers

Paper #1
Page 3, last paragraph of Section 3: “$w^k$” should be “$\omega^k$”

Paper #3
The line between (8) and (9) should read: “Maximizing (7) now gives...”
Last paragraph of Section 7: “30% timber flexibility” should be “100% timber flexibility”
References


