The Laffer Curve for Top Incomes in Sweden

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Abstract

This study estimates the effects on tax revenue due to a removal of the top marginal labour income tax *Värnskatten* and find at what tax rate on top marginal labour income Sweden will maximize its tax revenue. This is done by estimating the Laffer curve using top labour income distribution data in Sweden gathered from 2017 from Statistics Sweden and estimations of labour and income elasticities gathered from previous studies in the field. Our findings suggest that Sweden’s effective tax rate is higher than the optimal when it comes to maximizing tax revenues and that it would be profitable to remove *Värnskatten*.

*Keywords: Laffer curve, Taxes, Labour income, Income elasticity.*
# Table of Contents

1. Introduction ........................................................................................................... 1

2. Background ............................................................................................................. 3
   2.1 The Concept of the Laffer Curve ................................................................. 3
   2.2 Historical Tax Rates in Sweden ................................................................. 4
   2.3 Previous Studies ............................................................................................ 5

3. Theory ................................................................................................................... 7
   3.1. The Theory of Labour Elasticity ................................................................. 7
   3.2. The Theory of Income Elasticity ............................................................... 7
   3.3. Different Elasticities ..................................................................................... 8
   3.4. Theory of Pareto Distribution ................................................................. 11
   3.5. Calculating the Effective Tax .................................................................... 12
   3.6. Elasticity Concept ...................................................................................... 14
   3.7. Derivation of the Laffer Curve Peak ....................................................... 16
   3.8. Derivation of Laffer Curve Equation ....................................................... 18

4. Method ................................................................................................................. 19

5. Data ...................................................................................................................... 19

6. Result ................................................................................................................... 22
   6.1. Calculation of Pareto Values ................................................................... 22
   6.2. The Peak of the Laffer Curve ................................................................ 23
   6.3. Calculating the Revenue Gain .................................................................. 24
   6.4. Using Different Pareto Values .................................................................. 25
   6.5. Optimal Tax Rate with Different Elasticities ......................................... 27
   6.6. Historically Optimal Tax Rates ................................................................ 30

7. Discussion ............................................................................................................ 32

8. Conclusion ........................................................................................................... 35

References ............................................................................................................... 36
1. Introduction

The efficiency of taxes has always been an important subject in the political discussion. A way of measuring this efficiency is by looking at the change in tax revenue due to a tax change. However, the intuition that a tax increase raises tax revenue might not always be true. The famous theory of the Laffer curve, named after Ronald Reagan’s economic advisor, Arthur Laffer, shows the relationship between tax rate and tax revenue. The curve takes the form of an inverted U-shape and shows that tax revenue increases with a higher tax rate. This is true only until a certain point is reached, which is the peak of the curve. Raising the tax rate after reaching this point would instead result in a decrease in tax revenues (Laffer, 2004). Trabandt & Uhlig (2012) examine how the Laffer curve differs between USA and fourteen European countries when tax on labour income, capital income and consumption are considered. The purpose was to find the optimal tax level to maximize tax revenue and they concluded that most countries have not reached their peak. The reason why tax revenue decreases after a certain tax rate has to do with how individuals change their behaviour due to that tax change. Ericson, Flood & Islam (2015) shows that the elasticity varies depending on the demographics of the people in the samples.

Sweden has one of the world’s highest marginal tax rates for top income earners. The idea that it would be beneficial to lower the top marginal tax rate, in the context of raising tax revenue has been brought up in the political discussion. According to Lundberg (2017), Sweden would increase its tax revenue from lowering their top marginal tax rates for top income earners. The Swedish election in 2018 led to a change of government. The new government proposed to remove the top marginal tax rate called Värnskatten, which is an add-on tax of 5 percentage points for the top earners on labour income. This change was accepted and will take place in January 2020 (Gabrielsson, 2019).

This report will analyse whether Sweden will benefit from removing Värnskatten in the context of higher tax revenues. This report will also examine at which tax rate on labour income (förvärvsinkomst) Sweden would optimize its tax revenues. This is done to discover policy relevant information and strengthen the conclusion of previous studies in field.
The questions that will be answered are:

- Will the tax revenue increase with the removal of Värnskatten in Sweden?
- At which top marginal labour tax rate will Sweden maximize its tax revenues?

The hypothesis is that a reduced top marginal tax rate will generate more tax revenue from the income bracket from the population being subject to Värnskatten. This is a hypothesis that is in line with the consensus of reports from Lundberg (2017), Trabandt & Uhlig (2012), Sørensen (2010) and Holmlund & Söderström (2007). This means that Sweden would be situated to the right of the peak of the curve and would therefore benefit from a tax reduction in the long run, given that other variables are held constant.

This report only uses income data from 2017 to calculate Pareto values. The Pareto value explains the distribution of incomes and is used to calculate the shape of the Laffer curve. This value will be further explained in section 3.4. This report also ignores the tax reduction (jobbskatteavdraget) that is phased in for the top income earners and later phased out. The reason for this is because our dataset is not precise enough for us to make that distinction.
2. Background

2.1 The Concept of the Laffer Curve

As briefly mentioned in the introduction, the concept of the Laffer curve was introduced by the economist Arthur Laffer. This concept of tax efficiency is, however, not a new concept. The theory of tax efficiency can be dated back to the 14th century and Arthur Laffer acknowledged this himself (Laffer, 2004).

To begin explaining the concept, basic assumptions need to be presented. The core idea of the Laffer curve is that increases in tax rates generates increased tax revenue up until a point where the tax revenue starts to decline. The tax revenue eventually reaches 0 at a 100% tax rate, therefore giving rise to the inverted U shape. At the peak of the curve we find the optimal tax rates needed for maximizing tax revenue as can be seen in Figure 1.

*Figure 1. Laffer curve, the relationship between tax rates and tax revenues.*

The reason that the Laffer curve starts to decrease after a certain point is because taxes have an effect of “penalizing participation in the taxed activities”, as Laffer himself puts it (Laffer, 2004). The assumption of no tax revenues at a tax rate of 100% is straightforward. If all your income was taken as taxes, then according to a rational utility maximizing individual, no one would work. However, one might assume that people will start working in the untaxed informal
sector. Furthermore, the assumption of zero tax revenues at a 0% tax rate is also clear. Given these assumptions, a point at which the optimal tax revenues are collected should be somewhere between 0 and 100% (Laffer, 2004). Furthermore, in the case of top income earners, leisure time should be valued more than for low income earners. This assumption is argued for in Reynolds (2012) and Saez & Gruber (2002) where top income earners are more elastic than low income earners. This could be because top income earners are far above the subsistence levels, and therefore value leisure time more. Another aspect is that high tax rates for the top income earners could make them move abroad to another country with lower taxes.

2.2 Historical Tax Rates in Sweden

By looking more specifically on the case of Sweden, it is of interest to go through the context behind the top marginal tax rates in Sweden. The tax reform in 1991 drastically decreased the government income tax down from 45% to 20% (NE, n.d.). This government income tax of 20% would be paid for the part of the income that exceeded 170 000 SEK/year (SCB, 2019). With a municipality tax of 30%, the idea was to be able to keep half of your income when you summed up both taxes. With a government shift in 1995, the new prime minister introduced Värnskatten which was an increase of the government income tax of 5% points from 20% to 25% for people earning more than 209 000 SEK/year at the time. The extra 5% points were officially argued to strengthen the government’s finances for the coming 5 years and would thereafter be removed. This never happened and the tax is still intact (NE, n.d.).

In 1999, the additional 5% points tax was separated from the government income tax and was only to be paid if the income exceeded 360 000 SEK/year. At the time, the government income tax of 20% was paid on the part of the income that exceeded 219 000 SEK/year. Today, the income that exceeds 490 700 SEK/year, is subject to the government income tax. An income from labour that exceeds 689 300 SEK/year is thereafter targeted with an additional 5%, which is the today's Värnskatten (Skatteverket, n.d.). With an agreement in January 2019 between 4 political parties in the parliament, it was decided that on the first of January 2020, Värnskatten was going to be removed (Gabrielsson, 2019). The reason for this was to stimulate the population to work more and to make it more desirable to educate yourself. It would also make Sweden's labour market more attractive to high educated workers from foreign countries. (Liberalerna, n.d.).
2.3 Previous Studies

Lundberg’s (2017) report on the Swedish Laffer curve, uses the main idea of the Laffer curve as being a function of the Pareto parameter, which measures the distribution of income for top income earners, with a given elasticity. This being the case, you would only need to know the labour income data for the top labour income bracket to calculate the optimal tax rate. In a previous study from Badel (2013), an expression for the equation of the Laffer curve is derived. However, this equation does not take into account the number of high-income payers and thus only holds for the representative household as Lundberg (2017) argues. Lundberg (2017) derives a model which lands on the following expression:

\[
R(T) = N_0(z_b - b)\tau(1 - \tau)^{\alpha \varepsilon},
\]

where \(N_0\) is the number of people in the tax bracket, \(z_b\) is the average income of the people in the bracket, \(R(T)\) is the government's tax revenue and \(b\) is the bracket threshold. \(\tau, \alpha, \varepsilon\) is the tax rate, Pareto value and elasticity respectively.

The method of using the Pareto parameter to calculate Laffer curves is derived from the work of Saez (2001). Saez (2001) derives an equation for the optimal top tax rate, showing why it is equal to:

\[
T = 1/(1 + \alpha \varepsilon).
\]

Equation (2) will be further explained in the theory section of this report. Looking at this equation you can see that the only parameters that dictates the characteristics of the curve is the Pareto value and the elasticity. The Pareto value and the elasticity constant are therefore the most important factors in this report, which we will go into more depth on. Saez’s (2001) equation (2) is linked to equation (1) which is derived from Lundberg (2017). What differentiates them is that Lundberg’s (2017) equation adds the number of individuals in the bracket and average incomes in the bracket to the equation. This is done to see how the government's revenue changes when tax rates are varied. This then gives rise to the Laffer curve.

As we will discuss in the following part, there are other approaches to derive the Laffer curve. In Trabandt & Uhling (2012), the Laffer curve was estimated using a neoclassical model on
the US and EU14. They derived the steady state values for several different variables along the balanced growth path to trace the steady state tax revenue when the tax rate was varied. Their calculation is based on the assumptions of different constant elasticities for household income, labour and utility functions. The report did not only cover the average income taxes, but also the consumption and capital taxes. Their method to calculate the estimated curves is complex but it is a result that would indicate a narrower margin of uncertainty in the conclusions made. Since we have decided to mainly analyse the top marginal tax rate of labour income, we will not use this report as a primary source.

Looking at previous studies in this field, we find that different methods and assumptions has been used. Several studies analysing the Swedish optimal tax rate has been made. Our hypothesis is in line with most of them. Reports such as Lundberg (2017), Trabandt & Uhlig (2012), Holmlund & Söderström (2007) as well as Sørensen (2010) have come to the conclusion that the removal of Värnskatten would be self-financing.
3. Theory

3.1. The Theory of Labour Elasticity

The elasticity and the Pareto parameter are the two variables that affect the shape of the Laffer curve. The elasticity is the parameter that estimates an individual's marginal propensity to choose between working and spending the time on leisure activities. Since the labour elasticity is the worker's behavioural response of a tax shift, workers in this economy can change the labour elasticity in several ways. Either by choosing to work or not work at all, by working more hours or less hours or by choosing between accepting and not accepting a job that pays more but require higher workload. We can see that wages will most certainly affect labour supply since an individual's behavioural response is linked to the utility maximization of that individual. All things equal, a wage decrease in a certain field of work will decrease the incentives to work and a wage increase will increase incentives to work according to the substitute effect. On the other hand, the income effect will have the opposite effect since a person will have to work more in order to reach the same income after a wage decrease (McClelland & Mok, 2019). We can assume that a shift in taxes will have the same effect on an individual's utility as the wage shift has. A tax change can therefore be seen as a change in wages. This leads to that tax rates affects the labour elasticity (Saez, 2001).

3.2. The Theory of Income Elasticity

As mentioned earlier, the elasticity of labour is one of the central pieces in this report. We can extend the model with income elasticity as well. Lundberg (2017) uses a model to derive an equation for the Laffer curve where the income effect is included. The income elasticity of demand measures the change of demand of a good or service with a change in income (NE, n.d.). Then the change in taxes will not only affect labour supply but also have an effect on consumption.

The function for the Laffer curve with income effects in Lundberg (2017) is given by the equation:

\[ R(T) = N_0(z_b - b)\tau(1 - \tau)^{\epsilon\alpha + \eta}, \] (3)
where $\eta = (1 - t)\partial z / \partial m < 0$ and $\varepsilon$ in this case is the compensated elasticity and will be denoted as $\zeta^c$ later in this report.

The income effect does not seem to have a high impact on top marginal incomes. Lundberg (2017) mentions that this will have a small effect on where the Laffer peak would situate itself. The reason being is because the income effect, parameter $(n)$, is not amplifying the Pareto value and labour elasticity. The compensated elasticity, on the other hand, has a big impact because it is amplified by the Pareto parameter when $\alpha > 1$. The optimal tax rate was 2 percentage points higher when including income effect. Because of the small effects, we will not focus on income effects in this report but rather mentioning its possible effects on the results (Lundberg, 2017).

3.3. Different Elasticities

The model for calculating labour elasticities is based on studying the individual income data. An elasticity is then estimated over a time period by looking at what the resulting income change and hence, behavioural change is when a tax shift occurred. A simulation is then set up to estimate what a tax shift would result in.

Studies written in the 1980s focus mainly on estimations using continuous labour supply models based on Hausman (1981). In the last one or two decades, the studies on the Laffer curve has intensified in a new era described as “the new tax responsiveness literature” (Ericsson et al., 2015). Since the elasticities can change significantly over the course of time, the period of observation is an important aspect to consider. The choice of elasticity is also a critical aspect because their implication for economic policy can be large. As we in this report, use elasticities from previous reports unlike the Pareto value which we calculate, the elasticity is of greater importance for the result. Many reports have made micro calculations on households in Sweden to estimate the elasticity of the labour supply when tax policies have changed. Ericson et al. (2015) made a summary of previous reports on the labour elasticity in Sweden, summarizing the result from 6 previous studies. The result is shown in the Table 1.
Table 1. Elasticities in Sweden. Swedish data.

<table>
<thead>
<tr>
<th>Author</th>
<th>Data (Year)</th>
<th>Tax Change Description</th>
<th>Sample</th>
<th>Elasticity Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selén (2002)</td>
<td>HINK 1988 &amp; 1992</td>
<td>1991 Men age 25-55</td>
<td></td>
<td>Taxable Income (other income sources are included in the analysis): 0.2-0.4</td>
</tr>
<tr>
<td>Hansson (2007)</td>
<td>LINDA 1989 &amp; 1992</td>
<td>1991 Labor Income &gt;0 1989 &amp; 1992 Unchanged marital status and family size Age 25-60</td>
<td></td>
<td>Earned Income: Men 0.29 and 0.49 Women 0.47 and 0.76</td>
</tr>
<tr>
<td>Gelber (2014)</td>
<td>LINDA 1989-91</td>
<td>1991 Married/Cohabitation Age 18-65 Labor Income First Year &gt;0 Not self-employed</td>
<td></td>
<td>Earned Income: Men 0.41 Women 0.47</td>
</tr>
</tbody>
</table>

Note: Elasticities measured from Swedish reports covering the years 1981-2011. Ljunge & Ragan (2006) which was contained in Ericson et al. (2015) has been excluded since it was not able to be accessed.

Source: Modified Table from Ericson & Flood (2015).

Table 1 is a modified version taken from Ericson et al. (2015) where Ericson et al. (2015) result is also taken into account. The reports measure the labour elasticity for all income brackets when tax changes occur. Most of the reports cover the large Swedish tax reform in 1991. The column Data (Year) shows which database the data is taken from as well as which years the study covers. The column Tax Change shows which years of tax changes the studies covered. Most of the reports cover the large Swedish tax reform in 1991 even though there has been smaller tax changes before and after that.

By using these reports, we can construct an average elasticity for Sweden. This is done by calculating the mean from the elasticities in Table 1. Since Selén (2002) has calculated the elasticity with all income, we have chosen to exclude this report from our average calculation of the elasticity in Sweden. In Hansson (2007), 2 different methods are used which gives 2 different results for both men and women. The mean from these methods are used when calculating the average Swedish labour elasticity. In Gelber (2014) and Blomquist & Selin (2010), the mean of men and women are used. In Ericson et al. (2015), the mean between single males and married cohabited males is used as well for the mean of single females and married cohabited females. The average elasticity calculated from Table 1 is as follows:
The average labour elasticity in Sweden is 0.42.

Ericson et al. (2015) has the lowest elasticity in comparison to the reports used to calculate the elasticity average. The time span of the data used in their report, however, does not cover the major tax change in 1991 which could be a reason for the lower elasticity according to Ericson et al. (2015). Another difference in this report is that they exclude income earners with an income higher than 600 SEK/hour. However, Ericson et al. (2015) argue that since the sample size of those income earners is so small, this does not have a significant impact on the result. As previously mentioned in the introduction. Reynolds (2012) and Saez & Gruber (2002) shows that top income earners are more elastic than low income earners. Since the elasticities in the reports in table 1 is calculated with observed elasticity from all income brackets, these elasticities are conservative in this regard and the elasticity could be higher if only top income earners were observed.

In Lundberg (2017), an elasticity of 0.2 is used. Lundberg (2017) comments that a conservative approach is taken to avoid overestimating the effect of the elasticity. Lundberg (2017) also mentions that the consensus on the elasticity tends to show higher results then 0.2, which is strengthened by our own calculations above resulting in an average elasticity of 0.42 based on previous Swedish studies. Looking at our mentioned studies, the elasticities seem to vary substantially. It seems like the elasticities depend strongly on which type of household being analysed, as well as in which given time period. This creates some problems with the way the average elasticity of 0.42 was calculated. Firstly, some of the reports have observed the elasticities of individuals with different marital status which is not weighted for. Nonetheless, some of the reports in Table 1 shows a higher elasticity for married women, the opposite relationship is also shown in the Table. Secondly, the reports in Table 1 have included observed elasticities from all income brackets. However, as previously mentioned in the introduction, Reynolds (2012) and Saez & Gruber (2002) shows that top income earners are more elastic than low income earners. Hence, these elasticities are conservative in this regard. Even if the top income earners elasticity would be shown separately, we would have another problem with the fact that the samples are not weighted for the distribution of gender, which would have been needed for a correct comparison.
The elasticity data from Table 1, imply that group differences may exist. It is however, difficult to get a precise value for the elasticity, as we have argued for. For this reason, we will use several elasticities to compare with. The average elasticities on the Swedish reports, made mainly during the 90s, we calculated to 0.42. In Saez et al. (2009) estimation of the long run elasticity is mentioned to most likely be 0.12 - 0.4, which would have an average of 0.26. This is also the elasticity Saez uses in his calculations in Saez (2001). In our calculations of the Laffer curve later in this report, we will therefore use the elasticities of 0.42, 0.26 and 0.2. The elasticity of 0.2 is the most conservative and will therefore be used in our base calculation.

3.4. Theory of Pareto Distribution

The Pareto distribution is a theory that can be found in the realm of statistical probability. This theory is a distribution that first was described by Vilfredo Pareto. He identified that the inequality of incomes among populations followed a power law distribution which later became the Pareto distribution. He claimed that the wealth and the income in a society was disproportionately distributed, with a large accumulation of the wealth and income at the top (Corporate Finance Institute, 2019). The Pareto distribution is characterized by a power law, which means that the fraction of people with incomes greater than some cut-off level is proportional to the cut-off level raised to some power (Jones, 2015). The Pareto distribution can be calculated as follows:

\[ F(x) = 1 - \left(\frac{k}{x}\right)^\alpha \],  

where \( x \) is a random variable (the variable of the distribution). \( k \) is the lower bound on data and is the shape parameter (Corporate Finance Institute, 2019). This means that the Pareto distribution explains the proportionality between different levels of incomes and can then be used to give us a Pareto value for a certain income bracket. This value then explains the distribution in that income bracket. If the Pareto value were to be 3 for example, 1/3 of the average top bracket tax payer’s income would be subject to the top marginal tax rate. Further, that would mean that a proportional tax would yield a Pareto value of 1, i.e. a single tax rate for all incomes. Furthering a progressive tax system would yield a higher Pareto value (Lundberg, 2017).

The effect of a change in tax rate is divided up into two parts. The mechanical part is the part that is based on the proportional increase in revenues a tax rate increase would yield,
considering there are no behaviour changes. A mechanical Laffer curve would therefore be a straight line. The other part is the behavioural response which shrinks the effect of the mechanical increase when rates are increased. When tax rates are to the right side of the peak, an increase in tax rate would have a greater behavioural decrease in tax base than the mechanical tax increase would generate (Lundberg, 2017). The mechanical change can be expressed as the difference between average income and the tax threshold, while the behavioural change in revenue depends on average income. The relationship between these two is expressed as the Pareto value (Lundberg, 2017).

In Jones (2015), we can read about Piketty's theory on inequality. He explained that when returns on assets are greater than the economies average growth (GDP/worker growth), inequality increases. Piketty’s complete formula is that when equation (5) holds true, inequality grows:

\[
\frac{r-g-t-a}{n+d} > 0 .
\]

(5)

\( r \) is return on assets, \( g \) is the growth in the economy, \( t \) are taxes on wealth, and \( \alpha \) is the fraction of wealth that gets spent on consumption. Population growth \( n \) and death rate \( d \), also affect inequality by deluding wealth (Jones, 2015). This theory on wealth inequality is important when explaining the historical changes in Pareto values which will be brought up in the result. The link wealth inequality has with income inequality will also be discussed later in the discussion section of this report.

3.5. Calculating the Effective Tax

When calculating the Laffer curves in this report, the resulting optimal tax rate to maximize the tax revenue will have to be compared to the actual top marginal tax rate. More specifically, we need to establish the effective tax rate which is the tax rate the top marginal tax payers are subject to. To calculate the top marginal income tax rate, we use the following equation:

\[
\tau_t = \tau_m + \tau_g + \tau_v .
\]

(6)
where \( \tau_m \) is the municipal taxation which ranged between 29-35\% and had an average of 32.12\% in 2017 (Lundberg, 2017). \( \tau_g \) is the government tax which is 20\% and \( \tau_v \) stands for Värnskatten which is 5\% (Skatteverket, n.d.).

However, these are not the only taxes a top income earner is subject to. The employer of the top income earner has to pay a payroll tax which is 31.42\% in 2017 (SCB, 2017). Furthermore, there are consumption taxes that the top income earner has to pay when consuming the wage that is left after the income tax is paid. There are different ways to calculate the consumption tax. We have, for example, Du Rietz, Johansson & Stenkula (2015) where they divided excise tax revenue and value added tax (VAT) by the total private consumption. Their consumption tax landed at 25\% (Du Rietz et al. 2015). Sørensen (2010), however, made an even more thorough calculation where he accounted for VAT and excise duties on both the final goods as well as the VAT-exempt sectors. Capital gain taxes on real estate, property taxes and VAT on new residences as a tax on the consumption of housing services were also included. By multiplying the income that is left after paying the marginal income tax, we get the income that is subject to the consumption tax. He also landed on a consumption tax at 25\% (Sørensen 2010). Therefore, we have chosen to use their results in our calculations. The top marginal income tax, consumption tax and the payroll tax are the taxes the top marginal tax payer is subject to and is referred to as the effective tax, \( e \), and the equation for this tax is:

\[
\tau_e(z) = \frac{\tau_p + \tau_i(z) + \tau_c(1 - \tau_i(z))}{1 + \tau_p},
\]

where \( \tau_p \) is the payroll tax, \( \tau_i \) is the income tax, \( \tau_c \) is the consumption tax and \( z \) is the taxable income. To obtain the increase in tax payments caused by one unit increase in taxable income, we differentiate the tax liability with respect to the taxable income. We then put the tax revenue from an increase in \( z \), in relation to the increase in total employee compensation which is \( 1 + p \) (Lundberg, 2017). This is defined as the tax wedge, which is the ratio between the amount of taxes a worker pays and the actual cost for the employer. The reason why this is important to take into consideration is because the tax wedge measures to what degree the tax on labour income holds back employment (OECD 2019). The reason why we do not take into account the social benefits from the payroll taxes is because workers paying Värnskatten do not receive any additional benefits and therefore it can be seen as a pure tax (Lundberg, 2016).
3.6. Elasticity Concept

In Saez (2001), explanations for the way to calculate the elasticity for the top bracket income earners is provided.

First, we set up the utility function for an individual in this economy as:

\[ u = f(z, c), \tag{8} \]

which depends negatively on taxable income \( z \) because spending time working lowers your leisure time, and positively on consumption \( c \) as shown in equation (9):

\[ u_c = \frac{du}{dc} > 0 \quad u_z = \frac{du}{dz} < 0. \tag{9} \]

An individual in this economy has the following consumption constraint:

\[ c = z(1 - \tau) + R, \tag{10} \]

where \( \tau \) is the top marginal tax rate and \( R \) is the non-labour income. The first order condition for individual utility maximization is:

\[ (1 - \tau)u_c + u_z = 0. \tag{11} \]

Using the utility constraint, we end up with an equation for the uncompensated elasticity, \( \zeta^u \), which is:

\[ \zeta^u = \frac{1-\tau}{z} \frac{\partial z}{\partial (1-\tau)}. \tag{12} \]

Saez (2001) sets up a new expression for finding the optimal tax rate:

\[ \bar{\tau} = \frac{1-\bar{\theta}}{1-\bar{\theta} + \zeta^u + \zeta^c(\alpha - \tau)}. \tag{13} \]

The optimal rate in this case is dependent on a Pareto value, which is calculated according to the formula:
\[ \alpha = \frac{e_b}{x_b-b} \]  

(14)

As Saez (2001) explained, when the ratio of average income in the threshold to the average income is above a certain level, the ratio goes towards a constant number. This is the value we denote as the Pareto value, calculated with income data for a given year. An example of this can be shown in Figure 2 from Lundberg (2017).

*Figure 2. The distribution of incomes in Sweden compared to the Pareto distribution.*

Figure 2 shows how the income is distributed over the population. The Pareto distribution as we can see matches with the income distribution for top income earners. This is an important factor to consider. We need the Pareto values to be constant over the individuals that are located over the threshold. If this falls true, we can use the Pareto distribution for the top marginal incomes. As we see in the Figure above, over the threshold for central income tax, the observed data matches the Pareto distribution, which is why we can use it to calculate top marginal Laffer curves with it.
In equation (15) we can see Slutsky equation which shows the relationship between the uncompensated elasticity and compensated elasticity:

$$\zeta^c = \zeta^u - \eta.$$  \hspace{1cm} (15)

Assuming the uncompensated state when income effects are not being included, we put $\eta = 0$ this makes the elasticity a single expression. Furthermore, $g$ in equation (13) is the government's ratio of utility between maximizing income earners $z$ utility and utility of public funds. In our base assumption, $g = 0$ because we assume government is indifferent to top income earners utility. I.e. 1 dollar is worth as much in the government's bank account as it is in the high-income earners account.

Given our assumptions of $g = 0$ and $\zeta^c = \zeta^u$, equation (8) become a much simpler equation which we can use to calculate the optimal top tax rate which is derived in Saez (2001). The simplified version is shown in equation (16):

$$\bar{\tau} = 1/(1 + \alpha \zeta),$$  \hspace{1cm} (16)

where $\bar{\tau}$ is the optimal tax rate. We will be using equation (16) and changing the elasticity figure from $\zeta$ to $\varepsilon$, and the optimal tax rate $\bar{\tau}$ to $T$. The equation will then look like the following:

$$T = 1/(1 + \alpha \varepsilon).$$  \hspace{1cm} (17)

This equation is the equation we introduced in the introduction and we have now shown how to derive it.

3.7. Derivation of the Laffer Curve Peak $T = 1/(1 + \alpha \varepsilon)$

The central piece in this report will be based on the Laffer equation from Lundberg (2017), we will now show how Lundberg (2017) derived the equation (12). First of all, tax revenue from the top tax bracket is given by:

$$R = (\bar{z}_b - b)\tau N.$$  \hspace{1cm} (18)
The elasticity, simply put, is the behavioural response to changes in income or wages. Lundberg (2017) shows that the standard measure of this elasticity of taxable income with respect to the net of the tax rate can be put as:

$$
\varepsilon = \frac{dz/z}{d(1-\tau)/(1-\tau)} = -\frac{dz}{d\tau} \frac{1-\tau}{z} \cdot
$$

(19)

However, Lundberg shows that when considering a small tax reform, the number of people in the tax bracket is not the most important aspect. The derivative with respect to taxes is shown to be:

$$
\frac{\partial R}{\partial \tau} = N([Z_b - b] + \tau \frac{dZ_b}{d\tau}) = N([Z_b - b] - \frac{\tau EZ_b}{1-\tau}) ,
$$

(20)

where the first term of this equation shows the mechanical effect of a tax shift. In other words, the change in revenue when there is no behavioural change i.e. when the tax base is kept constant. The second term is the behavioural response to the tax shift. By dividing the second term by the first term, we get a ratio of the mechanical effect to the behavioural effect. When these two ratios are equal, the mechanical revenue loss is regained by the behavioural response when a tax cut is carried out. Thus, the revenue does not change at all. This ratio is the degree of self-financing (DSF) and can be expressed as the following equation:

$$
DSF = -\frac{dR}{dx} \frac{dR}{dx} \frac{1}{z} = -\frac{dZ_b}{d\tau} \frac{\tau}{\tau - \tau b} = \frac{a\tau}{1-\tau} .
$$

(21)

The DSF is fundamental as Lundberg (2017) puts it, the ratio of mechanical revenue loss that is regained by the behavioural response when a tax cut is carried out. Hence, by using DSF, we get a percentage of how much 1 tax point in $t_{(0)}$ will be worth/gained in the next period $t_{(1)}$. Following that, the DSF ratio of the mechanical effect to the behavioural effect, perfectly cancels each other out when we are at the optimal tax rate. The DSF is 1 or (100%). Putting equation (21) equal to the revenue equation (18), we get an expression for the Laffer curve peak:

$$
T = 1/(1 + a\varepsilon) .
$$

I.e. the equation derived in Saez (2001).
3.8. Derivation of Laffer Curve Equation \( R(T) = N_0(z_b - b)\tau(1 - \tau)^{e\alpha} \)

If we follow the assumption, that potential incomes that appear greater than a bracket threshold, \( b \), will follow a Pareto distribution. Which was explained in the theory section about Pareto distribution. The cumulative distribution function of potential incomes, \( F_0 \), is given by:

\[
1 - F_0(z_0) = N_0 \left( \frac{b}{z_0} \right)^{\alpha}, \tag{22}
\]

where \( N_0 \) has been multiplied in, and is the number of people who would be in the tax bracket if the tax rate was zero. The individual maximizes utility to the budget following constraint:

\[
c = (1 - \tau)z + y. \tag{23}
\]

Using the budget constraint, the taxable income supply equation is:

\[
z = z_0(1 - T)^e. \tag{24}
\]

By inverting equation (24) to get \( z_0 \) on the left side, we can get an expression for the potential labour income. This represent the income that would be present in absence of taxation. We then find that the people who will be in the top tax bracket is given by the number of people whose potential income exceeds \( b_0 \):

\[
b_0 = \frac{b}{(1 - \tau)^e}. \tag{25}
\]

Putting equation 25 into equation 22, we get:

\[
N(\tau) = 1 - F_0(b_0) = N_0(1 - \tau)^{ae}. \tag{26}
\]

Substituting this into equation (18) we end up with our final equation for the Laffer curve:

\[
R(T) = N_0(z_b - b)\tau(1 - \tau)^{ae}. \tag{26}
\]

This equation, as previously mentioned, was taken from Lundberg (2017). This is subsequently what we use for calculating the Laffer curves.
4. Method

The method in this report will be based extensively on the previous work of Lundberg (2017) and Saez (2001). The reason for basing this report on these previous studies is because of their relevance in this field of calculating Laffer curves, and partly because micro databases were not able to be accessed. Lundberg (2017) and other reports such as Sørensen (2010), calculated Swedish Laffer curves. This reports goal is to complement their findings by using different elasticities and historical Pareto values. This is to discover policy relevant information and strengthen and confirm previous results. This report will use Pareto values calculated from income distribution data in Sweden and use estimations of labour and income elasticities gathered from previous studies in the field. Additionally, to improve the result, the most plausible ranges on elasticities will be analysed. This report also analyses historic records of Pareto distributions to estimate the historic ranges using the Pareto values taken from the World Inequality Database. Calculations for the Laffer curves is created using the equation (1):

\[ R(T) = N_0(z_b - b)\tau(1 - \tau)^{\alpha\varepsilon}, \]

where \( N_0 \) is number of people in the tax bracket, \( z_b \) is the average income of the people in the tax bracket and \( b \) is the threshold of the bracket. Entering this equation into the statistical program STATA, Laffer curves can be extracted by estimating how \( R(T) \) (the tax revenues) changes when the tax rate \( \tau \) is varied. To calculate the optimal tax rates for maximizing tax revenue we use equation (2):

\[ T = 1/(1 + \alpha\varepsilon). \]

The optimal tax rate for maximizing tax revenue is then compared with the effective tax rate, as derived in section (3.5) and is shown in equation (7). In the result section of this report, we begin with calculating Pareto values using labour income data taken from 2017. This Pareto value is then used together with our base elasticity of 0.2 taken from Lundberg (2017), to calculate a Laffer curve. We then calculate the resulting tax revenue change due to a removal of Värnskatten. Further, we use different Pareto values depending on the number of people in the last bracket. After this, different elasticities and different historic Pareto values are used to calculate optimal tax rates for maximizing tax revenue.
5. Data

We used income data including labour income gathered from Statistic Sweden (Statistiska centralbyrån, 2019, see Table 2). The income data includes income from wages, pension, sick leave, parental leave, disability leave, unemployment benefits and business activity income. Income from capital is excluded. The income data is divided into different income brackets with their respective share of the population. For the different taxes and tax rates, we used Statistics Sweden and Swedish tax agency (Skatteverket). The consumption tax rate is extracted from Rietz et al, (2013) and Sørensen (2010). We use 3 different elasticities, (0.2, 0.26, and 0.42), which were taken from earlier studies which we discussed in the theory section.

Table 2. Amount of people in different income brackets from labour income 2017 for ages 20-65.

<table>
<thead>
<tr>
<th>Income (tSEK)</th>
<th>Average Income (tSEK)</th>
<th>Population</th>
<th>Tax Base (tSEK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>450-499</td>
<td>475</td>
<td>362 007</td>
<td>171 993 325</td>
</tr>
<tr>
<td>500-549</td>
<td>525</td>
<td>229 134</td>
<td>120 295 350</td>
</tr>
<tr>
<td>550-599</td>
<td>575</td>
<td>158 349</td>
<td>89 900 675</td>
</tr>
<tr>
<td>600-649</td>
<td>625</td>
<td>109 669</td>
<td>68 543 125</td>
</tr>
<tr>
<td>650-699</td>
<td>675</td>
<td>76 045</td>
<td>51 330 375</td>
</tr>
<tr>
<td>700-749</td>
<td>725</td>
<td>54 147</td>
<td>39 256 575</td>
</tr>
<tr>
<td>750-799</td>
<td>775</td>
<td>39 966</td>
<td>30 969 775</td>
</tr>
<tr>
<td>800-849</td>
<td>825</td>
<td>29 612</td>
<td>24 429 900</td>
</tr>
<tr>
<td>850-899</td>
<td>875</td>
<td>23 084</td>
<td>20 198 500</td>
</tr>
<tr>
<td>900-949</td>
<td>925</td>
<td>18 287</td>
<td>16 915 475</td>
</tr>
<tr>
<td>950-999</td>
<td>975</td>
<td>14 382</td>
<td>14 023 460</td>
</tr>
<tr>
<td>1 000-1 049</td>
<td>1025</td>
<td>11 522</td>
<td>11 810 050</td>
</tr>
<tr>
<td>1 050-1 099</td>
<td>1075</td>
<td>9 334</td>
<td>10 034 050</td>
</tr>
<tr>
<td>1 100-1 149</td>
<td>1125</td>
<td>7 591</td>
<td>8 539 875</td>
</tr>
<tr>
<td>1 150-1 199</td>
<td>1175</td>
<td>6 206</td>
<td>7 292 050</td>
</tr>
<tr>
<td>1 200-1 249</td>
<td>1225</td>
<td>5 182</td>
<td>6 347 950</td>
</tr>
<tr>
<td>1 250-1 299</td>
<td>1275</td>
<td>4 334</td>
<td>5 525 850</td>
</tr>
<tr>
<td>1 300-1 349</td>
<td>1325</td>
<td>3 593</td>
<td>4 760 725</td>
</tr>
<tr>
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<td>1375</td>
<td>2 991</td>
<td>4 112 625</td>
</tr>
<tr>
<td>1 400-1 449</td>
<td>1425</td>
<td>2 544</td>
<td>3 825 200</td>
</tr>
<tr>
<td>1 450-1 499</td>
<td>1475</td>
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<td>3 287 775</td>
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</tr>
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<td>1875</td>
<td>876</td>
<td>1 642 500</td>
</tr>
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<td>1 900-1 949</td>
<td>1925</td>
<td>670</td>
<td>1 299 750</td>
</tr>
<tr>
<td>1 950-1 999</td>
<td>1975</td>
<td>667</td>
<td>1 317 325</td>
</tr>
<tr>
<td>2 000-2 499</td>
<td>2250</td>
<td>4 272</td>
<td>9 612 000</td>
</tr>
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<td>2750</td>
<td>2 072</td>
<td>5 698 000</td>
</tr>
<tr>
<td>3 000-</td>
<td>3500</td>
<td>4 099</td>
<td>14 346 500</td>
</tr>
</tbody>
</table>

Note: Population is number of people earning that specific income. Tax base is average income multiplied by population. “(tSEK)” is Swedish crowns (SEK) in thousands.
Source: Modified Excel Table of statistic from SCB (2017).
In Table 2, we see the amount of people in different income brackets starting from 450 000 SEK/year. Since we don’t know the distribution in the different brackets, we have assumed that the average income is in the middle of every bracket range. Our assumption for the highest income bracket (3 000 000- SEK/year) is that it has an average of 3 500 000 SEK/year in Table 2. We have also calculated the different Pareto values if the average would be 5 000 000 SEK/year and 10 000 000 SEK/year. By multiplying the average income with the population in every bracket, we receive the tax base.

For estimating historic optimal tax rates, we used Pareto values gathered from Atkinson, Piketty & Saez (2010). They calculated Pareto values based on values from World Inequality Database. The historical values are used to see how the Pareto value has changed over the years. This means that the optimal tax rate to maximize tax revenue has changed which is of interest to examine. We summarized the data on historic Swedish Pareto values taken from the report from Atkinson et al. (2010) which can be seen in Table 3 below.

Table 3: Estimated Pareto Parameters for the years 1903-2005. Sweden.

<table>
<thead>
<tr>
<th>Year</th>
<th>Pareto</th>
<th>Year</th>
<th>Pareto</th>
<th>Year</th>
<th>Pareto</th>
<th>Year</th>
<th>Pareto</th>
</tr>
</thead>
<tbody>
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<td>1.94</td>
<td>1929</td>
<td>2.56</td>
<td>1955</td>
<td>3.58</td>
<td>1981</td>
<td>3.58</td>
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<td>1930</td>
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<td>1956</td>
<td>3.50</td>
<td>1982</td>
<td>3.50</td>
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<tr>
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<td>1931</td>
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<td>1957</td>
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<td>1983</td>
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<tr>
<td>1906</td>
<td>1.92</td>
<td>1932</td>
<td>2.90</td>
<td>1958</td>
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<td>1984</td>
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<td>1934</td>
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<td>1960</td>
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<td>1986</td>
<td>3.65</td>
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<td>2.93</td>
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<td>1991</td>
<td>2.48</td>
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<tr>
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<td>2.99</td>
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<td>2.39</td>
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<tr>
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<td>1973</td>
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<td>2.32</td>
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<td>1974</td>
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<td>2000</td>
<td>1.96</td>
</tr>
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<td>1949</td>
<td>3.25</td>
<td>1975</td>
<td>2.02</td>
<td>2001</td>
<td>2.02</td>
</tr>
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<td>2.45</td>
<td>1950</td>
<td>3.47</td>
<td>1976</td>
<td>2.11</td>
<td>2002</td>
<td>2.11</td>
</tr>
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<td>1925</td>
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<td>1951</td>
<td>4.07</td>
<td>1977</td>
<td>2.04</td>
<td>2003</td>
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<tr>
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<td>1952</td>
<td>3.72</td>
<td>1978</td>
<td>2.08</td>
<td>2004</td>
<td>2.08</td>
</tr>
<tr>
<td>1927</td>
<td>2.47</td>
<td>1953</td>
<td>3.92</td>
<td>1979</td>
<td>2.07</td>
<td>2005</td>
<td>2.07</td>
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<tr>
<td>1928</td>
<td>2.48</td>
<td>1954</td>
<td>3.82</td>
<td>1980</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Shows the Pareto values for the years 1903-2005 in Sweden. Blanks are years were data is missing.
Source: Own Excel table with data extracted from Atkinson et al. (2010)
6. Result

6.1. Calculation of Pareto Values

To calculate the Pareto value, we will be using the data from Table 1. We can then calculate the average income for the population that is subject to the tax we are looking at $z_b$. This is used to calculate the Pareto value according to the equation (14) which is mention in the theory, where $b$ is the threshold for the tax we are looking at. In this case, to calculate the Pareto value on the *Värnskatten*, we use the threshold of 638 500 SEK/year which was the threshold of *Värnskatten* in 2017 after deductions are made (Skatteverket, n.d.). Since our data is not specific enough, we will calculate the average income for people earning more than 650 000 SEK/year. This is done by adding the tax revenue of the income brackets with earners over 650 000 SEK/year. This adds up to a total of 311 914 250 000 SEK. This is with the assumption that the last bracket has an average income of 3 500 000 SEK/year. This is then divided by the amount of people in those brackets: 311 914 250 000 / 333 131 $\approx$ 936 311 kr. The Pareto value $\alpha$ is:

$$\frac{936 311}{936 311 - 650 000} = 3.27 .$$

Even if 638 500 SEK/year were to be used as the threshold of *Värnskatten*, it would also change the average in that bracket, $z_b$, which cannot be calculated with the data used in this report.
6.2. The Peak of the Laffer Curve

With the Pareto value calculated from the last section and the elasticity from Lundberg (2017), we can estimate the peak of the Laffer curve shown in Figure 3.

*Figure 3. Tax revenue as a function of tax rate with Värnskatten as the tax threshold.*

![Laffer Curve - Top tax bracket (Värnskatten)](image)

Note: The Laffer curve for Top income earners in Sweden, using Pareto of 3.27 and an elasticity of 0.2.
Source: Own calculations. Figure created with STATA.

Figure 3 shows where Sweden is on the Laffer curve as well as the optimal tax rate with the tax rate on X-axis and the tax revenue on the Y-axis. Using the maximizing tax rate equation (2), we get a peak at 60.45%:

$$T = \frac{1}{1 + 3.270.2} = 0.6045 = 60.45\%$$

The effective marginal tax rate, which can be compared to the optimal tax rate, is calculated according to equation (7), and gives us the effective tax rate of:

$$\tau_e = \frac{0.3142 + 0.5712 \times (1 - 0.5712) \times 0.25}{1 + 0.3142} = 73.63\%.$$  

Since the current effective tax rate is 73.63%, we get a 13.18% percent difference.
The Degree of self-financing in this case is calculated according to equation (21) and gives us a Degree of self-financing of:

\[
\frac{(3.27 \times 0.2 \times 0.7363)}{(1 - 0.7363)} = 183\%.
\]

As illustrated in Figure 3, the current tax rate is beyond the peak to the right. This would then mean that tax revenue loss is occurring. The add-on tax rate, *Värnskatten*, is an add-on of 5%. Therefore, this tax could be removed without going over the top. A removal of this tax suggests that tax revenues would increase.

### 6.3. Calculating the Revenue Gain

Using our base calibration of the elasticity 0.2, the revenue gain can be calculated according to a formula from Lundberg (2016). The change in revenue is equal to the behavioural revenue change subtracted to the mechanical change in revenue, according to:

\[
\Delta R = \Delta R_D - \Delta R_S ,
\]

(26)

Where \(\Delta R_D\) is the change in tax revenue due to the behavioural change and \(\Delta R_S\) is the change in tax revenue due to the mechanical change. Equation (27) shows how the mechanical revenue changes:

\[
\Delta R_S = N_0 \times (z_b - b) \times \tau_v \times (1 - \tau_c).
\]

(27)

The mechanical change is simply the tax revenue loss when reducing the tax rate, and in our case, the *Värnskatten* of 5%. By multiplying with what is left after the consumption tax, the income gained by an individual after the removal of *Värnskatten*, will be spend on consumption. Hence, the income returns to the government’s tax revenue. In equation (28) the behavioural revenue change is shown:

\[
\Delta R_D = N_0 \times z_b \times \left[\left(\frac{1-(\tau_i-\tau_v)}{1-\tau_i}\right)^\xi - 1\right] \times ((\tau_i - \tau_v) + \tau_p + (1 - (\tau_i - \tau_v)) \times \tau_c).
\]

(28)

The behavioural change is more complex and will be affected by the effective taxes the individual has to pay, and by the elasticity which captures the behavioural change of a tax
shift. $\tau_i$ is the top marginal tax rate. $(\tau_i - \tau_p)$ is the top marginal tax rate subtracting the Värnskatten of 5%. $\tau_p$ is the payroll tax. $\tau_c$ is the consumption tax.

Entering in our own values in equation (23) and (24) we get the following:

$$\Delta R_S = 333131 \times (936311 - 650000) \times 0.05 \times (1 - 0.25) = 3576715115.$$  

$$\Delta R_D = 333131 \times 936311 \times \left[ \left( \frac{1 - 0.5212}{1 - 0.5712} \right)^{0.2} - 1 \right] \times (0.5212 + 0.3142 + (1 - 0.5212) \times 0.25) = 6644437204.$$  

Entering the resulting values in equation (22), the change in revenue for the government is calculated as follows:

$$\Delta R = \Delta R_D - \Delta R_S = 6644437204 - 3576715115 = 3067722089.$$  

A removal of Värnskatten would increase the tax revenue to the government by more than 3 billion SEK/year. The total taxes collected from all sectors by the Swedish government in 2018 was 1072 billion SEK/year (Ekonomistyrningsverket, 2019). Of the 1072 billion SEK, the government income tax was 59.6 billion SEK. Further the government’s income from Värnskatten in 2016 was 5.2 billion SEK Lundberg (2016), indicating that these taxes are proportionally small.

### 6.4. Using Different Pareto Values

Due to the unknown mean of the individuals earning more than 3 million SEK/year, the Pareto value used in the base calibration is not certain. With changing the last brackets average income, the Pareto value will be affected. By using an average income of 5 million SEK/year, the Pareto value will be calculated as:

$$\frac{955000}{(955000 - 650000)} = 3.13.$$  

By using an average income of 10 million SEK/year, this gives a Pareto value of:
\[
\frac{1016000}{(1016000 - 650000)} = 2.78 .
\]

Lundberg (2017) received a Pareto value of 3.18 and Sørensen (2010) calculated a Pareto value of 3.185, which indicates an average in the last bracket of less than 10 million. By using the different Pareto values and the elasticity of 0.2, the optimal tax rate for Sweden when we use equation (1) is shown in Figure 4.

*Figure 4. Laffer curve with different types of Pareto values.*

![Laffer Curve - Different Types of Pareto Values](image)

Note: Different Laffer curves when the Pareto value is varied depending on the average income in the last bracket in the income dataset.

Source: Own calculations using STATA.

Here we can see the Laffer curve with different Pareto values depending on how we calculated the last bracket in our dataset. The difference is not massive but still important to mention. For the Pareto value to be in line with the results from Lundberg (2017) and Sørenson (2010), the average income in the last bracket has to be below 10 million.
6.5. Optimal Tax Rate with Different Elasticities

By using the elasticity from Lundberg (2017) along with Saez’s (2001) long run elasticity average and our own calculated average based on previous Swedish reports, we can calculate the optimal tax rates using the 3 different Pareto values from previous section. This is done by using equation (2). The optimal tax rate for the different elasticities and Pareto values are shown in Table 4.

Table 4. Optimal tax rate for different elasticities and Pareto values.

<table>
<thead>
<tr>
<th>Pareto Value</th>
<th>Elasticity</th>
<th>Optimal Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0,2</td>
<td>0,26</td>
</tr>
<tr>
<td>2,78</td>
<td>64%</td>
<td>58%</td>
</tr>
<tr>
<td>3,13</td>
<td>61%</td>
<td>55%</td>
</tr>
<tr>
<td>3,27</td>
<td>60%</td>
<td>54%</td>
</tr>
</tbody>
</table>

Note: Optimal tax rates in (%) using different Pareto values and elasticities.
Source: Own calculations using Excel.

The optimal tax rate ranges from 42% to 64%. The elasticity has a large impact since the difference in the optimal tax rate from using the elasticity in Lundberg compared to the average elasticities in the Swedish reports are around 18%-points. Also, none of the optimal tax rates exceeds the effective tax rate of 74%.
Figure 5. Pareto values over time in Sweden.

Note: Pareto values over time in Sweden, from World Inequality Database.
Source: Pareto values taken from Atkinson et al. (2010).

Figure 5 shows how the Pareto value has changed over time in Sweden. The following data is taken from Atkinson et al. (2010). Using the historical data on the Pareto values, we are able to achieve two things. Firstly, we can determine if the Pareto values differ over time. Secondly, we are able to see how this range of values would affect the optimal tax rates and to what degree.

The calculation method is the same as extracted from the World Inequality Database. The Pareto value taken from this database is calculated on all incomes. Thus, the Pareto value is lower compared to the calculations that were based on labour income only, which is the method we used for calculating our own Pareto values. In Lundberg (2017), a Pareto value was calculated with micro data. We replicated that in our report with the income bracket data from SCB. We did not have access to any microdata ourselves, so we used data from the World Inequality Database, which calculates on all income to get a comparison over time. We wanted to use this value to symbolize the rough change that Pareto distribution in income has had. We see a long trend of increase which ends in 1980 and decreasing after. The latest value, in 2005, is below the average mean.
Looking at Figure 5, we can see wide changes over time. Analysing this, we know from the theory that a low Pareto value could mean that the distribution of incomes is more unequal and that high Pareto values could indicate more income equality. However, changes in tax policy, i.e. choosing between a more progressive or proportional tax system can also affect the Pareto value.

With an effective tax rate in 2017 at 73.63%, it is possible to use the Pareto values, taken from World Inequality Database, to examine whether the peak of the Laffer historically has been located to the right of today’s effective tax. Instead of the equation (2) being a function of optimal tax rate $T$, we rewrite it as to be a function of the Pareto value which can be seen in equation (29) below:

$$\alpha = \frac{1}{\frac{T}{\epsilon}}.$$  \hspace{1cm} (29)

By using elasticities 0.2, 0.26 and 0.42 we find that the Pareto values are needed so that it will be at today’s peak shown in the following 3 calculations:

**Elasticity 0.2**

$$\frac{1}{\frac{0.7363}{0.2}} = 1.79$$

**Elasticity 0.26**

$$\frac{1}{\frac{0.7363}{0.26}} = 1.38$$

**Elasticity 0.42**

$$\frac{1}{\frac{0.7363}{0.42}} = 0.85$$

With an elasticity of 0.2, it gives incentives to reduce the tax when the Pareto value is over 1.79. According to Figure 5, this means that historically, the only time it can be policy-relevant to keep the add-on tax of 5%-points, was during the early twentieth century. But for the majority of time, it is highly beneficial in a tax revenue maximizing context to remove such a tax.
Lundberg (2017) comments that World Inequality Database Pareto values tend to be smaller compared to LIS Pareto values because of the nature of how the values is calculated. The World Inequality Database values include all income which differs from LIS, since it only takes into account labour income. Lundberg (2017) argues that since capital income and labour income are taxed separately, it would be more accurate to use Pareto values that only take into account labour income. The World Inequality Database Pareto is also smaller than his own calculations based on microdata. This implies that we can argue more strongly that over time, the tax cut would have been more beneficial. However, using elasticities of 0.26 and 0.42, the Pareto value of all income has always been over 1.377 and therefore it has been beneficial historically to reduce the marginal tax rate we have today.

6.6. Historically Optimal Tax Rates

Using the historical Pareto values based on World inequality Database together with our three chosen elasticities estimates, we can get optimal tax rates from a historical perspective which is shown in Table 4.

Table 5. The optimal tax rate with different Elasticities and historical Pareto values

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>0,2</th>
<th>0,26</th>
<th>0,42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Value</td>
<td>Optimal Tax Rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,45</td>
<td>78%</td>
<td>73%</td>
<td>62%</td>
</tr>
<tr>
<td>2.63 (Mean)</td>
<td>66%</td>
<td>59%</td>
<td>48%</td>
</tr>
<tr>
<td>4,07</td>
<td>55%</td>
<td>49%</td>
<td>37%</td>
</tr>
</tbody>
</table>

Note: Optimal tax rates for different elasticities and the different historic Pareto values in Sweden.
Source: Own calculations using Excel.

The historical mean of the Pareto value on the standard elasticity in this report, $\varepsilon = 0.2$, gives us the optimal rate of 66%. This is higher than our original calculations that generated a 60% peak. This is because today’s Pareto value being lower than the historical peak. Looking at Table 5 above, we can see that at one instance, we have optimal rates above today’s effective tax rate, where $\alpha = 1.45$ and $\varepsilon = 0.2$ we get a 78% optimal tax rate. However, as mentioned previously, the World Inequality Database values should be taken with a degree of uncertainty
because of the way it’s calculated. For most of the history, the optimal tax rates have been lower than today’s effective tax rate and a reduction can be argued for.
7. Discussion

One of the main points in our assumptions is the elasticity. The Pareto value is fairly precise in its calculation and consistent in the distribution for top income earners. However, the elasticity still has to be assumed. Throughout this report, we used different kinds of elasticities that were chosen in an analysis of the results of previous studies. When establishing a range of plausible optimal tax levels, the results were in line with several previous studies as Lundberg (2017), Saez (2001) and Trabandt & Uhlig (2012).

We found it difficult to gather data since micro databases were unavailable to us and the Statistics Sweden data set, that we relied on in this report, do not have any specific historical data available further back than 1999. Furthermore, the data was not complete for all brackets of income earners. To make the analysis more complete, we tried to estimate both the plausible range of Pareto values and historical Pareto values gathered from World Inequality Database. The reasoning was that the results would strengthen the hypothesis in this report as well as the work from previous studies.

Another important point of discussion is the problem of not having data for the last percentile of income earners. As mentioned earlier, we used an estimate of what the average income could be. While this seems unreliable, we did calculations on what the effect of a significantly large and small value would have on the optimal tax level to maximize tax revenue. Our restrictions on not being able to access micro data made us only use the World Inequality Database Pareto data in the historic analyses. This is not optimal since the World Inequality Database takes all incomes into account. Therefore, micro data for only labour incomes should have been used. However, it gave us insight about the historical aspects.

In Ericson et al. (2015), a summary of different Swedish reports shows how different elasticities can be assumed. However, it is important to notice that many of these reports only use a time span of around 3 years. A central piece of the theory is that the effects of labour elasticity only work in its fullest in the long run since the behavioural change takes time. If this assumption is correct, this would mean that the elasticities measured from the Swedish reports would be higher if the time spans were longer. This argues for an even lower optimal tax rate than the ones presented in the result.
In this report, we used data on labour income, where income from business activity is included. This differs from Lundberg (2017) since he excludes income from business activity. Sørensen (2010), on the other hand, included income from business activity and came to an almost identical conclusion as Lundberg (2017). Therefore, it does not seem to matter too much if we include or exclude income from business activity. Since Statistic Sweden published more specific data on labour income, including business activity, we decided to use that dataset.

As mentioned previously, we decided not to include income effects in our calculations. As Lundberg (2017) showed, it had an effect on changing the optimal tax rates with around 2 percentage points, from 61% without effects to 63% with the income effects. Furthermore, the Swedish calculated elasticity of 0.42 already includes the income effect. The only values that would be affected are the cases for elasticity of 0.2 and 0.26. If we added the income effect on our most conservative approach, in the case of elasticity 0.2 and Pareto 2.78, we get an optimal tax level of 66% (64% before income effect). Therefore, it does not change our overall conclusion.

The only time we receive a result showing that optimal tax rates are higher than today's effective rate is when we assume our conservative elasticity of 0.2 and the lowest historical Pareto value 1.44. However, considering the case of our own calculated Pareto value and the average elasticity from the Swedish reports, we land on an optimal tax rate of 42%. Since the effective tax rate is 73%, a removal of the whole government income tax of 25% might even be possible without revenue loss. The total tax revenue for the Swedish government was 1072 billion SEK in 2018, where the revenue from the government income tax was 59.6 billion SEK. Furthermore, the revenue from Värnskatten was only 5.2 billion SEK in 2016. Considering the revenue from Värnskatten, as well as the revenue from the government income tax, it is important to take into account that these taxes is small in comparison to the total tax revenues the government collects.

Looking at Figure 5, we can deduce that the Pareto values have changed over time. As mentioned in the theory, the Pareto value is linked to inequality. Piketty states in Jones (2015) that when returns on assets are greater than the economies average growth (GDP/worker growth), inequality increases. However, there are complexities to this. Policies, regulations and financial factors can affect the inequality. Considering this, high increases in taxes \( \tau \), could then consume a lot of the return on assets. A high enough tax could possibly remove inequality.
completely. Hence, changes in economic policies could then, in the long run, greatly affect the distributions of wealth. Consequently, more equality equals lower optimal tax rates as \( \alpha \) gets bigger. High taxes could then have an effect of reducing the optimal tax rates even more (Jones, 2015). This reasoning should stand true for income inequality as well. High taxes will decrease the inequality by reducing incentives to earn more. In the long run, the decreased inequality would result in higher Pareto distribution values. However, the tax system will also affect the Pareto value. A more progressive system for example, will have an effect of increasing the Pareto value. Since top income earners are Pareto distributed, higher taxes for top bracket earners or a more progressive tax system would, everything else held constant, decrease the optimal tax levels.

Even though the optimal tax rate, as we have derived in this report, is giving us the highest possible tax revenue for the government, it is not giving us the optimal profit for society. The taxes induce a welfare loss effect for every tax rate increase. Society might, however, want a certain level of taxes in order to function, but it will come with the expense of a deadweight loss.
8. Conclusion

Our calculations show that for the reasonable elasticities used in this report, the Laffer curve for Sweden's top marginal labour tax rate has a peak to the left of the present day’s effective tax rate. Sweden could therefore remove *Värnskatten* to increase the total tax revenue. Using our base calibration of elasticity 0.2, the optimal tax rate lands on 60.45%. With today’s effective tax rate of 73.63%, we can remove *Värnskatten*. The removal would increase the tax revenue for the central government with around 3 billion Swedish crowns / year. This is due to the behavioural change. We also showed the different results when varying the elasticity with respect to three different elasticities. The results included one conservative approach, one for long run estimates from Saez (2001) and one for averages of Swedish studies on this subject in the 1990s. For all three of these, we showed that the resulting optimal tax rate was lower than the effective tax rate, resulting in the same conclusion as before.

Looking at the historical context, using the lowest historical Pareto value, we see that the lowest elasticity to get the optimal tax rate to equal the effective tax rate of 73.63% is 0.246, which is higher than our base calibration of 0.2. We did, however, show that the most plausible estimates for elasticities in Sweden is much higher at an average rate of 0.42. The historical values are based on World Inequality Database, which we argued is systematically lower than our own Pareto calculations due to inclusion of all incomes.

In conclusion, we have showed in this report that *Värnskatten* could be removed without revenue loss for the government. The optimal tax level to maximize tax revenue is therefore to the left of today's tax rate. The result is in line with most previous report, as Lundberg (2017), Sorensen (2010) and Trabandt & Uhlig (2012). The Contribution of this report is a strengthen claim to the hypothesis that *Värnskatten* in Sweden could be removed. For further studies in this area, we recommend a continued research into the income elasticities as well as research into weather an optimal tax rate for maximizing tax revenue can be found for all incomes.
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