Non-Linear Analysis and Remaining Fatigue Life of Reinforced Concrete Bridges
Background document D4.5
This report is one of the deliverables from the Integrated Research Project "Sustainable Bridges - Assessment for Future Traffic Demands and Longer Lives" funded by the European Commission within 6th Framework Programme. The Project aims to help European railways to meet increasing transportation demands, which can only be accommodated on the existing railway network by allowing the passage of heavier freight trains and faster passenger trains. This requires that the existing bridges within the network have to be upgraded without causing unnecessary disruption to the carriage of goods and passengers, and without compromising the safety and economy of the railways.

A consortium, consisting of 32 partners drawn from railway bridge owners, consultants, contractors, research institutes and universities, has carried out the Project, which has a gross budget of more than 10 million Euros. The European Commission has provided substantial funding, with the balancing funding has been coming from the Project partners. Skanska Sverige AB has provided the overall co-ordination of the Project, whilst Luleå Technical University has undertaken the scientific leadership.

The Project has developed improved procedures and methods for inspection, testing, monitoring and condition assessment, of railway bridges. Furthermore, it has developed advanced methodologies for assessing the safe carrying capacity of bridges and better engineering solutions for repair and strengthening of bridges that are found to be in need of attention.

The authors of this report have used their best endeavours to ensure that the information presented here is of the highest quality. However, no liability can be accepted by the authors for any loss caused by its use.

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Summary

Background

For a sustainable development in Europe, there is a need to at least double the railway transports in the coming 20 years. In order to reach this, the residual service lives of existing concrete bridges need to be extended, at the same time as they are subjected to higher axle loads, higher railway speeds and heavier traffic intensity. Today, many concrete bridges are replaced because their reliability cannot be guaranteed based on the structural assessments made. This leads to environmental stresses and a bad usage of the society’s resources. If more correct structural assessment can be performed in the future, a substantial part of the cost can be avoided. Many old bridges also have a great cultural and historical value and the preservation of these is essential.

Non-linear analysis is commonly used in structural engineering research today. It has also been used successfully for assessment of existing concrete bridges in several cases. In most cases a considerably higher load carrying capacity compared to conventional methods has been shown. However the methods need to be further developed and questions need to be clarified in order for the methods to be generally applicable in engineering practice.

The residual life time of railway bridges is often limited by fatigue. However, current fatigue provisions rely on a narrow knowledge basis, compared to most other domains of structural concrete. Fatigue damage mechanisms for reinforced concrete are not yet well understood and codes are often based on pure experimental data with little scientific background. Consequently, improved methods for fatigue assessment are needed, and are likely to be able to provide a considerably increase in residual lifetime.

Aim and objectives

The aim of the work is to provide enhanced assessment methods that are able to prove higher load carrying capacities and longer fatigue lives for existing concrete railway bridges. The work is focused on non-linear analysis and remaining fatigue life.

The main objectives are:

- To facilitate the use of non-linear analysis for structural assessment. Subjects that are treated are:
  - Principles and rules for assessment of concrete bridges by non-linear analysis
  - FE modelling of corrosion
  - Evaluation of material properties
  - Simplified force and moment redistribution based on linear FE analysis
  - Bending-shear-torsion interaction, both for cracking and ultimate capacity

- Improved knowledge about the fatigue behaviour of concrete bridges and the development of realistic methods for evaluation of remaining fatigue life of existing bridges. A methodology for the assessment of fatigue safety will be developed.

Achievements

The work provides methods for non-linear structural analysis and fatigue safety evaluation in assessment of existing railway bridges. The use of non-linear analysis leads to higher load carrying capacities, but also to improved understanding of the structural response, forming a better basis for decisions in the assessment. The methods for fatigue assessment lead to increased remaining service life of reinforced concrete railway bridges.
The achievements of the work are:

- Models for assessment of concrete bridges by non-linear analysis are described, and examples and general recommendations for practical application of the methods are given.
- An overview of the effect of corrosion on the bond properties has been developed through non-linear analyses and tests found in literature. Recommendations for practical assessment of anchorage capacity in corroded railway concrete bridges are presented.
- The determination of reliable in-situ material properties in old concrete bridges is described, including properties for fully probabilistic analysis. A particular aim was to develop better methods to describe development of concrete properties with age and degree of hydration.
- Recommendations are given regarding re-distribution of sectional forces and moments, obtained through linear structural analysis (with FEM). In a case study, a typical slab bridge was designed using different methods, and its response evaluated through non-linear analyses.
- A good understanding of the load-carrying mechanisms in combined bending-shear-torsion gives a possibility to make more accurate assessments than with standard code methods. Methods and examples are presented.
- A state-of-the-art review of fatigue behaviour of reinforced concrete railway bridges is presented and a methodology to assess the remaining fatigue life of existing reinforced concrete bridges subjected to railway traffic is developed.

**Contributors**

Responsible and contributing authors for each chapter are presented together with the reviewers in the table below. The report was compiled and edited by Mario Plos, Chalmers University of Technology. An overall review of the background document was made by Lennart Elfgren, LTU.

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1 Introduction

This is the background document from the work group on concrete bridges in work-package 4 of Sustainable bridges. It covers the main research activities WP4-C-R-001 Non-Linear Analysis of Concrete Bridges and WP4-C-R-002 Remaining fatigue life of reinforced concrete bridges.

1.1 Background

In order to reach a sustainable development of the railways in Europe, it is of great importance that the huge investments made in railway bridges can be utilized during the entire lifetime of the structures. Many old bridges also have a great cultural and historical value and the preservation of these is essential. Today many structures are replaced because their reliability cannot be guaranteed based on an assessment following traditional design codes. Sometimes also unnecessary strengthening procedures are undertaken due to too conservative assessments. This is not an optimal use of the resources of the society. With more enhanced structural assessments a substantial saving of costs and environment can be achieved.

By using non-linear analysis for structural assessment of concrete bridges, the intrinsic load carrying capacity of more of these structures can be shown to be satisfactory, without costly investments. In cases where the structure has to be retrofitted, the costs can be kept at a minimum. By extending the lifetime of existing structures, great environmental benefits will be achieved, with less raw-material consumption, reduction of transportation and energy consumption, decreased pollution and less deposits. At the same time substantial savings are made, both for the society and for the owner or administrator of the structure.

1.2 Objective

The objective of the work is to facilitate the use of non-linear analysis for structural assessment of railway concrete bridges and to present improved methods for evaluation with respect fatigue. The aim is to be able to prove higher load carrying capacity and longer fatigue lives for existing concrete railway bridges through enhanced assessment. At the same time, improved understanding of the structural response will be obtained, forming a better basis for decisions in the assessment.
2 Assessment of concrete bridges by non-linear analysis

2.1 Introduction

A primary objective of structural engineering is to verify the strength and serviceability of civil works, such as bridges. To achieve that it is necessary to predict the behaviour of structures subjected to imposed actions, which requires the use of computational models. In case of reinforced concrete structures the models are generally based on principles of structural mechanics and material science. However, due to the large size, variability of forms and uncertainty of loading and environmental effects they also involve certain simplifications and approximations. This includes many significant assumptions, such as reductions of geometrical forms and stress states from three to two or one dimensions, simplified shapes and boundary conditions, and simplified material laws. It is reasonable to seek such models, which have a balanced approximation on all levels of engineering design: loading and environmental actions, structural analysis, performance check. Furthermore, a need of efficient but reliable design requires the computational models, which are "as simple as possible, but not more than that", implying that simplifications have limits.

Reinforced concrete is a composite material whose complex behaviour includes a low tensile strength leading to cracking even at the service load levels, a confinement effect causing the increase of compressive strength and an interaction of concrete and reinforcement and many other effects. Through the history of concrete structures many researchers devoted large effort to the development of models suitable for design of concrete structures. In early years, due to the complexity of concrete behaviour and lack of its understanding these models often did not reflected good enough the reality.

However, with the progress of structural-material research in and with advances in the modern information technology, more powerful models are recently available. These new models are based on the finite element method combined with the non-linear material laws based on smeared cracks, fracture mechanics and plasticity approaches. They are offered as commercially available tools for design and analysis, such as the computer programs ATENA (Cervenka V. 2002) and DIANA. These new methods bring a new quality in engineering design, namely by making possible a consistent approach to the use of constitutive laws of material behaviour for both, the analysis of internal forces and the check of resistance. These methods are often referred as computer simulations of structural behaviour.

The availability of these new models, commonly called as "non-linear analysis methods", requires also a new safety approach. It became clear, that the present system of partial safety factors, which fits to the inconsistent approach based on the "linear analysis" combined with local checks of cross sections cannot be simply extended to the non-linear analysis. Therefore, new safety formats, more suitable for a non-linear analysis are being developed and introduced in codes, Eibl and Hurtiene 1995.

A brief review of the models is treated in the initial part of this chapter, where only those models relevant for practical applications are mentioned. The emphasis is put on the FE-based models and relevant constitutive approaches. The next section summarizes recommendations for the practical application of non-linear analysis in the structural assessment. These recommendations include a summary of cases when nonlinear analysis may help to discover additional load-carrying capacity of the analysed structure. The suitable safety formats are described in Section 2.3, which connect the nonlinear approach to standard design and assessment methods. Suitable example and validation problems are described in the remaining sections of this chapter. The presented example problems can be used by the reader for validation of numerical tools or they document certain aspects or problems usually encountered during nonlinear analysis.
2.2 Review of models

We shall demonstrate the course of development and the present state of modelling of structural concrete on an example of the shear model, which represents one of the difficult subjects in concrete design, and which was in past (and often still is) not very well understood. We shall compare several approaches used in the history of concrete structures, without going to details. Off course, there are many other models, for example plates in bending, punching of plates, prestressing elements, etc., which are for brevity not discussed here. A more detail treatment of the subject can be found in the book by Collins and Mitchell 1987.

2.2.1 Truss models

The earliest model of cracked reinforced concrete beam subjected to shear introduced by Ritter 1899 is based on a truss analogy. This model, further developed by Mörsch (1912), assumes the concrete struts inclined under 45\(^\circ\), see Figure 2.1. The flow of forces is based on equilibrium only. The longitudinal reinforcing bars are treated as discrete struts, while concrete diagonals need not to connect the top and bottom points of the truss, but can be visualized as a continuous compression field. Until recently this model was the basis for the majority of building codes. However, this original truss model was found as too conservative when confronted with experiments. It does not take into account the ability of the web to redistribute the stresses in such a way that a higher load can be carried with a lower strut inclination. Assuming elastic behaviour, Kupfer 1969 showed, that the optimum direction of the concrete compression, corresponding to minimum complementary energy, is considerably flatter, and a model based upon the elastic strut inclination was proposed by Moosecker 1979. In application of Braestrup 1995, the angle of inclined concrete struts is considered variable. The model can be applied to the conventionally reinforced as well as prestressed beams, whether the tendons are bonded or not. Some of the principles of the original truss model are being used until today, for example in the smeared crack models.

In summary, the truss model is based on equilibrium of forces, neglects the concrete tensile strength and does not take into account deformations.

2.2.2 Plasticity-based models

The plasticity theory works with both conditions: equilibrium of stresses and compatibility of deformations. However, it offers also bound solutions, when only one condition is satisfied. This principle was used with advantage in many plasticity approaches. In this approach the truss model is regarded as a statically admissible stress distribution, defined by the strut inclination, which is optimised to give the highest lower bound solution for the shear load.

In 1973 a research group headed by M.P. Nielsen at the Technical University of Denmark in Copenhagen tackled the problem of beam shear by means of the classical theory of plasticity, using the modified Coulomb failure criterion as a yield condition for concrete, with the associated flow rule (normality condition). Already in 1967 Nielsen applied the lower bound
approach, assuming an uniform, inclined compression field in the web concrete, and an uniform tension in the stirrups, the compression zone and the main reinforcement being idealised as stringers. The strut inclination was determined by minimising the total volume of reinforcement, thus yielding of all steel was assumed. In a subsequent discussion reply Nielsen derived the shear strength governed by the inclined concrete struts (later termed the web crushing criterion). The Copenhagen shear group set out to verify the lower bound solution theoretically, by finding a corresponding upper bound, and experimentally, by determining the effective concrete strength (Braestrup 1994).

Contemporarily with the Copenhagen Shear Group the plasticity approach to beam shear was pursued by a research group headed by Thürlimann at the Federal Technical University in Zürich, who focused on a rigorous application of the lower bound method, Thürlimann 1978, Marti 1980. Based upon the truss model statically admissible stress distributions are assumed, corresponding to the reinforcement provided.

In the work of Maier 1985, 1988 the plasticity model was verified using results of extensive experimental investigations. The plasticity approach can be illustrated on an example of the wall analysed by Maier 1988. A basic step of this approach is assumption of the statically admissible stress field, which is constructed from triangular stress sub-regions with a constant stress, see Figure 2. The stress field must satisfy the conditions: equilibrium, yield function and boundary forces. The calculation of stresses-fields is relatively simple. Starting from boundary forces stresses can be directly calculated using equilibrium equations. Then yield conditions in each region are checked. The role of reinforcement is considered within the yield function, which covers the resistance of the composite material including the action of concrete in compression and of steel in tension. This method includes only few mathematical operations comparing to more exact solutions, such as the finite element method.

![Figure 2.2](image)

**Figure 2.2** Stress fields by Maier 1988 applied to plastic analysis of wall.

### 2.2.3 Strut and tie models

Another variation of the plasticity approach is “the strut-and-tie model" developed in Stuttgart by Schlaich and co-workers 1991, illustrated by a wall model shown in Figure 3. Stress state in a continuous structure is approximated by a truss with concrete compression struts and reinforcement ties. Similarly as in the case of plasticity, the analysis involves a determination of forces in the truss and then a check of the individual truss elements. The advantage of this approach is evident. The available truss programs can be readily used to solve the distribution of forces in the truss system based on the assumption of elastic material behaviour. In this way even complex continuum problems can be reduced to the truss systems, which are more easily understood by engineers. It is a “transparent design tool" (Schäfer et. al. 1991). Another advantage is, that it is also possible to solve both, stress and strain fields. However, the disadvantages are also evident. The success of the model is based on a judgement of the designer, how realistically he determines the sizes of concrete struts. Since this step is
rather arbitrary, the results are not very objective. Another problem is the determination of the truss forces, which is usually based on elastic analysis. If the truss system is statically indeterminate, these forces depend also on the stiffness and a solution beyond the elastic limit is not realistic. Nevertheless, the strut and tie model represents often an efficient solution for practical cases, when other more rational methods are not possible.

2.2.4 Compression field theory

Collins at the University of Toronto and his research group applied Wagner's 1929 approach in developing the compression field theory (Collins 1978). He recognised, that for further improvement of models the strain state must be taken into consideration together with equilibrium and constitutive relations. He focussed his research on the most frequent structural elements comprising two-way reinforced membranes, which can be a part of bridge girders, shear walls or off-shore structures. Vecchio and Collins 1982 tested reinforced concrete panels such as shown in Figure 2.4.

By analyzing experimental data of orthogonally reinforced members they found, that the only explanation for shear strength of such elements is a reduction of compressive strength. Based on these experiments they developed a constitutive law for response of the cracked...
concrete in compression acting in direction parallel with cracks. Testing rigs used for panels of various sizes are shown in the top-right corner of Figure 2.4. The compression field theory is based on the set of following conditions:

- equilibrium of stresses,
- compatibility of strains,
- coaxiality of principal stresses in concrete with principal strains,
- stress-strain relations for concrete and reinforcement.

Figure 2.4 shows the law for reduction of the maximum stress in concrete (compressive strength of cracked concrete) due to lateral tensile strain proposed by Vecchio and Collins and other authors. The model includes also a contribution of tensile stresses in concrete (tension stiffening) and a local check of the concrete shear stress and reinforcement stress on crack planes (within the Modified Compression Field Theory). The compression field theory represents a rational and consistent design model suitable for both, design check as well as numerical analysis. It was met by large interest from industry and found its way to the codes of practise. Meanwhile, a similar but slightly different model was presented by Hsu (1984, 1993), (ASCE-ACI 445 1998), who performed also similar experiments.

2.2.5 Non-linear finite element analysis

The finite element method became one of the most frequently used tool for the structural analysis. It is also a numerical base of many CAD packages. Since introduction of this method in design many authors tried to extend it also to the range of non-linear behaviour of concrete. First developments are due to Rashid 1968 and Cervenka 1972. It evolved in a numerical tool for simulation of the real behaviour of concrete structures. A complete response of a structure to the given imposed loading can be obtained by such analysis including stages of crack propagation in the pre-peak serviceability state, the failure load and failure mode and the post-peak behaviour. The model can be described by three levels as shown in Figure 2.5, each involving certain approximations:

- **Structure.** In the stiffness approach geometrical form is reduced in a system of finite elements, boundary conditions and loading. The structural response is described by the equilibrium matrix equation, where \( \mathbf{U} \) are discrete displacements, \( \mathbf{K} \) is a stiffness matrix and \( \mathbf{P} \) are loading forces.

- **Finite element.** A shape of the displacement function in terms of nodal displacements (reflected in the matrix \( \mathbf{B} \)) is assumed and used together with the material stiffness \( \mathbf{D} \) to calculate the element stiffness matrix \( \mathbf{k} \).

- **Constitutive relations.** They define behaviour of material in terms of stress-strain relations, function \( F(\sigma, \varepsilon) \) in a material point and corresponding material stiffness \( \mathbf{D} \). They reflect the non-linear material effects and failure, such as the concrete cracking and the reinforcement yielding.

The above formulation is typically incremental. The forces, displacements, strains and stresses are linearized increments within a load step.

The first two levels of the structural model in Figure 2.5 are well known from applications in other fields of engineering and can be solved with required accuracy, just providing sufficiently fine meshes and adopting reasonable shape functions in the finite elements. The third level, the constitutive modelling of specific properties of reinforced concrete, especially their derivation from experiments represents a difficult task, because material behaviour can not be easily separated form its structural context. In order to verify the validity of nonlinear models, the performance of programs is often confronted with experiments in bench mark tests Bonnard & Gardel 1994, Margoldova et. al. 1998.
Since cracking is the most important material property, a variety of crack models was proposed: the discrete cracks, the embedded cracks and the smeared cracks (see e.g. [21]). In practical applications the smeared crack model was found as most feasible. Therefore, we shall discuss this model in more detail in the next chapter.

### 2.2.6 Overview of material model types for brittle materials

This chapter provides a brief overview of the main aspects of the various material models for brittle cementitious materials such as concrete, masonry or rock.

**Plasticity models:** The stress increment \( \Delta \sigma \) in this model is calculated using the following expression

\[
\Delta \sigma = D \left( \Delta \varepsilon - \Delta \varepsilon_p \right)
\]

where \( D \) is the constitutive elastic stiffness, \( \Delta \varepsilon \) and \( \Delta \varepsilon_p \) is the increment of strain and plastic strain respectively. The plastic strain increment is calculated such that an appropriate plasticity function/criterion is satisfied. These models are applicable to steel, soil. These class of models can be used also for concrete, but in this case it should be noted that the elastic unloading inherent to these models is not very realistic for describing the tensile cracking.

**Damage models:** are derived based on thermodynamic consideration from an energy potential. The typical expression for stress calculation is:

\[
\sigma = (1 - \omega) D \varepsilon
\]

where \( \omega \) is the damage parameter, which represents the damage state, 0 – no damage, 1 – full damage. This class of models is applicable to brittle materials such as concrete cracking, rock and masonry.

**Orthotropic smeared crack models:** are quite closely related to the damage models. In many cases a damage model can be considered to be a special case of the smeared crack model. The stress calculation for a typical orthotropic 2D model may look as:

\[
\sigma = D_s \varepsilon
\]
However, contrary to the damage models, the secant constitutive $D_S$ is derived intuitively, but directly defining laws for the evolution laws of the individual components. For instance in case of the program ATENA (Cervenka V. 1985), the matrix $D_S$ has the following form for the plane stress case:

$$D_S = H \begin{bmatrix} \xi & v \xi^2 & 0 \\ v \xi^2 & 1 & 0 \\ 0 & 0 & G \end{bmatrix},$$

$$\xi = \frac{E_1}{E_2}, \quad H = E_1 (1 - \xi v^2)$$

This class of models is suitable for concrete, rock, masonry and other brittle materials. It is described in more detail in the subsequent chapter.

*Figure 2.5  Comparison of unloading in plasticity models (left) and smeared crack/damage models (right).*

**Microplane models**: in this class of models the material laws are formulated for an arbitrary surface inside the material (Bažant, Z.P et. al. 2000), and then they are integrated over a unit sphere to obtain a macroscopic behavior. The idea behind this approach is that the formulation of the material laws for an individual surface is quite simple and the model can nicely capture in interaction of microcracks in various directions. On the other hand, the necessary input parameters are usually not directly linked to the known macroscopic properties such as compressive and tensile strength. The microplane models are applicable to many materials ranging from steel, to concrete as wells as to biomedical tissues.

**Other models**: in recent years new approaches for the simulation of material cracking have been proposed, such as for instance X-FEM, elements with embedded discontinuities, gradient plasticity, Cosserat continuum and various nonlocal approaches. For a detailed comparison of these methods see Jirasek, M. (1998) and (2000). Currently these methods are still an active area of research, and they are usually not available in commercial packages.
2.2.7 Smeared Crack Model

The smeared crack model is the by far the most developed model that is present in some form in most commercial finite element codes.

The idea of smeared cracks was already used on the original truss model and in the compression field theory discussed above. Instead of considering each individual crack, which is a model incompatible with continuous displacement fields of the finite elements, the cracks are considered finely and uniformly distributed within a given material volume. One of the first such models was developed by Cervenka & Gerstle 1971 and was later successfully applied for the analysis of Toronto panels.

The Toronto experiments became widely known in connection with The Toronto Prediction Competition launched in June 1981 (Colins et al 1983). Cervenka 1985 submitted the best entry, see Figure 2.6, using a smeared crack model. His solution was based on a stiffness-formulated numerical constitutive model of composite material, in which the strains are common for the composite and the each material component, reinforcement and concrete, behaves according to its stress-strain relation. These relations for cracked concrete (compressive strength reduction, tension stiffening) were similar to those in the compression field theory.

The original smeared crack formulation Cervenka & Gerstle 1971 assumed a complete stress release after crack formation. It was soon found that such model is sensitive to the finite element mesh and therefore is not objective. Therefore, further improvements were sought by introducing a crack band model, based on the fracture energy concept. In this model a real discrete crack is simulated by a strain localisation in a continuous displacement field, which is enabled by a strain softening of material, Bazant and Oh 1983. The crack behaviour is idealised by a model of the cohesive fictitious crack according to Hillerborg 1976, where the crack opening law is governed by the three parameters: tensile strength ft, fracture energy Gf, and the shape of the softening curve. A significant contribution to this development is due to Rots 1988. Such model is also implemented in the commercial program ATENA, where the exponential function derived by Hordijk 1991, see Figure 2.7 (left), is used for the crack-opening (stress-crack width) law.

Objectivity of the crack band model with respect to mesh size and direction was treated in detail in Cervenka et. al. 1995. A comparison of crack patterns in a web of the T-beam tested by Braam 1991 and analyzed numerically by Cervenka & Margoldova 1995 is shown in Figure 2.9. The ability of the model to simulate the stiffness of cracked concrete is demonstrated on a case of reinforced bar subjected to tension (experiments by Hartl 1977). It is remarkable, that the model simulates well the effect of tension stiffening, i.e. contribution of concrete

![Figure 2.5](image)

Figure 2.5 Fundamentals of the microplane model (a) surface tractions (b) micromechanical explanation of the microplane concept (c) unit sphere for the microplane integration
in tension. Note, that a bond action between the concrete and the reinforcement is not explicitly modelled. However, a debonding is exhibited by cracking of concrete layer adjacent to bar. This debonding is stronger in the rotated crack model (Cervenka & Margoldova 1995).

Figure 2.6 Prediction of shear response of Toronto panels, Cervenka (1985).

Figure 2.7 Crack opening law (left). Strain softening law (middle). Crack band L (right).

Figure 2.8 Tension of R.C. bar tested by Hartl 1977, analysis by Cervenka & Margoldova 1995.
A real discrete crack is simulated by a band of localized strains as illustrated in Figure 2.10. Due to the energy formulation this model is objective and its dependency on the finite element mesh size is substantially reduced (Cervenka et al. 1995). This was confirmed by numerous studies, for example by those about shear failure published in Cervenka 1998.

Two crack models are recognized. In the fixed crack model the crack direction is determined and fixed at the time of crack initiation. In the rotating crack model the crack direction is identical with a principal strain direction and rotate if the strain direction changes. The main difference in these crack models is the absence of shear stresses on the crack plane in the rotating crack model due coincidence of principal strain directions with the crack orientation, which makes the rotating crack model more simple. In the fixed crack model the shear resistance of the cracks is modelled by means of the variable shear retention factor, which reflects the aggregate interlock effect of cracked concrete.

Concrete in plane stress condition can be well described by a damage model such as the one used in the ATENA, see Figure 2.11. It is based on the “equivalent uniaxial law”, which covers the complete range of the plane stress behaviour in tension and compression. The effect of biaxial stress state on the concrete strength is captured by the failure function due to Kupfer et al 1969. For the tensile response (cracking) the crack band method described
above is applied. Similar method is applied for the compressive softening. Thus complete softening behaviour is based on an objective and mesh independent approach.

Concrete under confinement in three-dimensional stress state can be described by the theory of plasticity with a non-associated flow rule according to Menetrey-Willam 1995. The failure function is illustrated in Figure 2.12 (left) in the space of principal stress. The strength is increasing with the hydrostatic compressive stress $\sigma_o$. The failure occurs when the deviatoric shear stress to reaches the failure function. An example of strength increase in the case of Menetrey-Willam failure function is shown in Figure 2.12 (right) and is compared with the similar function recomended by the Model Code 90. Strength is the maximum stress in $\sigma_3$ under the assumption of confining pressure $\sigma_2 = \sigma_1$. This graph represents a section through the surface shown in Figure 2.12 (left).

![Figure 2.11 Equivalent uniaxial law. (left) Bi-axial failure function by Kupfer [11]. (right)](image)

The plasticity theory describes also the plastic flow and volume change due to a distorsion and can model the volume increase of concrete under plastic deformations. The predictor-corrector scheme, Figure 2.5, is used for this purpose. The plastic volume change can be defined in the rendulic section, Figure 2.13, where $\xi$ is the hydrostatic axis and $\rho$ the deviatoric axis. Parameter $\beta$ defines the return direction form the predicted stress point $\sigma_{ij}$ back to the failure surface. This direction is used in the plastic flow rule to define the direction of plastic flow (in the principal stress space) and describes the volume change.

![Figure 2.12 Concrete failure in 3D stress state (left). Strength increase under confinement effect (right).](image)
Figure 2.13 Return mapping algorithmus.

It should be noted that the general three-dimensional behaviour of concrete can be also described by the microplane models, which are based on a micro-structural damage idea. Microplane model due to Bazant et al 2000 is implemented in ATENA and can be used for solution of practical cases.

Of course many other material models are needed for completeness of a successful simulation (reinforcement, steel, bond slip of reinforcing bars, interface, soil, etc.). Out of these models we shall mention only bond since it represents a specific feature of reinforced concrete mechanics.

Bond serves to transfer the stresses from reinforcement to concrete and assures the integrity of reinforced concrete structures. Bond mechanics is very complex and include all behaviour modes of concrete: friction, tensile and splitting cracks, compressive crushing. Theoretically it should be possible to solve the bond with available solid models. However, such approach leads to very large tasks and is not practically useful.

High attention was paid to develop more simple bond models, see for example the FIB report (CEB 1993). These models are usually cast in form of bond stress-slip diagrams and include influencing parameters (concrete properties, confinement, surface properties). They can be directly used in the finite element models. Examples are shown in Figure 2.14. It is realized, that some features, which are covered by the simplified bond laws are already covered by the concrete models. This is especially true for concrete splitting, which is the main source of the softening behaviour of bond slip laws. Such aspects must be carefully considered in particular cases. For example, in case of analysis, which sufficiently covers the splitting of concrete by a detail analysis of cracks in concrete it may be adequate to choose the plastic bond model.

Figure 2.14 Examples of bond slip law.
2.2.8 Elements of reinforced concrete model

Finite element modelling of reinforced concrete structures requires special tools for modelling of all types of reinforcement. Most of these models are illustrated in Figure 2.15 and 2.16 for two- and three-dimensional solids. In addition there is a family of similar axisymmetric elements, which is not shown here.

Concrete is modeled by solid elements. As examples quadrilateral and brick elements for 2D and 3D solids are shown. Low order elements are recommended for nonlinear analysis, since they are well validated. Concrete to concrete, or concrete to other material interfaces can model the frictional type of interaction between structural elements. The mesh type of reinforcement can be represented as smeared reinforcement. In this element the individual bars are not considered, while reinforcement is considered as a component of the composite material.

Individual bars can be modelled by truss elements embedded in concrete elements with axial stiffness only. In this technique the mesh is generated first for concrete. Then the bar elements are embedded in this mesh. The bar element can be considered as any other element but its nodes are made kinematically dependent on concrete nodes. Thus the reinforcing is not affecting the mesh generation. Cables, which are connected with the concrete structure only in limited number of points can be modelled as external cables. The described family of finite elements make possible a to cover most practical cases of reinforced concrete structures.

Figure 2.15 Two-dimensional elements of reinforced concrete.

Figure 2.16 Three-dimensional elements of reinforced concrete
2.3 Recommendations for structural assessment based non-linear analysis

Numerical simulation is another way how to check structural performance and is an alternative to physical testing in laboratory or on site. Unlike in real testing, simulation is not limited by structural size, load magnitude and testing facility. However, the power of computer simulation should not be overestimated and reasonable checks should be exercised to verify the results and conclusions.

Finite element method is typically used for numerical solution of continuum problem. Depending on the type of formulation (stiffness, compliance and mixed methods) the results are by definition different from exact solution.

In case of the most widely used stiffness method, the shape of displacement field is assumed and equilibrium is satisfied only in integral sense. That means, the best possible equilibrium is found for a given approximation (finite element type and size). The internal stresses are lower, compared with an exact solution. Thus, independently of material model, the approximations introduced by the finite element formulation only, can be a significant source of errors in numerical analysis.

An engineer should verify that the used numerical tools meets the following requirements:

(1) If finite element formulation is used, it should satisfy the requirement of convergence to exact solution by reducing element size (and increasing number of degrees of freedom). Similarly, in other numerical methods, the errors due to approximations should be adequately checked.

(2) Material models that are used for concrete, soil, rock, masonry, reinforcement and their interaction, should capture all significant and relevant features of material behaviour for the problem under consideration.

(3) Constitutive laws must be based on principles of continuum mechanics and failure and must ensure objectivity of solution in the context of numerical methods as described in more detail in Section 2.2. Models for material softening, i.e. materials exhibiting decrease in strength after reaching certain ultimate value of stress, should include appropriate regularization techniques.

Nonlinear analysis always provides additional insight into the structural behaviour, and allows engineers to better understand their structures. On the other hand nonlinear analysis is almost always more demanding than a linear analysis, therefore an engineer should be aware of its limits as well as benefits that can be obtained. Table 2-1 summarizes benefits of nonlinear analysis for different structure types and failure modes. It specifically concentrates on the possibility that nonlinear analysis may help to discover sources increasing the structural resistance.

2.3.1 Validation of methods and models

Before applying non-linear analysis to the assessment of bridges or other structures, it is the responsibility of the engineer to validate that the selected numerical tool meets the requirements specified in introductory remarks in Section 2.3. Also it should be verified that the numerical tools is suitable for the analysed problem and expected failure model. This verification can be performed either by studying the documentation of the used software tool, where various example problems are usually presented. It is however preferable that an engineer performs several simple example tests to verify the program behaviour on problems closely related to the expected behaviour of the analysed structure. Such a series of tests should typically consist of:

(1) Basic material tests (they should verify that the material model can capture the expected main features of the materials that are used in the analysed structure)
Table 2-1: Analysis type suitability for different structural types and failure modes

<table>
<thead>
<tr>
<th>Structure type</th>
<th>Failure Mode</th>
<th>Analysis Type</th>
<th>Source of Additional Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determinate</td>
<td>Bending</td>
<td>Probabilistic methods</td>
<td>More accurate statistical distribution</td>
</tr>
<tr>
<td></td>
<td>Shear/</td>
<td>Non-linear analysis,</td>
<td>Better utilization of tensile strength</td>
</tr>
<tr>
<td></td>
<td>Torsion/</td>
<td>Probabilistic methods</td>
<td>and failure mechanism</td>
</tr>
<tr>
<td></td>
<td>Punching</td>
<td></td>
<td>More accurate statistical distribution</td>
</tr>
<tr>
<td></td>
<td>Service-</td>
<td>Non-linear analysis</td>
<td>Tensile strength</td>
</tr>
<tr>
<td></td>
<td>ability</td>
<td>Probabilistic methods</td>
<td>More accurate statistical distribution</td>
</tr>
<tr>
<td>Indeterminate</td>
<td>All failure modes</td>
<td>Plastic analysis</td>
<td>Force redistribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-linear analysis</td>
<td>Force redistribution + tensile strength + 3D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probabilistic methods</td>
<td>effects + failure mechanism</td>
</tr>
<tr>
<td></td>
<td>Service-</td>
<td>Non-linear analysis</td>
<td>Tensile strength + 3D effects</td>
</tr>
<tr>
<td></td>
<td>ability</td>
<td>Probabilistic meth.</td>
<td>More accurate statistical distribution</td>
</tr>
</tbody>
</table>

(2) Mesh sensitivity tests (they should verify if the numerical model is sufficiently fine)

(3) Structural tests (they should verify that the software can accurately simulate the necessary class of structures)

Validation can be also accepted from other published and reliable sources. Various validation examples are presented at the end of this chapter. They contain experimental results, and they can be used for validation purposes.

2.3.2 Safety formats for non-linear analysis

In order to apply non-linear analysis in the assessment of practical engineering structures it is important to connect it to some code provisions by an appropriate safety format. Detailed information about safety formats can be found in other documents of the Sustainable Bridges project Casas J.R., Wisniewski D.F., Cervenka J.,(2006). This report contains only a summary of available approaches. Before describing the individual methods it is important to introduce the notion of design variables of resistance.

In case of non-linear analysis definition of basic variables are changed from local level of material point (or cross section) to global resistance (of member or structure). In analogy with variables for material resistance in partial factor method, the following definitions are introduced:

\[ R = r(f, \ldots) \] resistance function representing numerical model, in which \( R \) is basic design variable and \( f \) are material (or other) parameters, but not basic design variables,
mean value of resistance,

$R_k$ characteristic value of resistance,

$R_d$ design value of resistance.

Probabilistic analysis

Probabilistic analysis is a general tool for safety assessment of reinforced concrete structures and can be applied also in case of non-linear analysis. The limit state function in Eq. (4.3-3) can be evaluated by means of numerical simulation. In this approach the resistance function $r(r)$ is represented by non-linear structural analysis and loading function $s(s)$ is represented by action model.

Safety can be evaluated with the help of reliability index $\beta$, or alternatively by failure probability $P_f$ taking into account all uncertainties due to random variation of material properties, dimensions, loading, and other.

Probabilistic analysis can also be used for determination of design value of resistance $R_d$. In this way partial factor of resistance $\gamma = \frac{R_m}{R_k}$ can be calculated based on probabilistic approach.

Probabilistic analysis based on numerical simulation include following steps:

1. Numerical model based on non-linear finite element analysis. This model describes the resistance function $r(r)$ and can perform deterministic analysis of resistance for a given set of input variables.

2. Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects of actions, which are not in the action function $s(s)$ (for example pre-stressing, dead load etc.). Random properties are defined by random distribution type and its parameters (mean, standard deviation, etc.). They describe the uncertainties due to statistical variation of resistance properties.

3. Probabilistic analysis of resistance and action. This can be performed by numerical method of Monte Carlo-type of sampling, such as LHS sampling method. Results of this analysis provide random parameters of resistance and actions, such as mean, standard deviation, etc. and the type of distribution function for resistance.

4. Evaluation of safety using reliability index $\beta$ or probability of failure.

Probabilistic analysis can be also used for determination of design value of resistance function $r(r)$ expressed as $R_d$. Such analysis involves the steps (1) to (3) above and $R_d$ is determined for required reliability.

Global resistance factor – probabilistic estimate

It is assumed that random distribution of resistance is according to lognormal distribution. Mean and characteristic values of resistance shall be calculated using corresponding values of material parameters:

\[ R_m = r(f_m, ..., ) \quad R_k = r(f_k, ...) \]

Where $f_m$ , $f_k$ are mean and characteristic values of input material parameters, respectively.

The coefficient of variation $V_R$ of resistance shall be determined from:

\[ V_R = \frac{1}{1.65} \ln \left( \frac{R_m}{R_k} \right) \]

Partial factor of resistance for mean resistance shall be determined from:
\[ \gamma_R^m = \exp(\alpha_R \beta V_R) \]

where \( \alpha_R \) is sensitivity (weight) factor for resistance reliability and \( \beta \) is reliability index. The design resistance for the purpose of checking design condition is then calculated as 
\[ R_d = R_m / \gamma_R^m. \]

The factor \( \gamma_R^m \) includes only effects of random material properties. The effect of model uncertainties should be considered by separate factor \( \gamma_{r,u} \), which shall be used for reduction of design resistance. It is possible to assume \( \gamma_{r,u} = 1 \), if numerical model sufficiently reflects possible uncertainties.

This method allows to introduce reliabilities of all parameters, whose random variability can be expressed by mean and characteristic values and thus is not restricted to any particular mode of failure, stress state, reinforcement and structural form.

**Global resistance factor – partial factor estimate**

Design resistance \( R_d \) can be estimated using design material values as 
\[ R_d = r(f_d, ...) \]

In this case structural analysis is based on extremely low material parameters in all locations. This may cause deviations in structural response, e.g. in failure mode. It may be used as an estimate in absence of a more refined solution.

**Global resistance factor – EN1992-2 estimate**

Design resistance is calculated from
\[ R_d = r(f_{3m}, f_{ck}^{red}, ...) / \gamma_R \]

Material properties used for resistance function are used as follows:
\[ f_{3m} = 1.1 f_{yk} \text{ - mean value for steel yield stress,} \]
\[ f_{ck}^{red} = f_{ck} / \gamma_{c2} \text{ - reduced characteristic strength for concrete, } \gamma_{c2} = 1.15, \]

Partial factor of resistance shall be \( \gamma_R = 1.3 \)

This method is limited to cases where tensile strength of concrete is not a major parameter influencing the limit state.

**2.4 Basic material test validation – shear panel**

Geometrical model of the panel tested by Cervenka and Gerstle (1971) is shown in Figure 2.17(a). The reinforcement is made by no.3 deformed bars (USA) placed as shown in Figure 2.17 (b).
Material properties:

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$</td>
<td>$E_s$</td>
</tr>
<tr>
<td>20250 MPa</td>
<td>190000 MPa</td>
</tr>
<tr>
<td>$v$</td>
<td>$f_y$</td>
</tr>
<tr>
<td>0.2</td>
<td>353 MPa</td>
</tr>
<tr>
<td>$f_c$</td>
<td>$E_h$</td>
</tr>
<tr>
<td>26.75 MPa</td>
<td>700 MPa</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td></td>
</tr>
<tr>
<td>-0.00264</td>
<td></td>
</tr>
<tr>
<td>$w_d$</td>
<td></td>
</tr>
<tr>
<td>-0.5mm</td>
<td></td>
</tr>
<tr>
<td>$f_t$</td>
<td></td>
</tr>
<tr>
<td>3.1 MPa</td>
<td></td>
</tr>
<tr>
<td>$G_f$</td>
<td></td>
</tr>
<tr>
<td>58 N/m</td>
<td></td>
</tr>
</tbody>
</table>

Notation: $f_c$ – average cylinder compressive strength, $\varepsilon_c$ – compressive strain at peak, $w_d$ – total deformation of compressive softening zone, $f_t$ – tensile strength (direct), $G_f$ - fracture energy. (Calculated using the CEB MC90 formula).

2.4.1 Finite element model

Plane stress analysis was performed using 4-node quadrilateral isoparametric finite elements with 4 integration points. Constitutive relation for concrete was the fracture-plastic model, see Cervenka (2002). In this approach the tensile behavior is described by the fracture energy-based smeared crack model and the compressive behavior by non-associated plastic flow theory. Softening of concrete behavior in tension as well as compression is considered.

Note, that the tension stiffening effect (contribution of cracked concrete to the stiffness of reinforcement) is modeled indirectly through localization of strains. No specific tension stiffening parameter appears in the model. Here, the same constitutive model is used for plain as well as reinforced concrete. Perfect bond between concrete and reinforcement is assumed.

This study shows the effects of element size and type. Three meshes with 5, 10 and 20 elements over the panel length with square elements were used for the mesh sensitivity study. The mesh with 10 elements was also analyzed with triangular elements. To observe the effect of the element type (shape function).
The behavior of the panel includes the propagation of cracks from early stages of loading up to failure. At the maximum load the horizontal bars in bottom reach the yield stress and deform plastically and the concrete in the top right corner fails in compression. The failure mode is of bending type.

The study shows that all analytical models can reflect the real behavior with accuracy, which may be acceptable for engineering solutions. It also shows the trends of various effects, which can be utilized in practical cases.

(a) Quadrilateral elements.                        (b) Triangular elements.

Figure 2.18 Finite element meshes M10.

First, we observe the response during the crack propagation before maximum load is reached (ascending branch of load-displacement diagram). In this stage the response is dominantly effected by interaction of partially cracked concrete and reinforcement, so called “tension stiffening effect”. This behavior can be observed on the load displacement diagrams, Figure 2.19 and 2.20, and crack patterns, Figure 2.21 and 2.22.
The study shows, that larger elements give lower stiffness in this stage and that small elements reflect better individual discrete cracks due to better strain localization. It is clear, that if we continue to increase the number of elements so that the element size is several times smaller then the crack spacing, the crack pattern would be close to the experimental discrete cracks. However, in practical cases we cannot use extremely large numerical models and the tension stiffening effect may be underestimated.
If we use a triangular element for the same mesh size, the response is stiffer, see Figure 3. This clearly shows that the same behavior can be modeled by various ways. The reason for the stiffer response of the triangular mesh is the shape function of the triangular element with a constant strain field, while the quadrilateral element has a semi-linear strain field.

Next we examine the effect of the mesh size on the maximum load at failure. The load carrying capacity is exhausted by reaching the yield stress of reinforcement in the bottom part of the panel and compressive strength of concrete in the top right corner. This is a typical bending mode of failure, and larger elements give higher load response. The higher response is also reached by triangular (more stiff) elements. In this particular case the quadrilateral meshes M10 and M20 give identical response under maximum load, which indicates, that mesh M10 is sufficiently fine, while M5 and triangular mesh M10 give higher maximum loads.

The descending shape of the load displacement diagrams after the peak indicates a limited ductility, which is caused by the use of softening model for concrete in compression according to uni-axial tests. This limited ductility is not observed in experiments, where concrete behavior in the panel corner may be effected by confinement effect of rib in the third direction. Obviously, this difference can be easily reduced by assuming a perfectly plastic behavior of concrete, but such assumption is not rationally founded, since it applies only to a small region of a panel, and must be used with care.

The above study illustrates some general features valid for numerical simulation of reinforced concrete structures:

1. Two distinct constitutive models for concrete (smeared cracks, fracture energy-based crack band) and for reinforcement (bi-linear) are capable to simulate reasonably well the behavior of reinforced concrete panel.
2. Finite element size and type can significantly influence the structural response. A mesh sensitivity study is recommended to show the objectivity of results.
3. Coarse mesh underestimates the loads (structural resistance) in the stage of crack propagation and overestimates the ultimate load bearing capacity.
4. Low order elements overestimate the load response in the whole range.

The above conclusions are true for reinforced concrete structures with more-less uniformly distributed reinforcement, such as the wall panel shown. The structures with discrete reinforcement (beams without web reinforcement, etc.) are less sensitive to the underestimation of tension stiffening (item 2 above), see ref. CERVENKA (1998).

2.5 Material test validation - bond

The present formulation can describe the bond response by two models, which act as a series (chain) mechanical system. The bond force is first transferred from the bar body to its surface and then to surrounding concrete. In both models the same bond force exists. On the bar surface this force generates the bond stress and in the surrounding body of concrete it generates a three-dimensional stress state and cracks as schematically illustrated.

The bond slip can be generated by two different actions. If the bond stress $\tau_b < \tau_{b,\text{max}}$ (below the bond strength) then the bond force is transferred to continuum model and the bond behaviour is exhibited by cracks and damage in concrete. If the bond stress reaches the bond strength $\tau_b = \tau_{b,\text{max}}$ then a bond slip on the bar surface can occur. This may prevent a damage in concrete, and bond behaviour is exhibited by the interface model. Activity of each model depends on model parameters as well as element size. If the size of the zone with damage due to bond, $\Omega$ in, is considerably larger then the mesh size, the continuum model cannot well capture the bond behaviour.
The purpose of the proposed model is to reduce the mesh size effect on response of bond-dependent reinforced concrete structures. This was validated by the case study in which the specimen P.20.16.1 tested by Bigaj [3] was simulated by ATENA. The specimen is concrete cylinder with diameter 0.5m, length 1.2m and with centrally located bar with diameter 20mm. Concrete strength was 27.6 MPa and steel yield stress was 526 MPa. The simulation was performed with three finite element meshes shown in Figure 1. The figure shows the radial section of the specimen with the axis of rotational symmetry is on left where the bar is located. Three meshes were analysed with bond model and with perfect connection. The resulting load-displacement diagrams are shown in Figure 2. Unfortunately, report [3] publishes only the points around the maximum load and not those for initial load stages. In spite of this the performance of models can be evaluated.

Let us observe first the results obtained with rigid bond (without bon model). In this case only continuum-based bond behaviour is active. As can be seen from the load-displacement curves, the response is strongly dependent on the mesh size. For large elements the slip is much smaller showing too stiff behaviour. In contrast, the simulations with the bond model show consistent results for all sizes of mesh.

Figure 2.31 Meshes in pull-out test simulation.
The behaviour is illustrated by graphical results of bond stress distribution. It shows that in case of rigid bond the bond stress is much high than in cases analysed with bond model, in which the bond stress in limited by the bond constitutive law.

The study confirmed that the proposed bond model reduces effect of mesh size and can be offered as an efficient tool for finite element analysis of reinforced concrete structures.

2.6 Mesh sensitivity validation – material model objectivity

The purpose of this study was to investigate the influence of element size on the results of behavior of the simple reinforced concrete beam subjected to bending. Two material post-peak stress softening models were introduced: Fracture mechanics model and the so-called Local Strain model.

2.6.1 Material Models

Fracture mechanics model assumes fracture energy dissipation needed for generation of a unit area of stress-free crack. The stress-strain diagram is described by an exponential function for crack opening law, by the fracture energy and tensile stress.

The Local Strain softening model is based only on the stress-strain relationship without any energy consideration. The post-peak behavior is described by a curve defined by the strain parameter corresponding to complete release of tensile stress in the crack.

2.6.2 Geometry and Finite Element Models

The analysis was carried out on the concrete beams that had dimensions 100x100x840mm and the span was 800mm. The load was situated in the mid-span.

Three sizes of finite element meshes were used in this study: the coarse mesh with element size 80x16.7mm, the fine mesh with element size 26.7x16.7mm and finally the finest mesh with element size 15.4x16.7mm. It has to be noted that it was confirmed that further mesh refinement had no effect on results.

2.6.3 Material Characteristics

The experiment we are comparing our results to was performed by Uchida. We used the same material characteristics as he had measured on the tested beams. We also used some other material variables needed for the softening models.
Figure 2.23  Static Scheme

(a) Coarse mesh (80 x 16.7 mm)

(b) Fine mesh (26.7 x 16.7 mm)

(c) Finest mesh (15.4 x 16.7 mm)

Figure 2.24  Finite element meshes:

<table>
<thead>
<tr>
<th>Material Variable</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>E</td>
<td>21300</td>
<td>MPa</td>
</tr>
<tr>
<td>Compressive Strength</td>
<td>$f_c$</td>
<td>33.9</td>
<td>MPa</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>$f_t$</td>
<td>1.47</td>
<td>MPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>$\nu$</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Compressive Strain at $f_c$</td>
<td>$\varepsilon_c$</td>
<td>$2\times10^{-3}$</td>
<td>-</td>
</tr>
<tr>
<td>Max. Aggregate size</td>
<td>$d_{max}$</td>
<td>15</td>
<td>mm</td>
</tr>
<tr>
<td>Fracture Energy</td>
<td>$G_f$</td>
<td>$5\times10^{-5}$</td>
<td>MN/m</td>
</tr>
<tr>
<td>Softening Parameter</td>
<td>$C_3$</td>
<td>0.00125</td>
<td>-</td>
</tr>
</tbody>
</table>
2.6.4 Results
The problem was computed by the computer program for nonlinear finite analysis of reinforced concrete structures ATENA 2D v3.0.

![Image of Exponential Model](image1)

Figure 2.25

![Image of Local Strain Model](image2)

Figure 2.26

2.6.5 Discussion on results
From the resulting graphs is obvious that the convergence of the Local Strain model of softening based purely on stress-strain relationship without any further energy approach is completely dependent on the finite element mesh refinement. Whereas using the Exponential
Model of cracking introducing the fracture energy material variable can fix up this lack of convergence.

2.7 Structural test validation – code design and non-linear analysis

2.7.1 Introduction and Objectives
The purpose of this analysis was to observe the behavior of the model of the Bahen Center Beam undergoing loading at the mid-span. Nonlinear finite element method software ATENA 2D v3.0 and ATENA 3D v3.0 was used. The beam was also assessed by EC2 code of practice and the results were compared. The main question of our study was to find the failure mode of the analyzed structure. The codes of practice take into account only two types of them in this case: bending and shear failure. We wanted to assure us about the truth of this assumption in this case. As it will be shown later this problem was not so straightforward and involved the assessment of the critical place at the mid-span under the central column. This was possible by using the 3D version of the ATENA software.

2.7.2 Material Model
The real Bahen Center Beam is made from concrete that corresponds to “EC concrete” C35/45. So we used material parameters of this one: mean elastic modulus \( E_{cm} = 33500 \text{MPa} \); characteristic cylinder compressive strength \( f_{ck} = 35 \text{MPa} \); characteristic tensile strength \( f_{tk} = 2.2 \text{MPa} \). The reinforcement was made from the structural engineering steel with the characteristic value of the yield stress \( f_{yk} = 413.4 \text{MPa} \). Hardening of the steel was neglected. The material model of concrete called Cementitious2 was used in analyzing by ATENA.

2.7.3 Geometry and Finite Element Models
The Bahen Beam is 13.76m long, 2.5m wide and 1.8m high. It is supported by two circular columns with diameter 0.76m. The span of the beam is 12.2m and the load is transferred to the beam by the 0.76 diameter column situated in the mid-span. Because of the symmetry of the beam we modeled only one quarter of it in 3D analysis and one half in 2D analysis.

Structural models in the study:

- **2D Plane Stress**
  This case is modeled in ATENA 2D using the plane stress idealization. The longitudinal reinforcement is modeled by three bars which area is corresponding to the area in three layers of the real reinforcement. Stirrups are modeled by the same way.

![Figure 2.27 FE model of Bahen beam.](image)
• **2D Plane Strain**
  This case is solved by ATENA 2D and plane stress idealization where confinement against transversal deformation is introduced. The reinforcement is modeled by the same way as in the case above.

• **3D**
  Here we model a quarter of a beam in ATENA 3D. The columns are modeled with a circular shape. The reinforcement was simulated in a realistic way only a number of the longitudinal bars is reduced twice – the area of them of course corresponds to the real event.

• **3D – Upper Slab**
  There was introduced a part of the upper-floor slab which co-operates with the beam in this simulation. The slab is not permitted to have any transversal horizontal displacement and so it confines the transversal deformation of the beam.

### 2.7.4 Results

As it was said before the problem had been solved by software ATENA. From the graph in the Figure 1 is obvious that there are differences in the behavior of the structural models. First difference is in the type of failure. In models where no transversal deformation confinement is introduced (2D-Plane Stress and 3D) brittle failure occurs. In the 2D-Plane Stress model there is short plastic deformation (about 6 millimeters long) where yielding of longitudinal reinforcement bars occurs. Whereas the failure of the 3D model is strictly brittle – in about 2 millimeters of vertical deformation the support reaction decreases of about 2.1 MN and no yielding of the longitudinal bars is observed.

The behavior of the other two models (2D-Plane Strain and 3D-Upper Slab) after reaching the peak value of the load is much more plastic than the behavior of the previous ones. This is due to preventing the compressed zone of the beam from being transversally expanded. That is why the brittle compressive failure could not occur in these cases.

Figures 2 & 3 illustrate the deformation of the central lateral section of the beam after reaching the peak load. They show the critical importance of the upper floor slab to confining the deformation of the compressed upper part of the beam.

Another part of our study was to compare calculated results to ones of codes of practice. Eurocodes were used for this purpose.

There was dead load 1650 kips and live load 550 kips acting at the mid-span of the beam (together 9.79 MN). The support reaction from this load and the self weight of the beam was \( V_{sk} = 5.65\text{MN} \). When calculating the design values of the load we used \( \gamma_G = 1.2 \) (coefficient of reliability for dead load) and \( \gamma_Q = 1.4 \) (coefficient of reliability for live load). That gave the design value of the support reaction \( V_{sd} = 7.1\text{MN} \). The bending moment at the mid-span caused by the design load was \( M_{sd} = 40.1\text{MNm} \).

Using the EC2 code of practice for the assessment of concrete structures was calculated the shear resistance of the beam \( V_{Rd} = 7.75 \text{MN} \) that was greater than the design shear force \( V_{sd} = 7.1 \text{MN} \) and so the shear resistance was satisfied. Whereas the bending resistance was not satisfied according to EC2 in this case: bending moment capacity of the section was calculated as \( M_{Rd} = 37.5\text{MNm} \) which is smaller than \( M_{sd} = 40.1 \text{MNm} \).
Figure 2.28  Behavior with Characteristic Material Parameters

Figure 2.29  Behavior with Design Material Parameters
2.8 Structural validation – mesh sensitivity, comparison of safety formats, bending, statically determinate structure

The presented example problem (see Figure 2.1) is used to demonstrate two aspects of nonlinear analysis. The problem is analysed using several meshes with different element sizes as shown in Figure 2.2. to evaluate the effect of mesh size in nonlinear finite element analysis. The second objective of this simple example is to compare the resistance values calculated by using different safety formats as described in Section 2.3.2.

Figure 2.30  Deformation of central lateral section without (left) and with (right) confinement.

Figure 2.1  Beam geometry for safety formats comparison
Figure 2.2  Finite element meshes used in the bending example

Figure 2.3  Typical result from finite element analysis using the program ATENA (Cervenka Consulting)
The results from the mesh sensitivity analysis are shown in Figure 2.4. The figure clearly shows that if objective material models are used the ultimate load calculated by the coarse and fine mesh are not far from each other. This should be contrasted with the results described in Section 2.8, where huge mesh dependency appears if purely stress-strain based material models are used.

The results from the current problem even suggest that the coarsest mesh give more conservative values than the fine mesh, which is in contradiction to the classical idea of mesh size effect. This well known effect is also clear from Table 2-2, which contains the comparison of the highest compressive stresses calculated by using the different meshes.

**Table 2-2 Compressive stress comparison for different meshes and EN1992-1 resistance load level**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Highest concrete compressive stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very coarse</td>
<td>3.37</td>
</tr>
<tr>
<td>Coarse</td>
<td>6.72</td>
</tr>
<tr>
<td>Medium</td>
<td>8.35</td>
</tr>
<tr>
<td>Fine</td>
<td>9.82</td>
</tr>
</tbody>
</table>
Figure 2.5  Comparison of safety formats for the bending problem

Figure 2.5 contains the comparison of response curves as well as design resistance values by the different safety approaches. The numerical values of design resistance are for clarity also summarized in Table 2-1. The table demonstrates that for this simple case that can be easily checked by hand calculation all the methods give identical results. This is to be expected since the advantages of the more advanced methods will appear namely in cases when hand calculation and standard approaches are not applicable or introduce large simplifications. Such simplifications usually lead to significant underestimation of design resistance values.

Table 2-3  Comparison of calculated values for design resistance using various safety formats

<table>
<thead>
<tr>
<th>Safety Format</th>
<th>Design Resistance $R_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard partial factor based on EN1992-1</td>
<td>93</td>
</tr>
<tr>
<td>Global resistance estimate using EN1992-2</td>
<td>96</td>
</tr>
<tr>
<td>Probabilistic approach</td>
<td>95</td>
</tr>
<tr>
<td>Global resistance probabilistic estimate, Cervenka &amp; Holicky</td>
<td>101</td>
</tr>
</tbody>
</table>
2.9 Structural validation – comparison of safety formats, shear, statically indeterminate structure

The objective of the example presented in this section is to compare the results obtained by various safety format approaches on a more complicated problem of statically indeterminate structure with shear failure. The analyzed example has been also tested experimentally (see Figure 2.6) thus it is also possible to check the analytical results with an experimental proof.

The beam geometry with dimensions and material properties is shown in Figure 2.7 and the subsequent Figure 2.8 depicts the used numerical model and boundary conditions.

Several safety formats, as they are described in Section 2.3.2, are used to analyzed this shear beam. The used methods and the necessary material properties are listed in Table 2-4.

![Shear wall tested in the laboratory (TU Delft)](image1.png)

**Figure 2.6** Shear wall tested in the laboratory (TU Delft)

![Geometry of the shear wall example](image2.png)

**Figure 2.7** Geometry of the shear wall example
Table 2-4  Material properties used for various safety formats

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete 30/37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_c$ GPa</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>4.1</td>
<td>lognrm.</td>
</tr>
<tr>
<td>$f_c$ MPa</td>
<td>20</td>
<td>25</td>
<td>38</td>
<td>4.9</td>
<td>lognrm.</td>
</tr>
<tr>
<td>$f_t$ MPa</td>
<td>1.3</td>
<td>1.7</td>
<td>2.9</td>
<td>0.6</td>
<td>Weibull</td>
</tr>
<tr>
<td>$G_f$ N/m</td>
<td>35</td>
<td>44</td>
<td>66</td>
<td>13</td>
<td>Weibull</td>
</tr>
<tr>
<td>Steel 500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{sy}$ MPa</td>
<td>434</td>
<td>550</td>
<td>550</td>
<td>31</td>
<td>lognrm.</td>
</tr>
</tbody>
</table>

Figure 2.8  Finite element model for shear wall example
Step 38,
Scalars: rendering, Basic material, in nodes, Principal Strain, Max., <-1.330E-04;1.741E-02>[None]
Cracks: in elements, <2.000E-04;...>, opening: <-2.738E-05;1.769E-03>[m], Sigma_n: <-8.546E+00;2.473E+00>[MPa], Sigma_T: •

**Figure 2.9**  Final failure mode calculated by nonlinear analysis

**Figure 2.10**  Typical result from laboratory experiment
Figure 2.11  Reinforcement yielding at failure from nonlinear analysis

Figure 2.12  Set of load-displacement curves calculated by full probabilistic nonlinear analysis
Various safety formats are used to determine the design resistance of the analyzed structure. The calculated resistance values are compared in Table 2-5. The table shows that again all the methods give similar resistance values. The only difference is in the case of probabilistic methods where a strong sensitivity with respect to the selected shapes of distribution functions. In case of very realistic lognormal distribution, the calculated design resistance 512 is quite similar to those calculated by PSF and global resistance method. For other statistical distribution functions the calculated value could be either lower or higher. This demonstrates the importance of sufficient knowledge of statistical properties of input parameters.

Table 2-5  Comparison of calculated design resistances by various safety methods

<table>
<thead>
<tr>
<th>Method</th>
<th>$R_d$ [kN]</th>
<th>$R_d / R_d^{PSF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial factors (PSF)</td>
<td>501</td>
<td>1.0</td>
</tr>
<tr>
<td>Global factors EN 1992-2</td>
<td>490</td>
<td>0.98</td>
</tr>
<tr>
<td>Probabilistic using normal $R$</td>
<td>512</td>
<td>1.02</td>
</tr>
<tr>
<td>Probabilistic using Weibul $R$</td>
<td>620</td>
<td>1.24</td>
</tr>
<tr>
<td>Prob. all normal materials and $R$</td>
<td>465</td>
<td>0.93</td>
</tr>
</tbody>
</table>
3 FE modelling of corrosion

3.1 Introduction

3.1.1 Background
Since the beginning of the 1990s, non-linear FE analysis has been used to evaluate structural performance and reliability with encouraging results. Within the field of structural assessment of concrete bridges, FE analyses have given great benefits in most cases when used. Through non-linear analyses, strengthening or replacement of existing structures have been avoided. However, the method needs to be developed in several aspects, and questions need to be clarified, in order to be generally applicable. One important aspect is to be able to include deterioration mechanisms in the analysis, such as reinforcement corrosion and its effect on the concrete structure.

3.1.2 Aim and objectives
The aim of this part of the work is to use and further develop existing models to describe structural response with respect to reinforcement corrosion. The aim is to give an overview of how corrosion affect structures, which can be used in a practical way at assessment of existing structures.

3.1.3 Scope of Work
The model of reinforcement corrosion developed by Lundgren (2002); Lundgren (2005b) was used to give an overview of how corrosion affects the bond behaviour for different cases. Results from experiments described in literature were compiled.

3.2 Modelling the effect of corrosion on bond

3.2.1 General
Modelling of the effect of corrosion on bond was done by Lundgren (2002) and Lundgren (2005b). The modelling method used is specially suited for detailed three-dimensional finite element analyses, using solid elements for both the concrete and the reinforcement. Special interface elements were used at the surface between the reinforcement bars and the concrete to describe a relation between the traction and the relative displacement in the interface. The interface elements include a frictional bond model and a corrosion model, which can be viewed as two separate layers around a reinforcement bar, see Figure 3.1.

![Frictional bond model and corrosion model can be viewed as two separate layers around a reinforcement bar.](image)

*Figure 3.1: Frictional bond model and corrosion model can be viewed as two separate layers around a reinforcement bar.*
3.2.2 Bond

A model for the bond between ribbed reinforcement and concrete was used, for details see Lundgren (2005a). For analyses of smooth bars, adhesion was added to the model. The reason is that for ribbed bars, the adhesion contributes to only a small part of the bond, while for smooth bars, the ability to develop normal stresses during slip is much smaller. Thereby, the bond capacity due to friction is strongly reduced, and thus, the adhesion is no longer negligible, see also Gustavson (2002). Thus, the used model describes the basic mechanisms of bond such as:

- adhesion
- friction
- ability to cause normal stress at slip
- upper limit determined by failure of the concrete between the ribs

The stresses are limited by two functions, see Figure 3.2. One function ($F_1$) describes the friction, including the adhesion. The other function ($F_2$) describes the upper limit, which is determined from the stress in the inclined struts that results from the bond action, limiting the compressive and tensile stresses. As can be seen in Figure 3.2, the maximum bond capacity that can be obtained is roughly half the compressive stress that can be carried in the inclined struts; i.e. roughly half the compressive strength of the concrete. When the stresses are limited by the function describing the friction, normal stresses are generated.

By the use of this local model, and by modelling the bar itself and the surrounding structure with solid elements, the global behaviour with different failure modes are obtained as a result of the analyses. Examples of such failure modes are pull-out failure, splitting failure, or loss of bond due to yielding of the reinforcement.

![Figure 3.2: The yield surface in the frictional bond model.](image)

3.2.3 Corrosion

The corrosion products occupy a larger volume than the steel it was formed of, which leads to splitting stresses acting on the concrete. The volume increase of the corrosion products compared to the virgin steel was modelled in a corrosion layer, as described in Lundgren (2005b). As the rust is not free to expand, it is important to include the mechanical behaviour of the rust itself. Following the results in Lundgren (2005b), it was assumed that the rust behaves like a granular material; i.e. its stiffness increases with the stress level. Furthermore, for ribbed bars, it was assumed that corrosion affects the friction between the steel and the concrete. However, rather large corrosion penetrations were assumed to be needed before the friction decreased; this corresponds to that only corrosion attacks of such magnitude that they will influence the size of the ribs will influence the friction.
3.2.4 Finite element models
To investigate different typical cases, axisymmetric analyses were carried out. The geometry in these analyses is shown in Figure 3.3. The reinforcement had a diameter of 20 mm, while the cover was varying, see Table 3.1. In analyses where transverse reinforcement was present, 2 Ø6 were placed as shown in Figure 3.3, with the distance $a$ as tabulated in Table 3.1. In some of the analysed cases, meshes of varying density were used, see examples in Figure 3.3b and c. The results in these analyses corresponded well; thus, no mesh dependence was found.

![Figure 3.3: a) Dimensions and boundaries in the finite element analyses. Measurements in mm. b) and c) Examples of meshes used; one coarse and one dense mesh for the same geometry.](image)

The concrete was modelled with a constitutive model based on non-linear fracture mechanics, using a rotating crack model based on total strain; see TNO (2002). Axisymmetric models were used, assuming four radial cracks. For the tension softening, the curve by Hordijk et al. was chosen, as described in TNO (2002). In compression, an ideal plastic behaviour was used, assuming a compressive strength of 40 MPa. Other necessary material data for the concrete were estimated according to the expressions in CEB (1993). Thus, the used Young’s modulus was 34.2 GPa, Poisson’s ratio 0.15, tensile strength 3.0 MPa, and fracture energy 79.2 N/m. The elastic modulus of the reinforcement was assumed to be 200 GPa, and Poisson’s ratio was 0.3.

The input for the bond model for ribbed bars was as described in Lundgren (2005a). The input used for smooth bars is based on the work in Gustavson (2002). All chosen parameters are tabulated in Table 3.1.
3.3 Effect of corrosion on the bond for ribbed bars

The effect of corrosion on the bond behaviour is strongly influenced on whether there is transverse reinforcement present or not. Furthermore, it is separated between whether the cover will crack at pull-out of non-corroded reinforcement or not. This depends on the confinement from the surrounding concrete and the boundaries. These various combinations result in four different cases:

- with transverse reinforcement; cover cracks at non-corroded pull-out
- with transverse reinforcement; cover does not crack at non-corroded pull-out
- without transverse reinforcement; cover cracks at non-corroded pull-out
- without transverse reinforcement; cover does not crack at non-corroded pull-out

In the following, each of these different cases is described more in detail, with analyses and references describing the behaviour.

3.3.1 With transverse reinforcement; cover cracks at non-corroded pull-out

Commonly for ribbed bars, the covers will crack at non-corroded pull-out for small covers combined with large reinforcement bars, unless the boundaries provide restraint to prevent cracking. It is also common that transverse reinforcement is provided.

Analyses studying this situation were carried out using axisymmetric models as described in the section “Finite element models”. The concrete cover (b in Figure 3.3a) was 40 mm, thus the cover to diameter ratio was 2.0. The results from these analyses are shown in Figure 3.4. For non-corroded pull-out, a splitting crack reached the outer surface of the concrete when the bond stress was around 14 MPa. This decreased the stiffness; however, the bond stress could continue to increase until a maximum of 15.5 MPa was reached. The transverse reinforcement kept the structure together; thus causing a ductile behaviour. Corrosion alone cracked the cover at a corrosion penetration around 50 μm. Corrosion increased the initial stiffness in the bond-slip behaviour; thus, the more corrosion, the smaller the slip when the maximum bond stress was reached. Regarding the bond capacity, a small increase could be noticed for corrosion penetrations smaller than the one cracking the cover, and a small decrease for larger corrosion penetrations. These changes in capacity were very small.

These results can be compared to what has been measured in tests including transverse reinforcement and cover cracking at non-corroded pull-out. Al-Sulaimani et al. (1990) made beam tests with stirrups confining the main reinforcement. They measured a small increase for corrosion penetrations smaller than the one cracking the cover, and a small decrease for larger corrosion penetrations. Fang et al. (2004) measured almost constant bond capacities in pull-out tests. Lee et al. (2002) conducted pull-out tests and report of reductions in bond capacities for very large corrosion levels. Coronelli (1998) combined transverse reinforcement with special arranged so called skin reinforcement, and measured increasing bond strength with increasing corrosion level, even after corrosion cracking. Shima (2002) reports pull-out tests with a rather large loss of bond; however in these tests the transverse reinforcement was only arranged on one side of the bar with a rather small cover on the other side.

To conclude, for the case with transverse reinforcement and where the cover cracks at non-corroded pull-out, corrosion has only a minor effect on the bond behaviour. As the cover cracks for pull-out already without corrosion, the transverse reinforcement is already for non-corroded specimens keeping the structure together. If the cover cracks due to corrosion, this does not influence to any larger degree. For large corrosion penetrations, a small decrease in bond capacity can be seen, probably mainly due to that the ribs are being corroded. This behaviour can be seen both in analyses and in experimental results.
Table 3.1: Input in the analyses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ribbed bars</th>
<th>Smooth bars</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>40 and 80 mm</td>
<td>20 and 40 mm</td>
<td>Concrete cover, see Fig. 3.3a.</td>
</tr>
<tr>
<td>$a$</td>
<td>25 mm</td>
<td>15 mm</td>
<td>Distance to transverse reinforcement, see Figure 3.3a.</td>
</tr>
<tr>
<td><strong>Bond model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.04</td>
<td>0.015</td>
<td>Dilation parameter; controls the ability to generate normal stresses at slip.</td>
</tr>
<tr>
<td>$D_{22}$</td>
<td>$2.05 \times 10^{11} \text{ N/m}^3$</td>
<td>$2.05 \times 10^{11} \text{ N/m}^3$</td>
<td>Stiffness in the elastic stiffness matrix describing elastic bond-slip stiffness.</td>
</tr>
<tr>
<td>$f_a$</td>
<td>0</td>
<td><img src="image" alt="Adhesive strength" /></td>
<td>Adhesive strength (see Figure 3.2), depends on the hardening parameter which is approximately equal to the slip.</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td></td>
<td><img src="image" alt="Coefficient of friction" /></td>
<td>Coefficient of friction (see Figure 3.2) for uncorroded reinforcement, depends on the hardening parameter which is approximately equal to the slip.</td>
</tr>
<tr>
<td>$D_{11}$</td>
<td>$D_{11}/E_c$ [m$^3$]</td>
<td></td>
<td>Stiffness in the elastic stiffness matrix describing elastic normal stress – normal deformation stiffness.</td>
</tr>
<tr>
<td>$c$</td>
<td>$c/f_{cc}$ [-]</td>
<td></td>
<td>Maximum stress in the inclined compressive struts (see Figure 3.2), depends on the hardening parameter which is approximately equal to the slip.</td>
</tr>
<tr>
<td><strong>Corrosion model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>2.0</td>
<td>Volume of the rust relative to the uncorroded steel.</td>
<td></td>
</tr>
<tr>
<td>$K_{cor}$</td>
<td>14.0 GPa</td>
<td>Parameters describing the mechanical behaviour of the rust.</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>7.0</td>
<td>Function describing the effect of corrosion on the coefficient of friction; depends on corrosion penetration divided with reinforcement radius.</td>
<td></td>
</tr>
</tbody>
</table>
3.3.2 With transverse reinforcement; cover does not crack at non-corroded pull-out

For large covers combined with small reinforcement bars, the cover will not crack at non-corroded pull-out. This can also be true for larger bars or smaller covers, if the boundaries provide cracking restraint as for example when there is support pressure acting. Both of these cases can be combined with transverse reinforcement.

No experiments have been found in the literature for this combination; with enough confinement to avoid cracking at non-corroded pull-out combined with transverse reinforcement. The main reason for this is most likely that when transverse reinforcement has been arranged in tests, the cover used has been rather small to be similar to existing structures. However, as mentioned above, this is a possible combination, for example at supports or if small bar diameters are used.

Analyses studying this situation were carried out using axisymmetric models with a concrete cover of 80 mm, giving a cover to diameter ratio of 4.0. The results from these analyses are shown in Figure 3.5. For non-corroded pull-out, the maximum bond capacity close to half the compressive strength was reached, i.e. around 20 MPa. Corrosion cracked the cover at a corrosion penetration around 270 \( \mu \text{m} \). Corrosion increased the initial stiffness in the bond-slip behaviour; thus, the more corrosion, the smaller the slip when the maximum bond stress was reached. The maximum bond capacity remained almost unaffected by the corrosion for corrosion penetrations smaller than the one cracking the cover. For larger corrosion penetrations, the capacity decreased to the same level as for the case with transverse reinforcement but with a smaller cover, i.e. around 15 MPa. For further increase of the corrosion, the bond capacity remained almost unaffected. Also the initial stiffness decreased for larger corrosion penetrations, compared to the maximum stiffness which was obtained just before the cover was cracked. As the transverse reinforcement kept the structure together; a ductile behaviour was obtained in all analyses.

Concludingly, for the case with transverse reinforcement and where the cover does not crack at non-corroded pull-out, the effect of corrosion will be slightly different depending on if the corrosion penetration will crack the cover or not. For corrosion penetrations that do not cause cracking of the cover, the maximum bond capacity will remain almost unaffected, or even increase slightly. At the corrosion penetration that causes cracking of the cover, the maximum bond capacity will decrease to a smaller level, which will decrease only slightly for larger corrosion penetrations. This smaller level depends on the amount of transverse reinforcement.

Figure 3.4: Results from analyses with ribbed bars, with transverse reinforcement; cover cracks at non-corroded pull-out. Cover 40 mm, bar diameter 20 mm. a) Bond stress versus slip; b) maximum bond stress versus corrosion penetration.
3.3.3 Without transverse reinforcement; cover cracks at non-corroded pull-out

For small covers combined with large reinforcement bars, the cover will crack at non-corroded pull-out. If no transverse reinforcement is present, the bond capacity will be limited already for non-corroded bars.

This situation was analysed using axisymmetric models with a concrete cover of 40 mm, i.e. similar as in the corresponding case with transverse reinforcement. The results from these analyses are shown in Figure 3.6. For non-corroded pull-out, a splitting crack reached the outer surface of the concrete when the bond stress was just below 12 MPa. As there was no transverse reinforcement keeping the structure together, this resulted in a brittle failure. Corrosion alone cracked the cover at a corrosion penetration around 40 μm. Corrosion increased the initial stiffness in the bond-slip behaviour. The maximum bond capacity remained almost unaffected by corrosion for corrosion penetrations that did not crack the cover; however, for larger corrosion penetrations, the bond capacity decreased rapidly to almost zero.

These findings can be compared to experimental results on bond in corroded specimens without transverse reinforcement, and where the cover cracks at non-corroded pull-out. Mangat and Elgarf (1999) tested bond in beam tests according to RILEM’s recommendations. They found a small bond increase until the corrosion penetration reached a critical level; thereafter the bond capacity decreased. Auyeung et al. (2000) report similar findings from pull-out tests. Stanish et al. (1999) tested beams and found decreasing bond capacity for increasing corrosion levels. Ghandehari et al. (2000), Lee et al. (2002), Shima (2002), and Fang et al. (2004) all carried out pull-out tests and report that bond capacity decrease drastically with the corrosion level.

To conclude, for the case without transverse reinforcement and where the cover cracks at non-corroded pull-out, very limited corrosion will crack the cover. Thereafter, corrosion will decrease the bond capacity in a detrimental way. Similar findings can be found both in analyses and in experiments.
3.3.4 Without transverse reinforcement; cover does not crack at non-corroded pull-out

For large covers combined with small reinforcement bars, the cover will not crack at non-corroded pull-out. As discussed before, non-corroded pull-out might also not lead to cracking of the cover even for larger bars or smaller covers, if the boundaries provide cracking restraint as for example when there is support pressure acting. If designing without taking corrosion into account, no transverse reinforcement is needed in these situations.

This case was analysed using axisymmetric models with a concrete cover of 80 mm, i.e. similar as in the corresponding case with transverse reinforcement. The results from these analyses are shown in Figure 3.7. In the analysis without corrosion, a pull-out failure was obtained with a maximum bond capacity close to half the compressive strength, i.e. around 20 MPa. Corrosion cracked the cover at a corrosion penetration around 215 μm. For corrosion penetrations larger than that, the bond capacity decreased abruptly. For a corrosion penetration close to that level, the maximum bond capacity remained almost the same, but the failure mode changed from pull-out to splitting failure, thus resulting in a brittle failure instead of the ductile behaviour obtained for smaller corrosion penetrations, see Figure 3.7a and compare the curves for $x = 0$ and $x = 200$ μm. Corrosion increased the initial stiffness in the bond-slip behaviour; thus, the more corrosion, the smaller the slip when the maximum bond stress was reached.

Several experimental results exist where the cover does not crack at non-corroded pull-out, and no transverse reinforcement is supplied. Almusallam et al. (1996) made cantilever bond tests; in these tests the loss of bond at the corrosion level which caused cracking of the cover was very pronounced. For smaller corrosion levels, the bond capacity was rather high, while it became very low when corrosion had cracked the cover. Pull-out tests by Al-Sulaimani et al. (1990), Cabrera and Ghoddoussi (1992), Ghandehari et al. (2000), and Hussein et al. (2002) all show similar behaviour.

Thus, for the case without transverse reinforcement and where the cover cracks at non-corroded pull-out, the bond capacity will be decreased in a detrimental way when the corrosion cracks the cover. For smaller corrosion penetrations, the bond capacity remains almost unaffected or even increases slightly. As the analyses presented here are axisymmetric, they can be expected to be more sensitive to cracking than is to be expected in real structures, as the concrete surrounding the reinforcement there does not crack in all directions at the same time. However, similar findings were found also in cantilever bond tests.
3.3.5 General comments

The behaviour for all different cases is summarised in Figure 3.8. The scales in the bond-slip curves are varying, to make all graphs clearly visible. The scales in the maximum bond stress versus corrosion level graphs are, however, intended to be the same, to enable comparisons. Naturally, this summary is a simplification; for example, if the amount of transverse reinforcement is small, the behaviour will become close to that of specimens without transverse reinforcement. Also the transverse reinforcement can of course corrode; however, in general, larger corrosion penetrations are needed to substantially change the bearing capacity of the transverse reinforcement than to effect the bond of the main reinforcement. Considering these limitations, still the summary in Figure 3.8 is believed to be of help to understand the mechanisms, and at assessment of existing structures. One general observation for all cases is that corrosion increases the initial stiffness for all cases. The effect on the bond capacity varies more, depending on the presence of transverse reinforcement and the failure mode for uncorroded pull-out.

This variation on the effect on the bond capacity can be seen in Figure 3.9, where the maximum bond stress from the various analyses with ribbed bars is plotted versus corrosion penetration. As can be seen, transverse reinforcement makes the bond much less sensitive to corrosion.

3.4 Effect of corrosion on the bond for smooth bars

Also for smooth bars, it is the same parameters that influence the bond as for ribbed bars. Again, therefore, it is separated between four different cases:

- with transverse reinforcement; cover cracks at non-corroded pull-out
- with transverse reinforcement; cover does not crack at non-corroded pull-out
- without transverse reinforcement; cover cracks at non-corroded pull-out
- without transverse reinforcement; cover does not crack at non-corroded pull-out

It should be noted, that for smooth bars, the cover generally does not crack at non-corroded pull-out. Still, here it was chosen to include these cases in the study. The main reason for this choice was to ease comparison to ribbed bars.
Deformed bars

Transverse reinforcement

No transverse reinforcement

Transverse reinforcement

Smooth bars

Transverse reinforcement

No transverse reinforcement

At uncorroded pull-out

Cover cracks

No cracks

Cover cracks

No cracks

Cover cracks

No cracks

Effect of corrosion

Small bond decrease or appr. equal

Small bond increase until cover cracks, then appr. equal or slight decrease

Bond decrease already for low corrosion

Small bond increase until cover cracks, then abrupt decrease

Bond increase

Large bond increase until cover cracks, then appr. equal or slight increase

Bond decrease

Large bond increase until cover cracks, then decrease

Bond stress versus slip:
- uncorroded
- corr., precracking
- corr., postcracking

Maximum bond stress versus corrosion level (arrow indicates cover cracking)

Figure 3.8 Overview of effect of corrosion on bond
Figure 3.9: Maximum bond stress versus corrosion penetration; results from axisymmetric analyses with ribbed bars.

Experiments of smooth corroded bars by Cairns et al. (2002a; Cairns et al. (2002b) show that there is a large difference between top cast and bottom cast bars. This is more important for smooth bars than for ribbed bars. The bond capacity for uncorroded smooth bars is lower for top cast than for bottom cast bars. Furthermore, there is also a difference in the tendency to split the cover due to corrosion. Cairns et al. (2002b) found that the top cast bars could withstand a higher corrosion level before cracking of the cover than the bottom cast bars.

It should be noted that even though the bond model has been calibrated and used for smooth bars in Gustavson (2002), it has not earlier been used together with the corrosion model for smooth bars. Therefore, the experience of how these models work together is far less for smooth bars than for ribbed bars. Nevertheless, the results seem to be reasonable when comparing with available test results. Thus, even if the results cannot be trusted in the same way as for ribbed bars, they give a good indication of the expected behaviour in various situations.

In the following, each of the different cases is described more in detail, with analyses showing the behaviour. Generally, there are not as many experimental results available for smooth bars as for ribbed. For the cases where references have been found, these are presented.

3.4.1 With transverse reinforcement; cover cracks at non-corroded pull-out

As smooth bars generate far less splitting stresses than ribbed bars, it is not common for smooth bars that the cover will crack for non-corroded pull-out. For very small covers, this is possible; however, such small covers are generally not used. To enable an overview, this case was anyhow studied here, using axisymmetric models with a concrete cover of only 20 mm, i.e. with a cover to diameter ratio of 1.0. The results from these analyses are shown in Figure 3.10. As adhesion was included in the analyses with smooth bars, the bond-slip curves have an initial very steep branch, where the bond stress is increased with very small slips. In the analysis without corrosion, the bond stress could be increased until about 1.75 MPa before the yield line was reached. Thereafter, the stiffness decreased; still the bond stress could be increased until about 4.8 MPa, when maximum was reached. At a slip of about 0.7 mm, a splitting crack reached the outer surface of the concrete. The transverse reinforcement kept the structure together; thus causing a ductile behaviour. Corrosion alone cracked the cover already at a corrosion penetration around 20 μm; however, due to the
transverse reinforcement, the crack width was limited, thus the cracks were about 0.1 mm wide first at a corrosion penetration of around 450 μm. Corrosion increased the initial stiffness in the bond-slip behaviour; thus, the more corrosion, the smaller the slip when the maximum bond stress was reached. The maximum bond increased with increasing corrosion penetrations to a level of about 13 MPa. To conclude, for the case with smooth bars, transverse reinforcement and where the cover cracks at non-corroded pull-out, corrosion will increase the bond capacity.

No tests have been found in the literature with smooth bars including transverse reinforcement and cover cracking at non-corroded pull-out. This is most likely due to that this is not a very common case.

### 3.4.2 With transverse reinforcement; cover does not crack at non-corroded pull-out

As smooth bars do not generate any large splitting stresses, the covers don’t need to be so large to prevent cracking. The situation where the cover does not crack at non-corroded pull-out is therefore a common situation in real structures, which can be combined with transverse reinforcement.

Analyses studying this situation were carried out using axisymmetric models with a concrete cover of 40 mm, giving a cover to diameter ratio of 2.0. The results from these analyses are shown in Figure 3.11. For non-corroded pull-out, the maximum bond capacity is much lower than for the corresponding situation with ribbed bars; around 6 MPa compared to 20 MPa. Corrosion cracked the cover at a corrosion penetration around 55 μm. The maximum bond capacity increased quite a lot, to around 15 MPa, for corrosion levels smaller than the one cracking the cover; this is due to the positive effect of the corrosion induced pressure. For larger corrosion levels, the bond capacity remained approximately constant at this high level. Corrosion also increased the initial stiffness in the bond-slip behaviour; however, slightly decreased the ductility.

These results can be compared to test results. Cairns et al. (2002b) carried out beam end eccentric pull-out tests. In these tests, the cover was only 20 mm to 16 mm bars; still no cracking occurred at non-corroded pull-out. For the top cast bars, the bond strength increased to the level of bottom cast bars (i.e. roughly by a factor of 2) already for small corrosion levels. For higher corrosion levels, the bond capacity remained about the same. For bottom cast bars, the bond capacity was almost unaffected by corrosion. Cairns et al. (2002a) also carried out concentric pull-out tests with transverse reinforcement with larger

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**Figure 3.10** Results from analyses with smooth bars and transverse reinforcement; cover cracks at non-corroded pull-out. Cover 20 mm, bar diameter 20 mm. a) Bond stress versus slip; b) maximum bond stress versus corrosion penetration.
covers. In these tests, they measured an increase in bond strength for increasing corrosion levels, and report as much as 7.5 times the bond capacity of uncorroded specimen. Fang et al. (2004) made similar tests, and found that the bond strength increased to 2-3 times the capacity of uncorroded specimens already for rather low corrosion levels. This higher capacity was maintained also for further increase of the corrosion level, see also Figure 3.12.

Concluding, for the case with smooth bars, transverse reinforcement and where the cover does not crack at non-corroded pull-out, corrosion will increase the bond capacity, in some cases quite a lot.

3.4.3 Without transverse reinforcement; cover cracks at non-corroded pull-out

For very small covers, the cover might crack at non-corroded pull-out. If no transverse reinforcement is present, the pull-out failure will become rather brittle due to the splitting failure.

This situation was analysed using axisymmetric models with a concrete cover of 20 mm, i.e. similar as in the corresponding case with transverse reinforcement. The results from these analyses are shown in Figure 3.13. For non-corroded pull-out, a splitting crack reached the outer surface of the concrete when the bond stress reached its maximum around 4.2 MPa. As there was no transverse reinforcement keeping the structure together, this resulted in a rather brittle failure. Corrosion alone cracked the cover at a corrosion penetration around 15 μm. Corrosion increased the initial stiffness in the bond-slip behaviour. The maximum bond capacity increased slightly by corrosion for corrosion penetrations that did not crack the cover; however, for larger corrosion penetrations, the bond capacity decreased to almost zero.

To conclude, for the case without transverse reinforcement and where the cover cracks at non-corroded pull-out, very limited corrosion will crack the cover. Thereafter, corrosion will decrease the bond capacity.

No tests have been found in the literature with smooth bars and cover cracking at non-corroded pull-out. This is most likely due to that this is not a very common case.
3.4.4 Without transverse reinforcement; cover does not crack at non-corroded pull-out

The situation where the cover does not crack at non-corroded pull-out is a common situation in real structures, and it is not always that transverse reinforcement is provided.

This case was analysed using axisymmetric models with a concrete cover of 40 mm, i.e. similar as in the corresponding case with transverse reinforcement. The results from these analyses are shown in Figure 3.14. In the analysis without corrosion, a pull-out failure was obtained with a maximum bond capacity around 5.5 MPa. Corrosion cracked the cover at a corrosion penetration around 40 μm. At this corrosion penetration, the bond strength increased to about 10 MPa. For corrosion penetrations larger than that, the bond capacity decreased abruptly, and as the failure mode changed from pull-out to splitting failure, a brittle failure instead of the ductile behaviour obtained for smaller corrosion penetrations was obtained. Corrosion increased the initial stiffness in the bond-slip behaviour; thus, the more corrosion, the smaller the slip when the maximum bond stress was reached.

These results can be compared to results by Fang et al. (2004), who made concentric pull-out tests without transverse reinforcement. An initial increase in bond strength for small corrosion levels changed into a decreasing capacity for larger corrosion levels. However, in these tests, the bond capacity was not smaller than for the uncorroded specimens even for corrosion levels as high as 9%. Cairns et al. (2002b) carried out eccentric pull-out tests. Even though the scatter in these tests was rather large, some tendencies could be seen. For the top cast bars, the bond strength increased to the level of bottom cast bars for corrosion level around 50 μm. For higher corrosion levels, the bond capacity decreased gradually, both for top cast and bottom cast bars.

Thus, for the case with smooth bars, without transverse reinforcement and where the cover cracks at non-corroded pull-out, the bond capacity will be decreased when the corrosion cracks the cover. For smaller corrosion penetrations, the bond capacity increases. As the analyses presented here are axisymmetric, they can be expected to be more sensitive to cracking than is to be expected in real structures, as the concrete surrounding the reinforcement there does not crack in all directions at the same time.

Figure 3.13 Results from analyses with smooth bars without transverse reinforcement; cover cracks at non-corroded pull-out. Cover 20 mm, bar diameter 20 mm. a) Bond stress versus slip; b) maximum bond stress versus corrosion penetration.
### 3.4.5 General comments

The behaviour for all different cases is summarised in Figure 3.8. Generally, the bond capacity of smooth bars is less than for ribbed bars; however, for corrosion penetrations that do not crack the cover, the bond capacity can be increased to almost the same level as for ribbed bars.

The variation on the effect on the bond capacity can be seen in Figure 3.15, where the maximum bond stress from the various analyses with smooth bars is plotted versus corrosion penetration. As can be seen, transverse reinforcement makes the bond much less sensitive to corrosion.

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**Figure 3.14** Results from analyses with smooth bars, without transverse reinforcement; cover does not crack at non-corroded pull-out. Cover 40 mm, bar diameter 20 mm. a) Bond stress versus slip; b) maximum bond stress versus corrosion penetration.

**Figure 3.15** Maximum bond stress versus corrosion penetration; results from axisymmetric analyses with smooth bars.
3.5 Recommendations for assessment of existing corroded concrete structures

From the overview presented in the previous sections, recommendations were worked out for judgements concerning how serious corrosion is when assessing existing concrete structures. From drawings etc., it is supposed to be known what type of reinforcement that has been used, and whether transverse reinforcement crossing potential splitting cracks is provided or not. In Figure 3.16, the four various cases this leads to are listed in the order of how serious it is if corrosion takes place in anchorage regions; i.e. both at end anchorage and at splices. Each case is graded compared to statements when corrosion will become critical related to cracking of the cover due to corrosion: before the cover cracks, when the cover cracks, or first at a later stage.

In the following, recommendations regarding the various cases are given. They are meant to give guidance for bridge owners, of how to judge results from inspections and corrosion measurements. For detailed information about corrosion monitoring systems, see report D5.2.S3 Corrosion monitoring systems for railway concrete bridges.

It was concluded that ribbed bars without transverse reinforcement is most sensitive to corrosion. In this case, the anchorage capacity can be reduced already before the cover is cracked due to corrosion. And, especially if the cover is thick, as soon as cracking of the cover occurs, the anchorage capacity will be very much reduced, and thus become critical. As cracking of the cover can occur between inspections, it is in new design advised to use transverse reinforcement in zones where ribbed bars are anchored. This advice is also given in Berra et al. (2003). In existing structures, it is advised to inspect at close intervals if ribbed bars are anchored without any transverse reinforcement and if corrosion is likely to occur. For bridges where the anchorage capacity may become critical for the load carrying capacity, it is advised to follow the risk of reinforcement corrosion due to carbonation or chloride ingress. This can be made by using a corrosion monitoring system, see report D5.2.S3 Corrosion monitoring systems for railway concrete bridges, or through regularly taken material samples from concrete cover. Measures need to be taken as soon as there is any risk of corrosion.

Also for smooth bars without transverse reinforcement, the anchorage capacity will decrease when cracking of the cover occurs. However, the decrease will not be so abrupt as for ribbed bars, and furthermore, the anchorage length of smooth bars will be long already in the original design. Most often, smooth bars are provided with end hooks, providing most of the anchorage capacity. The orientation of the end hooks is most likely of importance; if the hook is oriented into the structure, the anchorage is less sensitive to corrosion.

In cases with smooth bars without end hooks oriented into the structure, and without transverse reinforcement, is it advised to inspect at close intervals if corrosion is likely to occur. For bridges where the anchorage capacity may become critical for the load carrying capacity, it is advised to detect the onset of corrosion. This can be monitored ..., see .... For this situation, it is judged that measures must be taken when there is any indication of corrosion.

For ribbed bars with transverse reinforcement, the anchorage of corroded reinforcement will depend on the amount of transverse reinforcement. It is also important that the transverse reinforcement crosses the potential splitting cracks. Normally the transverse reinforcement corrodes as well as the main reinforcement; naturally it is then only the uncorroded part of the transverse reinforcement that can be accounted for. If the transverse reinforcement is sufficient and satisfactory oriented, the anchorage will not be immediately critical, even if cracking of the cover occurs. In this case it is advised to do regular inspections in order to detect splitting cracks. Measures must be taken before the cover spalls off in order not to lose the anchorage capacity.
Finally, when smooth bars with transverse reinforcement corrode, the anchorage is not a very critical issue as long as the transverse reinforcement crosses the potential splitting cracks. Of course, as for ribbed bars, one needs to consider corrosion of the transverse reinforcement. Furthermore, if significant spalling of the cover occurs, investigations are needed. This case becomes critical mainly through reduction of reinforcement area. It is advised to do regular inspections to detect splitting cracks and cover spalling. Measures must be taken when the concrete cover has spalled off.

It is important to point out that these recommendations concern how critical corrosion is for the anchorage. At assessment of the load-carrying capacity, it is also important to take the effect of the area reduction into account. This includes reduction of the reinforcement area, of both the main and the transverse reinforcement, but also reduction of the concrete cross-section, for example when corrosion has caused spalling of the concrete cover in the compression zone. The effect of the area reduction becomes more important when transverse reinforcement is present, as the anchorage is not as critical then.

Figure 3.16  Overview of how critical corrosion is for the anchorage capacity. Observe that it is important that the transverse reinforcement crosses potential splitting cracks in order to be effective.
4 Evaluation of material properties

4.1 Introduction

4.1.1 Background
One very important aspect of the assessment of the performance of an existing structure is to evaluate resistance parameters in a consistent way. Assessment of the specific material properties of an existing concrete bridge can be based on information of the following types:

- Specifications of material qualities from the building documentation,
- Tests made of concrete and/or reinforcement from the time of construction,
- Direct tests on specimens taken from the structure at any age
- Non-destructive testing on the existing bridge.

This information has to be interpreted and transformed to give consistent input into the structural analysis employed in the assessment situation. The level of uncertainty associated with the material properties used for the assessment is also of great importance for an adequate assessment of safety and performance of the bridge. These requirements are especially important for assessment based on non-linear analysis.

4.1.2 Aim and objectives
The general objective is to improve the possibility to make a reliable determination of in-situ material properties, including their statistical variability, in existing concrete bridges. A particular aim is to develop better methods to describe development of concrete properties with age and degree of hydration.

4.1.3 Scope of Work
A general review of the state of art concerning the issues related to determination of in-situ concrete properties in old concrete structures will be made. Such issues are:

- Transformation from standard test results to strength in the structure
- Statistical treatment to handle different kinds of uncertainties
- Age dependence of concrete properties
- Transformations between different types of concrete specimens (size, shape, etc.)
- Transformations of concrete strength between different loading modes (e.g. between compression and tension)

The materials relevant for the resistance of concrete bridges are:

- concrete
- reinforcement
- prestressing steel

These materials will be discussed in the following sections.
4.2. Concrete

4.2.1 General
For concrete bridges built after 1945 some type of reference to concrete quality class related to a nominal value of standardised compressive strength is often available. This can be seen as a reference value, which after appropriate interpretation can be used as a basis for estimation of other material parameters. In other cases, where only information about the concrete mixture proportions may be available, rough estimates may also be made of compressive strength. If no information whatsoever is available, in-situ testing can be used if the concrete strength properties are important for the assessment of resistance. In-situ testing can also be used to get more reliable information about the current strength properties of concrete in old bridges and to exploit the beneficial effect of strength growth due to aging. Testing of drilled out cores gives the best basis for reliable assessment of the properties in existing concrete structures. For preliminary assessment and monitoring of concrete strength non-destructive and semi-destructive methods are available, such as the Smith Rebound Hammer, the Capo-test and the Ultrasonic Pulse Velocity method, see D3.15 (2006). Non-destructive methods are also very suitable as a method to select positions for drilling of cores.

A proper description of mechanical properties for concrete as a basis for structural analysis is a complex matter for the following reasons:

- A number of different strength parameters are needed
- Material properties change with age, due to continuous hardening
- Results from testing of strength depend on size and design of the test specimens used.
- The strength in the finished structure is different from that obtained by testing of standardised specimens.

Uniaxial compressive strength is normally used to characterize the quality of concrete and most other strength parameters can be derived from compressive strength by empirical relations. Compressive strength is also used in codes to define strength classes. Once the strength class is specified all necessary design material parameters needed for prediction of load bearing capacities are given by Eurocode 2 (2004).

For an existing bridge structure, the strength class specified for the original construction is one basis for evaluation. However, this gives a rather imprecise prediction of the actual strength, especially if the concrete is old. The strength has normally increased significantly due to continued hydration. Also, deterioration may have occurred so that the actual strength need to be re-evaluated. In such cases, in-situ testing of the concrete in the structure may be used to obtain more reliable information about the concrete properties.

When the verification of capacity is performed via non-linear structural analysis in-situ material properties should be used as input. Also when the assessment of the bridge is made by reliability analysis the strength parameters should refer to in-situ concrete properties.

4.2.2 Definitions
The following definitions of strength measures are given in prEN13791 (2003).

Standard compressive strength is the measured compressive strength of a standard test specimen which is sampled, made, cured and tested in accordance with standard test methods such as those described in EN12390-1 (2000) and EN12390-3 (2001).
Core compressive strength is the measured compressive strength of a core taken from the structure and is determined according to a standard such as EN 12504-1 (2000).

In-situ compressive strength in a structure or in a structural element is expressed as the strength of a standard test specimen (cylinder or cube). The in-situ strength is estimated from tests on a number of cores drilled from the structure or the structural element or from indirect tests on the structure.

Test location is a limited area which is selected for measuring in-situ strength.

Test region is one or several structural elements produced under similar conditions. A test region contains several test locations. A test region can be a part of a structure in which

- the strength has to be assessed
- the properties of concrete may be assumed to be distributed in the same way

4.2.3 Effect of specimen size and shape

Standard compressive strength is determined from uniaxial tests usually at the age of 28 days. Two different types of specimens are commonly used

- Cylinders with length 300 mm and diameter 150 mm (strength \( f_{c,\text{cyl}} \))
- 150 mm cubes (strength \( f_{c,\text{cube}} \))

Conversion between these two strength measures can be made with the following empirical relation which can be used for \( f_{c,\text{cube}} < 80 \) MPa.

\[
f_{c,\text{cyl}} \approx 0.8 \cdot f_{c,\text{cube}}
\]

Alternative relations may be found in the literature, see e.g. Betonghandbok Material (1994), but Eq.(4.1) is consistent with the definition of strength classes in de 2.

Compression tests in earlier times were usually also performed on cube or cylinders but sometimes with other dimensions. 200 mm cube specimens were e.g. commonly used before 1950. This has some influence on the results. For cubes with side d or for cylinders with fixed ratio between height h and diameter d the influence of size can be determined from

\[
f_{c,150} = \frac{f_{c,d}}{\beta_1(d)}
\]

where \( f_{c,150} \) and \( f_{c,d} \) are compressive strength for diameter 150 mm and d respectively. The relation is valid for \( 50 \leq d \leq 200 \) mm and values of \( \beta_1(d) \) are given in table 4.1.

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>( \beta_1(d) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.10</td>
</tr>
<tr>
<td>100</td>
<td>1.05</td>
</tr>
<tr>
<td>150</td>
<td>1.00</td>
</tr>
<tr>
<td>200</td>
<td>0.95</td>
</tr>
</tbody>
</table>

For cylinders with different slenderness \( h/d \) conversion to the value \( f_{c,0} \) valid for \( h= 300 \) mm and \( d= 150 \) mm can be made according to the formula

\[
f_{c,0} = \frac{f_{c,\text{obs}}}{\beta_1(d) \cdot \beta_3(h/d)}
\]

where \( f_{c,\text{obs}} \) is the strength determined for a cylinder with diameter d and slenderness h/d, \( \beta_1(d) \) is given in Table 4.1 and \( \beta_3(h/d) \) is given in Table 4.2.
Table 4.2  Conversion factors for compressive strength with respect to slenderness h/d of cylindrical concrete specimens.

<table>
<thead>
<tr>
<th>h/d</th>
<th>0.6</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₃(h/d)</td>
<td>1.4</td>
<td>1.2</td>
<td>1.05</td>
<td>1.0</td>
<td>0.98</td>
<td>0.98</td>
</tr>
</tbody>
</table>

The conversion factors above are based on a large number of test results presented in the literature, see also SS 137207 (1988).

4.2.4 In-situ strength

The compressive strength in-situ will generally be less than that measured on standard test specimens taken from the same batch of concrete. The reason for this is e.g. the degree of compaction and curing in practical site conditions. The location in the member where in-situ strength is determined has significant influence.

Tests on in-situ concrete indicate that

- In-situ strength can vary within a member both randomly and in an ordered fashion.
- The variations of in-situ strength within structural members can vary from one member to another.
- In-situ strength decreases towards the top of the pour, even for slabs, and can be up to 25 % lower at the top than in the body of the member.

Figure 4.1 gives an illustration to the influence of position in structure and difference between standard specimens, core strength and in-situ strength. The numbers given in the figure are examples, but typical relative to each other.

The ratio between in-situ strength and standard strength can be set to 0.85 for all strength classes, prEN13791 (2003). The conversion factor is based on an empirical relation between on one hand strengths for cores taken from the top pour of structural elements that have been produced in accordance with standards of good practice and on the other hand...
In non-linear analysis of concrete structures the material parameters should be related to in-situ compressive strength expressed in terms of cylinder strength. This will give the most realistic representation of the material response of the concrete in the structure.

4.2.5 Assessment based on specified strength class

Documents from the construction of the bridge may be available where the concrete quality is specified in terms of compressive strength. Normally the concrete is specified through a standardised 28-day compressive strength for cylinder or cube specimens. For concrete produced according to modern standards the quality class is defined in terms of characteristic value or \( f_{cm} \), usually for cylinder or cube specimens. The definition of nominal value is different in different national standards and for different time periods, but is usually equivalent to a low fractile of the strength distribution, so that

\[
f_{cm} = f_{c,nom} + \Delta f_c
\]

where \( f_{cm} \) is mean compressive strength and \( \Delta f_c \) is the margin between nominal strength and mean value. This difference is often specified in concrete standards through control procedures of concrete quality. It is then required that the mean concrete strength from quality control tests shall be at least \( \Delta f_c \) higher than the nominal value \( f_c \) for the concrete quality specified. It is therefore reasonable to estimate the mean 28 day strength from Eq. (4.4) if information can be found in the old standard about the value of \( \Delta f_c \). If such information is not available a conservative value of \( f_{cm} \) can be obtained by using the value \( \Delta f_c = 4 \text{ MPa} \).

When the mean value \( f_{cm} \) of standardised compressive strength is known, the characteristic value defined as the 5th percentile can be determined assuming a log-normal distribution, i.e.

\[
f_{ck} = f_{cm} \cdot \exp(-1.64 \cdot V_{fc})
\]

where \( V_{fc} = \sigma_{fc}/f_{cm} \) is the coefficient of variation and \( \sigma_{fc} \) is the standard deviation for \( f_c \). \( V_{fc} \) and \( \sigma_{fc} \) depend on the concrete production conditions and degree of control. For concrete production under modern quality control procedures, the standard deviation is quite low. In that case a standard deviation of the order 5 MPa can be assumed, which is a conservative estimate in cases where no other information is available.

For concrete in old bridge structures one can expect that the quality control procedures have been less strict, so that higher values of \( \sigma_{fc} \) can be expected. The quality of concreting was early classified with respect to the level of control. It can generally be expected that strict control was applied when concrete bridges were built, but even so the variability in strength was higher than is valid today due to more primitive equipment for mixing, concreting and compacting. It is therefore recommended that \( \sigma_{fc} \) for concrete produced under the strictest manufacturing class can be set to 8 MPa.

Further information about assessment of old concrete can be found in section 4.2.6.

The 28 day in-situ strength \( f_{cm, is} \) can be determined as

\[
f_{cm, is} = \kappa \cdot f_{cm}
\]
where $\kappa$ is a parameter accounting for the difference between in-situ strength and standard strength, with mean $E(\kappa) = 0.85$. The background for this value is briefly given in section 4.2.3. The coefficient of variation (COV) for the parameter $\kappa$ is estimated to $V_\kappa = 0.06$ based on JCSS (2005). The coefficient of variation $V_{f_{c,is}}$ for the in-situ strength $f_{c,is}$ can then be determined from

$$V_{f_{c,is}}^2 = V_{f_c}^2 + V_\kappa^2$$

Eq. (4.7)

For existing structures the increased strength due to aging can be utilised. The strength increase over time is especially significant for old concrete with initially low quality due to low water/cement ratio. Further information about aging influence is given in section 4.2.7.

### 4.2.6 Assessment based on specified mix proportions

For concrete structures built before and just after the Second World War strength classes are often not specified. Instead, there can be information about proportions of the constituents of the concrete mix. If the water/cement ratio is specified this gives the best basis for estimation of compressive strength. As an example, Figure 4.2 shows the results from control tests at a ready mix concrete plant in the years after the Second World War. The 28 day compressive strength is strongly dependent on the w/c ratio. There is also a considerable scatter in strength, which shows that the production technology at that time was on a lower level than is used today. The dependency is of similar type for different type of cements and at different ages. The strength level for a given w/c ratio varies with age, cement type and hardening conditions but is more or less independent of amount and type of aggregate.

Figure 4.3 shows the corresponding relation from modern laboratory made concrete with standard Portland cement. The strength at given w/c ratio is higher here although it should be born in mind that 150 mm cubes were used in this case, which in itself results in higher strength values.

In old structures the w/c ratio may not be specified, but only the mix proportions between cement, sand and coarse aggregate. The consistency of the fresh concrete may also be known, but since the amount of water in the mix is unknown, any estimation of the strength is uncertain. In such a case the amount of cement per unit volume can give a rough indication of the strength. Figure 4.4 shows a relation between strength and cement content in kg/m$^3$.

![Figure 4.2. Relation between compressive strength (28 day 200 mm cube strength) and w/c ratio for concrete produced in the 1940ies, Tengvik (1947).](image)
Figure 4.3  Empirical relation between 150 mm cube strength and w/c ratio for concrete. Laboratory tests with concretes made with cement from three different plants and sand from four different locations.

specified in the Swedish concrete regulations from 1926, SOU (1926) for concrete with moist and wet consistency respectively. The values are minimum requirements for control of strength measured on 200 mm cubes. 150 mm cube values would be about 5% higher. Figure 4.4 may be used to make a conservative estimate of the 28 day strength of old concrete. The cement content can be estimated from the proportions between cement, sand and coarse aggregate.

Figure 4.4 Minimum values for 200 mm cube strength as a function of cement content according to regulations for concrete production from 1926.
4.2.7 Age dependence of concrete properties

Due to continuing hydration of the cement, concrete properties depend significantly on age after casting. The compressive strength at an age \( t \) depends on the type of cement, temperature and other curing conditions. The age dependence is usually described under the assumption of a mean temperature of 20 °C and standard curing in accordance with e.g. EN 12390-2 (2000). This means that the specimens are stored in water or at very high humidity at 20 °C after removal of the moulds. Eurocode 2 (2004) and CEB-FIP Model Code, CEB (1993) recommends the following formula for mean compressive strength \( f_{cm}(t) \) at age \( t \) (in days)

\[
f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm}
\]

Eq. (4.8a)

with

\[
\beta_{cc}(t) = \exp\left[ s \left( 1 - \left( \frac{28}{t} \right)^{1/2} \right) \right]
\]

Eq. (4.8b)

where

- \( f_{cm} \) is the mean compressive strength at the age 28 days
- \( s \) is a coefficient which depends on the type of cement:
  - =0.20 for rapid hardening high strength cements (R)
  - =0.25 for normal and rapid hardening cements (N)
  - = 0.38 for slow hardening cements (S)

The cement type is defined in standards supporting Eurocode 2.

For conditions other than the reference conditions at 20 °C and standardised curing, the age dependence according to Eq. (4.8) must be adjusted. The most important parameters are temperature and relative humidity.

Another expression for the time dependence of short term concrete compressive strength is proposed in JCSS (2005), but this appears to be valid only for concrete of moderate age. Also Eq. (4.8) is mainly intended to predict age development in the earlier stages of the concrete life.

Few tests are available to verify the predictions of the empirical aging formula in Eq. (4.8) at very high ages of the order several decades, which is of interest for assessment of railway bridges. For 30 year old concrete Eq. (4.8) predicts a relative increase in strength from the 28 days value of 27% for normal cement and 43 % for slow hardening cement. Tests by e.g. Walz (1976) and Washa & Wendt (1975) give however clear indications that the growth in strength is significantly larger at such high ages. Furthermore, the relative aging effect seems to depend significantly on water-cement ratio. The reason for this is explained by e.g. Fagerlund (1987). For low strength concretes with higher w/c ratio the relative increase in strength is significantly larger than for concretes with lower w/c ratio. Figure 4.5 displays results from Walz (1976) confirming this. It is seen that for w/c ratios in the range 0.8-1.0 the strength is doubled after 30 years for normal Portland cement (diamond symbols) and more than tripled for slow hardening cement (square symbols). Also for lower w/c ratios the strength growth is 50 % and more. It is clear that Eq. (4.8) gives very conservative predictions at high ages. This is also confirmed by data published by Washa and Wendt (1975), who tested concrete at the age of 50 years. Test results from old Swedish bridges are given in Thun (2006).
Relative increase in strength from 28 d. to 30 years

![Graph showing the relative increase in strength from 28 days to 30 years for concrete with normal cement (Portland) and slow hardening cement (Hoch).](image)

**Figure 4.5** Ratio between compressive strength at age 30 year and at age 28 days as a function of w/c ratio for concrete with normal cement (Portland) and slow hardening cement (Hoch). From tests by Walz (1976).

Uncertainty in the prediction of growth of strength with age can be considered by estimating the COV for the relative increase in strength $\beta_{cc}(t)-1$. There is no information available concerning this uncertainty at very high ages, but a reasonable estimate is 0.3 and the standard deviation $\sigma_{\beta_{cc}}$ for $\beta_{cc}(t)$ then becomes

$$
\sigma_{\beta_{cc}} = 0.3 \cdot (\beta_{cc}(t) - 1)
$$

and the COV for $\beta_{cc}(t)$

$$
V_{\beta_{cc}} = \frac{\sigma_{\beta_{cc}}}{E(\beta_{cc})}
$$

where $E(\beta_{cc})$ is the mean value of $\beta_{cc}(t)$ determined from Eq. (4.8b) or estimated in some other way.

The coefficient of variation $V_{fc,t}$ for strength $f_c(t)$ at age $t$ is then

$$
V_{fc,t} = \sqrt{V_{fc,28}^2 + V_{\beta_{cc}}^2}
$$

where $V_{fc,28}$ is COV for 28 day compressive strength.

### 4.2.8 Assessment based on in-situ testing

Testing of cores provides a direct and the best method to estimate in-situ compressive strength. Methods other than tests on cores do not directly provide the in-situ compressive strength and must therefore be calibrated against core tests. Before an indirect test method is used for the assessment of in-situ strength, a relationship has to be established.

In-situ compressive strength can be determined by testing of concrete cores with length=diameter in the interval 100-150 mm. The test value can then be assumed to be equal to the strength of a 150 mm cube, prEN13791 (2003). Other sizes and lengths can be used if the test value is converted to standard cube or cylinder. Methodology for sampling and testing of cores is given in EN12504-1(2000).
Assessment of strength shall generally be based on at least 9 cores taken from one test region, but assessment of a specific structural element can be based on specimens from 3 cores or more, provided that the cores have a diameter ≥ 100mm.

The results from core tests can be interpreted as in-situ strength. In case of core specimens with length and diameter in the range 100-150 mm taken from the top pour this gives

\[ f_{\text{cm,is,cube}} = f_{m(n),\text{core}} \]

where \( f_{m(n),\text{core}} \) is the mean strength for \( n \) specimens (cores) and \( f_{\text{cm,is,cube}} \) is the mean in-situ compressive strength expressed in terms of standard 150 mm cube strength.

If the cores are sampled from other parts of the structure than the top pour, the in-situ strength to be used for analysis of resistance should be reduced in relation to the strength measured for the core specimens. A reasonable estimate is then

\[ f_{\text{cm,is,cube}} = 0.85 \cdot f_{m(n),\text{core}} \]

The in-situ cube strength can be transformed to in-situ cylinder strength by use of Eq. (4.1).

The coefficient of variation \( V_{fc,is} \) for in-situ strength can be estimated directly from the test results, provided that at least 9 cores have been taken from the test region in a representative manner, but the COV should be minimum 10 %, i.e.

\[ V_{fc,is} = \text{max}( V_{fc,\text{core}},0.10 ) \]

where \( V_{fc,\text{core}} \) is the COV determined from minimum 9 specimens taken from different cores.

### 4.2.9 Non-destructive methods as tools for assessment of in-situ strength

Non-destructive test methods for concrete strength are described in D3.15 (2006) can be used as a preliminary screening of the structure and as a basis for selecting locations for drilling of cores. Quantitative estimation of concrete strength can then be done directly from the core test results.

Indirect methods also provide alternatives to core tests for estimating in-situ strength. By an indirect test method a quantity other than strength is measured and it is therefore necessary to use a relationship between the indirect test result and the compressive strength of cores.

Two alternative procedures can be used:

1. Direct correlation with core test results
2. Calibration with cores for a limited strength range using a basic relationship (only for rebound hammer tests, ultrasonic pulse velocity tests and pull-out tests)

For assessment of characteristic strength, at least 15 strength estimates from the same test region by indirect method are needed. The evaluation may only be made according to criteria A, which is described in section 3.2.3.

### Direct correlation with core test results

An empirical relation must be derived between the in-situ strength and the indirect test result (NDE-parameter) for structural elements of a particular concrete. The relationship shall be based on at least 18 pairs of results covering the range of interest. Rules for deriving this relationship concerning number of tests, sampling etc. are given in prEN 13791(2003). The regression relation between strength and NDE-parameter is established and the standard error of estimate is calculated. The relation to be used is defined on a safe level so that only 10 % of the strength values are expected to fall below the curve defining the relationship.

The relationship shall only be used for direct estimation of in-situ strength for the type of concrete and the conditions for which it was established.
Calibration with cores for a limited strength range

This method can be used only for rebound hammer tests, ultrasonic pulse velocity tests and pull-out tests. Basic relationships are predefined for the three methods, but these have to be calibrated with core tests for at least 9 test locations. The results from the core tests are evaluated and used to make corrections of the basic relationship. This correction is made according to detailed rules given in prEN 13791(2003). As an example, the basic relationship between rebound hammer test results and initial rebound strength value is shown in Figure 4.6. Similar basic relationships for ultrasonic velocity testing and pull-out testing are given in prEN13791(2003).

For each of n test locations a correction factor $\delta_f$ is calculated as

$$[1] \quad \delta_f = f_{is} - f_R$$

where $f_R$ is the strength reading from the basic curve based on the indirect test result $R$ for the specific test location, and $f_{is}$ is the in-situ compressive strength from the corresponding core test at the same location.

The correction factor $\Delta f$, which is used for estimation of in-situ strength, is given by

$$[3] \quad \Delta f = \delta_f \cdot \text{mean}(n) - k \cdot s$$

where $\delta_f \cdot \text{mean}(n)$ is the mean value of the n corrections $\delta_f$ calculated by Eq. (4.14) and $s$ is the standard deviation of the n corrections. The value of the coefficient $k$ in Eq. (4.15) depends on the number of test results n, see Table .

![Figure 4.6. Basic relationship between rebound test result R and initial rebound strength value $f_R$. Source: prEN13791(2003).](image)

<table>
<thead>
<tr>
<th>Number of test results, n</th>
<th>Coefficient k</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.67</td>
</tr>
<tr>
<td>10</td>
<td>1.62</td>
</tr>
<tr>
<td>11</td>
<td>1.58</td>
</tr>
<tr>
<td>12</td>
<td>1.55</td>
</tr>
<tr>
<td>13</td>
<td>1.52</td>
</tr>
<tr>
<td>14</td>
<td>1.50</td>
</tr>
<tr>
<td>$\geq$15</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Table 4.3. Values for the coefficient $k$ in Eq. (4.15)
4.2.10 Other concrete strength properties

When the concrete quality is described via compressive strength, other mechanical properties may be assessed from empirical relations. These relations are summarised in Table 4.4.

The empirical formulas given in Table 4.4 are mainly taken from Eurocode 2. Many other relations can be found in the literature but the Eurocode formulas have been calibrated against extensive test data recently and it is logical to use them. The most uncertain parameters to estimate are fracture energy, bond strength and ultimate strain values, where the available data has been more limited.

Eurocode 2 gives a general schematic representation of the stress-strain relation in uniaxial compression to be used in non-linear analysis. It is expressed in terms of uniaxial compressive strength \( f_{cm} \), elastic modulus \( E_{cm} \), the strain \( \varepsilon_{c1} \) at peak stress, and the ultimate strain \( \varepsilon_{cu1} \). The idealised stress-strain relation is expressed as

\[
\frac{\sigma_c}{f_{cm}} = \frac{k \eta - \eta^2}{1 + (k-2)\eta}
\]

where \( \sigma_c \) is uniaxial compressive stress, \( \eta = \varepsilon_c / \varepsilon_{c1} \), \( \varepsilon_c \) is the uniaxial compressive strain and

\[
k = 1.1 E_{cm} \cdot \left| \varepsilon_{c1} \right| / f_{cm}
\]

Eq. (4.16a)

Eq. (4.16b)

Eq. (4.16) is valid only in the interval \( 0 < \varepsilon_c < \varepsilon_{cu1} \), and is illustrated in Figure 4.7.

![Figure 4.7](image)

Figure 4.7. Idealised, uniaxial stress-strain relation for non-linear analysis, Eurocode 2.

Strength and stiffness properties may in general be modelled by a log-normal statistical distribution.

Properties such as tensile strength, elastic modulus and fracture energy can also be determined from direct testing of the material in an existing structure. Tensile strength can be determined from cores either by axial tension tests or splitting tensile tests.

Eurocode 2 (2004a) states that “an approximate value of the axial tensile strength \( f_{ct} \)” may be taken as

\[
f_{ct} = 0.9 \cdot f_{ct,sp}
\]

Eq.(4.17)
Table 4.4. Relations between concrete properties and compressive strength. \( f_{cm} \) = mean cylinder strength.

<table>
<thead>
<tr>
<th>Property</th>
<th>Mean</th>
<th>COV %</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial tensile strength</td>
<td>( E(f_{ct}) = 0.20 f_{cm}^{2/3} ) [MPa]</td>
<td></td>
<td>CEB-FIP Model Code 1990</td>
</tr>
<tr>
<td></td>
<td>for strength class C50/60</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( E(f_{ct}) = 2.12 \cdot \ln\left[ 1 + \left( f_{cm} / 10 \right) \right] ) [MPa]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>for strength class &gt; C50/60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>( E(E_c) = 22000 \left\lfloor \frac{f_{cm}}{10} \right\rfloor^{0.3} ) [Mpa]</td>
<td>0.20</td>
<td>Eurocode 2</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>( E(G_F) = \alpha f_{cm} f_{cm0}^{0.7} ) [N/mm]</td>
<td>0.30</td>
<td>CEB-FIP Model Code 1990</td>
</tr>
<tr>
<td></td>
<td>( f_{cm0} = 10 ) MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic bond strength*, ( f_b )</td>
<td>( E(f_b) = k \cdot f_{ct} )</td>
<td>0.30</td>
<td>Eurocode 2</td>
</tr>
<tr>
<td></td>
<td>( k ) depends on bond conditions (position in the structure in relation to concreting) and bar diameter, see Eurocode 2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultimate compressive strain, ( \varepsilon_{cu1} ) (Figure 2)</td>
<td>( E(\varepsilon_{cu1}) = 3.5 % ) for ( f_{cm} ) &lt; 58 MPa</td>
<td>0.20</td>
<td>Eurocode 2</td>
</tr>
<tr>
<td></td>
<td>( E(\varepsilon_{cu1}) = 2.8 + 27 \left\lfloor \frac{98 - f_{cm}}{100} \right\rfloor^{3/4} % ) for ( f_{cm} ) \geq 58 MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strain at peak compressive stress, ( \varepsilon_{c1} ) (Figure 2)</td>
<td>( E(\varepsilon_{c1}) = 0.7 f_{cm}^{0.31} ) &lt; 2.8 % ( f_{cm} ) in MPa</td>
<td>0.20</td>
<td>Eurocode 2</td>
</tr>
</tbody>
</table>

* The basic bond strength is valid for straight bars in unconfined concrete with normal cover. For effect of cover, confinement and shape, see Eurocode 2.

where \( f_{ct,sp} \) is the splitting tensile strength. Eq. (4.17) can be applied for mean values of \( f_{ct} \) and \( f_{ct,sp} \). A relation of the same form can be used for characteristic values, but the conversion factor can be expected to be somewhat lower in this case.

When interpreting tensile strength results and using them in analysis, it is important to be aware of the fact that the spatial variability of tensile strength is normally larger than that for compressive strength. The effective tensile strength in a structure is also affected by scale (size-effects) as well as stress distribution in the tensioned volume.

For concrete strengths between 80 and 120 MPa test results and recommendations are given in HPCS (2000).

### 4.2.11 Creep and shrinkage

Neither creep nor shrinkage can be seen as distinct material properties for a certain concrete. They depend on the ambient humidity, dimensions of the structural element and the composition of the concrete; Creep also depends on the maturity of the concrete when the load is first applied and depends on the duration and the magnitude of the applied loading. Any estimation of creep and shrinkage should take these parameters into account.
The creep deformation of concrete $\varepsilon_{cc}(\infty, t_o)$ at time $t=\infty$ for constant compressive stress $\sigma_c$ can be estimated from

$$\varepsilon_{cc}(\infty, t_o) = \varphi(\infty, t_o) \cdot \left( \frac{\sigma_c}{E_{co}} \right)$$  \hspace{1cm} \text{Eq. (4.18)}

where $t_o$ is the age at first loading and $E_{co}$ is the secant modulus of elasticity at time $t_o$. According to Eurocode 2 (2004a) the linear formulation in Eq. (4.18) can be considered valid for stresses up to $0.45 f_{ck}(t_o)$. Simplified methods to determine the creep factor $\varphi(\infty, t_o)$ are presented in Eurocode 2.

For existing structures it is mainly drying shrinkage which is of interest. The drying shrinkage strain develops slowly, of the order of decades for concrete structures with large dimensions. According to Eurocode 2 (2004a) the final value $\varepsilon_{cd,\infty}$ of drying shrinkage strain can be determined as

$$\varepsilon_{cd,\infty} = k_h \cdot \varepsilon_{cd,0}$$ \hspace{1cm} \text{Eq. (4.19)}

where $k_h$ is a coefficient depending on the size of the structural element, and $\varepsilon_{cd,0}$ is a reference shrinkage value depending on concrete quality and relative humidity of the surrounding air. Numerical values for these parameters are given in Eurocode 2 (2004a).

More complicated, but more accurate methods to estimate creep and shrinkage in concrete can be found in ACI 209 (1992) and Bazant & Murphy (1995).

### 4.2.12 Dynamic effects on concrete properties

The influence of stress rate $\dot{\sigma}$ and strain rate $\dot{\varepsilon}$ on concrete properties in compression is illustrated in Figure 4.8, displaying the ratio between dynamic properties $x_{dyn}$ and quasi-static properties $x_{stat}$. The static values correspond to a testing rate of the order 1 MPa/s.

The change in a property due to dynamic effects is usually expressed as a relation between the relative value of the property and logarithm of strain rate. For compressive strength a bilinear relation is valid, with moderate linear increase in strength at strain rates up to about $30 \text{ s}^{-1}$, whereas for higher strain rates the strength increases at a much higher rate. For most vibration problems the strain rate is significantly lower than $30 \text{ s}^{-1}$, so that expected changes in properties are moderate for this class of problems.

![Figure 4.8. Relative influence of stress rate and strain rate on concrete properties in compression, CEB Bulletin 209 (1991).](image-url)
The elastic modulus $E_c$ and ultimate strain $\varepsilon_{cu}$ both show a linear increase with the logarithm of strain rate over the whole range. Empirical formulas for calculation of strain rate effects are given in Table 4.5.

Table 4.5. Dynamic influence on concrete properties in compression. CEB Bulletin 209 (1991). Static properties are defined for Error! Objects cannot be created from editing field codes.

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
<th>Valid for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength, $f_c$</td>
<td>$\frac{f_{c, dyn}}{f_{c, stat}} = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}<em>0} \right]^{-1.026 \alpha}$ with $\alpha = \frac{1}{5 + 3 \cdot f</em>{cm, cube}}$; $f_{cm, cube}$ is the mean static cube strength</td>
<td>$\dot{\varepsilon} \leq 30 , s^{-1}$</td>
</tr>
<tr>
<td>Modulus of elasticity, $E_c$</td>
<td>$\frac{E_{c, dyn}}{E_{c, stat}} = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right]^{-0.026}$</td>
<td>-</td>
</tr>
<tr>
<td>Ultimate strain, $\varepsilon_{cu}$</td>
<td>$\frac{\varepsilon_{cu, dyn}}{\varepsilon_{cu, stat}} = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right]^{-0.020}$</td>
<td>-</td>
</tr>
</tbody>
</table>

Corresponding relations for properties in tension are given in Table 4.6. Note that tensile strength is more sensitive to strain rate than compressive strength, whereas the opposite is valid for modulus of elasticity.

Table 4.6. Dynamic influence on concrete properties in tension. CEB Bulletin 209 (1991). Static properties are defined for Error! Objects cannot be created from editing field codes.

<table>
<thead>
<tr>
<th>Property</th>
<th>Formula</th>
<th>Valid for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength, $f_t$</td>
<td>$\frac{f_{t, dyn}}{f_{t, stat}} = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}<em>0} \right]^{-1.016 \delta}$ with $\delta = \frac{1}{10 + 0.5 \cdot f</em>{cm, cube}}$; $f_{cm, cube}$ is the mean static cube strength</td>
<td>$\dot{\varepsilon} \leq 30 , s^{-1}$</td>
</tr>
<tr>
<td>Modulus of elasticity, $E_c$</td>
<td>$\frac{E_{c, dyn}}{E_{c, stat}} = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right]^{-0.016}$</td>
<td>-</td>
</tr>
<tr>
<td>Ultimate strain, $\varepsilon_{cu}$</td>
<td>$\frac{\varepsilon_{cu, dyn}}{\varepsilon_{cu, stat}} = \left[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right]^{-0.020}$</td>
<td>-</td>
</tr>
</tbody>
</table>

The influence of stress rate on bond of smooth bars and strands is negligible, but the opposite is valid for deformed bars. The dynamic effect on bond properties for ribbed bars de-
pends on relative displacement (slip) between steel and concrete. The following relation can be used for the bond stress $\tau$.

$$\frac{\tau_{\text{dyn}}}{\tau_{\text{stat}}} = \left[ \frac{\dot{\tau}_{\text{dyn}}}{\dot{\tau}_o} \right]^\eta$$

with

$$\eta = 0.7 \cdot (1 - 2.5 \Delta) (f_{cm,\text{cube}})^{0.8}$$

$\Delta = \text{slip in mm}$

$\dot{\tau}_o = 0.1 \text{ MPa/s}$

Eq. (4.20) is valid for $0 < \Delta < 0.2 \text{ mm}$.

### 4.3 Steel reinforcement

#### 4.3.1 General

The basic mechanical property for reinforcing steel is the static yield strength $f_y$ defined at proof strain 0.2 %. The grade of reinforcing steel is usually defined through a nominal value of yield strength, which is close to but not fully equivalent with the characteristic value $f_{yk}$.

Reinforcing steels in bridges built after the mid 1960:ies have a nominal value $f_{y,\text{nom}}$ for the yield strength in the interval 400-600 MPa. From about 1945 to the mid sixties smooth bars with yield strength in the interval 220-400 Mpa were most common. Before the Second World War smooth bars with nominal yield strength 220-260 Mpa were used. Other properties of interest are the tensile strength $f_t$ and the strain $\varepsilon_u$ at maximum load.

For modern reinforcing steel with yield strength in the interval 400-600 MPa, a probabilistic description of reinforcing steel, the yield stress, here denoted by $X$, can be modelled as a sum of three independent random variables

$$X = X_1 + X_2 + X_3$$

where

$X_1 \in N(\mu_1(d), \sigma_1)$ represents the variations in global mean of different mills,

$X_2 \in N(0, \sigma_2)$ the variations in a mill from batch (melt) to batch,

$X_3 \in N(0, \sigma_3)$ the variations within a batch and $d$ is the nominal bar diameter.

For high standard steel production the values $\sigma_1 = 19 \text{ MPa}$, $\sigma_2 = 22 \text{ MPa}$ and $\sigma_3 = 8 \text{ MPa}$ can be used, JCSS(2005). This gives an overall standard deviation $\sigma_s = 30 \text{ MPa}$. The variation of strength within the same batch is very small, which means that the actual strength for all bars from the same batch and with a given diameter can be determined with high precision from tests on a few specimens.

#### 4.3.2 Assessment based on specified grade

The yield stress $f_y$ can be modelled by a log-normal distribution. If the nominal or characteristic value for strength of the specified grade is defined in statistical terms, e.g as a fractile, it can be used in assessment situations. The standard deviation $\sigma_s$ for a general population representative for the grade considered can be set to 30 MPa.

If the nominal value $f_{\text{nom}}$ is not defined in statistical terms the overall mean value $E(f_y)$ can be estimated from
\[ B(f_y) = f_{nom} + 2\sigma_s \quad \text{Eq. (4.17)} \]

This is slightly on the conservative side compared with data from Degerman (1981) who analysed a great number of quality control tests from Swedish mills concerning the grades Ks40 and Ks 60. The yield strength decreases somewhat with increasing diameter for the same grade. This effect is illustrated in Figure 4.8 by data from Degerman (1981) based on tests from Swedish steel mills.

Mean yield strength for reinforcement

![Graph showing mean yield strength for reinforcement](image)

Figure 4.9. Effect of bar diameter on yield strength for ribbed reinforcing steels, Degerman (1981).

### 4.3.3 Assessment based on test results from the structure

A better estimate can be found if test data on the specific reinforcement are available either from tests during construction or tests made for specimens taken from the structure in connection with the assessment. The average value from the tests can be used as mean value for reinforcement yield strength provided that the test sampling is representative for the population considered. If it is known that all reinforcement comes from the same mill or the same batch the standard deviation can be reduced accordingly.

### 4.3.4 Other properties

For tensile strength \( f_{st} \) or strain at ultimate load \( \varepsilon_{su} \) in non-linear modelling it is recommended to use the lower limits for ductility requirements in Eurocode 2 for steel of different ductility classes. These are reproduced in Table 4.7. Note, however, that the ratio \( f_{t}/f_y \) can in reality be significantly higher for reinforcing steel in existing structures. Especially for old reinforcing steel with low yield strength, the ratio between tensile strength and yield strength can be quite high. For instance smooth steel with nominal yield strength 220 MPa can have a nominal tensile strength, which is twice the yield strength. The actual tensile strength as well as the ultimate strain \( \varepsilon_{su} \) can be easily determined from tests on reinforcement taken from the structure.

The tensile strength \( f_{st} \) has a larger variation than the yield strength \( f_{sy} \). JCSS (2005) suggests that the standard deviation can be set to 40 MPa.
Table 4.7  Requirements for ductility class of reinforcing steel according to Eurocode 2. Characteristic yield stress in the range 400-600 MPa.

<table>
<thead>
<tr>
<th>Property</th>
<th>Ductility class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio tensile strength to yield strength $f_t/f_y$</td>
<td>$\geq 1.05$ $\geq 1.08$ $\geq 1.15$ $&lt; 1.35$</td>
</tr>
<tr>
<td>Strain at maximum load $\varepsilon_{su}$ (%)</td>
<td>$\geq 2.5$ $\geq 5.0$ $\geq 7.5$</td>
</tr>
</tbody>
</table>

4.3.5 Dynamic effects on properties of steel reinforcement

The modulus of elasticity remains unchanged under dynamic loading, but yield stress $f_y$ and tensile strength $f_t$ increases with strain rate. Empirical relations for the effect of dynamic loading are given in Table 4.8, where index $\text{dyn}$ stands for property under dynamic load and $\text{stat}$ for property under quasi-static conditions. Other notations are

$\dot{\varepsilon} = \text{strain rate}$

$\dot{\varepsilon}_o = \text{strain rate under quasi-static loading} = 5 \cdot 10^{-5} \text{ s}^{-1}$

For high quality steel and prestressing wires the influence of strain rate on strength properties is almost negligible.

Table 4.8.  Strength properties for reinforcing steels under dynamic loading with strain rate

<table>
<thead>
<tr>
<th>Hot rolled steel:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress $f_y$</td>
</tr>
<tr>
<td>$f_{y,\text{dyn}} = f_{y,\text{stat}} + 6.0 \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right)$</td>
</tr>
<tr>
<td>Tensile strength, $f_t$</td>
</tr>
<tr>
<td>$f_{t,\text{dyn}} = f_{t,\text{stat}} + 7.0 \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cold worked steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 % proof stress $f_{0.2}$</td>
</tr>
<tr>
<td>$f_{0.2,\text{dyn}} = f_{0.2,\text{stat}} + 4.3 \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right)$</td>
</tr>
<tr>
<td>Tensile strength, $f_t$</td>
</tr>
<tr>
<td>$f_{t,\text{dyn}} = f_{t,\text{stat}} + 6.5 \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right)$</td>
</tr>
</tbody>
</table>

4.4 Prestressing steels

Existing prestressed concrete structures are normally designed with any of the following three types of prestressing steel:

- cold drawn wires
- strands, which consists of a number of (most often 3 or 7) cold drawn wires spun together in a helical configuration.
- high-strength bars
The following reference mechanical properties can be defined for pre-stressing steels.

- Tensile strength $f_p$
- Effective elastic modulus $E_p$
- Strain at maximum load, $\varepsilon_{pu}$

These variables can be assumed to be uncorrelated. Characteristic or nominal values for tensile strength, here denoted $f_{pk}$, associated to a particular steel grade, are generally specified by manufacturers of pre-stressing steel products.

Statistical parameters of these random variables are given in JCSS (2005), and reproduced in Table 4.9. Further data on prestressing steels can be found in Almunia (1993), Devalapura (1992) and Harrop (1985).

A typical stress-strain diagram for prestressing steel is shown in Figure 4.10. The 0.1 % proof stress $f_{p,0.1}$ is defined in Eurocode 2 as a property which characterises the stress strain relation. This property is strongly correlated with the tensile strength and can therefore be expressed via $f_p$. According to JCSS (2005)

$$ f_{p,0.1} = k_{0.1} \cdot f_p $$

where $k_{0.1}$ is a deterministic parameter which can be obtained from the steel supplier. If such information is not available $k_{0.1}$ can be set to 0.85. Appropriate analytical descriptions of the stress-strain curve are proposed in JCSS (2005).

Table 4.9: Mechanical properties for prestressing steels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard dev.</th>
<th>Coeff. of variation</th>
<th>Reference</th>
</tr>
</thead>
</table>
| $f_p$    | 1.04 $f_{pk}$ or $f_{pk}$ + 66 [MPa] | - 40 MPa | 0.025 | Mirza et al (1980)
|          |      |               |                     | FIP (1976) |
| $E_p$    | 200 GPa (wires) | 195 GPa (strands) | 200 GPa (bars) | 0.02 | Mirza et al (1980) |
| $\varepsilon_{pu}$ | 0.05 | 0.0035 | - | Mirza et al (1980) |

Figure 4.10 Schematic stress-strain diagram for prestressing steel
In some cases, results from strength tests of the pre-stressing steels used during construction are available. Such results can be used to estimate the mean values of strength, proof stress and ultimate strain. The variability may also be estimated from such data.
5 Redistribution of moment and forces from linear FE analysis

5.1 Introduction

5.1.1 Background
The finite element method (FEM) is increasingly used for design and assessment of bridges. It can rationalise and simplify the calculations, in particular for complicated geometries where three-dimensional (3D) analyses are required. Due to the complex load situation for bridges, with moving loads and many possible load combinations, the structural analyses are for practical reasons normally linear. However, linear analyses lead to concentrations of sectional moments and forces in 3D models, e.g. where there are concentrated supports or loads, in corners of slab frame bridges etc. These concentrations often do not correspond to reality. Since concrete bridges normally crack long before the ultimate load is reached, in reality the sectional moments and forces will distribute over larger parts of the structure. In ultimate limit state (ULS), the reinforcement will yield, leading to even larger redistributions.

Consequently, there is a need to be able to redistribute the linearly calculated concentrated sectional moments and forces, without having to perform a non-linear analysis. In a design situation, such a redistribution can be made as a basis for the choice of reinforcement distribution. In an analysis situation, the real distribution of the sectional moments and forces needs to be approximated reasonably well after such a redistribution in order not to underestimate the load carrying capacity.

5.1.2 Aim and objectives
The aim of this work was to facilitate the use and ease the quality review of linear finite element (FE) analysis in design and assessment of concrete bridges. The work should result in general recommendations for how unrealistic concentrations of sectional forces and moments, obtained through linear structural analysis (with FEM), can be smeared out over a larger part of the bridge structure.

5.1.3 Scope of Work
The study is focused on slab bridges, and the concentration of cross-sectional moments and shear forces at concentrated supports. It consists of a state-of-the-art review followed by modelling and linear as well as non-linear analysis with FEM to study the possibility for redistributions. Comparisons between different design and analysis methods are made on a typical slab bridge used as a case study. Finally, general recommendations are established for redistribution of concentrated sectional moments and forces, obtained from linear structural analysis.

5.2 Review of methods for structural analysis
In this section, different approaches for structural analysis of reinforced concrete slab bridges are presented. The feasibility for bridge design and assessment are discussed together with the approximations involved in each of them.
5.2.1 Linear elastic frame analysis

Today, concrete slab bridges are normally analysed and designed under the requirement that the cross-sectional forces and moments should be determined according to the theory of elasticity. If the geometry is not extremely complicated, the forces and moments are usually calculated through two-dimensional (2D) linear frame analysis. To obtain the forces and moments in both the longitudinal and the transversal direction, 2D analyses are made for the two directions separately. Often, the loads on the main (longitudinal) frame model are assumed to be transferred to the model in transverse direction, which in turn distributes the loads on, for example, a number of columns or bearings.

This means that the loads that are applied to the 2D models are determined to fulfil equilibrium for the three-dimensional structure. However, the compatibility requirements are disregarded. Consequently, the requirements from the theory of elasticity are not fulfilled regarding the response of the structure in 3D. Instead, the approach follows the lower bound theorem of the theory of plasticity. The approach can be seen as a simplified strip method, where the cross-sectional forces and moments in each strip are determined according to the theory of elasticity.

Bridges are designed for moving traffic loads and for a variety of load combinations. This means that each strip is analysed for a large number of load cases, with different load distributions along the strip, and each cross-section is designed for the maximum cross-sectional forces and moments that occur in that section. Altogether, this means that, for a particular load case with a particular traffic load position, the bridge will obtain an over-capacity in reality.

Existing bridges were often not designed the way they would be designed today. If the original design was based on other principles, the load-carrying capacity may become underestimated if the structural analysis is made as if the bridge was to be designed. However, the method used offers some possibilities for different load distributions between strips in the different directions. In addition, a limited redistribution of the moments along the strips may be considered.

5.2.2 Linear elastic 3D analysis

For more complicated geometries, 3D linear elastic analyses can preferably be used. For this purpose, FE analyses with shell or slab elements are suitable. In such an analysis, the common requirement that the cross-sectional forces and moments should be determined according to the theory of elasticity is fully complied with. Both the equilibrium and the compatibility requirements are fulfilled. The results from such an analysis contain more of information compared to a linear elastic 2D frame analysis. However, the problem with such an analysis is that high stress concentrations can be obtained locally. These are usually expressed as concentrated cross-sectional moments and shear forces. Such a response does not correspond to the real behaviour of the concrete bridge. In reality, the concrete will crack and the reinforcement will eventually yield, leading to redistributions of the stresses.

Another reason that can lead to high concentrations of moments and shear forces are simplifications made in the geometrical modelling. For example, for practical bridge analysis, a column is often modelled using beam elements connected to the slab in a single node, and a bearing is often modelled as prescribed degrees of freedom for a single node. When refining the element mesh close to such a node, the cross-sectional moments and shear forces will tend to go to infinity.

A 3D slab analysis will give a moment field consisting of bending moments in two directions and a torsional moment in the slab plane. The tensile reinforcement should be designed to balance the complete moment field, including the torsional moment. According to e.g. Concrete Structures, Euro-Design Handbook, Eibl (1995), the positive and negative reinforce-
ment moments, giving the reinforcement in the top and bottom of the slab, are calculated according to:

\[ m_{st,\text{pos}} = m_x + \mu_1 \cdot t \]  \hspace{1cm} \text{Eq. (5.1a)}

\[ m_{st,\text{neg}} = m_x - \mu_2 \cdot t \]  \hspace{1cm} \text{Eq. (5.1b)}

\[ m_{y\text{,pos}} = m_y + \frac{1}{\mu_1} \cdot t \]  \hspace{1cm} \text{Eq. (5.1c)}

\[ m_{y\text{,neg}} = m_y - \frac{1}{\mu_2} \cdot t \]  \hspace{1cm} \text{Eq. (5.1d)}

where \( m_x \) and \( m_y \) are the bending moments in the x and y directions, \( t \) is the torsional moment and \( \mu_1 \) and \( \mu_2 \) are factors that can be chosen with respect to practical considerations, usually close to 1.

In addition to the moments, the 3D slab analysis will provide the cross-sectional shear forces in two directions, as well as normal forces in two directions and the in-plane shear force. For slab bridges without normal forces or prestress, the three last cross-sectional forces need not to be taken into account.

In a 3-D analysis, the moments will vary continuously in both directions of the slab. This means that the moment determining the reinforcement in a certain direction will vary not only in the reinforcement’s longitudinal direction but also in the transversal direction. To design the reinforcement so that it follows the design moment variations completely is not realistic since it would require a continuous variation of the distances between the reinforcement bars. This is not possible from a practical point of view.

Instead the slab has to be divided in strips with constant reinforcement spacing. A solution on the safe side would be to design the reinforcement for the highest moment within the strip width. However, this will lead to poor reinforcement economy, in particular for strips containing moment concentrations, for example over a column or a support point. In practice, it may not even be possible to reinforce for these high moments.

The reinforcement within a strip may instead be designed for the average reinforcement moment over the strip width. This approach is based on the theory of plasticity in the same way as the choice of strip widths when designing with 2D frame analysis. The work presented in this report deals with how this averaging should be made and the proper choices of strip widths.

When evaluating an existing bridge based on a linear elastic 3D analysis, there is an obvious risk that the load-carrying capacity will become under-estimated if the bridge slab was not designed the same way. However, it may serve as an initial evaluation, and if it shows sufficient capacity it can be used as a solution on the safe side. In this case, the distribution of the torsional moment to the reinforcement moments, and the strip widths with constant reinforcement spacing should be chosen based on the actual reinforcement distribution in the slab.

### 5.2.3 The strip method

The strip method developed by Hillerborg (1974, 1996) is based on the lower bound theorem of the theory of plasticity. This means that it leads to sufficient safety at the ultimate limit state (ULS), provided that the slab has an adequately plastic behaviour. If a moment distribution can be found which fulfils the equilibrium, and the slab is able to carry these moments, the slab has sufficient safety in ultimate state if it has sufficient rotational capacity. This is the
case for most non-prestressed reinforced concrete slabs subjected to static loads, provided they are under-reinforced (designed so that the reinforcement yields before the concrete reaches the ultimate strain in compression).

The strip method is primarily a design method; a moment distribution that fulfils the equilibrium is found and the reinforcement is then designed for these moments. Practically, the load on the slab is distributed on strips in the different directions. Following the method, the moments in the strips has only to be in equilibrium with the applied loads, and the compatibility conditions need not to be fulfilled. This means that many different moment distributions are possible. However, some distributions are more suitable then others, minimizing the need of plastic redistribution and giving a good response for service loads. In general, the moment distribution should not differ too much from the linear elastic solution. In his handbook, Hillerborg (1996) gives recommendations regarding how the load distribution should be made in order to fulfil these requirements. Following the recommendations, the method also gives shear forces that can be used in shear and punching design.

When analysing the load-carrying capacity for a slab with a given reinforcement layout, the strip method is not straightforward. An initial load distribution has to be assumed, based on the reinforcement distribution, and the capacities of the strips have to be checked under this assumption. If the capacity is not sufficient for some of the strips, the initial load distribution can be changed and improved solutions can be obtained through an iterative process. Consequently, following the lower bound theorem, if the capacity eventually is shown to be high enough in all strips, the slab has sufficient safety in ULS, provided it has sufficient rotational capacity.

5.2.4 The yield line method

The yield line method is based on the upper bound theorem of the theory of plasticity. A failure mechanism that fulfils the requirements of compatibility in ULS is found, and the magnitude of the load that leads to this failure is determined. This means that the method will provide the correct load carrying capacity if the most critical mechanism is found, provided that the slab has sufficient plastic deformation capability. However, for other possible mechanisms, it leads to unconservative estimates of the capacity. This means that the method gives solutions that are not on the safe side and that it needs to be combined with other methods, such as the strip method, except for simple cases where the critical mechanism is known.

The yield line method is primarily an analysis method, and it is practical to use for evaluation of existing slabs. It is a simple method and, for an experienced engineer, a rough estimate of the load carrying capacity can be determined quickly. For design, the method is less well suited since it gives no guidance to how the reinforcement should be distributed.

5.2.5 Non-linear analysis

In order to take full advantage of the inherent capacity of an existing structure, a non-linear analysis is often necessary. Fully non-linear analyses are usually performed with general FE programs, using non-linear material models for the concrete and the reinforcement. The principles for such analyses are described in Chapter 2.

Such non-linear analyses are suitable for accurate assessment of existing structures. They are less useful in the design process since they require that the design is defined in detail before the analysis is made, but can be used to verify or evaluate the performance of a chosen design. In such an analysis the response of the structure is reflected in a realistic way and different design solutions can be compared. In the case study of a typical slab bridge, reported in section 5.5, non-linear FE analyses are made to evaluate the response of the
bridge for different reinforcement detailing, designed with linear elastic 2D frame analysis, linear elastic 3D FE analysis and with the strip method, respectively.

Simplified non-linear analysis, where the cross-sectional response is defined in terms of e.g. moment-curvature relations, can be used instead of linear analysis for improved system level analysis. However, the response for different loads cannot be superimposed into load cases. On the other hand, a more correct estimation of the distribution of sectional moments and forces are obtained.

5.3 Stress concentrations from geometrical simplifications

Simplifications made in the geometrical modelling often leads to very high concentrations of moments and shear forces locally. This occurs, for example, when a slab modelled with shell elements is supported by columns modelled with beam elements or by bearings modelled by boundary conditions in a single node. In such a connection node, the cross-sectional moments and shear forces in the slab will continue to increase when the element mesh is being refined.

5.3.1 Mesh dependency

However, such singularities are local disturbances of the moment and force fields, and do not influence the cross-sectional moments and shear forces a short distance from the point where the singularity appear. This was illustrated by Davidson (2003), who performed analyses of a slab with a point load modelled with different mesh densities and with different element orders. In the loaded node, the difference in results is huge, but only one element length away (in the sparse mesh) the difference in small, and two element lengths away it is negligible.

Different commercial design programs, based on FE slab analysis, deal with this problem in different ways. For example, the program FEM-Design, see StruSoft (2004) applies a type of peak smoothing, see Figure 5.1. A circular peak smoothing region is generated by the post-processor, based on the slab thickness and the cross-section of the object causing the singularity, e.g. a column or a vehicle tyre. A constant value equal to the mean value of the studied cross-sectional moment or shear force is assumed within this region. However, since a mesh refinement will lead to a higher peak value and, accordingly, a higher mean value over the region, the result will still be mesh dependent, although to a lower degree.

![Figure 5.1](image-url) Peak smoothing in the design program FEM-Design. From StruSoft (2004).
5.3.2 Comparison to geometrically detailed models

When comparing the results from a model with simplified geometry to corresponding results obtained with the geometry modelled in detail, it becomes apparent that the maximum cross-sectional moments and shear forces does not appear inside connection region, for example where a column is connected to a slab. This was shown for frame corners by e.g. Plos (1995), Johansson (2000) and Lundgren (1999). It was also illustrated by Davidson (2003) for a slab bridge supported by columns.

It must also be pointed out that cross-sectional moments and forces, defined as integrals of the stresses over the cross-sectional height, do not have a clear interpretation inside a connection region. Instead, these regions must be seen as disturbed regions where beam or slab theory is not valid.

5.3.3 Critical cross-sections

Critical cross-sections for the reinforcement moments and the shear forces must be determined based on possible failure modes in the reinforced concrete slab. For cast connections between slabs and columns or walls, the critical bending crack will form along the surface of the column of the wall, and the tensile reinforcement will start to yield here. Consequently, the critical cross-section for a bending failure in the slab is along the surface of the column or wall, see Figure 5.2. This is in accordance with the Eurocode 2 (2004a).

A critical shear crack, which leads to a shear (or punching) failure, can generally be assumed to have an inclination not steeper than 45 degrees, see also Eurocode 2 (2004a). It will start as an interior "web" shear crack or, more likely, as a bending crack that deviates due to the shear stresses inside the slab. At a cast connection between a slab and a column or a wall, it will meet the compression zone in the corner at the intersection, see Figure 5.2. Consequently, the critical cross-section for the shear force can be assumed to be at a distance equal to the inner level arm, $d$, from the surface of the column or wall.

![Figure 5.2 Critical sections for determination of cross-sectional moment (left) and shear force (right) at the connection between a slab and a column or wall.](image)

5.3.4 Modelling and analysis

To study the influence of the mesh density around a column, linear FE analyses were made of a simply supported circular slab, supported along the edges and in a single central point. The slab was modelled with first order shell elements with different mesh densities. In Figures 5.3 and 5.4 the moment variations in the two main directions are shown for a path through the centre of the slab, for different mesh densities. It can be seen that, for each mesh refinement made, the moment will only change a little one element away from the support point in the coarser mesh. Since the moment resistance that is required is evaluated
from the area under the curve, a mesh density with only one element between the support point and the critical section for moments is enough in many engineering applications. Furthermore, the moment two elements away from the support point, in the coarser mesh, does not change noticeable when refining the element mesh. The same thing was observed only one element away from the support point, when second order shell elements were used. Consequently, a mesh density corresponding to two first order elements, or one second order element between the support point and the critical section for which the moments are sought is needed, in order for the result not to be mesh dependent.

**Figure 5.3** Influence of the element size on the bending moment along a path parallel to the reinforcement that is to be designed (perpendicular to the moment vector). From a linear analysis of a simply supported circular slab with a central support point.
Figure 5.4  Influence of the element size on the bending moment for a path perpendicular to the reinforcement that is to be designed (parallel to the moment vector). From a linear analysis of a simply supported circular slab with a central support point

5.3.5 Conclusion

It was found that if the finite element mesh around a support point is dense enough with respect to the extension of the support and the slab thickness, the design moments and shear forces in the critical cross-sections are unaffected by the geometrical simplification. Smoothing of the peak values are not necessary; since the critical cross-sectional moments and shear forces appears outside the peak region, they are unaffected by any mesh refinement.

Based on this, the following recommendations can be given:

- The mesh density should be chosen such that there are at least two element lengths between the support point and the critical cross-sections.
- The moments and shear forces in the critical cross-sections should be used for design and assessment.

5.4 Stress concentrations from material simplifications

The stress concentrations obtained through a linear analysis of a concrete bridge, often expressed as concentrations of cross-sectional moments and forces, do normally not exists in reality. The concrete will crack already for service loads, leading to redistribution of moments and forces. In the ultimate limit state, the reinforcement will start to yield, leading to even larger redistributions. The material simplification introduced through the assumption of linear elastic response will lead to higher cross-sectional moments than in reality, e.g. around a column or a concentrated support, since cracking and subsequent yielding in the reinforcement is not included.
5.4.1 Redistribution of moments due to material non-linearity

When the concrete cracks, the linear torsional moments in the slab will redistribute to bending moments that can be resisted by the cracked reinforced concrete slab. Redistribution of moments will also take place from one slab cross-section to another, e.g. from a support to a span section. Redistribution will also occur within each cross-section, in the direction perpendicular to the reinforcement direction. The amount and distribution of the reinforcement in the slab determines how and to what extent moment redistributions will take place in ULS.

When designing a slab, a moment distribution can first be chosen and the reinforcement thereafter designed to resist these moments. Due to the ability of the slab to deform plastically, the redistributions required can take place provided the slab has sufficient deformation capacity. The moment distribution can be obtained, for example with the strip method, with a linear analysis or through redistribution of the linear moments. The principle is that the linear moments can be redistributed as long as equilibrium is fulfilled, provided that the deformations required for this do not violate the structural integrity.

Similarly, when analysing an existing slab, the redistribution of moments will depend on the available reinforcement in the slab. Here, the principle is that the linear moments will become redistributed due to cracking and yielding until it corresponds to the moment distribution originally designed for, provided that the required deformations do not violate the structural integrity.

5.4.2 Lateral re-distribution of moments in slab cross-sections

The main task for this work is to provide guidelines for redistribution of moments from linear analysis in the lateral direction, within each cross-section, perpendicular to the reinforcement direction. Here, the moments need to be redistributed so that they can be resisted by reinforcement practically arranged in strips with constant bar spacing. In this section, recommendations from handbooks regarding how the widths of these strips can be chosen and, if applicable, how the total moment should be distributed between the strips.

Recommendations from design methods for flat slabs

There are several “handbook methods” for flat slabs described in the literature. The term flat slab is normally used for a slab supported by columns arranged in a regular pattern. In these methods, the average moment in a cross-section is distributed on a column strip and a middle strip. Examples on recommendations from such methods are found in Eurocode 2 (2004a) and are given by ACI, see e.g. McGregor (1992). In Eurocode it is pointed out that the reinforcement distribution should reflect the behaviour under working conditions, with a concentration of moments over the column. Unless rigorous checks are made for serviceability, half of the total top reinforcement should be concentrated into a column strip with the width:

\[ w = \frac{l_1}{8} + \frac{l_2}{8} \]  

Eq. (5.2)

where \( l_1 \) and \( l_2 \) are the distances from the column of the strip to the adjacent columns, in the direction perpendicular to the reinforcement.

In the method given by ACI, see McGregor (1992), the column strip width should be chosen to be:

\[ w = \frac{l_{\text{min},1}}{4} + \frac{l_{\text{min},2}}{4} \]  

Eq. (5.2)

where \( l_{\text{min},1} \) and \( l_{\text{min},2} \) are the distances from the column of the strip to the adjacent columns, in the direction with the shortest column distances.
In the support section, 75% of the total moment should be taken by the column strip and 25% of the moment by the middle strip. In the span section, 60% of the total moment should be taken by the column strip.

**Recommendations in the strip method**

In the strip method, presented by Hillerborg (1996), the total moments determined for the main strips are subdivided on column and mid strips, see Figure 5.5. The strip method is more general and covers more cases than just flat slabs. Consequently, the guidelines given are less rigorous and need more of engineering judgement to be applied to different cases. The recommendations given are in many cases differentiated depending on the requirements on crack widths in serviceability limit state (SLS) and on the possibility to prioritise a practical reinforcement layout for the resistance in ULS. In Figure 5.5, some alternatives for strip widths and moment distributions for strips supported on columns method are shown. For stricter requirements on limited crack widths in SLS, a larger part of the moment should be resisted by the column strip.

**Swedish recommendations for bridges**

In Swedish handbooks and regulations, some recommendations of interest for application on bridges can be found. In Handboken Bygg, Byggnästarens förlag (1969), recommendations regarding distribution of bending moments to resist a wheel pressure are given, see Figure 5.6. A similar expression is found in the Swedish regulations for assessment of existing bridges, Vägverket (1998):

\[
b_{ef} = b_x + t_{bel} + t_0 + t_{lb} + \min(0.75 \cdot l, 2.70 \ m) \quad \text{Eq. (5.3)}
\]

where

- \(b_x = 0.20 \ \text{m}\) is the width of the wheel,
- \(t_{bel}\) is the thickness of the asphalt layer
- \(t_0\) is the thickness of fill material on the slab
- \(t_{lb}\) is the thickness of the concrete cover and
- \(l\) is the theoretical span length
Figure 5.5  Different possible distributions of the total moment according to the recommendations given for the strip method. The total moment is resisted by a column strip and a mid strip, for support (s1 - s4) as well as span (f1 – f2) cross-sections. From Hillerborg (1996).

Figure 5.6  Width for distribution of the moment required to resist a wheel pressure, according to Vägverket (1998). From Davidsson (2003).

In the Swedish regulations for bridge design, Vägverket (2004), recommendations are given regarding redistribution of moments obtained through 3D FE analysis. These are based on a conservative interpretation of the recommendations given in the Swedish concrete code BBK 94, Boverket (1994), regarding lateral distance between a tensile reinforcement bar and the
corresponding concrete compression zone. In ULS, the maximum width of strip in which the moment can be redistributed and resisted by reinforcement with constant spacing is:

\[ w = \min(3h ; l/10) \]

Eq. (5.4)

where \( h \) is the cross-section height and \( l \) is the theoretical span length between two sections with zero bending moments.

For check of the performance in SLS, the widths for redistribution of moments are even more limited:

\[ w = \min(2h ; l/10) \]

Eq. (5.5)

Moment re-distribution in current design of bridges

Today, at least in Sweden, most slab bridges are designed using linear 2D frame analysis, see section 5.2.1. In these bridges, the reinforcement is usually not concentrated to column strips, in the way it is recommended when designing flat slabs, when using the strip method or when designing a slab bridge based in a linear 3D analysis. On the other hand, no problems have been reported regarding too large crack widths or with the safety for such bridges. Consequently, the very strict limitations on the widths for redistribution of moments from 3D analyses can be questioned. Furthermore, it can be questioned if not a certain redistribution of the moments to column strips is preferable, independent of the method for structural analysis used in the design.

5.5 Example: A two-span slab bridge supported by columns

To study how different assumptions in the design of a bridge affect the reinforcement distribution and the bridge response, a typical slab bridge was used as a case study. The reinforcement was designed using different methods for structural analysis, and then evaluated through non-linear FE analysis. The example illustrates the effect on the response of the redistribution made from a linear analysis and compares it to the response with other structural analysis methods.

5.5.1 The bridge

A typical two-span slab bridge with a simplified design was chosen for the case study. The bridge had a total length of 25 m and it was 11 m wide. It was supported by three roller bearings at each end, and by three columns in the middle of the bridge opening, see Figure 5.7. The slab had a thickness of 0.6 m and the columns had square cross-sections with 0.6 m sides and a length of 5.0 m. The two spans had a length of 12.0 m and the bearings and columns were positioned with 4.0 m spacing in transversal direction.

The bridge was designed to have a concrete grade of C35/45 and a reinforcement quality of B500B. The following characteristic material properties were assumed:

Concrete grade of C45/35:

- Young’s modulus \( E_c = 34.0 \) GPa
- Poisson’s ratio \( \nu = 0.2 \)
- Compression strength \( f_{cc} = 33.5 \) MPa
- Tensile strength \( f_{ct} = 2.1 \) MPa
- Fracture energy \( G_f = 105 \) Nm/m²
Steel bars B500B:
Young’s modulus \( E_s = 200 \text{ Gpa} \)
Tensile strength \( f_{ss} = 500 \text{ MPa} \)

Figure 5.7: Geometry of the slab bridge used as case study.

5.5.2 Loads
Bridges are normally designed for a variety of load combinations and possible traffic load positions, but in this study we only designed and analysed the bridge for one critical load case. The bridge was assumed to be loaded with the self weight and with traffic loads only. The critical load case was determined to give a maximum support moment in one of the columns closest to the edge.

The loads were chosen according to the Swedish bridge code, Bro 2004, see Vägverket (2004). The self weight of the concrete, including the reinforcement, was 25 kN/m³. The critical traffic load was determined to be of equivalent load type 1 according to Vägverket (2004), with load positions according to Figure 5.8. The traffic load consisted of both distributed and concentrated loads in 3.0 m wide traffic lanes. The characteristic traffic loads were:

- Distributed load
  \[ q_1 = 4 \text{ kN/m}^2 \]
  \[ q_2 = 3 \text{ kN/m}^2 \]
  \[ q_3 = 2 \text{ kN/m}^2 \]

- Concentrated load
  \[ P_1 = 250 \text{ kN} \]
  \[ P_2 = 170 \text{ kN} \]
The following partial safety coefficients were used for design in ULS:

- **Self weight**: \( \psi_{\text{selfweight}} = 1.0 \)
- **Traffic load**: \( \psi_{\text{traffic}} = 1.5 \)

### 5.5.3 Reinforcement design

The design of the reinforcement was made in three different ways, based on different methods for the structural analysis. These resulted in three different distributions of cross-sectional moments to design the reinforcement for. In all three cases, the required amount of reinforcement needed to resist the moments was calculated using non-linear cross-sectional analysis in ULS according to Eurocode 2 (2004a). A simplified stress distribution with constant compression stress for the concrete, see Figure 5.9, was used and the contribution from the compression reinforcement was not taken into account:

\[
0.8 \cdot b \cdot x \cdot f_{cc} = A_s \cdot \sigma_s \quad \text{Eq. (5.6)}
\]

\[
M = 0.8 \cdot b \cdot x \cdot f_{cc} \cdot (d - 0.4 \cdot x) = A_s \cdot \sigma_s \cdot (d - 0.4 \cdot x) \quad \text{Eq. (5.7)}
\]

![Figure 5.9](image)  
**Figure 5.9** Reinforced rectangular concrete section, a) general compression stress distribution, b) simplified compression stress distribution.

In the design, characteristic values were used for the material parameters. Furthermore, the reinforcement was designed for the critical load case according to 5.5.2 only, meaning that all other loads and traffic load positions were disregarded in the design. This was made in
order to be able to make direct comparisons between the loads designed for and the load levels reached in the non-linear analyses. Characteristic material properties were preferred instead of design values in the non-linear analyses, since this will give a more realistic representation of the bridge response. Consequently, direct comparisons between realistic responses with different reinforcement design can be made. However, the bridge will not obtain a reinforcement design that would be realistic for a real bridge.

The three methods used for structural analysis were:

- Two-dimensional (2D) linear frame analyses, according to common practise for bridges.
- Three-dimensional (3D) linear FE analysis using shell and beam elements. The moments obtained were redistributed in lateral direction in accordance with the recommendations given in Vägverket (2004).
- The strip method presented by Hillerborg (1996). The method is based on the lower bound theorem of plasticity. The principles and recommendations given in Hillerborg (1996) were followed, e.g. regarding choices of support to field moment ratio and strip widths.

Reinforcement design based on linear 2D frame analysis

In this case, separate two-dimensional frame analyses were made for the two main directions in the bridge. The linear 2D frame program STRIP STEP 2, commonly used by bridge engineers in Sweden, was used for the analyses. The load carrying system, slab widths for each frame analysis etc was chosen according to common practise among Swedish bridge engineers.

Firstly, three individual one meter wide strips in the longitudinal direction of the bridge were analysed, one for each traffic load lane, see Figure 5.10. In the transverse direction three separate strips, over the bearing and column rows respectively, were analysed. The transverse strips were loaded with the support reactions from the longitudinal strips. Over the two rows of bearings, the strip widths were chosen to be one meter. Over the columns, the strip was chosen to be three meters wide, i.e. 5 times the thickness of the slab.

The reinforcement was designed for the cross-sectional moments obtained from the frame analyses and placed with constant spacing within each strip. In between the strips, the minimum reinforcement required according to Vägverket (2004) was provided. This resulted in a reinforcement layout according to Figure 5.11.
Reinforcement design based on linear 3D FE analysis

A 3D linear FE analysis of the bridge was made using the general finite element program Diana, see TNO (2004). The FE model consisted of 8-node second order shell elements for the slab and 3-node second order beam elements for the columns, see Figure 5.12. In the model, the columns were connected to the slab in a single node. The shell element size was approximately 0.5 x 0.5 m, but the mesh was refined around each column so that there were two elements lengths between the connection point and the edge of the column in the plane of the slab. Each column was modelled by ten beam elements.

The boundary conditions provided by the bearings were modelled through prescribed displacements in the supported nodes, see Figure 5.12. Here, displacements were prevented in the vertical direction and, for the mid node in each row of bearings, also in transversal direction. In the support node for the columns, all displacements were prevented together with the torsional moment.

From the analysis, reinforcement moments according to section 5.2.2, equation 5.1, was obtained, see Figure 5.13. The reinforcement in each layer and direction was calculated according to the following procedure:

1. Critical sections through parts with maximum moments were identified from the contour plots. Critical sections were identified both perpendicular to and parallel to the reinforcement direction, see Figure 5.13.

2. For the critical sections, the reinforcement moments were extracted to diagrams.

3. Firstly, the required moment capacities in the critical sections perpendicular to the reinforcement direction were studied. For the parts of the slab that were located over the real extent of the column, a linear analysis gives unrealistic high moments due to the simplified geometrical modelling (i.e. the slab is connected to the column in
Figure 5.12  FE model with boundary conditions for the linear 3D analysis of the bridge.
Longitudinal reinforcement, bottom layer:

![Diagram of longitudinal reinforcement, bottom layer]

Longitudinal reinforcement, top layer:

![Diagram of longitudinal reinforcement, top layer]

Transversal reinforcement, top layer:

![Diagram of transversal reinforcement, top layer]

Figure 5.13 Reinforcement moment contour diagrams from the linear FE analysis of the bridge (moments in Nm/m). Critical sections identified and used in the reinforcement design are shown as red lines. In areas with the darkest blue colour, only minimum reinforcement is required. For the bottom reinforcement in transversal direction, there is no need for more than the minimum reinforcement.
a single node, see section 5.3). Here, the maximum moment occurring in the adjacent critical section, around the border of the column, was used instead. As an example, the required moment capacity for the longitudinal top reinforcement in a cross-section over the columns is shown in Figure 5.14.

4. Based on the recommendations given in Vägverket (2004), widths for strips with equal reinforcement spacing were chosen.

5. For each such strip, the total required moment capacity for the strip was calculated by integrating the area under the curve. This required moment capacity was then used to design the provided amount of reinforcement so that it was larger than or equal to the required. In Figure 5.14 the provided reinforcement is expressed with the bar diameter and the spacing for each strip.

6. **After this**, the required moment capacities in the **critical sections parallel to the reinforcement direction** were studied. The moment curves in the diagram were shifted a distance $a_i = d$, according to Eurocode 2 (2004a), to account for the influence of inclined cracking. As an example, the required moment capacity for the longitudinal top reinforcement is shown for one critical section over the most loaded column, parallel to the reinforcement direction, in Figure 5.15.

7. Based on these diagrams, the anchorage and curtailment of the reinforcement was designed. The lengths of the reinforcement bars were designed so that the provided moment capacity, with respect to the anchorage lengths, became larger than the required moment capacities, see Figure 5.15. However, since the capacity in the sections with maximum moments was covered through redistribution of the moment in lateral direction, the provided moment capacity does not have to cover the required capacity curve of the diagram in these parts. To simplify the procedure and to obtain a conservative design, lateral redistribution was not used to optimise the curtailment. Furthermore, equal lengths for the reinforcement bars were aim at in order to obtain as simple reinforcement layout as possible.

8. In the regions where no reinforcement was needed with respect to the required moment capacities, minimum reinforcement according to Vägverket (2004) was provided.

The resulting reinforcement layout is shown in Figure 5.16.
Figure 5.14 Required and provided reinforcement moment capacity for the top reinforcement in longitudinal direction. The diagram shows the variation of reinforcement moment along the critical section in the slab over the columns, perpendicular to the reinforcement direction (line A in Figure 5.13).

Figure 5.15 Required and provided reinforcement moment capacity for the top reinforcement in longitudinal direction. The diagram shows the variation of reinforcement moment along the critical section in the slab over the most loaded column, parallel to the reinforcement direction (line B in Figure 5.13).
Reinforcement design based on the strip method

The reinforcement of bridge was also designed based on the strip method, according to Hillerborg (1996). The strip method is a lower bound approach for plastic analysis of reinforced concrete slabs, which is well suited for design of reinforcement in ULS. The slab was designed following the recommendations for flat slabs, and for additional concentrated loads, according to Hillerborg (1996).

The slab was divided in regions by load dividing lines, i.e. lines with assumed zero shear force, see Figure 5.17. These regions formed continuous one-way strips in x- and y-directions, supported by the columns. Each such strip was then treated as a continuous beam, and the support and span moments were determined to meet equilibrium. Following the recommendations in Hillerborg (1996), the relation between support and span moments becomes close (but not equal) to the linear elastic distribution. The analysis was made separately for the distributed load and for the concentrated loads and the moments were then added. This led to a small overestimation of the span moments, as the maximum moments from the two loading cases do not appear in the same cross-section.

The moment in each main strip was distributed transversely to a “column strip” and a “mid strip” with equal widths and constant reinforcement spacing. In support cross-sections, 80% of the total support moment was resisted by the “column strip” and in span cross-sections, 60% of the total span moment was resisted by the “column strip” (distributions s2 and f2 in Figure 5.5). The moments from adjacent “column” or “mid strips” were averaged to obtain equal reinforcement spacing. Where the strip method resulted in lower reinforcement amounts, the minimum reinforcement according to Vägverket (2004) was provided. This resulted in a reinforcement layout according to Figure 5.18.

5.5.4 Non-linear FE models

The slab bridge was modelled using the general finite element program DIANA, TNO (2004). The reinforced concrete in the slab was modelled using shell elements with embedded reinforcement. This means that the concrete and the reinforcement material properties are mod-
elled separately, with individual non-linear material models, but that complete interaction between them is assumed (i.e. no slip occurs).

![Load dividing lines](image1)

**Figure 5.17** Load dividing lines (dash-dotted) and strips, 1 to 6 in longitudinal direction and A to F in transversal direction, for the strip method analysis. Point loads and traffic lanes with different distributed load intensities (in gray).

![Reinforcement layout](image2)

**Figure 5.18** Reinforcement layout based on analysis with the strip method. (Bar distances has been rounded.)

**Geometry and reinforcement**

The slab was modelled with four nodes, first order shell elements (type Q20SH) with embedded reinforcement. The elements had 9 integration points (according to the Simpson rule) over the slab thickness to account for the stress variation in the concrete. The slab consisted of between 1100 and 1512 elements, depending on the reinforcement design. The element sizes did not exceed 0.5 x 0.5 m, but were adjusted to fit the regions with different reinforcement content. Each column was modelled by five beam elements (type CL18B), with three
nodes each, and was connected to the slab at one single node. The columns were assumed to have a linear response, and had no embedded reinforcement.

The reinforcement was defined as embedded reinforcement grids in the shell elements. With this approach the reinforcement has the same degrees of freedom as the element it is embedded in. However it has separate material properties and the response is evaluated in separate integration points. This means that perfect interaction is assumed and that no bound slip can occur between the reinforcement and the surrounding concrete. Consequently the concrete cracking that appears in the analyses does not represent individual cracks, rather cracked areas with a certain crack direction.

Reinforcement grids were defined for the top and bottom reinforcement, respectively. This means that both reinforcement layers at each side of the slab had the same eccentricity, taken as a weighted mean value with respect to the concrete cover, bar diameters and amount of reinforcement. Each reinforcement layer was defined by giving its direction and the reinforcement cross-sectional area per slab width according to section 5.5.3, see Figures 5.11, 5.16 and 5.18. However, here, the calculated reinforcement amounts were used without any rounding.

**Boundary conditions**

The boundary conditions were assumed in the same way as in the linear analysis in section 5.5.3. This means that all displacements were prevented together with the torsional moment at the bottom of the columns. Furthermore, at the nodes supported by bearings, the vertical displacements were prevented and, at the mid support nodes, also the transversal displacements.

![FE model with reinforcement grids (orange) and boundary conditions (black) for non-linear analysis of the bridge, with reinforcement designed based on 2D frame analysis. To the left, the load distribution system described below is shown.](image-url)
Material models

Non-linear material models, available in DIANA, were used for the concrete and the reinforcement. For the columns, linear material properties for concrete were assumed. Characteristic material parameters according to section 5.1.1 were used in the analyses.

The concrete was modelled with a constitutive model based on non-linear fracture mechanics. A rotating crack model based on total strain was used, see TNO (2004). For the tension softening, the curve proposed by Hordijk was used, see Figure 5.20, with the fracture energy smeared over a length, $h$. Since the reinforcement was modelled with complete interaction to the concrete, all concrete elements in a cracked region will become cracked, and the length $h$ corresponds to the mean crack spacing. Here, the mean crack spacing was approximated to be $h = 0.13 \text{ m}$. The hardening of concrete in compression was described by the expression of Thorenfeldt, see Figure 5.20. For the reinforcement, the response is defined by the uniaxial elastic-ideal plastic stress-strain relations, see Figure 5.21.

Loading

The bridge was loaded in three subsequent steps in the non-linear analyses:

1. The bridge was first subjected to its self weight, using gravity loading.
2. The distributed and concentrated traffic loads were then applied as external loads up to their design values.
3. Finally, the concentrated traffic loads were further increased, using a prescribed deformation \((u)\). A load distribution system was introduced to distribute the resulting reaction force from the node given the prescribed deformation to the bridge.

The load distribution system consisted of a statically determined system of simply supported, very stiff beams. In Figure 5.22, the principle of the load distribution system is shown. The beams are connected to each other through constraints giving the coupled nodes equal vertical displacements. In the same way, the “bottom nodes” of the load distribution system are connected to the nodes of the bridge slab that should be loaded by concentrated loads. This means that the reaction force, occurring in the “loading node” when given the prescribed deformation, was distributed through the beam system to the nodes on the bridge that should be loaded by the concentrated traffic load.

![Figure 5.22 The principal design of the load distribution system.](image)

In order to avoid reaction forces in the "loading node" when the bridge is loaded with gravity load and design traffic loads (in step 1 and 2) a dummy spring element was introduced between a "displacement control node" and the "loading node" on the "top" beam of the load distribution system. The non-linear dummy spring was given a negligible stiffness when loaded in tension, and a very high stiffness when loaded in compression. In step 2, the concentrated traffic loads were applied through an applied force in the "loading node". In Step 3, the concentrated traffic loads were further increased through a prescribed vertical displacement in the "displacement control node". To reach zero deformation of the dummy spring, the step was started by giving the "displacement control node" a deformation corresponding to the elongation of the spring in the first two increments.

The truss elements shown in Figure 5.22 were only used to check that the loads were distributed as intended and was not included in the final model. Furthermore, in the final model, the beams of the load distribution system were not oriented as in Figure 5.22, but were instead placed parallel to each, see Figure 5.19.

5.5.5 Non-linear FE analyses

General

In order to determine the response and the load carrying capacity of the bridge with different reinforcement design, non-linear FE analyses were made. In the analyses, the loads were increased successively according to the loading scheme described in section 5.5.4. In each step, the loading was applied incrementally with prescribed increment sizes. The BFGS secant iteration method, see TNO (2004), was used to find equilibrium within each increment. An energy convergence criterion was used with default parameters. If the convergence criterion was not reached within the maximum number of iterations allowed, usually 300 itera-

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tions, the analysis was continued. The quality and reliability of the results was evaluated after the analysis, and the analysis was rerun with changed parameters if the errors in the solution were found to be too large. If the analysis was converging, but did not fulfil the convergence criterion, the results from the increment not converged completely were disregarded, but the results from the following converging increments were used. The latter occurred for a number of increments in the analyses, close to and after the maximum load was reached.

Below, the analysis results are reported for the three different reinforcement designs. The response under loading up to and past the maximum load is described. The response of the bridge with reinforcement designed based on linear 2D frame analysis are shown in detail. The bridge had in general the same response also with the other reinforcement design. Therefore, detailed results are only shown when they differ from the first analysis results.

Reinforcement design based on linear 2D frame analysis

In Figure 5.23, the relation between the traffic point load $P_1$, see section 5.5.2, and the displacement of the “loading node”, see section 5.5.4, is shown. In the diagram, the response of the bridge under increasing loading can be followed and major observations of the response are indicated. In Figure 5.24, the deformation of the bridge for loading with self weight, with design load in ULS and after forming of a mechanism (last increment) are shown. The yield lines forming the mechanism can clearly be seen in the figure.

The design load for check of crack widths in SLS can be defined differently, possibly including a small part of the characteristic traffic load in addition to the self weight. In the analysis, the response of the bridge is initially linear, but the concrete starts to crack when loading with the self weight, see Figure 5.25. However, the maximum crack strain corresponds to a crack width that cannot be detected visually. When reaching a load level including 30% of the characteristic traffic load, a first major crack has formed across the most loaded span. The maximum crack strain, occurring in the bottom of the slab, at the end with lowest reinforcement content, is $\varepsilon_{cr} = 2.26 \cdot 10^{-3}$. This corresponds approximately to a crack width of

![Figure 5.23](traffic_point_load_p1_as_a_function_of_displacement_of_the_loading_node_for_the_bridge_designed_based_on_linear_2d_frame_analysis.png)
Figure 5.24  Deformed FE models of the bridge, for loading with (a) self weight, (b) design load in ULS and (c) after formation of a mechanism. (The deformations are scaled with varying factors, so that the maximum deformation is the same in all comparable figures.)

\[ w \approx h \cdot \varepsilon_{cr} = 130 \cdot 2.26 \cdot 10^{-3} = 0.29 \text{ mm} \]

This rather large crack width, compared to the requirements in SLS, is a consequence of that the bridge slab was designed in ULS for one traffic load position only; this resulted in a very low reinforcement amount at the side of the bridge where the largest crack width occur. At the other end of the crack, the crack width is approximately half of the maximum.

When the load is further increased, the main bottom reinforcement at the side with lowest reinforcement amount starts to yield at a load level of approximately 40\% of the characteristic traffic load \((P_t = 94 \text{ kN})\). At a load level of approximately 75\% of the characteristic traffic load \((P_t = 188 \text{ kN})\), the top reinforcement starts to yield above the column with the smallest load, in a region where the slab has a low reinforcement amount.

When the design load in ULS, according to Vägverket (2004), was reached, the slab was substantially cracked, see Figure 5.26, and yielding in the reinforcement had occurred in large parts along the critical cross-sections, see Figure 5.27.

After the design load level, only the point loads of the traffic load are increased. When approaching the maximum load, a yield line is formed across the heaviest loaded span. After this, the deformations increases, but the load is almost constant. A second yield line is formed when the traffic point loads reaches \(P_t = 402 \text{ kN}\), and a mechanism is formed. After this, the load is more or less constant with increasing deformation, before it starts to decrease slowly, see Figure 5.23.

To be able to evaluate how large plastic rotations that occur before a mechanism is formed, for the different reinforcement designs, the strains in the reinforcement was studied. In Figure 5.28 the stresses and strains in the reinforcement are shown for the maximum load level, just...
when the mechanism was formed. The maximum reinforcement strain occurred in the longitudinal bottom reinforcement, in the most loaded span at the end with lowest reinforcement content, $\varepsilon_{s,\text{max}} = 12.2 \cdot 10^{-3}$. The maximum reinforcement strain in the top reinforcement occurred in the longitudinal reinforcement over the most loaded column, $\varepsilon_s = 8.4 \cdot 10^{-3}$.

Figure 5.25 Crack pattern (primary cracks) for loading with self weight, on the bottom (left) and top (right) surface of the slab. (Views from above)

Figure 5.26 Crack pattern (primary cracks) for design loading in ULS, on the bottom (left) and top (right) surface of the slab. (Views from above)

Figure 5.27 Stresses (in Pa, above) and strains (below) in the longitudinal reinforcement for design loads in ULS, in the bottom (left) and top (right) layer. Red colour in the stress plots indicates that the reinforcement is yielding in the last increment ($\sigma_s = f_y = 500$ MPa). Light blue in the strain plots indicate that yielding has occurred before this load level is reached ($\varepsilon_s \geq \varepsilon_y = 2.5 \cdot 10^{-3}$). (Views from above)
Figure 5.28  Stresses (in Pa, above) and strains (below) in the longitudinal reinforcement for maximum load, just when the mechanism was formed, in the bottom (left) and top (right) layer. Red colour in the stress plots indicates that the reinforcement is yielding ($\sigma_s = f_{st} = 500$ MPa). Light blue in the strain plots (or higher in the colour scale) indicate that yielding has occurred before this load level is reached ($\varepsilon_s \geq \varepsilon_{sy} = 2.5 \cdot 10^{-3}$). (Views from above)

Reinforcement design based on linear 3D FE analysis

The response of the bridge is similar to that of the bridge designed based on 2D frame analysis. In Figure 5.29, the relation between the traffic point load $P_1$ and the displacement of the “loading node” is shown, and major observations of the response are indicated. The deformation shape of the bridge is shown in Figure 5.30 for the maximum load reached. It is very similar to the deformation of the bridge designed based on 2D frame analysis, but with a litter larger difference in deflections between the most loaded and the opposite edge.
Traffic point load, $P_1$ [kN]

**Figure 5.29** Traffic point load $P_1$ as a function of the displacement of the "loading node", for the bridge designed based on linear 3D FE analysis.

**Figure 5.30** Deformed FE model of the bridge for maximum load level reached, for the bridge designed based on linear 3D FE analysis.

The crack width for the self weight is undetectable visually. At a load level including 30% of the characteristic traffic load, the maximum crack strain is $\varepsilon_{cr} = 2.03 \cdot 10^{-3}$ in the slab above the column with the smallest load. This corresponds approximately to a crack width of

$$w \approx h \cdot \varepsilon_{cr} = 130 \cdot 2.03 \cdot 10^{-3} = 0.26 \text{ mm}$$

Yielding in the longitudinal top reinforcement is initiated above one of the columns at a load level of approximately 50% of the characteristic traffic load ($P_1 = 128$ kN). At a load level of approximately 1.25 times the characteristic traffic load ($P_1 = 312$ kN), the bottom reinforcement starts to yield in the most loaded span, at the most loaded edge of the bridge slab.

The analysis was not continued until a final failure mechanism was formed. However, the results show that the forming of a mechanism was very close and that the maximum load was reached. The maximum load level was $P_1 = 402$ kN. At this load level, yield lines had almost formed across the heaviest loaded span and over the line of columns, see Figure 2.31. Also in the span with lower loading, a yield line had started to form. The maximum
Figure 5.31 Stresses (in Pa, above) and strains (below) in the longitudinal reinforcement for maximum load, just when the mechanism was formed, in the bottom (left) and top (right) layer. Red colour in the stress plots indicates that the reinforcement is yielding ($\sigma_s = f_{st} = 500$ MPa). Light blue in the strain plots (or higher in the colour scale) indicate that yielding has occurred before this load level is reached ($\varepsilon_s \geq \varepsilon_{sy} = 2.5 \cdot 10^{-3}$). (Views from above)

reinforcement strain, $\varepsilon_s = 25.3 \cdot 10^{-3}$, occurred in the longitudinal top reinforcement over the most loaded column. The maximum reinforcement strain in the bottom reinforcement occurred in the most loaded span, at the most loaded edge, $\varepsilon_{s,\max} = 22.4 \cdot 10^{-3}$.

Reinforcement design based on the strip method

The response of the bridge when designed based on the strip method is similar to that of the other bridge designs. The relation between the traffic point load $P_1$ and the displacement of the "loading node" is shown in Figure 5.32, and major observations of the response are indicated. The deformation shape of the bridge immediately after forming of a mechanism is shown in Figure 5.33. In this case, the displacements in the least loaded span is a little larger than with the other reinforcement designs.

Also here, the crack width for self weight was visually undetectable. When loaded with 30% of the characteristic traffic load, the maximum crack strain was $\varepsilon_{cr} = 2.10 \cdot 10^{-3}$, occurring in the bottom of the slab in the most loaded span, at the edge with lowest reinforcement content. This corresponds approximately to a crack width of

$$w \approx h \cdot \varepsilon_{cr} = 130 \cdot 2.10 \cdot 10^{-3} = 0.27 \text{ mm}$$

Yielding is initiated in the longitudinal bottom reinforcement in the most loaded span, at the least loaded edge, at a load level of approximately 40% of the characteristic traffic load ($P_1 = 94$ kN). The bottom reinforcement starts to yield over one of the columns, at a load level of approximately 80% of the characteristic traffic load ($P_1 = 210$ kN). At a load level of approximately 1.4 times the characteristic traffic load ($P_1 = 355$ kN), a yield line is formed across the bridge slab, over the columns. This means that the first yield line occurs before the design load level is reached, for this reinforcement design.

The maximum load level, $P_1 = 406$ kN, was reached when a second yield line occurred across the most loaded span and a mechanism was formed, see Figure 5.34. The maximum reinforcement strain was then $\varepsilon_{s,\max} = 15.1 \cdot 10^{-3}$, in the longitudinal top reinforcement over the most loaded column. The maximum reinforcement strain in the bottom reinforcement was at

---

$\sigma_s = f_{st} = 500$ MPa

$\varepsilon_s \geq \varepsilon_{sy} = 2.5 \cdot 10^{-3}$

$\varepsilon_{s,\max} = 22.4 \cdot 10^{-3}$

$\varepsilon_{cr} = 2.10 \cdot 10^{-3}$

$h \cdot \varepsilon_{cr} = 0.27 \text{ mm}$

$P_1 = 94$ kN

$P_1 = 210$ kN

$P_1 = 355$ kN

$\varepsilon_{s,\max} = 15.1 \cdot 10^{-3}$
Figure 5.32  Traffic point load $P_1$ as a function of the displacement of the “loading node”, for the bridge designed based on strip step analysis.

Figure 5.33  Deformed FE model of the bridge directly after a mechanism was formed, for the bridge designed based on strip step analysis.

the same time $\varepsilon = 14.9 \cdot 10^{-3}$, occurring in the most loaded span, at the least loaded edge. After the mechanism had formed, some increments with a low convergence rate occurred, and the results for the following increments may also be doubtful.

5.5.6 Evaluation and discussion

Comparison of the different reinforcement designs

The three different reinforcement design methods resulted in approximately the same reinforcement amounts, if the total reinforcement amounts in the cross-sections above the columns and in the most loaded span are added, see table 5.1. However, the distribution between support and span moment amounts varied between the different designs. There is also a considerable variation between the reinforcement distributions in the lateral direction within each cross-section, see Figure 5.35.
Figure 5.34  Stresses (in Pa, above) and strains (below) in the longitudinal reinforcement for maximum load, just after the mechanism had formed, in the bottom (left) and top (right) layer. Red colour in the stress plots indicates that the reinforcement is yielding ($\sigma_s = f_{st} = 500$ MPa). Light blue in the strain plots (or higher in the colour scale) indicate that yielding has occurred before this load level was reached ($\varepsilon_s \geq \varepsilon_{sy} = 2.5 \times 10^{-3}$). (Views from above)

The design based on linear 3D analysis resulted in the most even distribution across the cross-sections, from one end to the other. In the support cross-section, this design resulted in a varying reinforcement content between “columns” and “mid” strips, which the design based on 2D frame analysis does not have. The strip method, on the other hand, resulted in an even more pronounced variation between column and mid strips, also in the span cross-section.

Load carrying capacity and deformations

All three reinforcement designs resulted in the same load carrying capacity for the bridge. The load carrying capacities in the non-linear analyses corresponded to traffic point loads approximately 7% higher than the design values. One reason contributing to that the capacity was higher than the design load in the non-linear analyses, is that the compression reinforcement was not taken into account when the reinforcement was designed. It is also important to remember that the traffic point loads are only a fraction of the total load.

Table 5.1  Provided total cross-sectional reinforcement areas, in the cross-sections where the yield line forms, above the columns and in the most loaded span, with the different reinforcement designs.

<table>
<thead>
<tr>
<th>Reinforcement designed with</th>
<th>$A_{s,tot}$ [10^3 mm²]</th>
<th>support</th>
<th>span 1</th>
<th>ratio</th>
<th>sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D frame analysis</td>
<td>23.3</td>
<td>11.7</td>
<td>2.0</td>
<td>35.0</td>
<td></td>
</tr>
<tr>
<td>3D FE analysis</td>
<td>21.2</td>
<td>13.9</td>
<td>1.5</td>
<td>35.1</td>
<td></td>
</tr>
<tr>
<td>Strip method</td>
<td>20.2</td>
<td>14.4</td>
<td>1.4</td>
<td>34.6</td>
<td></td>
</tr>
</tbody>
</table>
The good agreement between the analyses with different reinforcement designs confirms that it is correct to disregard the moment concentrations in the slab over the columns; when the slab and columns are modelled using shell and beam elements connected in one single node, the moments in the critical cross-section adjacent to the column can be used in accordance with the conclusions in section 5.3. Furthermore, the design procedure for reinforcement design based linear 3D FE analysis, described in section 5.5.3, can be used.

The load-deflection relations, for the "loading node" of the bridge, are very similar for the different reinforcement designs, see Figure 5.37. However, the deflections under high loads, during the forming of the mechanism, increases more for the bridge designed with 3D FE analysis. Furthermore, the deformed shape of the bridge is a little different for this case: The deflection at the most loaded edge is almost the double compared to the opposite edge, in the most loaded span, see Figure 5.30. For the other designs, the deflections at opposite
edges are almost equal. The yield line at mid-span becomes also more inclined to the bridge edges for the 3D analysis design, compared to the other reinforcement designs.

Crack widths for service loads
The maximum crack widths for the design load in SLS, according to Vägverket (2004), are almost the same for the different reinforcement designs. However, the maximum crack widths occur in different parts of the slab. With the reinforcement designed with 2D frame analysis and with the strip method, the maximum crack widths occur in the bottom of the span, at the unloaded edge. Here, these designs have a much smaller reinforcement amount than the design based on the linear 3D analysis. With the latter design, the maximum crack width occurs above the column closest to the unloaded edge instead. Here, the reinforcement amounts do not differ very much between the different designs.

The magnitudes obtained do not correspond to what they would be in a real bridge, designed with any of these methods. Firstly, the reinforcement was designed using characteristic values in ULS, leading to lower reinforcement amounts. Secondly, the reinforcement was designed for one traffic load position only, leading to low reinforcement contents in the part of the slab where the maximum crack widths in the span occur. Thirdly, the reinforcement was not designed with respect to the requirements in SLS. Nevertheless, the results indicate that the different ways of designing the reinforcement are equally good with respect to crack widths.

Plastic deformations in ULS
The plastic reinforcement strains in the yield lines when the mechanisms were formed were studied for the different bridge designs. These strains are related to the plastic rotations that occur under loading up to the formation of the mechanism. The strains observed in the nonlinear analyses, were recalculated to equivalent plastic strains with respect to the widths of the yield lines in the analyses, i.e. the element length, \( l_{elem} \), of the yielding elements, perpendicular to the yield line. In accordance with the model for rotation hinges in Eurocode 2

Figure 5.37 Comparison between load-displacement relations for the different reinforcement designs. Traffic point load \( P_1 \) is shown as a function of the displacement of the “loading node”.

![Figure 5.37](image-url)
(2004a), the equivalent plastic strain was assumed to be distributed along a reinforcement length of two times the slab height, \( h = 0.60 \text{ m} \):

\[
\varepsilon_{pl,eq} = \frac{l_{elem}}{2 \cdot h} \left( \varepsilon_i - \frac{f_{st}}{E_s} \right) = \frac{l_{elem}}{1.2 \text{ m}} \left( \varepsilon_i - 2.5 \cdot 10^{-3} \right)
\]

Eq. (5.8)

The calculated maximum equivalent plastic strains in the yield lines, when the mechanisms were formed, are shown in table 5.2.

<table>
<thead>
<tr>
<th>Reinforcement designed with</th>
<th>( \varepsilon_{s,\text{max}} [10^{-3}] )</th>
<th>( l_{el} [\text{m}] )</th>
<th>( \varepsilon_{pl,\text{eq,\text{max}}} [10^{-3}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear 2D frame analysis</td>
<td>12.2 8.4</td>
<td>0.5 0.5</td>
<td>4.0 2.5</td>
</tr>
<tr>
<td>Linear 3D FE analysis</td>
<td>25.3 22.4</td>
<td>0.3 0.5</td>
<td>5.7 8.3</td>
</tr>
<tr>
<td>Strip method</td>
<td>15.1 14.9</td>
<td>0.5 0.5</td>
<td>5.3 5.2</td>
</tr>
</tbody>
</table>

Table 5.2 Comparison of maximum plastic reinforcement strains, made equivalent with respect to the FE mesh, between the non-linear analyses with different reinforcement designs

The reinforcement designed with linear 3D analysis leads to larger plastic strains (i.e. larger plastic rotations) before the failure mechanism is formed, compared to the other reinforcement designs. The maximum plastic strains in the support yield line occur right over the most loaded column, while the maximum plastic strain in the span yield line occur on the edge at the most loaded side of the bridge, see Figure 5.31. This corresponds well with the deformed shape of the bridge, see Figure 5.30, and is a consequence mainly of the overall reinforcement distribution across the cross-sections. The reinforcement design based on linear 3D analysis has a quite evenly distributed main reinforcement across the bridge, while the other two reinforcement designs have the gravity centre of the provided reinforcement shifted more towards the loaded side of the critical cross-sections, see Figures 35 and 36. Consequently, for the reinforcement designed with linear 3D analysis, the yield lines start from the most loaded side of the bridge and penetrate towards the opposite edge. This leads also to larger plastic rotations (and larger deflections) before the mechanism has formed.

For the other two reinforcement designs, the support yield line starts over all three columns and interconnects and penetrates towards the edges after this, see Figures 5.27, 5.28 and 5.34. The span yield line starts first at the least loaded edge, but thereafter also on the opposite edge, and the two parts interconnects to a complete yield line. This leads to more equal deflections on both edges of the bridge, see Figures 5.24 and 5.33. Furthermore, smaller plastic rotations are needed for the yield lines to form.

For the reinforcement design based on the strip method, the support yield line formed at a quite early stage, before the maximum load level was reached. Consequently, additional plastic rotations occurred before the span yield line was formed, and the equivalent plastic strains reached the same level as for the case with the reinforcement designed based on linear 3D analysis. However, the size of the plastic rotations was more evenly distributed along the yield line for the strip method design.

The influence of the more “local” distribution between “column” and “mid” strips, that are obtained with the linear 3D analysis and even more pronounced with the strip method, does not seem to have the same large influence on the size of the plastic rotations. Instead, the smallest plastic rotations are obtained with the reinforcement designed with 2D frame analysis, without any concentration of reinforcement over the columns.
Altogether, the results show that the overall distribution of reinforcement across a yield line, and the distribution between support and span reinforcement amounts, has a larger influence on the maximum plastic rotation occurring than the “local” distribution between “column” and “mid” strips. These “global” distributions are for both the strip method and the 2D frame analysis a consequence of the choices that are made by the designer when setting up the structural model for the bridge, and not directly of the method itself. For the linear 3D analysis, on the other hand, the “global” distribution is given by the analysis. It is interesting to note that, despite that a linear 3D analysis often is regarded as “more correct”, it leads to a reinforcement distribution that obviously does not correspond better with the needs for moment resistance when approaching the failure load in this case.

5.6 General conclusions and recommendations

It can be concluded that unrealistic concentrations of cross-sectional moments and forces are obtained when performing linear 3D FE analyses of reinforced concrete slab bridges with concentrated supports. There are two different reasons for the unrealistic concentrations, both due to simplifications in the modelling:

- **Geometrical simplifications** originate typically from supports in single nodes, or when e.g. a column is connected to a slab in a single node.
- **Material simplifications** originate from that the materials are assumed to have a linear response while, in reality, both concrete and reinforcement have very non-linear behaviour with cracking and yielding.

The problem with **geometrical simplifications** can be overcome by using a sufficiently dense FE mesh and by using the cross-sectional forces and moments in the critical cross-sections. When a slab is connected to a column or supported in a single node the following recommendations are given:

- The mesh density around the support node should be chosen such that there are at least two first-order (or one second-order) shell elements between the node and the critical cross-section.
- The moments and shear forces in the critical cross-sections should be used for design or assessment. The higher peak values for moments and shear forces, inside the critical cross-sections, has no physical meaning and can be ignored. The location of the critical cross-sections depends on the failure mode. For example, for a cast connection between a column and a slab, the critical cross-section for the bending moment in the slab is along the surface of the column, and the critical cross-section for shear force is on a distance from the surface of the column equal to the inner level arm of the slab.

The concentrated cross-sectional moments and forces due to **material simplifications** need to be distributed. The torsional moments obtained through a linear analysis cannot be resisted by the cracked reinforced concrete slab. Instead they need to be resisted by bending moments in the reinforcement directions. Consequently, reinforcement moments need to be calculated from the linear results, taking both the bending and torsional moments into account, see section 5.2.2.

Furthermore, the reinforcement moments need to be re-distributed in lateral direction, within cross-sections perpendicular to the reinforcement direction. The reinforcement is normally arranged in strips with equal reinforcement spacing. The moments obtained from a linear 3D analysis of a slab, may be re-distributed so the total reinforcement moment within each strip is resisted by the total amount of reinforcement provided in the same strip, as long as the width of the strip is not too big. What limits a proper width is the plastic rotations needed in
the slab in ULS, for the moment to re-distribute when loaded up to its failure load. In section 5.5.3, a design procedure for design of reinforcement from linear 3D analysis is outlined.

Different recommendations regarding the widths of strips width equal reinforcement spacing are given in different handbooks. In practice, also different design methods will result in different widths with equal reinforcement spacing. In the project, a case study was performed where the reinforcement of a typical slab bridge was designed with different methods, and the different designs were analysed with non-linear FEM and compared. The case study showed that the overall lateral distribution of the reinforcement across the whole width of the bridge had a larger influence on the maximum plastic rotations required than the more local distribution between “column” and “mid” strips. On the other hand, this “global” distribution of moments depend more on the choices that are made by the designer when setting up the structural model for the bridge and not directly on the choice of widths for strips with equal reinforcement spacing. In fact, the design based on linear 2D frame analysis, with equal reinforcement spacing within very wide strips, gave the lowest requirements on plastic rotations. Furthermore, the distribution between support and span moments had a larger influence on the plastic rotations required than the choices of strip widths. For SLS, no difference was found between the different designs regarding the crack widths. Altogether, the results indicates that restrictions on the strip widths, used for redistribution of moments from linear 3D analysis, has a minor influence on the plastic rotations needed to form the failure mechanism in ULS.

When designing a slab bridge, it is reasonable to arrange the reinforcement in “column” and “mid” strips with equal reinforcement spacing, with widths chosen according to recommendations in handbooks. The study indicates that it is appropriate to choose the width of the column strip in the interval:

$$\frac{l_{\text{min},1} + l_{\text{min},2}}{8} \leq w \leq \frac{l_{\text{min},1} + l_{\text{min},2}}{4}$$  \hspace{1cm} \text{Eq. (5.2)}$$

where $l_{\text{min},1}$ and $l_{\text{min},2}$ are the distances from the column of the strip to the adjacent columns, in the direction with the shortest column distances,

with the mid strips covering the regions between the column strips. However, it is important that the slab has sufficient capacity for plastic rotations. It is recommended that the cross-sections at least fulfill the requirement for ductility given for linear elastic analysis with limited redistribution according to Eurocode 2 (2004a). The recommendations given in Vägverket (2004) regarding widths for moment re-distribution is likely to be conservative, both with respect to the response in SLS and in ULS.

When assessing a slab bridge, re-distributions of the reinforcement moments within the slab cross-sections are necessary to avoid excessive under-estimation of the load-carrying capacity. The moment should be redistributed so that the moment distribution corresponds to the available reinforcement in the existing bridge. Recommendations in handbooks regarding strip widths for re-distributions should not be used, since this would mean that too rigorous limitations on the re-distributions is introduced. Instead, the study indicates that substantial re-distribution can be allowed compared to the linear distribution, if the slab has sufficient capacity for plastic rotations. Consequently, the ductility should be checked with respect to the requirements in Eurocode 2 (2004a).
6 Bending-shear-torsion interaction – cracking and ultimate capacity

6.1 Background

Shear forces, torsion moments and bending moments are related and interact. This is a vital question for the behaviour of bridges and has caused much concern since the beginning of the use of reinforced concrete structures. A lot of work has been carried out in this area; see e.g. ASCE-ACI Committee (1998).

As a background and an introduction some examples are given below. In a box girder section, torsion (twisting) moments interacts with shear forces in the way outlined in Figure 6.1. They act together in one of the webs while they counteract each other in the other web.

\[ T = q_T \cdot z \cdot b + q_T \cdot b \cdot z \]

\[ q_T = \frac{T}{2bz} \]

\[ V = 2q_v \cdot z \]

\[ q_v = \frac{V}{2z} \]

**Figure 6.1. Shear flow due to a twisting moment, T and a shear force, V in a general cross section of a box girder beam.**

In Figure 6.2 the interaction is studied between the horizontal forces caused by a bending \( M \) and the ones caused by a shear force \( V \). This implies that adequate anchorage is provided for the longitudinal bars, a question which has recently been studied by Shave et al (2003).

The interaction can also be modeled according to the theory of plasticity, assuming yielding of all reinforcement before concrete compression failure. Such an analysis may give an interaction surface as outlined in Figure 6.5.

6.2 Aim and objectives

In this chapter, the analysis of bridges loaded in combined bending-shear-torsion will be presented. A simple model according to the theory of plasticity is first presented. Then the modified compression field theory (MCFT) is introduced as an alternative to the standard methods given in the Eurocodes. This theory gives a possibility to follow the behaviour (stresses and strains) from the un-cracked stage through cracking of the concrete and yielding of the steel up to the ultimate failure of the section. Examples will be presented on how the interaction between shear, torsion and bending may be treated. The crack widths in the serviceability limit state (SLS) will sometimes design the amount of reinforcement and this will also be studied.
6.3 Theory of plasticity

A general introduction to the theory of plasticity is given in section 2.2.2. A simple way of studying the interaction is to use the theory of plasticity, Lüchinger (1977), Thürlimann (1978), Hsu (1984, 1993, 1998) and Elfgren et al (1972, 1974, 1979). If all reinforcement yields before failure an interaction relation can be established for the failure mode in Figure 6.3, see Figure 6.5. This relation can also be established by a kinematical method according to Figure 6.4. The inner and outer work is then studied when the beam rotates around the compressed axis AD in Figure 6.4b. The work equation gives the relations 6.3.1-6.3.3. A test that illustrates the assumed failure mechanism is presented in Figure 6.6.
Figure 6.3. Failure surface used for the study of a beam loaded with a bending moment $M$, a twisting moment $T$ and a shear force $V$, Elfgren (1972).

The failure capacity for a pure bending moment, a pure torsion moment and a pure shear force can with notation from Figure 6.4 be written as

$$M_0 = h\left(P_3 + P_5\right)$$

$$T_0 = 2bh\sqrt{\frac{P_3 + P_5}{b + h}}p_s$$

$$V_0 = 2h\sqrt{\frac{P_3 + P_5}{h}}p_s$$

The interaction in Figures 6.4 and 6.5 can be written as

$$\frac{M}{M_0} + \left(\frac{T}{T_0}\right)^2 + \left(\frac{V}{V_0}\right)^2 = 1$$

The angles can be written as

$$\cot \alpha_2 = \left(\frac{T}{2bh} + \frac{V}{2h}\right)\frac{1}{p_s} = \cot \alpha_T + \cot \alpha_V$$

$$\cot \alpha_4 = \frac{T}{2bh}\frac{1}{p_s} = \cot \alpha_T$$

$$\cot \alpha_6 = \left(\frac{T}{2bh} - \frac{V}{2h}\right)\frac{1}{p_s} = \cot \alpha_T - \cot \alpha_V$$
Figure 6.4. Notation and ultimate deformations using a kinematic method according to the theory of plasticity, Elfgren (1979). When the reinforcement yields in the cracks ABC and DEF the beam rotates around the axis AD.
Figure 6.5. Interaction surface between torsion moments T, shear forces V and bending moments M in a rectangular beam according to the theory of plasticity, Elfgren (1972, 1974). Different surfaces are obtained for the cases when the compressed part of the section is located in the top, bottom or side of the beam.

Figure 6.6 Cracking and failure mechanism for a beam tested to illustrate the failure mode in Figure 6.4 (From above: top, front, bottom and back side). In the left part two cracks ABC and FED and the rotation hinge AD are marked. The beam is rectangular with $b \times h \times l = 0.1 \times 0.2 \times 3.3$ m. The capacity of the stirrups is $p_s = 0.236$ MN/m. In the ultimate stage the beam was loaded with shear, torsion and bending moment with the relation $0.2:0.5:1$. Elfgren (1979).
6.4 Modified Compression Field Theory

6.4.1 General

The cracked web of a reinforced concrete beam transmits shear in a relatively complex manner. As the load is increased, new cracks form while pre-existing cracks spread and change inclination. Because the section resists moment as well as shear, the longitudinal strains and the crack inclinations vary over the depth of the beam, see Figure 6.7.

Shear design procedures for reinforced concrete that determine the angle $\theta$ by considering the deformations of the transverse reinforcement, the longitudinal reinforcement, and the diagonally stressed concrete have become known as compression field approaches. With these methods, equilibrium conditions, compatibility conditions, and stress-strain relationships for both the reinforcement and the diagonally cracked concrete are used to predict the load-deformation response of a section subjected to shear.

Kupfer (1964) and Baumann (1972) presented approaches for determining the angle $\theta$ assuming that the cracked concrete and the reinforcement were linearly elastic. Methods for determining $\theta$ applicable over the full loading range, based on Wagner’s procedure within the tension field theory, were developed by Collins and Mitchell (1974) for members in torsion, and were applied to shear design by Collins (1978). This procedure was called the compression field theory, CFT. Comments on the method are also given in section 2.2.4.

Figure 6.8 summarizes the basic relationships of the CFT. The shear stress, $\nu$, applied to the cracked reinforced concrete causes tensile stresses in the longitudinal reinforcement, $f_{sx}$, and the transverse reinforcement, $f_{sy}$, and a compressive stress in the cracked concrete, $f_z$, inclined at angle $\theta$ to the longitudinal axis. The equilibrium relationships between these stresses can be derived from Figure 6.8 (a and b) as

$$\rho_x f_{sy} = f_{cy} = \nu \tan \theta \quad \text{(6.4.1)}$$

$$\rho_y f_{sx} = f_{cx} = \nu \cot \theta \quad \text{(6.4.2)}$$

$$f_z = \nu (\tan \theta + \cot \theta) \quad \text{(6.4.3)}$$

where $\rho_x$ and $\rho_y$ are the reinforcement ratios in the longitudinal and transverse directions.
Figure 6.8. The basic relationships of the Compression Field Theory, CFT. Mitchell and Collins (1974) cited from ASCE-ACI Committee (1998).

If the longitudinal reinforcement elongates by a strain of $\varepsilon_x$, the transverse reinforcement elongates by $\varepsilon_y$, and the diagonally compressed concrete shortens by $\varepsilon_2$, then the direction of principal compressive strain can be found from Wagner's (1929) equation, which can be derived from Mohr's circle of strain, see Figure 6.8 (d) as
\[
\tan \theta = \frac{\varepsilon_x + \varepsilon_2}{0.5\gamma_x} \quad \text{and} \quad \tan \theta = \frac{0.5\gamma_x}{\varepsilon_y + \varepsilon_2} \quad \text{gives}
\]
\[
\tan^2 \theta = \frac{\varepsilon_x + \varepsilon_2}{\varepsilon_y + \varepsilon_2}
\]

(6.4.4)

Before this equation can be used to determine \( \theta \), however, stress-strain relationships for the reinforcement and the concrete are required. It is assumed that the reinforcement strains are related to the reinforcement stresses by the usual simple bilinear approximations shown in Figure 7.8 (e and f).

 Based on the results from a series of intensively instrumented beams, Collins (1978) suggested that the relationship between the principal compressive stress, \( f_2 \), and the principal compressive strain, \( \varepsilon_2 \), for diagonally cracked concrete would differ from the usual compressive stress-strain curve derived from a cylinder test, Figure 6.8 (g). He postulated that as the strain circle becomes larger, the compressive stress required to fail the concrete, \( f_{2\text{max}} \), becomes smaller, Figure 6.8 (h).

It was suggested that the diagonally cracked concrete fails at a low compressive stress because this stress must be transmitted across relatively wide cracks. If the initial cracks shown in Figure 6.8 (a) formed at 45° to the longitudinal reinforcement, and if \( \theta \) is less than 45°, which will be the case if \( \rho_y \) is less than \( \rho_x \), then significant shear stresses must be transmitted across these initial cracks Figure 6.8 (b). The ability of the concrete to transmit shear across cracks depends on the width of the cracks, which, in turn, is related to the tensile straining of the concrete.

After the stirrups have yielded, the shear stress can still be increased if \( \theta \) can be reduced. Reducing \( \theta \) will increase the tensile stress in the longitudinal reinforcement and the compressive stress in the concrete. Failure will be predicted to occur either when the longitudinal steel yields, or when the concrete fails. Moment will increase the longitudinal tensile strain \( \varepsilon_x \), which will reduce the shear capacity.

The difference between the Compression field theory with no influence of the tensile stresses accounted in the cracked concrete and the Modified compression field theory can be seen in Figure 6.9.

![Figure 6.9. Stress fields in web of reinforced concrete beam, Collins and Mitchell (1997)](image)

### 6.4.2 Influence of Concrete Tensile Strength

The modified compression field theory, MCFT, is based on the research of Michael P Collins and his co-workers at the University of Toronto during the last 30 years. The method is used at the present in the American Codes, AASHTO (2002) and in the Swedish design handbook for High Performance Concrete Structures, HPCS (2000). It has recently been applied by Puurula (2004) in the study of a prestressed box girder bridge.

Before the web of the beam cracks the shear is carried by the diagonal tension and compression stresses in the direction of the principal stresses. The principal stresses acts at an inclined angle, \( \theta \), to the longitudinal axis of the beam, see Figure 6.10. After increasing the loading the tensile stresses exceed the tension capacity of the concrete and diagonal cracks
will form. This leads to rearrangement of the internal stresses in the section. The tensile stresses decrease and the shear reinforcement become activated. The tensile stress varies from zero in the cracks to the maximum value in the middle between the cracks. In the modified compression field theory the shear is carried by diagonal compression stresses, $f_2$, tension in the stirrups and diagonal tension stresses, $f_1$, in the concrete between the cracks, see Figure 6.8 (c) and Figure 6.11.

The shear stress $\nu$, in the web is assumed to be uniformly distributed

$$\nu = \frac{V}{b_i d_v}$$  \hspace{1cm} (6.4.5)

**Figure 6.10.** Stresses in a beam loaded in shear. (a) Stresses acting on an infinitesimal element in the web of the beam. (b) Mohr’s circle for shear stresses $\nu$ and principal stresses $f_1$ and $f_2$. Figure from Enochsson et al (2004), Gabrielsson (1999).

**Figure 6.11.** The equilibrium in the modified compression field theory. Figure from Gabrielsson (1999)
The maximum shear stress can be expressed with the Mohr’s circle as

\[ v = \frac{f_1 + f_2}{2} \sin 2\theta = \left( f_1 + f_2 \right) \sin \theta \cos \theta = \frac{f_1 + f_2}{\tan \theta + \cot \theta} \]  

(6.4.6)

where

- \( f_2 \) = principal compression stress
- \( f_1 \) = principal tension stress
- \( \theta \) = angle between diagonal strut and the longitudinal axis of the beam

The diagonal compression stresses pull the flanges apart but the tension stresses normal to the cracks keep the beam together. Because the compression stresses, \( f_2 \), can become much higher than the tension stresses, \( f_1 \), the difference must be taken by the tension in the stirrups. This equilibrium can be expressed as, see the left hatched area in Figure 6.11.

\[ A_v f_v = (f_2 \sin^2 \theta - f_1 \cos^2 \theta) b_v s \]  

(6.4.7)

where

- \( f_v \) = the average stress in the stirrups

with \( f_2 \) from the Eq.6.4.6 gives

\[ V = f_1 \cot \theta \cdot b_v d_v + A_v f_v \frac{b_v \cot \theta}{s} = V_c + V_s \]  

(6.4.8)

The equation expresses the load carrying capacity for shear as a sum of the concrete contribution, due to the tension in the concrete and, the shear reinforcement contribution due to the tension in the stirrups.

If there is no axial forces acting on the section the axial component of the diagonal compression shall be taken by tension in the axial reinforcement. The equilibrium in axial direction can be expressed as, see the right hatched area in Figure 6.11.

\[ N_v = A_{va} f_v + A_{vp} f_p = (f_2 \cos^2 \theta - f_1 \sin^2 \theta) b_v d_v \]  

(6.4.9)

where \( f_v \) and \( f_v \) are the average stresses in the axial, longitudinal reinforcement and the prestressed reinforcement. Inserting \( f_2 \) from the Eq. 6.4.6 gives

\[ N_v = A_{va} f_v + A_{vp} f_p = V \cot \theta - f_1 b_v d_v \]  

(6.4.10)

When the web of the beam cracks, the average stresses, \( f_1 \), decreases and the principal strains, \( \varepsilon_1 \), increases. The tensile stresses in the cracked concrete stiffen the beam, reduce the strains in the concrete and make it possible for the beam to resist larger shear forces before the beam fails.

The stresses in the cracks differ from the average stresses being calculated with in the equations above. The tension stresses in the concrete in cracks become zero but the stresses in the stirrups increase. The load carrying capacity for shear is limited by the capability of the crack to transfer the forces across the crack. When the shear is still low the tension is transferred over the crack by locally increased stresses in the stirrups. When the shear is increased the stirrups reach the yielding limit in the crack. At even higher shear forces the local shear stresses, \( V_{si} \), on the surface of the crack need to be activated to transfer the tension stresses across the crack, see Figure 6.12. When the principal strains and the crack widths increase the shear capacity in the web is limited by the ability of the cracks to transfer the stresses across the crack. Coarseness of the crack surface gives the upper limit for the load transferring capacity across the crack, see Figure 6.12a.

When the concrete has cracked the forces in the crack have to be equal in amount with the forces in the space between the cracks. Equilibrium gives the upper limit for the average tensile stresses, see Figure 6.12b.
\[ A_s f_y \left( \frac{d_v}{s \tan \theta} \right) + v_{ci} b_d d_v \sin \theta = A_v f_y \left( \frac{d_v}{s \tan \theta} \right) + f_i b_d d_v \cos \theta \]  

(6.4.11)

\[ f_i = v_{ci} \tan \theta + \frac{A_v}{s b_d} (f_{yy} - f_y) \]  

(6.4.12)

\[
v_{ci} = \frac{1}{\sin \theta \cos \theta} \left( v_y - v_v \right)
\]

\[
\tan \theta = \frac{2 (v_x - v_y)}{v_y + v_x}
\]

\[
\theta = \tan^{-1} \left( \frac{v_y}{v_x} \right)
\]

**Figure 6.12.** (a) Local shear stresses, \( v_{ci} \), in the crack, Collins, Mitchell (1997). (b) The two cross sections used in the Eq.6.4.12 to calculate the upper limit for the average principal tension stresses. In the crack, cross section A-A, the stirrups are yielding. Section B-B is located in the space between the cracks. Figure modified from Gabrielsson (1999).

---

**Figure 6.13.** Assumptions made in the Modified Compression Field Theory, MCFT, Bentz(2000).
Assumptions made in the Modified Compression Field Theory, MCFT, are summarised in Figure 6.13.

Evan Bentz (2000) have modified the tension stiffening relationship given in Figure 6.13 to better take the size effect into the consideration. The relationship used by Bentz is dependent on whether the cracked concrete is well reinforced or poorly reinforced. The tension stiffening is largely a bond phenomenon. That is, it is the bond between the reinforcing bar and the concrete that causes any tension to develop in the concrete between cracks. Thereby at locations where the concrete is reinforced with closely spaced array of small diameter bars, the average tensile stress in the cracked concrete can be expected to be higher than at locations reinforced with a widely spaced array of large diameter bars. An appropriate parameter to indicate the bond characteristics of different arrays of reinforcement is to divide the area of concrete in tension by the perimeter of all the reinforcing bars bonded to the area.

For well reinforced cracked concrete the following relationship is proposed:

\[
\frac{f_i}{1 + \sqrt{3.6m \cdot \varepsilon_i}} = \frac{f_{c,\text{ff}}}{m} \tag{6.4.13}
\]

\[m = \frac{\sum A_c}{d_b \pi}\]

where

- \(m\) = the bond parameter in millimetres
- \(A_c\) = the area of concrete effectively bonded to the bar
- \(d_b\) = diameter of bar in concrete stiffened area

The factor 3.6 \(m\) in Eq. 6.4.13 was originally given the value 200 (from tests on small elements with a thickness 0.07 m reinforced with \(\phi 6 \times 50 \text{ mm}\)). However, it was raised to 500 after tests of larger shell elements (1.515\(\times\)1.515\(\times\)0.285 m with \(\phi 20 \times 72 \text{ mm}\)). In this way a larger reduction was obtained of the strength at increased strain of large structures with sparse reinforcement. With Eq. 6.4.13 the value may increase still more in order to accommodate still larger structures with less dense reinforcement. For elements of the thickness 0.178 m tested by Hsu et al (1984, 1993) with \(\phi 20 \times 289 \text{ mm}\) we get 3.6 \(m\) = 1500.

### 6.4.3 Size effect in shear

In a paper by Lubell et al (2003), the size effect is discussed. Here some of the results in the paper are reviewed. According to the standard code of the American Concrete Institute, ACI 318-02, the basic expression for shear capacity, \(V\), is:

\[
V_c = 2\sqrt{f_c b_n d} \tag{6.4.14}
\]

It is intended to be a conservative estimate of the shear failure load of sections not containing shear reinforcement. It was not appreciated that for members without stirrups, the shear stress at failure decreases as the members become larger. This decrease in failure shear stress as member size increases is called the "size effect" in shear.

An extensive experimental investigation of the size effect in shear was conducted by Shioya et al in Japan. The main results of this work are summarized in Figure 6.14, where it can be seen that the shear stress at failure decreases, both as the member depth increases and as the maximum aggregate size decreases. The simplest explanation of the size effect in shear is the larger crack widths which occur in larger members.
Crack widths increase nearly linearly both with the tensile strain in the reinforcement and with the spacing between cracks. Shioya observed that the crack spacing at middepth of his beams was about equal to half the depth of the beams. Hence, for the same reinforcement strains, doubling the depth of the beam will double the crack widths at mid-depth.

Using the parameters identified by the Modified Compression Field Theory (MCFT), Collins and Kuchma suggested the following expression for the shear capacity of members not containing stirrups:

\[ V_c = \frac{115}{50 + s_e} \sqrt{f_c' b_w d} \]  \hspace{1cm} (6.4.15)

with \( \sqrt{f_c'} \) not to be taken greater than 100 psi = 0.7 MPa. The effective crack spacing parameter \( s_e \), accounts for the influence of the crack spacing \( s_x \), and the maximum aggregate size \( a \) in the following manner:

\[ s_e = \frac{1.38 s_x}{a + 0.63} \]  \hspace{1cm} (6.4.16)

The crack spacing \( s_x \), is taken as 0.9d for members that have only concentrated reinforcement near the flexural tension face, or as the maximum distance between the layers of longitudinal reinforcement if the members contain intermediate layers of crack control reinforcement. To be effective, each layer of such reinforcement should have a total area of at least 0.003b\( s_x \), and the individual bars should not be spaced further apart than 24 in. (600 mm).
Comparisons of test results from beams with and without intermediate layers of crack control reinforcement clearly demonstrate that it is distance away from the reinforcement rather than the total depth of the beam which dictates the magnitude of the size effect in shear. This point is illustrated in Figure 6.15, where it can be seen that the addition of three layers of intermediate longitudinal bars greatly reduced the crack spacing near mid-depth and increased the shear capacity by more than 50%. The ACI expression for $V$, provided as Eq. 6.4.15, predicts that both beams shown in Figure 6.15 would have a shear strength of 318 kN. The MCFT based expression, Eq. 6.4.13, predicts shear failure loads of 198 kN for the member without intermediate layers of reinforcement and 302 kN for the member with such layers.

![Figure 6.15 Influence of distribution of longitudinal reinforcement on cracking pattern and shear strength of two companion beams, Lubell et al (2003).](image)

### 6.5 Crack width calculation

#### 6.5.1 General

The crack width $w$ is calculated in the same way in the Modified Compression Field Theory as in European Codes, see e.g. CEB (1993) or Eurocode 2. (2004). The crack width is written as the product $w = \varepsilon_1 \cdot s_\theta$, of the principle strain in the crack $\varepsilon_1$ and the estimated crack distance $s_\theta$, see Figure 6.13. The tensile strain in the uncracked concrete between the cracks is hereby neglected. This gives somewhat larger cracks than in reality. As there is some variation of the distances between the cracks there will also be a variation in the crack widths. CEB (1993) for that reason defines a characteristic crack width $w_k$ which is larger than all but 5% of the cracks. This characteristic width is calculated as $w_k = 1.7 \cdot w_m$, where $w_m$ is the mean crack width.

In the program Response-2000, Bentz (2000), the crack distance is calculated with a simplified CEB-FIP-model as
where \( \theta \) is the inclination of the compressive struts
\( c_x \) is the largest perpendicular distance to a horizontal reinforcement bar in the section
\( c_y \) is the largest perpendicular distance to a vertical reinforcement bar in the section
\( d_b \) is the bar diameter
\( \rho \) is the reinforcement ratio in a distance of 7.5\( d_b \) on each side of the bar.

By choosing the largest perpendicular distance the model aims to calculate the maximum characteristic value \( w_k \).

The crack distance varies with the level which is studied. Between two layers of the reinforcement the term 0.1 \( d_b/\rho \) is interpolated for values obtained for different reinforcement bars.

If a section is subjected to a bending moment the crack distance is not allowed to take a larger value than half the effective height of the section.

An example is given in Figure 6.23 below, where a maximum crack width of 0.28 mm is obtained.

An important check in the program Response-2000 is that the mean stresses in a cracked part (Section B in Figure 6.12) also can be carried locally in a crack (Section A in Figure 6.12). The following condition should be fulfilled

\[
A_c \cdot f_{t} + A_s \cdot f_{sx} \leq A_s \cdot f_{sx\text{crack}}
\]

where
\( A_c \) is the concrete area
\( f_{t} \) is the principle mean tensile stress in the concrete between the cracks
\( A_s \) is the stirrup stress
\( f_{sx} \) is the stirrup strain between the cracks
\( f_{sx\text{crack}} \) is the stirrup strain in a crack

When the concrete cracks, it looks like the reinforcement is getting stiffer (tension stiffening). The concrete stresses are not reduced to zero at once as a crack develops but reduces successively according to Eq. 6.4.13. This helps the reinforcement to carry a higher load than the strain indicates. By this the reinforcement seems to get an increased stiffness. However, it is necessary to check that this stiffening not causes \( f_{sx\text{crack}} \) to exceed the yield stress and this is carried out in a special crack control.

The program also checks that the concrete shear stress in the crack \( v_{ci} \) does not exceed the value

\[
v_{ci} = \frac{0.18 \sqrt{f'_c}}{0.31 + \frac{24w}{a + 16}}
\]

where
\( f'_c \) is the concrete compressive strength [MPa]
\( w \) is the crack width as calculated above [mm]
\( a \) is the maximum diameter of the ballast [mm]
6.5.2 Recommendations regarding crack widths

Some recommendations can now be made based on the performed tests and calculations.

The crack width in a reinforced structure depends roughly on the reinforcement strain and the distance between the cracks according to the formulae

\[ w = \varepsilon_1 \cdot s_\theta \]

where

\( \varepsilon_1 \) is the principle strain in the crack, and \( s_\theta \) is the estimated crack distance \( s_{\theta} \), see Figure 6.13.

We want to avoid yielding of the reinforcement. For a normal steel with a yield stress of \( \sigma_y = 400 \) MPa we will have a yield strain of \( \varepsilon_y = \sigma_y / E_s = 400 / 200000 = 0.002 \). Depending on the crack distance we can now calculate the crack width for beginning yielding of the reinforcement according to Table 6.1.

Table 6.1 Approximate crack widths \( w \) [mm] causing reinforcement yielding for a steel with a yield stress of 400 MPa and different crack distances \( s_{\theta} \) [mm].

<table>
<thead>
<tr>
<th>Crack distance ( s_{\theta} ) [mm]</th>
<th>Crack width ( w ) [mm] for steel yielding</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.4</td>
</tr>
<tr>
<td>300</td>
<td>0.6</td>
</tr>
<tr>
<td>500</td>
<td>1.0</td>
</tr>
<tr>
<td>1000</td>
<td>2.0</td>
</tr>
</tbody>
</table>

If e.g. the minimum surface reinforcement in a bridge has a spacing of 300 mm this will determine the crack spacing to 300 mm implying that crack widths larger than 0.6 mm should cause concern as they indicate that the reinforcement is close to yielding.

6.5.3 Influence of temperature, restraint, creep and shrinkage

However, recent research and development have put focus on the fact that it is not only the mechanical load but also temperature decrease and the influence of the restraint from the adjacent construction as well as the transient mechanical properties of the hardening concrete and the autogenous shrinkage in modern concretes that are involved in a complex interaction in the formation of cracking of concrete.

Cracking in concrete can be classified according to Figure 6.16 and Table 6.2, TRB (2006). Some influencing factors are illustrated in Figure 6.17.
Figure 6.16. Different types of cracks in concrete, TRB (2006)

Figure 6.17. Factors influencing the formation of cracks during the hardening phase of the concrete. Modified from Emborg and Bernander (1994) and Springenschmid (1998).
With respect to the newly cast concrete, the risk of surface cracks at the heating stage of the temperature cycle for an ordinary civil engineering construction is usually highest within one to two days after casting in self-balancing parts of the structure. In some cases through cracking also may occur in this very early age if there are markedly different dimension changes, and in some cases also there might be a risk of through cracking in the adjacent structure during the heating stage. The main problem of surface cracks in this very early age is strongly dependent on the dimensions of the construction, and for the majority of the civil engineering structures, say dimensions up to about 1 or 1.5 m depending on the local situation, no extra measures have to be taken to avoid surface cracks. Besides, if the matter of very early age surface cracking is settled, the final state will end up in compressed surface layers in homogenous concrete. This latter situation is beneficial as it will counteract the start of cracks at later temperature variations or late shrinkage.

For the issue of “ordinary” through cracking for civil engineering structures at the temperature cooling stage, the situation is completely different. For the most common bridge constructions through cracks are formed weeks after casting of the new concrete. The crack starts in a section a short distance from the casting joint, and it continues from the joint as long as the degree of restraint is big enough. The cause of the cracking is contraction (temperature de-
crease and/or shrinkage) of the concrete from the reaching of the "zero stress level", which occurs in many cases about two to three days after casting. After this the concrete will remain in a tensile state, and if a crack is formed it will remain open. If the stress state is low enough compared with the tensile strength, there will be, due to long term creep in concrete, a residual stress level about 30 to 50 percent of the maximum stress level. So, the final situation is here a tensile stress state, which may influence the design process, especially when there will be a one-sided water pressure in the service life of the structure.


6.6 Example. The Källösund Bridge, North of Gothenburg

6.6.1 General

The Bridge over Källösund, see Figure 6.18 and 6.19, is one of the first bridges in Sweden built with the balanced cantilever construction method ("freivorbau"). It was completed in 1959. One cross section critical to shear, torsion and bending is given in Figure 6.20.

In a cross section there exist six section forces: three normal forces (an axial force and shear forces in two directions) and three moments (twisting moments and bending moments acting around two axes). These forces interact in both the reinforcement and the concrete. In Figure 7.1 the shear flow is studied in a cross section with the size $b \times z$. The shear force and the twisting moment interact in one web and counteract in the other.

6.6.2 Materials and loads

Properties of materials and loads were examined using deterministic and probabilistic methods; see Enochsson et al (2004) and Jeppsson et al (2004).

![Figure 6.18. Geometry of the Källösund Bridge to Tjörn on the west coast of Sweden.](image)
6.6.3 Capacity

The capacity was calculated using various methods for shear combined with torsion and bending among them Eurocode 2 (2002) and the Swedish Code BBK 94 (1994). The modified compression field theory (MFCT) was applied using the program Response developed by Bentz (2000). One example of a result is given in Figure 6. The results according to MFCT are considerably higher than the ones according to EC2 and BBK, see further Enochsson et al (2004) and Puurula (2004).

Hypothetical failure mechanisms are illustrated in Figure 6.22. A possible failure hinge in bending in span 2-3 is not a direct risk of collapse as there is a need for a second hinge at 3 in order to develop a mechanism, see Figure 6.22a. In the critical section for shear and torsion between 3 and 4 in Figure 6.22b, the deformation capacity is probably smaller. Failure will probably occur after extensive cracking in the top slab followed by concrete web crushing. A symmetric failure between 4 and 5 is also possible.

After the assessment, it was recommended to strengthen the bridge with externally bonded carbon fibre reinforced polymers, CFRP, see Täljsten (2002). An alternative might have been to install a measurement system and to follow the strain development, see further Enochsson et al (2004), Puurula (2004), Plos (2004) and Plos et al (2004).
Figure 6.21. Top: Load-deformation diagram for the cross section in Figure 6.18. Up to point A the section is uncracked and behaves linearly elastic, at C the maximum load is reached and at D failure occurs after a concrete compression failure. In the insert crack sizes at failure are given in mm, Enochsson et al (2004). Bottom: A more detailed presentation from Puurula (2004). In the insert first the inclination of the concrete compressive struts are shown (top), followed by crack patterns (middle) and concrete compressive stresses (bottom).
6.6.4 Conclusions
It was shown that refined condition assessment methods can be useful tools in the reas-
essment of the load-carrying capacity of bridges. It gives a better picture of the failure
mechanism and it also gives higher loads than many traditional methods based on codes.

6.7 Example. The Gröndal Tram Bridge, Stockholm

6.7.1 General
view is given in Figures 6.23 and 6.24. Soon after completion, diagonal cracks were ob-
served in the webs. The cracks grew for some time and the bridge was closed to traffic dur-
ing three months. The bridge was after that strengthened with bars which were drilled
through the webs in the most cracked sections and with Carbon Fibre Reinforced Polymers
in other sections.

Figure 6.23. Gröndal Tram Bridge. .Elevation of Central Part. Westerberg (2005)
6.7.2 Some results

Different ways to calculate the crack widths and the ultimate strength have been proposed. One method is to use the Response-2000 program developed by Benz (2000) and based on the Modified Compression Field Theory, MCFT. Some examples from such calculations are given below in figures 6.25 – 6.27. Some more examples are given in Malm (2006).

Figure 6.25. Illustration of the calculation of crack widths for the load step after cracking for Section 7.442 for long time loading. The vertical load on the web is at this occasion 3885 kN (marked with a red cross). Stirrups $\phi 16$ s 200 and horizontal bars $\phi 12$ s 200 in both sides of the web. The diagrams show starting from the left in the top row:
(a) distance between vertical cracks $s_x$ (ca 500 – 700 mm in the web)
(b) distance between horizontal cracks $s_y$ (ca 600 mm in the web)
(c) distance between diagonal cracks $s_\theta$ (ca 475 mm in the web)
(d) principle strain $\varepsilon_1$ (max 0.6 mm/m in the web)
(e) crack widths $w_k = \varepsilon_1 \cdot s_\theta$ (max 0.6 \cdot 0.475 = 0.28 mm in the web)
(f) crack pattern with crack widths at different levels.

Figure 6.26. Gröndal Bridge. Maximum load. Example on data given by the program Response-2000. In the left part of the figure a load-deformation graph is shown indicating that the maximum shear force is 7285 kN at a shear strain of about $\gamma_{xy} = 2.4$ mm/m. There is also a moment-curvature diagram which shows a moment of $M = -75232$ kNm and a curvature $\phi = -0.2$ rad/km. The present load step is indicated by a dashed cross. In the top row there is first a view of the analysed cross section. There are also figures which give longitudinal and transversal strains. In the middle row there is a figure illustrating the crack pattern with crack widths in mm. There are also figures illustrating the shear strains and the shear stresses. In the bottom row principal compressive and tensile stresses are shown together with maximum allowed stresses. In between, a figure illustrates the shear stress variation in the cracks.
6.8 The Luossajokk Bridge

6.8.1 General
The owner wanted to increase the maximum allowed axle load from 250 to 300 kN along the railway line "Malmbanan", between the Swedish and Norwegian coasts, Paulsson et al (1996, 1997). For one of the bridges, The Bridge over Luossajokk Creek, see Figures 6.28 and 6.29, an assessment showed that the increased load would exceed the allowed stresses in the reinforcement in three sections: (1) in the top of the short span due to longitudinal bending moment, (2) in the bottom of the short span due to transversal bending moment and (3) close to the mid support the shear transfer between the beam and the slab was too low in the transverse direction.

6.8.2 Measurements and Probabilistic Assessment
A probabilistic evaluation of the safety was made in 2001 together with in-situ monitoring of actual strain levels, tests of concrete properties and refined FE analysis, Enochsson et al (2002). The development of strain with time was periodically checked from 2001-2006.

It was found that the measured strains were quite small so according to them there were no problems either in bending or in shear. The probabilistic assessment of the bending capacity
in the most strained section confirmed this, Enochsson et al (2002) and the bridge could be used with the higher axle load.

Figure 6.28. Geometry of the Luossajokk bridge in Kiruna. Enochsson et al (2002)

Figure 6.29. Luossajokk Bridge in Kiruna, Sweden. Enochsson et al (2002)
7 Fatigue of reinforced concrete

7.1 General
The research work presented in this and the following chapter is summarised in chapter 6 of WP4’s main deliverable “Guideline for Load and Resistance Assessment of Existing European Railway Bridges”.

This chapter contains background information about the fatigue behaviour of reinforced concrete. The fatigue behaviour of steel reinforcement, concrete and reinforced concrete elements are described. In chapter 9 the content of this chapter is exploited in order to assess the remaining fatigue life of existing reinforced concrete bridges.

7.2 Introduction
Modern railway systems call for heavier traffic intensity, higher axle loads and higher speeds of trains. Existing bridges need to resist to these severe loading conditions with high reliability during their service life. Structural materials are subjected to repeated stress cycles which may lead to a reduction of their strength properties with increasing number of load application. This phenomenon is called fatigue. Also, the life load, i.e. the rail traffic load, is relatively high compared to the dead load of railway bridges which make them in principle more vulnerable to fatigue damage.

Short and medium span bridges are subjected to fatigue loading due to moving wheel loads, which is characterised by a high number of load cycles. Over the service life of a bridge the number of load cycles may exceed 100 million. Despite this fact, reinforced concrete bridges have commonly not been designed for fatigue until the first code provisions for fatigue of reinforced concrete were introduced about 20 years ago.

Current fatigue provisions rely however on a narrow knowledge basis when compared to most other domains of structural concrete. Fatigue damage mechanisms for reinforced concrete are not yet well understood and codes are often based on pure experimental data with little scientific background. Appropriate damage accumulation theory is still lacking, and thus a worst case scenario of fatigue action effect is considered in codes.

This conservative approach is acceptable for the design of new structures, but for existing structures it is inappropriate and may lead to unnecessary and costly strengthening. Using the code provisions to assess the fatigue safety of existing railway bridges in Switzerland, deck slabs are identified as the most fatigue vulnerable elements and shear loading of the concrete as the determinant failure mode (Brühwiler et al 1994).

From the above points follows that the examination (assessment) of existing (and also the design of new) railway bridges must consider the repeated fatigue action due to railway traffic. To reduce uncertainties in current engineering methods, knowledge about the fatigue behaviour of concrete bridges must be improved and realistic methods for the examination of existing bridges and for the determination of their remaining service life need to be developed.

7.3 Fatigue of reinforcement
The fatigue behaviour of steel reinforcement is similar to fatigue of elements in steel construction. For steel reinforcement, the fatigue relevant parameters are:

- the stress range,
- the number of stress cycles and
- discontinuities both in the cross section and the layout of the steel reinforcement, resulting in stress concentration at possible fatigue damage locations.

Fatigue life of steel reinforcement can be divided into a crack initiation phase, a steady crack propagation phase and fracture of the remaining section.

The fatigue behaviour of the reinforcement can be represented by means of the S-N-diagram (Wöhler line) in a double-logarithmic representation (Figure 7-1). The nominal fatigue strength is commonly defined by the stress range amplitude at 2 million cycles. This value is called fatigue category and refers to a given S-N-diagram.

The dotted line below the fatigue limit depends on the load history and is valid if there are stress ranges higher than the fatigue limit. The highest amplitudes of stress ranges due to rail traffic loads are in the domain of fatigue limit. But only a small number of fatigue test results are available in this domain. Consequently, the aim is to increase the knowledge of the domain of “quasi” fatigue limit. Therefore, Linear Elastic Fracture Mechanics Theory (LEFM) which takes into account the load history in a physical correct way is applied.

In a parametric study, the influence of the load history as well as the location of the stress spectra on the fatigue life will be studied in chapter 9. The relevant parameters are identified for a better assessment of the fatigue live below the nominal fatigue limit.

A literature review regarding the fatigue strength of various types of steel reinforcement has led to the results shown in Table 7-1.

The following findings can be deduced from this Table and from the literature (Martin and Schiessl 1981/82, Schwarzkopf 1995, Nürnberger 1982):

Reinforcing steel:
- The fatigue behaviour of reinforcing steel is investigated by means of tensile tests on bars and bars embedded in concrete.
- Crack initiation on ribbed bars usually starts at the root of a rib, which causes stress concentration. There are however no significant differences regarding the fatigue strength for ribbed bars of various types.
- The steel quality does not influence the fatigue strength since it depends mainly on discontinuities such as those introduced by ribs.
- There is a dependence of the fatigue strength from the bar diameter. Since no clear distinction can be made, it is usually distinguished between bars with \( \phi \leq 20 \text{mm} \) and those with \( \phi > 20 \text{mm} \).

![Figure 7-1: S-N-diagram for fatigue behaviour of steel reinforcement](image)
Table 7-1: Fatigue categories for reinforcing steel, prestressing steel and prestressing systems.

<table>
<thead>
<tr>
<th>Fatigue category Δσ_s_fat [N/mm²]</th>
<th>Description</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reinforcing steel</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>170</td>
<td>1. Bars: Ø ≤ 20 mm, straight</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Vertical stirrups: Ø ≤ 16 mm, bent in accordance with standards</td>
<td>For bent bars, the values are reduced as follows: Δσ_s_fat, red = kØ × Δσ_s_fat, where kØ = 0.35 + 0.026 d_i/Ø_s, where d is the diameter of the bar bender roller</td>
</tr>
<tr>
<td>140</td>
<td>Bars: 20 mm ≤ Ø &lt; 40 mm, straight</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>1. Butt welds and welded intersections between bars (e.g. reinforcing lattice).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Mechanical joints between bars.</td>
<td>If no results from specific studies are available, the standard value shown should be used.</td>
</tr>
<tr>
<td><strong>Prestressing steel and prestressing systems</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>Single strands and single-layer tendons in plastic ducts</td>
<td>Tendons may be bonded or unbonded friction has a significant effect on fatigue behaviour. Where prestressing is carried out using unbonded tendons, the anchorage is the determining factor.</td>
</tr>
<tr>
<td>170</td>
<td>1. Multi-strand and HiAm cables in plastic ducts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Straight tendons without ducts (e.g. in elements fabricated by the long-line method)</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>Multi-strand and HiAm cables in steel ducts.</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>Anchorages, cable couplers</td>
<td>For prestressing systems that comply with the standards.</td>
</tr>
</tbody>
</table>

- The fatigue strength of bent bars is reduced because the steel undergoes plastic deformation through bending leading to local eigenstresses and microcracks. The smaller the radius of curvature of the bending, the larger is the reduction in fatigue strength. With respect to straight bars the fatigue strength of bent bars is reduced by applying a coefficient as given in Table 8-1.

- Bending stirrups are however an exception. Tests have revealed that stirrups always fail in the shear crack and thus outside the zones where the bendings are located. Also, reduction of steel stresses due to local concrete confinement has been observed in the zones of bending.

- Straight laps of bars present the same fatigue strength as continuous bars if the standard lap joint length (as defined in codes) is respected.

- Welds have a strong detrimental effect on the fatigue behaviour of bars since microcracks are introduced in the steel material at the toe of the weld root. These microcracks present the starting point of fatigue crack growth. Consequently, the fatigue strength of bars with welds shows a significant reduction.

- Mechanical connections of bars present a low fatigue strength which is due to large stress concentrations in the threads. The fatigue strength of mechanical connectors depends on the given product and is difficult to classify.
Prestressing steel and prestressing systems:

- Prestressing steel shows in principle the same fatigue behaviour as reinforcing steel.

- Fretting between prestressing steel and duct as well as between single strands or wires has a significant effect on the fatigue strength. This kind of fatigue damage is called fretting fatigue. The main parameters are the material and the geometry of the duct, the amplitude of lateral pressures (due to cable curvature) and relative displacements between prestressing steel and ducts. Since there is also fretting between single units, the number of wires or strands in a duct is also of importance. A quantitative distinction between all these parameters is however not possible. Significantly reduced fatigue strength has been observed in all tests with fretting when compared to fatigue test on single prestressing steels.

- The use of plastic ducts instead of steel ducts has revealed a significantly improved fatigue behaviour which is due to lower friction properties at the contact surfaces and more flexible bond behaviour.

- Regarding the fatigue behaviour of anchorages and couplings, there are only a few test results available in the literature which are inconsistent and sometimes contradictory. In particular, there are no test results available in the domain of high cycle fatigue which is of main interest to railway bridges. From the few results, it can be concluded that anchorages and cable couplings present low fatigue strength varying depending on the various systems and products. In particular, reduction of fatigue strength is to be expected in zones where single wires or strands are opened out as well as in zones of force concentration due to tri-axial stress states. Anchorages and couplings usually are located in zones of small fatigue loading (in terms of amplitude of stress range).

7.4 Fatigue of concrete

7.4.1 Concrete in compression

The majority of fatigue tests are compression tests on cylinders or prisms where only a few test results exist at more than 5 million stress cycles. These investigations show that fatigue loading may lead to fatigue fracture of concrete and that the number of stress cycles depends on the magnitude of both the maximum and the minimum stress.

The fatigue strength of concrete may be described by means of a Goodman-diagram showing the relation between the stress couple \((\sigma_{\text{min}}, \sigma_{\text{max}})\) and the number of stress cycles \(N\) (Figure 7-2). However, dependence of the fatigue strength from the number of stress cycles is small, and the maximum stress due to the fatigue load needs to be larger than 50% of the compressive strength in order to observe fatigue failures in the relevant domain up to 100 million stress cycles.

Plain concrete under force-controlled compression or tension fatigue loading exhibits strongly increasing strains within a first short period (Dyduch et al 1994, Cornelissen and Reinhardt 1984), followed by a period of steady, but little increasing strains. During the last period, strains again increase significantly before the specimen fails (Figure 7-3). The stiffness decreases significantly during the test mainly due to crack formation on a microscopic level.

Under uniaxial compression, the concrete-matrix is extensively micro-cracked during this last period. An increasing number of cracks appear parallel to the loading direction on the outer surface of specimen with subsequent failure period (Dyduch et al 1994).

Concrete behaviour under tension fatigue loading is also dominated by crack propagation; early age micro-cracks in the cement matrix and at the interface between aggregates and cement matrix propagate steadily until the specimen fractures showing one discrete crack.
Concrete subjected to stress reversals deteriorates rapidly which is explained by the interaction of the differently oriented micro-cracks due to compression and tension loading (Weigler and Rings 1987, Cornelissen and Reinhardt 1982).

No tests have been carried out under pure shear fatigue loading because it is difficult to apply this type of loading without creating other stresses. During the last decade, the fatigue of plain concrete has been studied under variable amplitude fatigue loading, but no realistic damage accumulation model could be found.

### 7.4.2 Concrete in tension

It is known from investigations, see e.g. Gopalaratnam et al. (1985), that a loading cycle in the post-peak region, i.e. after peak-load is reached, causes energy to dissipate so that the envelope of monotonic loading is not reached at reloading. The energy dissipates due to damage caused by the un- and reloading, an idea initially introduced by Gylltoft (1983), see Figure 7-4a. This means that, for cyclic loading, the stress states do not follow the common monotonic envelope curve because of the repeated unloading-reloading cycles. Nevertheless, the envelope still bounds the admissible stress state. This means that the energy dissipated by a loading cycle decreases the fracture energy. Furthermore, the fracture energy
decreases due to increased temperature. It is also found in experiments that the deformation at unloading is partly reversible, but with a considerable irreversible part that increases with increasing total deformation; see Reinhardt et al. (1986) and Figure 7-4b.

From experiments it is also found that the un- and reloading stiffness decreases with every loading cycle. If the unloading stiffness at a certain loading cycle is normalized with the initial elastic stiffness, a steep declining curve can be seen in Figure 7-5. For example, a 70% loss of stiffness at a deformation of only 10μm; was reported by Reinhardt (1984). In addition, the reloading stiffness in a loading cycle seems to be smaller than the unloading stiffness of the same loading cycle. For post-peak response it can be assumed that the whole localized cross section is softening and damaged by microcracks, which is not the case for pre-peak response.

Cyclic loading in the pre-peak region means that the stress state must be yielding to cause any irreversible deformation. If the stress state is elastic, the response at unloading is accordingly elastic with only reversible deformation. On the other hand, if the stress state is yielding, but without reaching the peak load, the response can exhibit irreversible deformation causing hysteresis loops with consistent dissipation of energy and stiffness degradation (Figure 7-6a), keeping in mind the discussion in the preceding chapter that dissipation of energy is due to material damage. The increase of irreversible deformation with increased

Figure 7-5: Degradation of stiffness during cyclic loading, unloading path; Reinhardt (1984).
total deformation is continued with repeated loading cycles until the softening branch is reached, without reaching the monotonic tensile strength, see Hordijk (1991) and Figure 7-6b. This has been investigated by Reinhardt (1984), who made tests on pre-peak cyclic loading.

To explain this phenomenon of increasing irreversible deformation the stress distribution in front of a crack can be interpreted before and after a loading cycle (Figure 7-7) assuming, at an applied load, $F$, that only the crack-tip is softening, while the rest of the cross-section is still elastic. Before the loading cycle is initiated, the applied load, $F$, gives a stress distribution with a certain extension of the softening zone. When unloading, however, the elastic deformations may recover; because of softening at the crack tip, the material cannot recover the deformation totally. Part of the softened material that has deformed beyond yielding must be compressed to follow the unloading deformation, resulting in a self-equilibrating residual stress distribution when the external stresses are released. Consequently, the material is affected by unloading, and it can be concluded that parts of the material already affected by damage cannot resist the same stress when reloaded. Therefore, the stress distribution is

Figure 7-7: Illustration of the stress distribution in the cohesive zone, before and after a loading cycle, $N$. 
shifted into the material and, furthermore, the crack is extended; see (Reinhardt, 1984). This supports the theory that the irreversible deformation increases with total deformation.

The first report about concrete subjected to fluctuating load date back to the 1920s, with Clemmer (1922) followed by Crepps (1923). Thirty years later, Murdock et al. (1958) and Murdock (1965) presented a modified Goodman-diagram for concrete subjected to fluctuating load. The following reports found were by Tepfers (1978), Gylltoft (1983), Reinhardt (1984) and Reinhardt et al. (1986). The early investigations, although few, conclude that the response of cyclic loading changing from compression to tension is governed by the load which is the highest in comparison to the corresponding ultimate strength. In other words, if the stress in tension is almost the ultimate tensile stress, while the stress in compression is only a small portion of the ultimate compression strength, then the stress in tension governs the fatigue life of the detail, while the stress in compression is considered as zero. Tepfers (1978) put this hypothesis to the test and showed that the tendency to fatigue failure increases if the lower stress in a direct tension test is decreased into compression, which clearly contradicts the early hypothesis. Reinhardt (1984) contributed to the findings of Tepfers by concluding that the size of the hysteresis loops increases with increasing compression stress. The response that concrete exhibits when subjected to fluctuating load is clearly the same as for compression and tension, if one disregards the intermediate region where the active zone translates from one extreme to another. As discussed in the preceding section, when unloading in the post-peak region in tension, the response of a part of the softened material enters the state of compression dependent on the amount of irreversible deformation that has been developed. More irreversible deformation in tension means a higher deformation when the point changes state. Under compressive loading the deformation decreases continuously, with a very low stiffness. If the compressive load is further increased the stiffness increases more and more until the original stiffness is almost reached; see Reinhardt et al. (1986) and Figure 7-8.

![Experimental Results](image)

**Figure 7-8:** Example of a double notched specimen subjected to fluctuating load; (Hordijk, 1991).
7.5 Fatigue of reinforced concrete elements

7.5.1 Bending fatigue
Most investigations regarding the fatigue behaviour of reinforced concrete elements focused on the fatigue strength as expressed by S-N-curves, i.e. the relationship between stress range and number of load cycles at fatigue fracture.

The fatigue damage process can be summarized as follows. After first cracking the fatigue loading causes progressive deterioration of the bond between the reinforcement and the concrete adjacent to the cracks. Larger crack widths and a smaller contribution of the concrete in tension between the cracks result in larger deflections. Failure normally occurs due to bar fatigue fracture; another failure mechanism is spalling of concrete in the compression zone.

However, even over-reinforced beams (i.e. concrete compression failure under static loading) fail due to reinforcement fracture when subjected to fatigue loading. Due to the strain redistribution under compression, in most cases the concrete of a reinforced concrete member does not fail under bending stresses.

7.5.2 Shear fatigue
A number of shear fatigue tests were performed on beams without shear reinforcement (stirrups) (Markworth et al 1984, Frey and Thürlimann 1983, Ueda and Okamura 1983). The following fatigue behaviour has been observed. Beams without shear reinforcement develop a bending crack pattern after the first few cycles; deformation increases only slightly. A critical shear crack appears which crosses bending cracks. The large width of this crack does not allow any stress transfer and as a result, the load is transferred by a strut and tie model and by dowel action. The specimen fails due to bar fatigue failure, followed by fracture of the compression strut in the upper flange.

In old trough bridges the fatigue capacity may be dependent on the concrete shear capacity in the connection between the slab and the beam. This case has been studied by Thun (2006).

Beams with shear reinforcement show fatigue failure of stirrups or spalling of the surrounding concrete. The failure mechanism is ductile.

7.5.3 Slab-like elements
Several researchers investigated the fatigue behaviour of scaled deck slab elements under stationary pulsating and moving wheel loads (Sonoda and Horikawa 1982, Perdikaris and Beim 1988). These investigations were motivated by severely fatigue damaged deck slabs of highway bridges in Japan (Matsui 1986). Deck slabs have shown a punching shear failure mode. The moving wheel loads are leading to stress reversals that are more detrimental to fatigue strength than stationary pulsating loads.

The fatigue behaviour of slab-like beam elements has been investigated in (Schläfl 1999, Schläfli and Brühwiler 1997) considering slab-like elements under predominant bending and shear fatigue loading:

a) Slab-like elements under bending

The fatigue behaviour of reinforced concrete slab-like elements without shear reinforcement has been investigated by means of specimens representing – in a simplified way – transverse slab strips of girder bridges which typically have strong reinforcement (and thus a main load bearing behaviour perpendicular to the rail traffic direction).
Eighteen slab-like beams (without shear reinforcement) with a length of 3.2m were subjected to four-point bending. Three different reinforcement ratios were chosen: 0.68%, 1.37% and 1.6%.

Measurements during testing showed that strains and deflections increased significantly within the first thousand cycles followed by a period of constantly increasing strains and deflections at a much lower rate. The strain distribution over the depth of the tested elements was linear. Increasing strain and deflection was accompanied by crack propagation. Changes in the crack pattern were not very accentuated for cases where the initial loading was above 50% of the ultimate load.

Six beams had a fatigue life longer than 10 million stress cycles. Fatigue failure of the element, i.e. the element was no longer able to bear the fatigue load, was only observed when the superior fatigue load $P_{\text{sup}}$ was above 50% of static ultimate load. Failure always occurred due to fatigue fracture of bars. Concrete under compression never showed any failure mode like spalling at the surface, although the stresses were very high, i.e. often above 60% of the cylinder strength $f_{ck}$ of 35 MPa (Figure 7-9a). Even the unusually high reinforcement ratio of 1.6% did not result in concrete fracture.

The commonly used detail category of 170 MPa for non-welded high yield steel reinforcement seems to be slightly optimistic when compared to the test results (Figure 7-9b). This is explained by the difference between the calculated stresses (calculated with the equilibrium of internal forces and plane cross section) on which the points in Figure 7-9b are based and the measured stresses which were in fact higher due to additional bending stresses at locations of concrete cracks.

It was mentioned (Dyduch et al 1994) that the stiffness of plain concrete subjected to fatigue loading decreases significantly as a function of stress level and number of cycles. The results obtained from multi-level tests on specimens confirmed this behaviour. Due to this loss of stiffness a redistribution of stresses in the most deformed concrete fibres of the compression zone occurs (Weigler and Dillmann 1982), and the initial linear stress distribution in the concrete compression zone becomes parabolic. This process is more pronounced for slender elements (i.e. slabs) because of their high strain gradient; the concrete behaves more ductile if the compression zone is smaller. This may explain why even over-reinforced beams fail due to bar fatigue fracture before concrete failure.

One specimen was subjected to a static fracture test after 10 million cycles at two different load levels (with the higher superior fatigue load $P_{\text{sup}}$ at 49% of the static ultimate load). No

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**Figure 7-9:** Test results in comparison with fatigue strength of (a) concrete under compression and of (b) mild steel reinforcement.
further crack propagation was observed before the specimen started yielding. The yield and ultimate loads of the specimen were equal to the corresponding values obtained from the static reference test (without prior fatigue loading).

As a conclusion, under predominant bending fatigue loading, failure of the element is due to fatigue fracture of the steel reinforcement. Fatigue failure of concrete under normal compression forces is unlikely to occur. The fatigue strength of steel bars in reinforced concrete may be described by S-N-curves as obtained from fatigue testing of naked steel reinforcement.

b) Slab-like elements under shear

In order to investigate the shear fatigue behaviour, ten slab-like beams (without shear reinforcement) were subjected to eccentric three-point-bending (Schläfli 1999). After one quasi-static test to determine the reference static ultimate load $P_u (= 307 \, \text{kN})$ and the cracking behaviour, eight specimens were subjected to fatigue loading. In order to “force” shear failure of concrete, a high reinforcement content of 1.6% has been chosen (Figure 7-10).

![Slab-like beams](image)

**Figure 7-10:** Shear fatigue test on slab-like beams.

Five distinct phases and load bearing modes have been identified during the fatigue life of the specimens (Figure 7-11):

1. **Initiation:** In the first load cycle, the formation of flexural cracks is observed.

2. **Propagation:** Within approximately one hundred thousand cycles, the tips of the bending cracks propagate and incline to coalesce into a main diagonal crack which develops up to the specimen’s horizontal compressive strut.

3. **Shear crack opening:** After the formation of the main diagonal crack, the strains and (residual) deflection increase with continued fatigue loading at a very slow rate and result in a notable reduction of the specimen stiffness. The shear crack opening continues to increase steadily and slowly and the crack tip propagates into the compression zone.

![Crack propagation](image)

**Figure 7-11:** Typical crack propagation
(4) **Parallel crack initiation:** As the opening of the diagonal crack gets larger, a crack develops along the (horizontal) flexural reinforcement. This horizontal crack is caused by the dowel action of the reinforcement and separates the cover concrete from the specimen. Dowel action increases as stress transfer normal to and along the steadily opening diagonal crack decreases, until the opening of the diagonal crack is large enough to render stress transfer impossible.

(5) **Failure:** In the final phase, there is no longer stress transfer along the diagonal crack and the shear load is transferred by a strut and tie model and by dowel action. Specimen failure is caused by bar fatigue fracture, followed by fracture of the compression strut in the upper flange.

For 6 specimens, final failure was the result of fatigue fracture of the reinforcement, and in one case “peeling off” of the reinforcement due to dowel action occurred. Bar fracture was detected by measurements (deflection and strain) and by observing the crack pattern and crack openings. In one test, fatigue fracture of the reinforcement occurred due to tensile stresses in the bar at the flexural crack under the maximum bending moment.

The quasi-static tests showed the same cracking phases, load bearing modes and failure processes as the fatigue loaded specimens, although the failure was due to “peeling off” of the reinforcement from the specimens.

Regarding the fatigue strength, the three single and five multiple level fatigue tests gave the results represented in a S-N-diagram as shown in Figure 7-12. The fatigue tests showing no significant fatigue damage were stopped, in most cases, after ten million cycles; the corresponding results were considered as a “run-out”. The “run-out” specimens were subsequently subjected to a higher fatigue load. Fatigue failure was only observed when the superior load level $P_{\text{sup}}$ was above 40% of the reference static ultimate load $P_U$.

Shear loading may be expressed by the nominal shear stress $\tau_{\text{c,nom}} = P/A$, with $A$ being the concrete cross section (beam depth times width). In Figure 7-12, the results of this test program are compared to those of two other investigations by means of nominal shear stress at $P_{\text{sup}}$ as compared to the ultimate shear strength. As can be seen, the results of the three test series do not intersect; indicating that there is no direct relation between these investigations. Consequently, the nominal shear stress appears to be an unsuitable property to describe shear fatigue loading.

![Figure 7-12: Shear fatigue strength (\( \tau_{\text{c,nom}}: \text{nominal static shear strength} \))](image-url)
As a conclusion, under predominant shear fatigue loading, failure of the element is also due to bar fracture, after a distinct crack pattern in the concrete showing a main diagonal and a parallel crack along the horizontal steel reinforcement has occurred.

### 7.5.4 Failure behaviour of slab like elements under bending solicitation

From the above sections follows that reinforced concrete railway bridge elements under bending are the most fatigue relevant.

An analysis of bending fatigue tests described in (Schläfli 1999) and (Johansson 2004) led to following findings:

- Rebars break not all at the same time during fatigue solicitation. Due to the high scatter of fatigue resistance, the damage accumulation does not occur in every rebar at the same time and to the same extent.
- The fracture of two bars at the same time was only observed in beams with high number of reinforcement bars (10 and 12 bars).
- The cycle number between first rebar fracture and failure is rather high (+15-25%) in tests at low stress range ($\Delta \sigma \leq 200$ MPa).
- Deflection varies little after first rebar fracture and increases significantly only after the last rebar fracture before failure (caused by yielding of remaining rebars).
- The cross section is always ductile, unbroken bars are always present at failure.
- S-N curves are well suited to predict the first rebar fracture for stress ranges above the fatigue limit. S-N curves may be conservative in the domain near the nominal fatigue limit.
- The lower the fatigue solicitation, the higher is the scatter of fatigue resistance results (cycle number for fracture).

### 7.5.5 Prestressed girders

Fully prestressed girders (with bonded girders) show rather high fatigue strength because the relative displacements between prestressing steel and duct is small. Also, the concrete remains uncracked. Consequently the problem of fretting fatigue and possible stress variations (concentrations) due to cracking is significantly reduced.

Fatigue of prestressed girders is relevant if the section is decompressed under fatigue loading. This may occur in partially prestressed beams and under higher fatigue loading (in combination with temperature loading). Due to decompressing of cracked sections, the stress magnitude due to fatigue loading increases significantly (Figure 7-13). Consequently, the relative displacements between prestressing steel and duct becomes large and fretting fatigue between wires and/or wire and duct may occur leading to a reduction of fatigue strength.
The fatigue behaviour of partially prestressed girders (with large tendons) was investigated by several researchers. Eskola and Marti (Eskola, 1996) showed that the fatigue life increased by about 80 percent by using plastic ducts instead of corrugated steel ducts. Whereas strand to duct contact pressure was decisive for initiating fatigue cracks in tendons with steel ducts, fretting fatigue within individual strands played the major role for tendons with plastic ducts. Initial wire breaks had only little influence on the overall behaviour of the tested girders but fatigue failures on the non-prestressed reinforcing bars had a trigger effect on the further damage.

7.5.6 Fatigue of reinforced concrete structures

Regarding fatigue of reinforced concrete structures the main problem is the fatigue of the reinforcement bars due to highly localized stress in the bars at cracked sections in the structure. The normal design principle today is to disregard the influence of the concrete in tension when evaluating a reinforced concrete cross section with respect to fatigue. This result in larger stress widths in the reinforcement, than if the response of the concrete in tension had been taken into account. Furthermore, also the bond properties between the reinforcement and surrounding concrete influence the response in the reinforcement. Consequently, to be able to evaluate the stress width in the reinforcement more exactly, there is a need to include also the concrete and the bond properties.

From a more general point of view, different types of reinforcement combined with specific loading conditions rule the risk of fatigue. Observations of such conditions have yielded following conclusions:

- **Welded reinforcement in beams and columns**
  Always a risk for fatigue in welded sections due to introduced residual stresses at manufacturing.

- **Reinforced beams**
  Where bending failure or shear failure is an issue, fatigue failure of the bars should be considered as most probable.

- **Reinforced beams with stirrups**
  If the beam is delivered with stirrups and subjected to the risk for shear failure, fatigue of the stirrups is most likely.
**Table 7.2:** Risk for fatigue of beams and columns with different types of reinforcement and loading conditions.

<table>
<thead>
<tr>
<th>Reinforced</th>
<th>Prestressed</th>
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<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td>Bar fatigue fracture.</td>
<td>Bond deterioration close to crack surface</td>
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<tr>
<td>Fatigue failure of bars</td>
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</table>
8 Methodology for the fatigue safety assessment

8.1 General
The aim of performing a proof of fatigue safety is to demonstrate that the fatigue effects of rail traffic loads will not impair the safety of the structure during its intended service life. For railway bridges, proof of fatigue safety is generally required for all structural elements, in particular for those subjected directly to wheel loads.

In an effort to exploit current knowledge about fatigue of reinforced concrete (described in Chapter 8), a methodology to assess the remaining fatigue life of existing reinforced concrete bridges subjected to railway traffic is suggested. This methodology shall be used when examining existing railway bridges for future higher traffic demands.

In Chapter 8 the conclusion is drawn that the fatigue safety examination of reinforced concrete elements of existing railway bridges includes in principle a fatigue safety check of the steel reinforcement. For the fatigue safety evaluation of steel reinforcement, existing knowledge in fatigue behaviour of steel structures can be used. Fatigue failure of concrete is very unlikely to occur if the concrete is in good condition, i.e. concrete is not suffering from any deterioration mechanism (cracking) due to bar corrosion, frost or alkali-aggregate reaction.

Consequently, a rational methodology for the assessment of fatigue safety is based on the three following study areas taking advantage of the fact that the bridge exists already:

1) study of the bridge structure and evaluation of reinforcement detailing
2) inspection of the existing bridge and study of the past performance
3) fatigue safety check

In the following, each of the three study areas will be discussed.

8.2 Evaluation of the bridge structure and reinforcement detailing
If the principles of good fatigue design practice have been respected when the bridge was built and if the bridge is in good condition, then requirements on structural safety and serviceability are usually predominant. This may be different, and a fatigue safety check is essential, in cases where low fatigue strength is to be expected for the steel reinforcement.

The main objective of the study of the bridge structure and the detailing of the reinforcement is thus to detect fatigue vulnerable spots. Such fatigue vulnerable spots are predominantly present at locations where the rules of “good” fatigue resistant design have not been respected.

Grouping types of reinforcement into fatigue categories in accordance with Table 8-1 allows recognising types of reinforcement with low fatigue strength. Fatigue vulnerable reinforcement details include the following:

- All welded reinforcement is principally fatigue vulnerable, including welded wire mesh, tack welding to reinforcing steel bars, prestressing steel or ducts. Also, butt welds are more fatigue resistant than other load-bearing welds of bars.
- Mechanically connected reinforcing bars show also significantly reduced fatigue strength.
- If the radius of curvature of bars is smaller than the minimum values according to specifications, reduced fatigue strength is to be expected.
- Areas of high concentration of bars or a complex process of placing of the reinforcement might have led to a fatigue vulnerable spot since it was very difficult to obtain the required quality when pouring and working the concrete. Hence, load transfer between bars may not be optimal giving rise to stress concentrations.

- Anchorages for and coupler between prestressing elements show in general rather low fatigue strength although there might be some differences between the various prestressing systems. Anchorages and couplers (of bonded tendons) are fatigue vulnerable if they are not located in areas where the stress ranges are small.

- The fatigue strength of bars is also reduced when corrosion is probable or can be observed, paying special attention to pitting corrosion. However it is difficult to describe this reduction in terms of fatigue strength.

8.3 Bridge inspection and monitoring

The condition of a railway bridge is monitored by regular inspections at least every 5 years throughout its service life. Bridge elements with fatigue vulnerable reinforcement details or high level of stress range can be scanned for cracks (at the surface of the concrete). This gives fundamental information on the current fatigue behaviour of the embedded reinforcement.

*It can be assumed that there is no fatigue damage if the concrete is un cracked. If the bridge was already exposed to significant rail traffic (more than 100'000 trains inducing more than 1'000'000 load cycles) this state can be assumed to remain stable only when traffic (axle) loads are not increased.*

Measurements on a crack (variation of the crack opening) can be made. This allows a rough estimation on the real stress range in the reinforcement under known loading and the effect can be extrapolated to the effect under the fatigue load. The scope of this procedure is to find out if the stress range due to the fatigue load remains below the fatigue limit in the real structure.

Appropriate monitoring techniques (including detailed inspections) allow for determining the probability of detection of a fatigue damage indicator (usually a crack). Detailed information on monitoring can be found in background document: D5.2.S1 Guidelines for monitoring of steel railway bridges.
8.4 Fatigue safety check

8.4.1 Concept
The fatigue safety of a structure is deemed to have been proved if the following condition is satisfied:

\[ n_{\text{fat}} = \frac{R_{d,\text{fat}}}{E_{d,\text{fat}}} \geq 1.0 \]

where:

- \( n_{\text{fat}} \): fatigue degree of compliance
- \( R_{d,\text{fat}} \): examination value for the fatigue resistance (including a partial safety factor)
- \( E_{d,\text{fat}} \): examination value for the fatigue action effect

No partial safety factors for the fatigue action effect are used when proving fatigue safety.

The partial safety factor for fatigue resistance \( \gamma_{\text{fat}} \) may be set to 1.15 for taking account of differences between the actual load-bearing system and the system used for the calculation, simplifications and inaccuracies in the strength model, plus inaccuracies in the cross-section.

In the following, fatigue safety is proved separately for

- the reinforcing steel and
- the concrete.
- Additionally, a method is suggested using the overall structural response of an element.

8.4.2 Proof for reinforcing steel

Determination of fatigue effect (fatigue stress):
The determining parameters when calculating the examination values for the effect of fatigue are the stresses due to fatigue loading only. The stress range \( \Delta \sigma(Q_{\text{fat}}) \) is derived from the absolute value of the difference between the maximum stress \( \sigma_{\text{max}}(Q_{\text{fat}}) \) and the minimum stress \( \sigma_{\min}(Q_{\text{fat}}) \) induced by the effect of fatigue load in the relevant unfavourable positions.

Under bending moment the stresses are calculated at a cracked cross-section, assuming that plane sections remain plane and linear-elastic material behaviour. The behaviour of concrete under tension is ignored.

Under fatigue loading, the bond behaviour of reinforcing and prestressing steel is different. This may be considered by increasing the stress in the reinforcing steel using the following factor:

\[ k = \frac{A_r + A_p}{A_r + A_p \sqrt{\left( \phi_r / \phi_p \right)}} \]

with \( A_r \) and \( A_p \) being the area of reinforcing and prestressing steels respectively, \( \phi_r \) is the largest bar diameter and \( \phi_p = \sqrt{A_p} \) being the equivalent diameter of the prestressing steel.

The ratio \( \xi \) of the bond strength between prestressing steel (of bonded tendons) and reinforcing steel is approximately:
\[ \xi = \frac{6}{\sqrt[6]{A_p \cdot f_{ctm}}} \]  with \( f_{ctm} \) being the average value of the tensile strength of the concrete.

For prestressed structures with bonded tendons, stresses can only be calculated if the true prestressing force is known. If the load is smaller than the decompression moment, the stress range remains small. If, at maximum fatigue load, the decompression moment is exceeded, the stress ranges will increase significantly.

Where prestressing is by means of unbonded tendons, the stresses will have to be calculated iteratively.

For beams and slabs with shear reinforcement, the stresses in the main and shear reinforcement are calculated using a lattice model. The angle of inclination of the concrete compression struts to be assumed when calculating the stress differences in the reinforcement is as follows:

\[ \tan \alpha_{fat} = \sqrt{\tan \alpha} \leq 1.0 \]

With \( \alpha \) being the angle of inclination of the compression strut at ultimate limit state.

**Determination of fatigue resistance:**

Common types of reinforcement are divided into fatigue categories (\( \Delta \sigma_{s, fat} \)) according to their fatigue strength (see Table 7-1). The number designating the fatigue category is the fatigue strength in MPa for 2 million load cycles.

**Verification:**

*Proof with respect to the fatigue limit:* Initially, the fatigue safety of reinforcing steel is verified with respect to the fatigue limit. If all stress ranges due to service loads remain below the fatigue limit \( \Delta \sigma_{s, D} \) for the entire service life, fatigue failure will not occur:

\[ n_{fat} = \frac{\Delta \sigma_{sd, D}}{\Delta \sigma_{sd} (Q_{fat})} \geq 1.0 \]

The nominal fatigue resistance may be taken as: \( \Delta \sigma_{sd, D} = 0.8 \cdot \Delta \sigma_{sd, fat} \)

*Proof with respect to equivalent stress range:* If fatigue safety has not been verified with respect to the fatigue limit, proof of operational resistance shall be performed. When using this method, the aim is to demonstrate that the equivalent stress range does not exceed the fatigue strength:

\[ n = \frac{\Delta \sigma_{sd, fat}}{\Delta \sigma_{ed} (Q_{fat})} \geq 1.0 \]

The equivalent stress range \( \Delta \sigma_{sd} \) is the product of the operational load factor \( \lambda \) and the stress range induced by the fatigue load \( Q_{fat} \):

\[ \Delta \sigma_{ed} = \lambda \cdot \Delta \sigma_{sd} (Q_{fat}) \]

The operational load factor \( \lambda \) [EN 1991-2:2003] compares the fatigue effect of the load models with that of the effective fatigue load. This factor depends on traffic composition, traffic volume, load geometry, service life, number of load cycles, fatigue load and static system. It is based on linear damage accumulation theory.
8.4.3 Proof for concrete

In the determination of stresses in concrete due to fatigue loading it must be considered that such calculated stress values only represent an approximation of effective stresses. Also, reliable fatigue damage accumulation method is still lacking. Consequently, it is not possible to perform a rigorous and reliable fatigue safety check for concrete. Fortunately, proof of fatigue safety by calculation is not required for normal stresses in concrete if inspection shows that the concrete is in good condition.

Nevertheless, it is useful to determine and evaluate the level of fatigue solicitation. A simple proof of compression fields consists in showing that the maximum compressive stress due to permanent loads as well as frequent temperature and rail traffic fatigue loading are smaller than 50% of the nominal value of compressive strength of concrete. Besides the compressive stresses due to bending may be calculated using a stress block.

For deck slabs without shear reinforcement, fatigue failure of concrete may occur only when the stress level is higher than 40% of the ultimate load of the structural element. Also, the nominal shear stress is not the relevant parameter to describe fatigue failure. As a consequence, a fatigue safety check with respect to the ultimate load may be conducted.

Consequently, for the concrete no fatigue safety check appears to be necessary for normal concrete fulfilling usual quality requirements. A safety check is advisable only in cases of existing bridges with low concrete strength due to bad fabrication and deterioration (microcracking due to corroding bars and freeze-thaw cycles). In those cases, however, the verification of the structural safety and serviceability may also not be fulfilled.

8.4.4 Proof with respect to ultimate load

Fatigue testing revealed that relevant fatigue damage only occurs if the level of fatigue solicitation is beyond 50% and 40% of the ultimate load for predominant bending and shear fatigue loading respectively. This was found for test specimens where the effect of the dead load is rather small. This finding is increasingly valuable for bridge elements, containing typically a higher dead load part.

From this follows that no fatigue failure of the structural element will occur if the following conditions are fulfilled:

- under predominant bending fatigue: \( n_{fat} = \frac{0.5 \cdot F_{ult}}{F_{fat,max}} \geq 1.0 \)

- under predominant shear fatigue: \( n_{fat} = \frac{0.4 \cdot F_{ult}}{F_{fat,max}} \geq 1.0 \)

\( F_{ult} \) is the ultimate load of the structural element. It is determined by means of a non-linear structural analysis using nominal values of material properties and considering partial safety factors (resistance coefficients). [Detailed information on non-linear structural analysis can be found in Chapter 2]. \( F_{fat,max} \) is the corresponding maximum fatigue load due to permanent loads as well as frequent temperature and rail traffic fatigue loading.

8.5 Remaining fatigue life of reinforced concrete railway bridges

8.5.1 General

The remaining service live of reinforced concrete bridges or bridge elements is affected only by the reinforcement when inspection shows that concrete is in good condition.
For cases where the fatigue safety check of Section 8.4 for the reinforcement is not fulfilled, rebar fatigue may be expected and the service life of the bridge element may be limited due to fatigue crack growth in the reinforcement.

As the fatigue safety check is based on worst rebar test results (typically on 5% fractile values) only the weakest rebars of the determinant cross section are represented by the fatigue safety check result. As within the determinant cross section, provided that there is a distributed reinforcement, the same scatter of fatigue resistance may be expected, there will be only a part of rebars suffering a fatigue crack growth at the beginning.

The remaining service life calculation is again based on worst rebars properties. The point in time for presumable fracture of the weakest rebar may be predicted. The fracture of the weakest rebar does not lead to structural failure as most of remaining unbroken rebars will be undamaged (more than 90%) and show a ductile behaviour. That point in time may trigger surveying (after a special condition survey), such as crack and deflection monitoring.

A deterministic procedure for a reliable prediction of the remaining fatigue life of the steel reinforcement using LEFM (Paris law for stable crack propagation) is elaborated. LEFM is physically more realistic than linear damage accumulation with the (rigid) bilinear S-N curve, (which is a reasonable tool for new elements design) as the effect of the shape of the stress spectrum is better taken into account.

Fracture mechanic parameters of reinforcement bars are not well known. In order to be able to conduct crack propagation calculations, the presence of a (fictive) initial flaw $a_0$ on reinforcement bars leading immediately to stable crack growth is assumed.

### 8.5.2 Fracture crack size calculation

Fracture of reinforcement bars occurs when the size of one crack reaches a dimension $a = a_{fr}$ where the applied stress $\sigma_0$ is equal to the strength of the remaining cross section. In a fatigue test with constant stress range that stress corresponds to the superior stress level, thus $\sigma_0 = \sigma_{sup}$. Two different fracture modes exist; brittle fracture and fracture by yielding.

**Determination of crack size $a_{cr}$ for brittle fracture:**

Equation (8.1) gives the size of critical crack, leading to brittle fracture

$$a_{cr} = \frac{1}{\pi} \left( \frac{K_C}{Y_{cr} \sigma_{sup}} \right)^2$$

(8.1)

This is deduced from the definition of the stress intensity factor $K = Y \cdot \sigma_0 \cdot \sqrt{\pi \cdot a}$ putting $K = K_C$, $Y = Y_{cr}$ and solving for $a$.

$K_C$ is the critical value of stress intensity factor. It is a material constant and its determination is made experimentally on pre cracked specimens. There also exist empirical relations between the impact strength of the Charpy test and the value of $K_C$. A value of $K_C = 3000$ [Nmm$^{(-3/2)}$] may be taken.

$Y_{cr}$ is the correction factor depending on the crack size and geometry of the remaining cross section. Given that the correction factor depends itself on the dimension of the crack, the determination of the critical cracks size $a_{cr}$ is made through iteration varying $a_{cr}$. The formula for $Y_{cr}$ for semi circular surface flaws in round bars is given by Equation (8.11) [BS 7910: 1999].

**Determination of crack size for failure by yielding:** the crack size $a = a_y$ where the remaining cross section yields is determined by calculating the necessary area-reduction $A(a_y)$ leading to a stress $\sigma = f_y$. 
\[ A(a_y) = A_0 (1 - \frac{\sigma_{up}}{f_y}) \]  
(8.2)

\( A_0 \): rebar cross sectional area of undamaged rebar.

The lower value of both, \( a_{cr} \) and \( a_y \) determines the fracture crack size:

\[ a_f = \min(a_{cr}, a_y). \]  
(8.3)

### 8.5.3 Initial flaw size

From above section and fatigue test results, following parameters are known:

- crack size leading to fracture \( a_f \)
- number of stress cycles leading to fracture \( N \)
- magnitude of constant stress range \( \Delta \sigma \)

The simplifying assumption is made that initial flaw leads directly to stable crack propagation.

According to research by Paris [Paris P., Erdogan F. 1963] the range of stress intensity factor \( \Delta K \) characterises sub-critical crack growth under fatigue loading. As crack growth rate against range of stress intensity factor give straight lines on log-log scales, the following relation was established:

\[
\log \left( \frac{da}{dN} \right) = m \log(\Delta K) + \log D
\]  
(8.4)

Taking out the logs gives the so called Paris Law:

\[
\frac{da}{dN} = D \cdot \Delta K^n
\]  
(8.5)

\( D, n \): depend on the material

\[
\Delta K = Y \cdot \Delta \sigma \cdot \sqrt{\pi a}
\]  
(8.6)

Through replacement of \( \Delta K \) with \( Y \cdot \Delta \sigma \cdot \sqrt{\pi a} \) (equation (8.7)) and integration (equation (8.8)), the number of cycles taken place for the growth of crack from the size \( a = a_i \) to \( a_j \) can be obtained analytically. The assumption is made that \( Y \) is constant:

\[
N_{ij} = \int_{a_i}^{a_j} dN = \int_{a_i}^{a_j} \frac{1}{D \cdot \Delta K^n} da = \int_{a_i}^{a_j} \frac{1}{D \cdot Y^n \cdot \Delta \sigma^n \cdot \pi^{n/2} \cdot a^{n/2}} da
\]  
(8.7)

\[
N_{ij} = \frac{1}{D \cdot \alpha \cdot \pi^{n/2} \cdot Y^n \cdot \Delta \sigma^n} \left[ \frac{1}{a_i^n} - \left( \frac{a_i}{a_j} \right)^a \right]
\]  
(8.8)

\( N_{ij} \): number of cycles which increased the crack size from \( a_i \) to \( a_j \)

\( a \): crack size, with \( a_j > a_i \)

\( \alpha \): integration constant \( \alpha = n/2-1 \)

The subscript \( i \) is chosen for the smaller crack size of the increment. In order to be able to calculate the crack size previous to a given increment of \( N \) equation (8.8) is solved for \( a_i \):
\[ a_i(N_{ij}) = (a_i^{-a} + D \cdot \alpha \cdot \pi^{n/2} \cdot Y^n \cdot \Delta \sigma^n \cdot N_{ij})^{1/\alpha}, \quad \alpha = n/2 - 1 \]  

(8.9)

As \( Y \) varies as a function of the crack size, the calculation may be done by increments of \( N_{ij} \) with constant \( Y \), starting from \( a_i = a_{fr} \). Average values for \( Y \) are taken between \( a_j \) and \( a_i \). The increments of \( N \) have to be chosen small enough in order to obtain results agreeing well to theory. An increment size of 0.1 mm leading to a total of 150-200 increments is sufficiently fine.

### 8.5.4 Crack growth calculation under variable stresses

According to this approach, the service life under variable stresses until the rupture of the first rebar (of a distributed reinforcement) is determined. Figure 8-1 shows a rebar cross section with initial flaw \( a = a_0 \) and crack at fracture \( a=a_{fr} \).

The calculation procedure for rebar crack growth and fatigue life is schematically presented in Figure 8-2. The crack propagation calculation is based on equation (8.5). A numerical integration is difficult as correction factor \( Y \) depends on the crack size \( a \). In literature, \( Y \) is often given by complex formulas. However, the calculation may be conducted by crack increments. Constant stress intensity factor (SIF) ranges (one SIF range for each stress range class \( \Delta \sigma_i \)) are assumed. In a first step a reasonable increment size \( \Delta a \) is chosen. Starting from the initial flaw size \( a_0 \), the flaw (crack) size is increased by increments \( \Delta a \) until the fracture crack size \( a_{fr} \) is reached. An equivalent (constant) SIF range is calculated for each increment and each stress range class \( \Delta \sigma_i \) (equation (8.10)).

\[ \Delta K_{eq,i} = \left[ \frac{\Delta K_{inf,i}^n + \Delta K_{sup,i}^n}{2} \right]^{1/n} \]  

(8.10)

where

\[
\begin{align*}
\Delta K_{inf,i} &= Y_{inf,i} \cdot \Delta \sigma_i \cdot \sqrt{\pi a_{inf}} \\
\Delta K_{sup,i} &= Y_{sup,i} \cdot \Delta \sigma_i \cdot \sqrt{\pi a_{sup}}
\end{align*}
\]

The correction factor \( Y \) depends on the size of rebar and crack. BS 7910 (1999) gives formulas for \( Y \) (Equation (8.11)). Only membrane stress is considered as rebar solicitation (no bending). Equation (8.11) is valid for crack depth \( a \) not higher than 0.6 times the rebar diameter which covers already the domain of rebar crack growth calculations.
The crack propagates only if the stress intensity factor (SIF range) $\Delta K_{eq,incr,i}$ exceeds the threshold value $\Delta K_{th}$ (equation (8.12)). With increasing crack depth the same stress range leads to an increased SIF range.

$$\Delta K_{eq,i} > \Delta K_{th} \tag{8.12}$$

The cycle number of crack growth effective and non crack growth effective stress ranges is counted for each increment. The sum of all increment's cycle number corresponds to the fracture cycle number $N_{fr}$. 

$$Y = \frac{1.84}{\pi} \left[ \tan \left( \frac{\pi a}{4r} \right) \right]^{0.5} \cos \left( \frac{\pi a}{4r} \right) \left[ 0.752 + 2.02 \cdot \left( \frac{a}{2r} \right) + 0.37 \cdot \left( 1 - \sin \left( \frac{\pi a}{4r} \right) \right)^3 \right] \tag{8.11}$$
Figure 8-2: Calculation procedure for rebar crack growth and fatigue life
A typical crack growth curve (crack depth $a$ as a function of cycle number $N$) for variable amplitude loading condition is shown in Figure 8-3.

Figure 8-3: Typical crack growth curve for variable amplitude loading conditions

8.5.5 Parametric studies

The crack growth calculation input values (Figure 8-2 left) are affected by a certain scatter. The influence of variation of the values is investigated.

In current parametric study, only the highest stress ranges are above the fatigue limit (in other words, only the highest SIF ranges are above the SIF range threshold value). Therefore most stress cycles of the total stress spectrum don’t lead to crack growth at the beginning.

Influence of initial flaw size

At certain SIF range threshold values a small variation of the initial flaw size may strongly influence the fatigue life

Influence of crack increment size

Too high increment size may strongly falsify the cycle number to fracture for variable loading conditions. Good accuracy can be obtained with an increment size below or equal to 10 μm leading to about 15’000-20’000 increments.

Influence of the SIF range threshold

Even a small error of the threshold value (e.g. ± 5%) has a strong influence on the fatigue life, it may duplicate respectively halve the fatigue life.

Influence of the accuracy of stress range calculation

As well small errors of the stress range values (e.g. ± 5%) may duplicate respectively halve the fatigue life.

Influence of the width of the stress range class

Stress range classes for high stress ranges should be made as narrow as possible as too high class width may strongly falsify the result.

Influence of higher admissible axle loads

The introduction of increased maximum axle loads and / or the increase of the number of axle loads in the upper range lead to a significant reduction of the fatigue life for the same annual gross tonnage.

Influence of the fracture crack depth
As can be seen in Figure 8-3, the crack growth curve is rather steep during the last few cycles. A small error of the fracture crack depth does not alter the fatigue life.

**Conclusion**

Fatigue life of reinforced concrete bridges is strongly affected by errors of input values on the action effect side as well on the resistance side, which agrees well with the results of rebar fatigue tests close to fatigue limit, showing high scatter. Calculations should be done with fine crack increments and narrow stress range classes for high stress ranges. Accurate information about the heavy traffic is essential for a reliable fatigue life calculation.

Crack growth calculation results should be treated with care. A sensitivity analysis (upper and lower bound value consideration) is indicated. Amongst others, based on these findings, the fatigue safety concept is elaborated.

**8.5.6 Fatigue safety concept**

**a) Introduction**

The fatigue safety concept takes into account the high uncertainty of fatigue life predictions and the fact that fatigue failure is announced in advance.

The fatigue safety concept consists of the combination of fatigue analysis and condition survey. Due to the relative small live load of ballasted reinforced concrete bridges and their young age, no signs of fatigue damage can be observed in many bridges yet. However, the increase of maximum allowable axle loads may accelerate fatigue damage.

The fatigue safety concept is applied if fatigue safety check is not fulfilled. Figure 8-4 systematically shows the time axis. It begins with the examination condition survey that is intended to update the input values for the fatigue life calculation with variable stress ranges. At the examination condition survey, the bridge is checked for its state at the locations that are critical for fatigue. The crack pattern of the concrete is recorded. If there is a well-established crack pattern, some measurements may be conducted to update the action effect model for the reinforcement stress ranges. The description on how the action effect may be updated by crack width measurements is given below.

Then, with carefully chosen material parameters, a lower bound for the number of years until first rebar fracture is calculated. During this calculated period, only routine condition survey is conducted. A damage model is described with which the fatigue life of weak reinforcement bars may be calculated.

At the end of the period, a re-updating condition survey allows an updating of the fatigue life calculation that was calculated as a lower bound. With the results of that re-updating condition survey, the fatigue life may eventually be extended.

After the extended fatigue life, which is still a lower bound, there are two possibilities to proceed: (1) the bridge is reinforced or replaced, or (2) a monitoring system is introduced. The introduction of monitoring is only advisable for bridge elements with distributed reinforcement, such as slabs, containing a high redundancy for the fatigue resistance of the reinforcement bars.

The bridge element is monitored from the point in time where the lower bound of the re-updated calculated fatigue life of the weakest reinforcement bars ends in order to increase the safety on an acceptable level. During a limited period, a loss of one or eventually a higher, limited number of reinforcement bars may be accepted as there are still intact reinforcement bars present within the respective cross section due to the variation of fatigue resistance within a distributed reinforcement.
When a rebar loss is detected, a predefined intervention period starts during which the rehabilitation or reinforcement should be undertaken. The length of the intervention period is chosen as a function of the expected evolution of the fatigue damage of remaining rebars. During the intervention period, the required structural performance may be reduced. To keep the structural safety on the same level, a lower probability of occurrence of extreme axle loads may be considered or a temporary reinforced axle load control performed.

Subsequently, the steps of the fatigue safety concept are explained and justified in detail.

**Figure 8-4: Fatigue safety concept for reinforced concrete bridges (monitoring may be considered only for elements with distributed reinforcement)**

**b) Importance of inspection and updating for the fatigue examination**

The fatigue behaviour of fatigue tests is shortly described here first, and then it is carried over to bridge elements with their typical action effects.

**Fatigue behaviour of fatigue tests specimen with constant amplitude loading:**

In the bending fatigue tests of M. Schläfl [Schläfl 1999], it was observed that the deflection increase after the first loading during cyclic loading was small for high superior load levels (more than 60% of the ultimate resistance). The deflection increase was the most pronounced during the first thousands of load cycles. The deflection increase relative to the deflection increase during the first loading was much more pronounced for specimens with lower superior load levels.

The increase of deflection after the first thousands of cycles was explained with the increase of remaining strain of the concrete of the compression zone. The measurements of the concrete showed an increase even at high cycle numbers of several millions. The increase of the reinforcement stresses was less pronounced. The increase of the stress range in the reinforcement however was almost zero after the first thousands of cycles. This is due to the appearance of residual stress behaviour, increasing the lower stress level of the reinforcement.
Fatigue behaviour of bridge elements with variable loading:

Much more than in the fatigue tests, the fatigue behaviour of the reinforced concrete bridges is influenced by the concrete in tension. The cycling loading has a much lower frequency and the stress ranges are lower than the stress ranges of the test specimens. Due to the lower loading frequency, the fatigue damaging is not as fast as in tests and the bridge element may be still in a state corresponding to the first thousands of cycles of the described fatigue test specimens with constant amplitude loading, where the concrete works in tension. Additionally, the concrete, also the concrete in tension, may creep under permanent loads, which may prevent partially the formation of cracks.

From above made statements it may be expected for reinforced concrete bridges, where the live load has a smaller part than in the fatigue tests, that the concrete in tension is involved longer in the fatigue behaviour under service load conditions. In such a case, the stresses in the reinforcement are lower than calculated with traditional method on a fully cracked concrete cross section.

From this follows that it is indispensable to update the calculations through inspections before costly strengthening measures are planned and executed due to calculated reinforcement stress ranges.

(c) Updating of the action effect model through crack width measurements

The reinforcement stress range, or better, the reinforcement strain range cannot be measured directly in a non-destructive way, as the reinforcement bars are not attainable due to the concrete cover.

For stresses in the reinforcement that are high enough to be fatigue relevant, it is indispensable that the concrete is cracked. It must be possible to assess the reinforcement stress by the measurement of the crack width as the opening of a crack is accompanied by an elongation of the reinforcement. Variations of the crack opening rather than absolute crack openings are of interest, as the variation of the stress in the reinforcement is fatigue relevant only. Variations of the crack width are easy to measure width Ω-gauges or inductive displacement sensors by the recording of the relative displacements between the two concrete surfaces adjacent to the crack.

The main difficulty is the calculation of the reinforcement stress range from a known crack width variation. It depends strongly on the bond behaviour between the reinforcement bars and the surrounding concrete near the cracks. Chapter 3 gives information on the bond behaviour of reinforcement bars.

An upper bound value for the stress range may be obtained by the assumption of good bond behaviour as with good bond behaviour, the crack width variation is distributed on a relative short length on the reinforcement bar and therefore the strain is high.

(d) Relation cutting force - crack opening

The variation of the crack width under a loading with known axle loads and geometry is measured, e.g. with a locomotive. In parallel, the cutting force that corresponds to the location of the crack, and which is responsible for the crack width variation is calculated using an adequate structural model for the bridge and the same loads and load configurations.

By the assemblage of the measured crack width due to a load position and the calculated cutting force due to the same load position, the relation cutting force-crack width is obtained. With that relation, the crack width variation for other load configurations may be calculated.
e) Calculation of the fatigue life of the weakest reinforcement bar

The procedure that was presented in Chapter 8.5.4 is based on the theory for Linear Elastic Fracture Mechanics (LEFM). LEFM was applied in order to represent the evolution of the fatigue damage of a reinforcement bar in an as possible physically correct way (even though stable crack growth was assumed from the beginning on). With the LEFM approach, the relevant parameters could be found through the parametric study of Chapter 8.5.5. However, the input values, such as the (fictive) initial crack depth and the SIF-range threshold had to be calculated first based on the S-N curve.

A more practical method based on LEFM was developed by Peter Kunz [Kunz 1992]. It has the advantage that the well-known parameters of the S-N curves such as the nominal fatigue limit, the nominal fatigue strength and the slope are used as input. The damage of a single stress range is calculated by following equation:

$$d_i = \frac{\Delta \sigma_i^n - \Delta \sigma_D^n \cdot (1 - D)^n}{\Delta \sigma_c^n - \Delta \sigma_D^n \cdot (1 - D)^n} \cdot \frac{1}{2 \cdot 10^6}, \quad d_i \geq 0$$ \hspace{1cm} (8.13)

- $d_i$ : damage due to a single stress range of the stress range class $\Delta \sigma_i$
- $\Delta \sigma_i$ : stress range class i
- $\Delta \sigma_D$ : nominal fatigue limit
- $\Delta \sigma_c$ : nominal fatigue strength
- $D$ : accumulated damage

The whole procedure for the fatigue life calculation is shown in Figure 8-5. Above right, the input values (resistance side) for the fatigue life calculation are given. Below, the stress range histograms for each period of the traffic model are mapped schematically (action effect).

The damage $d_1$ of the first stress range (the lowest) is calculated with Equation (8.13). If the stress range is below the nominal fatigue limit $\Delta \sigma_D$ (or $\Delta \sigma_{s,D}$), a value for $d_1 < 0$ results and the damage $d_1$ is set to zero.

The “average damage” per cycle, is calculated by Equation (8.14). $d_{\text{cycle}}$ is the sum of the damage $d_i$ of each stress range class $\Delta \sigma_i$ multiplied with the part of the cycle number $R_i$ within the total cycle number of the histogram. $N_i$ is the cycle number of the stress range class i and $N_{\text{tot}}$ is the cycle number of all stress range classes together.

$$d_{\text{cycle}} = \sum_{i=1}^{m} R_i \cdot d_i$$ \hspace{1cm} (8.14)

where: $R_i = \frac{N_i}{N_{\text{tot}}}$

The damage $d_{\text{cycle}}$ is added to the previous total damage $D$ (initially $D$ is zero). If the damage is still lower than $D = 1.0$, the cycle number now amounts to $N = N+1$ ($N$ is 0 at the beginning).

$$D = d_{\text{cycle}} + D$$ \hspace{1cm} (8.15)

Alternatively, in order to safe computation time, good results may also be obtained when the calculation is done by increments for the cycle number (e.g. $N_{\text{incr}} = 10'000$). Then the damage amounts to:
\[ D = N_{\text{incr}} \cdot d_{\text{cycle}} + D \]  

The calculation continues if the damage \( D \) is below 1.0. The cycle number \( N \) will be equal to

\[ N = N + 1, \text{ or, alternatively: } N = N + N_{\text{incr}} \]  

If the total cycle number that corresponds to a period of the traffic model is reached before \( D = 1 \), the next period of the traffic model is chosen. The trains are counted.

The damage accumulation is conducted until \( D = 1 \). The total cycle number \( N \) then corresponds to the fracture cycle number \( N_{\text{fract}} \). The number of future trains is calculated as follows:

\[ N_{\text{fat}} = N_{\text{trains}} - N_{\text{today}} \]  

\( N_{\text{trains}} \) is the total number of trains until fracture (calculated with the traffic model for the past and the future), \( N_{\text{today}} \) is the number of trains until today (with the traffic model for the past).

**Figure 8-5: procedure for the fatigue life calculation for reinforcement bars**

Alternatively, the fatigue life may be calculated with the linear damage accumulation theory using the tri linear S-N curve [Haibach 1970]. The results are virtually the same, even if that theory does represent the real physical conditions less properly than the LEFM theory.

**f) How to detect a broken reinforcement bar through continuous measurements (monitoring)**

In fatigue tests, reinforcement fractures can be detected through continuous or periodic measurement on the concrete surface. A reinforcement fracture was detected by comparing two consecutive measurements over a crack under a given cutting force. The failure of one or two reinforcement bars lead to an abrupt increase of the measured value.
In real bridges, the fracture of the first rebars will not occur as fast as in a fatigue test, where the first fracture occurred due to the relative strong fatigue loading with constant stress range already after some hours or days. In real bridges, already the point in time where the fatigue damaging is initiated may last over several years. The time lag between different rebar fractures may consist in several years, too.

It is therefore not easy to compare two measurements that are spaced in time for several years. Many changes may occur, the climatic condition between two measurements may change, the load carrying behaviour may be altered, deterioration may occur et cetera. Therefore, it is important to measure continuously. The fracture of a reinforcement bar induces suddenly an abrupt change. In order to exclude other mechanism leading to changes in crack width variations, it is imperative to choose the measurement intervals carefully and interpret the measurements well. Well, for now, there is not enough experience in long term monitoring. In order to introduce successfully the fatigue safety concept, it is indispensable that reliable rebar loss detection is possible.
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