A Model-Predictive-Control Based Smart-Grid Aggregator

AMANDA PAULUS
A Model-Predictive-Control Based Smart-Grid Aggregator

AMANDA PAULUS
Abstract

Intermittent energy source usage, such as solar and wind power, is continuously increasing. Intermittent energy sources are highly dependent on prevailing weather conditions, resulting in stochastic electricity generation. The expected stochasticity in electricity generation will cause issues for the current power grid. Moreover, an expected issue for the Swedish power grid is higher peak loads. Thus, there is an emerging need for novel and smart power systems capable of shifting peak loads in the future electricity grid.

Model Predictive Control (MPC) is a sophisticated control method that is suitable for smart-grid aggregators. Hence, MPC can be used to optimally control the efficiency of energy use in a smart grid and shift peak loads.

The purpose of this thesis is to investigate optimal peak load-shifting and efficiency of electrical substation operation in a smart grid in Ramsjöäsen, Sweden, using an MPC based smart-grid aggregator. Furthermore, the purpose is also to contribute to the theoretical foundation for future peak load-shifting in smart grids.

Within the thesis project a mathematical model for the smart grid in Ramsjöäsen is developed, which is then used to simulate different scenarios. The simulated results indicate that an MPC based smart-grid aggregator improves the performance of the smart grid in Ramsjöäsen, as regards to both peak load-shifting and efficiency of electrical substation operation.
Sammanfattning

Användningen av intermittenta energikällor, såsom sol och vindkraft, ökar ständigt. Intermittenta energikällor är starkt beroende av rådande väderförhållanden, vilket resulterar i stokastisk elproduktion. Den förväntade stokasticiteten i elproduktion kommer att orsaka problem för det nuvarande elnätet. Dessutom förväntas högre toppbelastningar för det svenska elnätet. Således finns ett växande behov av nya och smarta kraftsystem som kan reducera toppbelastningar i det framtida elnätet.

Model Predictive Control (MPC) är en sofistikerad styrningsmetod som är lämplig för smart-näts aggregatorer. Därför kan MPC användas för att optimalt styra effektivitet av energianvändning i ett smart nät och minska toppbelastningar.

Syftet med detta examensarbete är att undersöka optimal reducering av toppbelastningar och drift-effektivitet av transformatorstationen i ett smart nät i Ramsjöåsen, Sverige, med hjälp av en MPC baserad smart-näts aggregator. Dessutom är syftet att bidra till den teoretiska grunden för framtida topplastskapning i smarta nät.

Inom examensarbetsprojektet utvecklas en matematisk modell för smart nätet i Ramsjöåsen, som sedan används för att simulera olika scenarier. De simulerade resultaten indikerar att en MPC baserad smart-näts aggregator förbättrar smart nätets prestanda i Ramsjöåsen, vad gäller både topplastsreducering och drifteffektivitet av transformatorstationen.
# Contents

<table>
<thead>
<tr>
<th>List of Tables</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>iv</td>
</tr>
</tbody>
</table>

## 1 Introduction

1.1 Background .......................... 1
1.2 Problematisation ..................... 2
1.3 Purpose ............................. 2
1.4 Research Question ................... 2
1.5 Delimitation ......................... 2
1.6 Thesis Structure ..................... 3

## 2 Literature and Theory

2.1 Swedish Grid ........................ 4
2.2 Electrical Substation ............... 5
2.3 Requirements for Future Swedish Grid 6
2.4 Ramsjöäsen Björklinge ............... 6
   2.4.1 Heat Pump ....................... 8
   2.4.2 Solar Cell ....................... 8
   2.4.3 Battery Storage ................. 8
   2.4.4 Electrical Vehicle .............. 9

## 3 Mathematical Theory

3.1 Model Predictive Control ............ 10
   3.1.1 MPC stability ................... 11

## 4 Model

4.1 Variables ................................ 12
4.2 Dynamic Equations ..................... 13
4.3 Time-step Constraints ................. 14
4.4 Objective Function ..................... 15
4.5 Mathematical Model ..................... 16

## 5 Method

5.1 Data .................................. 17
5.2 Scenarios ................................ 19
   5.2.1 Standard Case .................... 19
   5.2.2 Scenario 1 ......................... 20
   5.2.3 Scenario 2 ......................... 21
List of Tables

5.1 Assumptions on one-week’s charging behaviour; EV charging start times 18
6.1 Scenario 1 results, October 2017 23
6.2 Scenario 1 results, February 2018 24
6.3 Scenario 2 results, October 2017 26
6.4 Scenario 2 results, February 2018 26
6.5 Scenario 3 results, October 2017 29
6.6 Scenario 3 results, February 2018 29
List of Figures

2.1 Illustration of the Swedish electricity grid, analogue to blood-vessel anatomy (Göteborg Energi, 2018) ........................................ 4

2.2 Local grid area managed by Upplands Energi with Björklinge underlined (Upplands Energi, 2018b) ...................................... 7

5.1 EV charge profiles .......................................................... 18

6.1 Max power $P_{max}$, Scenario 1 ........................................ 25

6.2 Optimal electrical substation operation percentage $\phi$, Scenario 1 .......................................................... 25

6.3 Mean per half-hour standard deviation $\sigma$, Scenario 1 .......................................................... 25

6.4 Max power $P_{max}$, Scenario 2 ........................................ 28

6.5 Optimal electrical substation operation percentage $\phi$, Scenario 2 .......................................................... 28

6.6 Mean per half-hour standard deviation $\sigma$, Scenario 2 .......................................................... 28

6.7 Max power $P_{max}$, Scenario 3 ........................................ 31

6.8 Optimal electrical substation operation percentage $\phi$, Scenario 3 .......................................................... 31

6.9 Mean per half-hour standard deviation $\sigma$, Scenario 3 .......................................................... 31

A.1 The left figure shows convex sets. The right figure shows non-convex sets (Sasane and Svanberg, 2012) ........................................ 38

A.2 Convex function where $C = \mathbb{R}$ (Sasane and Svanberg, 2012) .......................................................... 39
Chapter 1

Introduction

1.1 Background

Global warming is putting pressure on today’s society and its energy consumption. This pressure has inter alia lead to the Swedish energy sector aiming at becoming carbon-dioxide neutral by 2050 (Naturvårdsverket 2012). To become carbon-dioxide neutral, today’s fossil fuel energy sources must be replaced by renewable energy sources. Thus, in a future fossil-fuel-free society electricity is expected to be the primary energy carrier (Halvgaard et al. 2014).

Renewable or intermittent energy sources are highly dependent on weather conditions. Intermittent energy sources, unlike fossil fuels, are thereby not constantly available for conversion to electricity, leading to stochastic electricity generation. Furthermore, contrary to fossil fuels, renewable energy is not directly controllable since it cannot be efficiently nor easily stored (Hedegaard and Meibom 2011). The expected stochasticity in electricity generation will cause issues for the prevailing power grid and calls for novel and smart power systems (Bitar et al. 2011).

One expected issue for the Swedish power grid is higher peak loads (Helbrink 2014). Accordingly, a smart power system capable of shifting peak loads in the grid is needed; a smart grid. Since the core issue will not be lack of energy but lack of efficient energy use, the smart grid must balance energy consumption with the fluctuating energy production. The energy consumers and producers in a smart grid will assist the grid and are therefore required to be flexible (Bitar et al. 2011). To achieve this flexibility, new control strategies are needed.

The use of Model Predictive Control (MPC) on various smart grid networks has been reported by Halvgaard et al. (2014). MPC can handle time-varying constraints and obtain optimal solutions for receding time horizons. This makes MPC a suitable control strategy for smart-grid aggregators; MPC can be used to optimally control the efficiency of energy use in a smart grid and shift peak loads (Bordons and Camacho 2007) (Halvgaard et al. 2014). Moreover, the MPC algorithm obtains optimal controls based not only on the model of a dynamical system but also on the systems predicted evolution (Bordons and Camacho 2007). Hence, predicted electricity prices and demand forecasts can easily be incorporated into the smart-grid controller.
1.2 Problematisation

As a consequence of increasing renewable energy source usage, such as solar and wind power, smart grids will be needed to shift peak loads in the future electricity grid.

Optimal control of power use in a smart grid requires an aggregator which is able to handle time-varying constraints and obtain optimal solutions on a receding time horizon. Thus, Model-Predictive-Control (MPC) based aggregators are suitable smart-grid aggregators.

In a residential district in Ramsjöåsen, located in Björklinge, Sweden, there currently exists a small number of homeowners connected to a smart grid. The homeowners are flexible consumers that assist the local grid during peak loads (Sustainable Innovation, 2014). The purpose is to investigate efficiency of electrical substation operation and optimal peak load-shifting in the smart grid in Ramsjöåsen, Sweden, using an MPC based smart-grid aggregator. By presenting an MPC based aggregator for the smart grid in Ramsjöåsen, this thesis will provide proof-of-concept and therefore be significant for future research and real world implementations of MPC based smart-grid aggregators.

1.3 Purpose

The purpose of this thesis is to investigate

- optimal peak load-shifting in the smart grid in Ramsjöåsen using an MPC based smart-grid aggregator
- efficiency of electrical substation operation in Ramsjöåsen using an MPC based smart-grid aggregator

and thereby contribute to the theoretical foundation for future peak load-shifting in smart grids.

1.4 Research Question

The following research questions will be answered to attain the purpose of this thesis.

1. How can peak loads in the smart grid, in Ramsjöåsen, be optimally shifted by an MPC based smart-grid aggregator?

2. How do different smart-grid components impact the smart grid, peak load-shifting and electrical substation operation in Ramsjöåsen?

3. What impact will an MPC based smart-grid aggregator have on electrical substation operation in Ramsjöåsen?

1.5 Delimitation

There currently exits research on the use of MPC on various smart grid networks. However, research on MPC based smart-grid aggregators in the context of Swedish electricity market is scarce. As the
investigation is limited to the smart grid in Ramsjöäsen, it may not be entirely applicable to other areas. Nevertheless, by reporting the use of MPC on the smart grid in Ramsjöäsen, Sweden, this work potentially provides a basis for much greater application of smart grids in Sweden.

1.6 Thesis Structure

The thesis structure is as detailed in the following. In Chapter 2, background theory is given on the Swedish grid and future requirements for it. Further, an introduction to the smart grid in Ramsjöäsen is given. Chapter 3 describes MPC and MPC stability. Chapter 4 presents the mathematical model produced and used in this work’s simulations. Chapter 5 explains the data processing and method of producing and performing the simulations. Chapter 6 presents the simulation results. Chapter 7 discusses the results presented in Chapter 6 and also draws conclusions based on them. Finally, Appendix A consists of complementary mathematical theory that might be helpful to the reader.
Chapter 2

Literature and Theory

2.1 Swedish Grid

The Swedish electricity grid is divided into three parts: the main transmission grid (stamnätet), the regional grid (regionalnätet) and the local grid (lokalnätet). The main transmission grid carries enormous amounts of electricity from the power plants to different parts of Sweden. The main transmission grid consists of high voltage power lines, carrying up to 400 $kV$, to enable long distance transfer of electricity with rather small losses. Where the main transmission grid ends, the electricity is transformed to approximately 100 $kV$ and passed on to the regional grid power lines. The regional grid branches out from the main transmission grid and carries the electricity for shorter distances, hence the lower voltage power lines. High energy consumers, e.g. big industries, are connected to the regional grid. Analogously, the local grid branches out from the regional grid and carries electricity with around 10 $kV$. Smaller industries and households are connected to the local grid. However, the electricity is transformed yet again for the last distance before it reaches the Swedish households' 230 $V$ power outlets (Konsumenternas Energimarknadsbyrå 2018) (Göteborg Energi 2018).

\[\text{Figure 2.1: Illustration of the Swedish electricity grid, analogue to blood-vessel anatomy (Göteborg Energi 2018).}\]

The Swedish grid market is a monopoly, issued by the Swedish government. Consequently, the grid is entirely governed by laws set by the Swedish government. The main transmission grid is owned...
by the Swedish government through the company Svenska kraftnät. The regional grids are owned by larger energy companies, such as Vattenfall AB or Ellevio AB. In turn, the local grids are either owned by private energy companies or by municipalities \cite{Konsumenternas_Energimarknadsbyrå_2018,Göteborg_Energi_2018}.

2.2 Electrical Substation

An electrical substation transforms voltage levels between different parts of the grid, e.g. it is through an electrical substation that the voltage is transformed from 400 kV in the main transmission grid to 100 kV in the regional grid.

A part of this thesis’ purpose is to investigate efficiency of electrical substation operation in Ramsjöåsen using an MPC based smart-grid aggregator. The aim is to produce results that take electrical substation losses into consideration to ensure that the local electrical substation is running optimally.

When shifting peak loads in the smart grid in Ramsjöåsen, one is continuously working on optimizing towards the smart grid’s superstructure. However, by considering electrical substation efficiency one is optimizing locally. Thus, the purpose is to investigate efficient operation of the local substation in Ramsjöåsen.

2.3 Requirements for Future Swedish Grid

Electricity is produced at the same moment as it is consumed. Thus, the insertion of electricity into the grid as well as the withdrawal of electricity from the grid has to be balanced, i.e. equal. As the use of intermittent energy sources increase, new ways of balancing energy production with energy consumption are needed.

Energy generation from intermittent energy sources, like wind and solar power, are expected to increase in Sweden \cite{Helbrink_2014}. Intermittent energy sources depend on prevailing weather conditions. Energy will therefore be generated whenever possible; energy will be generated when there is solar radiation and wind. Further, intermittent energy sources enable energy to be produced at lower levels in the grid, i.e. at the local grid level \cite{Elforsk_2014a,Helbrink_2014}. This differs from the current situation where central production facilities are connected to the main transmission grid. Since the grid and networks of today are dimensioned and built based on current conditions, changes caused by intermittent energy sources place new demands on the grid and on how energy should be balanced. The new demands are placed on all levels of the grid: the main transmission grid, the regional grid and the local grid.

On main transmission grid level, the expected issues are sealed power and price volatility. With price volatility the investors will face a higher investment risk. Further, the risk for the consumers will also increase as it will be more difficult to adjust energy consumption to avoid high prices. Sealed power is when intermittent energy generation results in energy supply which is higher than the energy demand. Sealed power can result in total price collapse. This puts pressure on the main transmission grid nationally and also requires better exchange with neighbouring countries. With better exchange the supply surplus can be utilised to avoid total price collapse. Increased demands are also expected on the regional grid level, however what these demands will be is not clear as
the regional grid is more difficult to generalise than the main transmission grid and the local grid (Elforskn 2014a) (Helbrink 2014).

On local grid level an increase in small-scale decentralised production, mainly from wind and solar power, is expected. In fact, a study by Svensk Energi (2014) shows that 25% of Swedish households are considering investments in own energy production. As mentioned, when decentralised local energy production increases, together with the fact that the production is intermittent, the conditions on the traditional grid change. If intermittent energy production on the local level is high the flow in the grid’s network will change directions, going from lower to higher levels in the grid.

Further, a large increase in electric vehicles (EVs) is expected. The EVs will most likely be charged through the local grid, adding to the current load (Elforskn 2014a). EV charging requires high amounts of energy. Moreover, charging patterns for most EVs are expected to be similar and also coincide with peak loads from other energy use. This will especially cause problems during the winter season when peak loads in Sweden are at their highest (Elforskn 2014a) (Helbrink 2014).

Peak load-shifting can solve the issues mentioned above and stabilise the grid networks. This is done by balancing energy consumption with energy production. Peak load-shifting is primarily useful when the intermittent energy production or the load is high. Through load-shifting and by utilising the inertia in e.g. heating of buildings, distinct parts of the grid are relieved and can be used more efficiently (Helbrink 2014).

Moreover, the future smart grid does not only require smart consumption, as in flexible consumers, but also requires smart technology. Such technologies can be increased measuring and monitoring of the grid network that provide accurate information in real time (Elforskn 2014b).

2.4 Ramsjöåsen Björklinge

This work will investigate efficiency of electrical substation operation and optimal peak load-shifting for a smart grid in Ramsjöåsen, located in Björklinge, Sweden. The main transmission grid is, as mentioned, owned and managed by the Swedish government through the company Svenska kraftnät. The regional grid for this case is managed by the energy company Vattenfall AB and the local grid in managed by the customer-owned energy company Upplands Energi AB. Upplands Energi delivers electricity to approximately 13,000 customers north of Uppsala, Sweden (Upplands Energi 2018b). The area managed by Upplands Energi is illustrated in Figure 2.2.
Upplands Energi purchases a certain amount of power (and pays an electricity tariff) from Vattenfall. The amount of power purchased by Upplands Energi is its so called subscribed power (abonnerad effekt). Today Upplands Energi’s subscribed power is $60\text{ MWh}$, meaning that the total power consumed, or the total load, by its $13\,000$ customers should not exceed $60\text{ MWh}$. However, if a peak load exceeds the subscribed power then a fee is incurred due to the stress this places on the regional grid.

During the winter of 2015 a peak load exceeded the subscribed power when it reached $66\text{ MWh}$. This resulted in a 2 million SEK fine. As Upplands Energi is customer-owned, the fine is paid by its customers (Lindborg and Ridenour, 2018a). With optimal peak load-shifting such situations could be avoided.

This work investigates a part of Upplands Energi’s local grid in Ramsjöäsen, Björklinge. Specifically, this work considers Ramsjöäsen’s electrical substation which provides electricity for 64 households, a recycling facility and street lightning. The electrical substation transforms electricity for the last distance before it reaches the area’s 230 V power outlets. The electricity from the substation is transferred through eight power lines, where six lines are for the 64 households, one line is for the recycling facility and one line is for street lightning.

The electrical substation in Ramsjöäsen has a $kVA$-rating of $630\,kVA$, which is the best operating point for a substation. Consequently, the substation in Ramsjöäsen is oversized for its current load, a not-uncommon situation for an area that expects to expand.

At 20 % overload the substation’s losses rise significantly. Thus, efficient electrical substation operation is attained when the substation is kept within a 20 % range of its $kVA$-rating. Since the considered substation is oversized for its current load, it will never reach a 20 % overload (Lindborg 2018). In order to gain insight into the affect of control on a substation operating near capacity,
the substation will be simulated as if it had a lower kVA-rating.

Half of the 64 homeowners are flexible consumers that assist the local grid during peak loads through Ngenic’s smart thermostat *Ngenic Tune* [Lindborg and Ridenour 2018a] [Tour of facilities 2018]. In this thesis work it is assumed that some homeowners also have solar panels, EVs and battery storage. Thus, optimal peak load-shifting in the smart grid in Ramsjöåsen will be attained by controlling (the Ngenic Tune connected) heat pumps together with the invented solar panels, EVs and battery storage, using the MPC algorithm.

### 2.4.1 Heat Pump

The heat pumps used (by the 64 homeowners) in Ramsjöåsen are for waterborne heating systems, where the heat is drawn from either the ground or the ambient air, and then transferred to water in the heat pumps [Lindborg and Ridenour 2018a].

As mentioned, 32 of the 64 homeowners in Ramsjöåsen are flexible consumers that assist the local grid during peak loads through Ngenic Tune. Ngenic Tune is connected to the home heat pump. Homeowners enter a desired indoor temperature, which Ngenic Tune receives. Further, Ngenic Tune measures sunlight entering through windows, relative humidity, outdoor temperature and indoor temperature. The data are then used to control the heat pump to optimize the indoor climate. Hence, Ngenic Tune reduces energy consumption by taking more parameters into account when heating a house, compared to regular heat pumps [Ngenic 2018].

### 2.4.2 Solar Cell

A solar cell, or a photo-voltaic (PV) cell, transforms solar energy into electricity. Although solar cells are most efficient during sunny and clear weather, they also function during cloudy weather [Mertens 2014]. This enables use of solar cells in countries like Sweden. In Sweden, inflow of solar energy is approximately 1000 kWh/m$^2$ for a regular year. The solar cells that are considered in this thesis convert about 10-15 % of the incoming solar energy to electricity, i.e. about 100-150 kWh/m$^2$ per year [Upplands Energi 2018a].

The 64 homeowners considered do not own solar panels, however, it will be assumed that they do. The investigation will be based on data taken from Upplands Energi’s own solar panels. Further, it is assumed that the solar system is connected to a battery storage system. Hence, surplus solar electricity production can not only be fed back into the local grid but can also be stored in the battery [Lindborg and Ridenour 2018a].

### 2.4.3 Battery Storage

Battery storage enables surplus energy to be stored when energy demand is low, and consumed later when the demand is higher. With this, peak loads can be shifted and thus battery storage is useful in smart grids as it assists the grid in controlling loads. Further, stochasticity in energy production (caused by increasing intermittent energy sources such as solar and wind power) can be counterbalanced by battery storage; the variations caused by the stochastic energy production can be fast and drastic and battery storage can easily and quickly restore this imbalance in the grid [Hansson 2016].
Battery storage also benefits consumers by enabling them to buy more electricity when electricity prices are low, and buy less or sell when prices are high. Moreover, consumers can use more own-produced solar energy since surplus energy is stored when the demand is low, and consumed later when the demand is high. This is especially beneficial in regions where the energy cannot be fed back (and sold) into the grid (Fitzgerald et al., 2015).

Analogous to the case with solar cells, the 64 homeowners considered do not have access to battery storage, however, it will be assumed that they do. The investigation will be based on Ferroamp Elektronik’s battery storage system EnergyHub. EnergyHub communicates with the network-enabled devices and optimizes the flow of energy between them. The EnergyHub system is powered by the Power Module. The Power Module is a 3.5 kW three phase bi-directional power inverter; the Power Module converts energy from solar cells and energy storage so that it can be used in buildings or fed back into the grid. The EnergyHub is scalable and can have up to four Power Modules, i.e. ranging from 3.5-14 kW (Ferroamp Elektronik, 2018a). Furthermore, the EnergyHub system contains energy storage, called Energy Storage Module, of type Li(FePO₄) and size 7.2 kWh (Ferroamp Elektronik, 2018b).

In Ramsjöåsen the EnergyHub system will be implemented in two ways: either there will be two distinct configurations of EnergyHub 7 kW installed with one Energy Storage Module, or one EnergyHub 14 kW installed with two Energy Storage Modules.

2.4.4 Electrical Vehicle

EV charging requires high amounts of power and since a large increase in EV usage is expected, peak loads will increase. As described in Section 2.3, the EVs will most likely be charged through the local grid and the EV charging is expected coincide with current peak loads (Elforsk, 2014a)(Helbrink, 2014). Thus, EVs are important to consider when investigating smart grids.

Analogous to previously described, the 64 homeowners considered do not own EVs, however, it will be assumed that they do. The investigation will be based on EV charge profiles provided by Chargestorm AB.
Chapter 3

Mathematical Theory

3.1 Model Predictive Control

Model Predictive Control (MPC) is a sophisticated control method that provides an optimal control strategy which is based on a dynamic model of a constrained system and its predicted future development (Bordons and Camacho 2007) (Han and Kwon 2005). MPC solves a finite-horizon optimization problem iteratively and thus repeatedly computes the control inputs. In each iteration, only the control input for the current time slot is implemented on the system. This yields a new current model state which is optimized again, repeatedly.

Since the prediction horizon is shifted forward for each iteration, MPC is a Receding Horizon Control (RHC) problem. The RHC principle and iterative optimization process of MPC results in closed-loop feedback, which enables compensation of model uncertainties and gives robustness to external disturbances (Lawrence and Lee 1967).

The MPC method is successful in practice and is used routinely in distinct industries (Qin and Badgwell 2003). Moreover, MPC is proven to be very robust in practice, as it manages to optimize a given problem even with great disturbances (Lindborg and Ridenour 2018a).

In this thesis work, MPC will be used to solve a problem of the form

$$\begin{align*}
\text{minimize} & \quad \sum_{t=0}^{N-1} [(x_t - x_{r,t})^T Q_1 (x_t - x_{r,t}) + u_t^T Q_2 u_t] + (x_N - x_{r,N})^T Q_f (x_N - x_{r,N}) \\
\text{subject to} & \quad x_{t+1} = Ax_t + Bu_t + Dw_t, \quad t = 0, \ldots, N - 1, \\
& \quad C^x x_t \leq c^x_t, \quad t = 0, \ldots, N - 1, \\
& \quad C^u u_t \leq c^u_t, \quad t = 0, \ldots, N - 1, \\
& \quad C^f x_N \leq c^f_f,
\end{align*}$$

(3.1)

where

- $x_t \in \mathbb{R}^{n \times 1}$ are the state variables, $x_{r,t} \in \mathbb{R}^{n \times 1}$ are the desired reference state variables,
- $u_t \in \mathbb{R}^{m \times 1}$ are the control inputs and $w_t \in \mathbb{R}^{\omega \times 1}$ are the uncontrolled inputs, all at time $t$.
- $Q_1 \in \mathbb{R}^{n \times n}$ are cost weights placed on $x_t$ and $x_{r,t}$, $Q_2 \in \mathbb{R}^{m \times m}$ are cost weights placed on $u_t$ and $Q_f \in \mathbb{R}^{n \times n}$ are cost weights placed on the final states $x_N$ and $x_{r,N}$. $Q_1 \geq 0, Q_2 > 0, Q_f \geq 0$ are symmetric.
• $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $D \in \mathbb{R}^{n \times \omega}$ are matrices that describe the system; $A$ describes how the system develops without control, $B$ describes how the system reacts to control and $D$ describes how the state variables are affected by uncontrolled inputs.

• In addition there are time-dependent constraints on the state ($C^x \in \mathbb{R}^{\nu \times n}$ and $c^x_t \in \mathbb{R}^{\nu \times 1}$), the control signal ($C^u \in \mathbb{R}^{\mu \times m}$ and $c^u_t \in \mathbb{R}^{\mu \times 1}$) and the final state ($C^f \in \mathbb{R}^{\nu \times n}$ and $c^f_t \in \mathbb{R}^{\nu \times 1}$).

• $N$ is the time horizon with which the optimization is performed.

The objective function of (3.1) penalises deviation of the states $x_t$ from the desired reference states $x_{r,t}$. In conclusion, given constraints, one wants to find a control sequence $u$ that minimizes the quadratic objective function for given state and control costs as defined by the weights $Q_1, Q_2$ and $Q_f$.

For further reading on problem (3.1) see Hovd (2004), Lindquist and Sand (2010) and Jönsson (2010).

### 3.1.1 MPC stability

Closed-loop stability of MPC can be attained by proper choice of the terminal weight penalty $Q_f$ to reflect future objective cost in (3.1) (Angeli et al., 2012) (Kenneth and Rawlings, 1993). However, more recent studies show that closed-loop stability of MPC can be achieved without the use of the terminal weight penalty. With some assumptions on controllability, which are guaranteed for most linearly constrained systems (Boccia et al., 2014), asymptotic stability can be attained by sufficiently large choice of prediction horizon length $N$ (Boccia et al., 2014) (Grüne, 2012) (Grüne et al., 2010).

In this thesis work, long prediction horizons $N$ will be used together with proper choice of values and constraints in the optimization model to ensure stability.
Chapter 4

Model

In this section a mathematical model for the smart grid in Ramsjöåsen is defined and presented. The model is an expansion of a previously defined (by the thesis provider Ngenic) model. The mathematical model considers

- Total power from the electrical substation in Ramsjöåsen, i.e. all eight power lines consisting of 64 households, street lightning and a recycling station.
- One Energy Storage Module (ES) battery of type $Li(FePO_4)$ and size 7.2 kWh.
- One solar PV collective consisting of 60 PVs with 37 kW capacity in total.
- One Tune collective with 2 kW operating power per Tune.
- Four electrical vehicles (EVs), where one EV has a maximum charge rate of 20 kW and 3 EVs have a maximum charge rate of 3.5 kW.

The Tune collective denotes the homeowners’ Ngenic Tunes in aggregate. The same applies for the PV collective.

4.1 Variables

The state vector variables $x_t$ denote the total power [$kW$] or total energy [$kWh$] at time $t$ and $x_{r,t}$ are the respective desired reference states. The $x_t$ variables are defined as:

$$x_t = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}_t = \begin{bmatrix} \text{Charge Level of ES Battery [kWh]} \\ \text{Charge Level of EV 1 [kWh]} \\ \text{Charge Level of EV 2 [kWh]} \\ \text{Charge Level of EV 3 [kWh]} \\ \text{Charge Level of EV 4 [kWh]} \\ \text{Operating Power of Tune collective [kW]} \\ \text{Sum of Power [kW]} \end{bmatrix}_t,$$  

(4.1)

and the $x_{r,t}$ variables are defined analogously.

\footnote{Note that this notation is slightly imprecise. The subscript $t$ denotes the time and the subscripts $i = 1, \ldots, 7$ denote different smart-grid components.}
The control input variables $u_t$ denote the control action [kW] taken at time $t$ and are thus given by

$$ u_t = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}_t = \begin{bmatrix} \text{Power to and from ES Battery} \\ \text{Power to EV 1} \\ \text{Power to EV 2} \\ \text{Power to EV 3} \\ \text{Power to EV 4} \\ \text{Power to Tune-collective} \end{bmatrix}_t. \quad (4.2) $$

The input variables $w_t$ denote the uncontrolled inputs of solar production [kW] and total consumption [kW] (i.e. background household usage, power to street lightning and power to recycling station) at time $t$ and are defined as

$$ w_t = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}_t = \begin{bmatrix} \text{PV Power Production} \\ \text{Power Consumption} \end{bmatrix}_t. \quad (4.3) $$

This thesis will consider the impact of each EV alone and hence the EVs are treated separately and not as a collective. Moreover, $x_6$ denotes the Tune collective’s deviation (in power) from total power used by the heat pumps. The total power used by the heat pumps is in turn contained in $w_t$.

Further, as $x_7$ is the sum of all power at each time $t$, $x_7$ enables peak load investigation.

### 4.2 Dynamic Equations

The system evolution and system dynamics are represented by

$$ x_{t+1} = Ax_t + Bu_t + Dw_t, \quad t = 0, \ldots, N - 1, $$

$$ A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{\tau}{60} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\tau}{60} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\tau}{60} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\tau}{60} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tau}{60} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\tau}{60} & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, $$

where $\tau$ is the time-step length, $A$ describes how the system develops without control $u_t$, $B$ describes how the system reacts to control $u_t$ and $D$ describes how the state variables are affected by uncontrolled inputs $w_t$. Further,

$$ x_{t+1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}_{t+1} = \begin{bmatrix} x_1 + \frac{\tau}{60} u_1 \\ x_2 + \frac{\tau}{60} u_2 \\ x_3 + \frac{\tau}{60} u_3 \\ x_4 + \frac{\tau}{60} u_4 \\ x_5 + \frac{\tau}{60} u_5 \\ x_6 + u_6 \\ u_7 + u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + w_1 + w_2 \end{bmatrix}_t. $$

In this mathematical model it is assumed that the power to (and from) the ES battery and to the EV is fully transformed into energy, without any loss. Namely, 1 kW per time-step length charges the battery with $\frac{\tau}{60}$ kWh and hence summations as e.g. $x_1 + \frac{\tau}{60} u_1$ are valid.

---

2The term $\frac{\tau}{60} u_i$, $i = 1, \ldots, 6$, converts the unit to kWh, $\forall \tau.$

---

13
4.3 Time-step Constraints

The state variables $x_i, i = 1, \ldots, 5$, are at each time $t$ constrained by

$$0 \leq x_1 \leq ES_{\text{capacity}},$$

$$0 \leq x_i \leq EV_{i, \text{capacity}}, \quad i = 2, 3, 4, 5,$$

since $x_i, i = 1, \ldots, 5$, represent the charge levels of the ES battery and EVs, respectively. Further, $x_6$ denotes the operating power of the Tune collective, and hence

$$0 < Tune_{\text{min capacity}} \leq x_6 \leq Tune_{\text{max capacity}}.$$

As $x_7$ is the sum of all power, no constraints need to be defined for it. Reformulated on vector form, the state constraints are

$$C^x x_t \leq c^x_t, \quad t = 0, \ldots, N.$$

The constraints will vary depending on different scenarios (presented in Section 5.2) and thus, $C^x$ and $c^x_t$ are scenario-dependent parameters.

Correspondingly, constraints are placed on the control inputs $u_t$. As $u_1$ is the power to and from the ES battery (charging and discharging speed)

$$ES_{\text{min power}} \leq u_1 \leq ES_{\text{max power}},$$

where

$$ES_{\text{min power}} < 0, \quad ES_{\text{max power}} > 0.$$

However, $u_i, i = 2, \ldots, 5$, denote the power to the respective EV and therefore

$$0 \leq u_i \leq EV_{i, \text{max power}}, \quad i = 2, 3, 4, 5.$$

Furthermore, since $u_6$ denotes the power to the Tune collective

$$Tune_{\text{min power}} \leq u_6 \leq Tune_{\text{max power}},$$

where

$$Tune_{\text{min power}} < 0, \quad Tune_{\text{max power}} > 0.$$

---

3 Power can both be drawn from and given to the ES battery, whereas power is only given to the EVs.

4 The operating power of the Tune collective $x_6 > 0$. Thus, power is only given to the Tune collective and not drawn from it. The constraint $Tune_{\text{min power}} \leq u_6 \leq Tune_{\text{max power}}$ regulates the amount of power to the Tune collective.
The control constraints in vector form are

\[
\begin{bmatrix}
1 & 0 & \ldots & 0 \\
-1 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 \\
0 & -1 & \ldots & 0 \\
\vdots & \ddots & \ddots & 0 \\
0 & 0 & \ldots & 1 \\
0 & 0 & \ldots & -1
\end{bmatrix}
\]

\[C^u u_t \leq c^u_t, \quad t = 0, \ldots, N - 1.\]

where \(C^u\) and \(c^u_t\) are parameters.

### 4.4 Objective Function

The objective is to minimize the total state cost and control cost, while penalising deviation of the states \(x_t\) from the desired reference states \(x_{r,t}\), namely\(^5\)

\[
\sum_{t=0}^{N} (x_t^T Q_1 x_t - 2x_t^T Q_1 x_{r,t}) + \sum_{t=0}^{N-1} u_t^T Q_2 u_t.
\]

The matrices \(Q_1\) and \(Q_2\) are costs placed on the states \(x_t\) and the control inputs \(u_t\), respectively\(^6\)

\[
Q_1 = \begin{bmatrix}
ES_{cost}^x & 0 & 0 & 0 \\
0 & EV_{cost}^x & 0 & 0 \\
0 & 0 & Tune_{cost}^x & 0 \\
0 & 0 & 0 & Peak_{cost}^x
\end{bmatrix}, \quad Q_2 = \begin{bmatrix}
ES_{cost}^u & 0 & 0 & 0 \\
0 & EV_{cost}^u & 0 & 0 \\
0 & 0 & Tune_{cost}^u
\end{bmatrix},
\]

\(Q_1\) and \(Q_2\) are parameters and will also depend on different scenarios. For instance, \(ES_{cost}^u\) affects how often the battery will be charged and discharged. A low \(ES_{cost}^u\) will render a volatile charging pattern, which in reality is inefficient as it wears out the battery. Further, \(Q_1 \geq 0, Q_2 > 0\) and symmetric, which ensures convexity.

Finally, the prediction horizon \(N\) and time-step length \(\tau\) will be chosen in accordance with the description in Section 3.1.1 to ensure stability.

\(^5\)This is the same objective as in (3.1). See Appendix A.
\(^6\)\(Q_f = Q_1\), as this system does not need to be penalised against a final state. See Section 3.1.1
4.5 Mathematical Model

The following optimization problem on the form of (3.1) is obtained.

\[
\begin{align*}
\text{minimize} \quad & \sum_{t=0}^{N} (x_t^T Q_1 x_t - 2x_t^T Q_1 x_{r,t}) + \sum_{t=0}^{N-1} u_t^T Q_2 u_t \\
\text{subject to} \quad & x_{t+1} = Ax_t + Bu_t + Dw_t, \quad t = 0, \ldots, N - 1, \\
& C^x x_t \leq c_t^x, \quad t = 0, \ldots, N, \\
& C^u u_t \leq c_t^u, \quad t = 0, \ldots, N - 1.
\end{align*}
\]

In the simulations, the problem above will be reformulated as a Quadratic Programming (QP) problem in the control inputs \( u_t \) only, see (A.5) in Appendix A. Application of MPC on the reformulated problem (A.5) with prediction length \( N \) and simulation length \( T \) is illustrated in Algorithm 1.

**Data:** \( x_0 \)

**Result:** \( U = (u_0(x_0), u_1(x_1), \ldots, u_{N-1}(x_{N-1})) \)

let \( t \leftarrow 0; \)

while \( t < T \) do

measure \( x_t; \)

if \( EV \) is plugged in the smart grid then

activate respective elements in reference state \( x_{r,t} \) and control matrix \( B; \)

else

deactivate respective elements in reference state \( x_{r,t} \) and control matrix \( B; \)

set respective elements in \( x_t \) to zero;

end

solve finite-horizon MPC problem on the form of (3.1) via the QP (A.5);

extract the obtained optimal control \( U^* = (u_t^*(x_t), u_{t+1}^*(x_t), \ldots, u_{t+N-1}^*(x_t)); \)

apply first element of \( U^*\): \( u_t = u_t^*(x_t) \);

let \( t \leftarrow t + 1; \)

end

**Algorithm 1:** MPC algorithm
Chapter 5

Method

This work intends to solve the problem at hand using open-source technologies. Therefore, all data processing is handled with Python programming language, mainly through the Python data analysis library Pandas. Additionally, the Python-based solver CVXOPT is used as it is suitable for solving convex QPs (CVXOPT User’s Guide 2018).

5.1 Data

Data for this thesis are provided by Upplands Energi AB and Chargestorm AB. Upplands Energi provided time series data of

- Total consumption power from the electrical substation in Ramsjöäsen
- PV power from their own solar power system

The data cover sample periods October 2017 and February 2018. These sample periods are chosen to compare one month with high peak loads (February) and one month with less exceptional peak loads (October). With added disturbances and units converted from W to kW, the total consumption and PV power data are used as the uncontrolled inputs $w_t$ in (4.3).

Chargestorm provided time series data of typical EV charge profiles. The data were provided with unit Ampere but are converted to kW with assumed voltage 230 V. A total of eight distinct charge profiles were provided. After performing an analysis of the data, three charge profiles are chosen to be part of the simulations. These comprise typical EV charging patterns. Namely, charging patterns which start with higher charging that later decreases, see Figure 5.1.

---

1 Upplands Energi’s solar power system is located close to Ramsjöäsen making the PV power data aligned with Ramsjöäsen’s weather observations.
Further a typical EV charging pattern is expected to start around four o’clock on weekdays and vary on weekends. Thus, assumptions on one-week’s charging behaviour are made. In Table 5.1 assumed start times for each simulated EV, for one week, are presented.

**Table 5.1:** Assumptions on one-week’s charging behaviour; EV charging start times.

<table>
<thead>
<tr>
<th></th>
<th>EV 1 (Charge Profile 1)</th>
<th>EV 2 (Charge Profile 2)</th>
<th>EV 3 (Charge Profile 1)</th>
<th>EV 4 (Charge Profile 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>16:00</td>
<td>16:30</td>
<td>16:45</td>
<td>17:00</td>
</tr>
<tr>
<td>Tuesday</td>
<td>16:05</td>
<td>16:25</td>
<td>16:50</td>
<td>16:55</td>
</tr>
<tr>
<td>Wednesday</td>
<td>15:55</td>
<td>16:35</td>
<td>16:40</td>
<td>17:05</td>
</tr>
<tr>
<td>Thursday</td>
<td>16:00</td>
<td>16:30</td>
<td>16:45</td>
<td>17:00</td>
</tr>
<tr>
<td>Friday</td>
<td>15:30</td>
<td>16:30</td>
<td>16:30</td>
<td>17:15</td>
</tr>
<tr>
<td>Saturday</td>
<td>12:00</td>
<td>15:00</td>
<td>17:00</td>
<td>17:00</td>
</tr>
<tr>
<td>Sunday</td>
<td>13:00</td>
<td>13:00</td>
<td>16:00</td>
<td>16:00</td>
</tr>
</tbody>
</table>

A time series data set is generated from Table 5.1 where each charge profile [kW] is inserted at corresponding start time. This time series is used in the simulations to compute the sum of power for the case when MPC is not applied, i.e. the sum of uncontrolled power (more about uncontrolled
power in Section 5.3). Furthermore, another data set is generated by converting the units to kWh in the previously mentioned data set. This data set is in turn used as the desired reference states \( x_{r,t} \), for \( x_2 - x_5 \), in (4.1). Finally, one wants to emphasise that the time-step length \( \tau \) is taken into account in all unit conversions.

### 5.2 Scenarios

#### 5.2.1 Standard Case

The issue at hand is investigated by simulating different scenarios. When defining the scenarios some assumptions are made. This section presents these assumptions and defines standard scenario values.

The ES battery’s maximal charging and discharging speeds are 6 kW and −6 kW, respectively. Charging speeds ranging between the interval \([-3, 3]\) kW are assumed to be beneficial and to not wear out the ES battery \([\text{Becker, 2018}]\). Therefore, ES battery charging speed is mostly kept within this interval but is allowed to deviate to higher charging speeds occasionally. Moreover, The ES battery capacity is 7.2 kWh \([\text{Ferroamp Elektronik, 2018b}]\).

Standard values and assumptions for the Tune collective are obtained by thesis supervisor (from Ngenic) Jonathan Ridenour. Ridenour obtained these values after an extensive analysis on the Tune’s behaviour. First, the operating power of one Tune is assumed to be 2 kW. Thereby, the desired reference states \( x_{r,t} \) for the operating power of the Tune collective, i.e. for \( x_6 \), are 2 kW · (Number of tunes). Second, \( x_6 \) is only allowed to deviate from its reference state with more than 3 kW between two and four times per day\(^2\). Other standard values for the Tune collective are presented in the bullet list below.

In the standard case the EVs are almost fully charged and only differ slightly from the charge profiles. This ensures that the peak loads are not shifted by simply not charging the EVs; one wishes to shift peak loads while still managing to charge the EVs. Additionally, EV capacities and charging speeds are obtained from the provided data.

As this work intends to optimally shift peak loads, the peak cost \( \text{Peak}_{x_{\text{cost}}} \) in each scenario is the maximal cost that does not interrupt desired scenario specific behaviour.

In summary, values for the standard case are:

- Time-step length \( \tau = 20 \) minutes
- Prediction length \( N = 5 \) days
- Simulation length \( T = \) one month
- 30 Tunes in Tune collective
- 60 PVs in PV collective
- 4 EVs (Charge Profile 1, 2, 1, 3)
- ES state cost \( ES_{x_{\text{cost}}} = 0 \) as there exists no desired reference charging pattern.

\(^2\) The reference states for \( x_1 \) and \( x_7 \) are set to zero.
• ES control cost $ES^u_{\text{cost}}$ is chosen so that charging speed $\in [3, -3]$ kW for most of time $T$.
• $ES_{\text{capacity}} = 7.2$ kWh
• $ES_{\text{max power}} = 6$ kW
• $ES_{\text{min power}} = -6$ kW
• EV state cost $EV^x_{\text{cost}}$ and control cost $EV^u_{\text{cost}}$ are chosen so that the EVs are almost fully charged. High $EV^x_{\text{cost}}$ in combination with low $EV^u_{\text{cost}}$ increases the EV charge level.
• $EV_{1,\text{capacity}} = 8.5$ kWh
• $EV_{2,\text{capacity}} = 14.5$ kWh
• $EV_{3,\text{capacity}} = 8.5$ kWh
• $EV_{4,\text{capacity}} = 4.5$ kWh
• $EV_{1,\text{max power}} = EV_{2,\text{max power}} = EV_{3,\text{max power}} = 3.5$ kW
• $EV_{2,\text{max power}} = 11$ kW
• Tune collective state cost $Tune^x_{\text{cost}}$ and control cost $Tune^u_{\text{cost}}$ are chosen so that $x_6$ deviates from $2$ kW $\cdot$ (Number of tunes) with more than $3$ kW between two and four times per day.
• $Tune_{\text{max power}} = 2$ kW $\cdot$ (Number of tunes) + (Number of tunes) $\cdot$ 1
• $Tune_{\text{min power}} = 2$ kW $\cdot$ (Number of tunes) + (Number of tunes) $\cdot$ (-1)
• $Tune_{\text{max capacity}} = Tune_{\text{max power}}$
• $Tune_{\text{min capacity}} = Tune_{\text{min power}}$

In the following sections each scenario is presented with respective values deviating from the standard case. The scenarios are deemed important and relevant as each scenario investigates a certain aspect of the difficulties of future smart grids (Lindborg and Ridenour, 2018b). As previously mentioned, EV charging requires high amounts of power and since a large increase in EV usage is expected, peak loads will increase. Thus, Scenario 1 and Scenario 3 investigate the affect of EV charging on the smart grid. Further, the ES battery is a smart-grid component with facilitating purposes and one wants to investigate the ES battery’s ability to alleviate the grid. Therefore, Scenario 3 investigates the affect of ES charging on the smart grid.

5.2.2 Scenario 1

Scenario 1 investigates how EV charging affects peak load-shifting and optimal electrical substation operation by altering the number of EVs in the smart grid. In Scenario 1, the following sub-scenarios are simulated over periods October 2017 and February 2018.

1A 0 EVs
1B 1 EV (Charge Profile 1)
1C 2 EVs (Charge Profile 1, 2)
1D 3 EVs (Charge Profile 1, 2, 1)
5.2.3 Scenario 2

Scenario 2 investigates how EV charging affects peak load-shifting and optimal electrical substation operation by modifying the EV charging speeds and charge levels. In Scenario 2, the subsequent sub-scenarios are simulated over periods October 2017 and February 2018.

2A Max EV charging speed, 0.3, low charge level (30%)
2B Max EV charging speed, 0.5, medium charge level (50%)
2C Max EV charging speed, 0.7, high charge level (70%)
2D Max EV charging speed, 1, full charge level (100%)

5.2.4 Scenario 3

Scenario 3 investigates how ES charging affects peak load-shifting and optimal electrical substation operation by altering the ES charging speeds. In Scenario 3, the sub-scenarios below are simulated over periods October 2017 and February 2018.

3A Very slow ES charging speed (standard ES\textsuperscript{u} \cdot 2)
3B Slow ES charging speed (standard ES\textsuperscript{u} \cdot 1.5)
3C Medium ES charging speed (standard ES\textsuperscript{u})
3D Fast ES charging speed (standard ES\textsuperscript{u} \cdot 0.5)
3E Very fast ES charging speed (standard ES\textsuperscript{u} \cdot 0.0001)

5.3 Simulation

The simulations are performed by running sub-scenario code-files. Each sub-scenario has its own Python code-file which is generated by altering a main code-file, consisting of the standard case scenario. Hence, a number of simulations are run for each scenario, e.g. in Scenario 1, the sub-scenarios 1A, . . . , 1E are simulated.

In addition to wanting to see how the different scenarios affect the smart grid, this work also wants to examine what impact different months have. Thus, all scenarios are simulated over periods October 2017 and February 2018 to compare one month with high peak loads (February) and one month with less exceptional peak loads (October).

After each simulation, two sets of results are saved: an uncontrolled and a controlled set of results. The uncontrolled case does not utilise all smart-grid components but merely adds total power consumption, PV power production (negative) and actual EV power (time series generated from charge profiles provided by Chargestorm) at each time $t$. The controlled case is the outcome of running an MPC simulation where all smart-grid components are utilised.
Further, a set of metrics are calculated and saved for every simulation. These metrics are presented in the following section.

## 5.4 Metrics

Three metrics are defined and used for the analyses of the simulation results. The defined metrics enable investigation of peak load-shifting and efficiency of electrical substation operation, in Ramsjöäsen when applying MPC. Hence, the metrics are defined in such a way that the desired purpose of this thesis can be attained through them. These three metrics are presented and defined below.

- **\( P_{\text{max}} \):** max power at the electrical substation in Ramsjöäsen
- **\( \phi \):** optimal electrical substation operation percentage, i.e. portion of total time that the electrical substation is operating within a beneficial interval
- **\( \bar{\sigma} \):** mean per half-hour standard deviation

Let \( X_t \) with \( t = 0, 1, 2, ..., T \) be a time-series resulting from an MPC simulation. Then

\[
P_{\text{max}} = \max \{X_t\},
\]

\[
\phi = \frac{\sum_{t=0}^{T} I(X_t \in \mathcal{O})}{T + 1},
\]

\[
\bar{\sigma} = \frac{\sum_{i=0}^{N} \sigma_i}{N + 1},
\]

where \( \mathcal{O} \) is the set of power values [\( kW \)] that lie within the beneficial range for the electrical substation in Ramsjöäsen, \( N \) is the number of half-hour intervals in \( T \), and

\[
\sigma_i = \sqrt{\frac{\sum_{t=0}^{T} (X_t - \bar{X}_t)^2}{T}}
\]

is the standard deviation of the \( i \)-th half-hour interval.

The electrical substation in Ramsjöäsen is oversized for its current load. Nevertheless, the question of keeping an electrical substation within a 20 % range of its \( kVA \)-rating is still relevant, as described in Section 2.2 and 2.4. Therefore, when analysing the results it is assumed that the electrical substation has a lower \( kVA \)-rating and that the power factor is 1, so that \( kVA = kW \). The beneficial range for the electrical substation in Ramsjöäsen is thus defined as \( (0.25, kVA \text{-rating} \cdot 1.2) \) with \( kVA \text{-rating} \) set to 200 \( kVA \) instead of its actual 630 \( kVA \)-rating.
Chapter 6

Results

6.1 Scenario 1

Scenario 1 investigates how peak load-shifting and optimal electrical substation operation are affected by the number of EVs connected to the smart grid in Ramsjöäsen. The results from Scenario 1 are summarised in Table 6.1 and Table 6.2.

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$ [kW]</th>
<th>$\phi$ [%]</th>
<th>$\bar{\sigma}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>Standard case</td>
<td>163.2231</td>
<td>99.2778</td>
</tr>
<tr>
<td></td>
<td>0 EVs</td>
<td>158.4362</td>
<td>99.2778</td>
</tr>
<tr>
<td></td>
<td>1 EV</td>
<td>158.4362</td>
<td>99.2778</td>
</tr>
<tr>
<td></td>
<td>2 EVs</td>
<td>158.4362</td>
<td>99.2778</td>
</tr>
<tr>
<td></td>
<td>3 EVs</td>
<td>158.4363</td>
<td>99.4444</td>
</tr>
<tr>
<td></td>
<td>4 EVs</td>
<td>158.4362</td>
<td>99.4444</td>
</tr>
<tr>
<td>Controlled</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Scenario 1 results, October 2017

Table 6.1 presents the results of Scenario 1 over sample period October 2017. From Table 6.1 one understands that the max power at the electrical substation in Ramsjöäsen $P_{\text{max}}$ decreases from approximately 163 kW in the uncontrolled case (when MPC is not used) to approximately 158 kW in the controlled cases (when MPC is used). It might appear that peak loads of equal size are shifted in the controlled cases as $P_{\text{max}}$ remains at 158 kW. Nevertheless, $P_{\text{max}}$ in the uncontrolled and controlled cases do not necessarily represent the same peak load; $P_{\text{max}}$ is a global maximum over the entire sample period. Additionally, adding EVs to the smart grid increases peak load magnitudes. This indicates that larger peak loads are shifted for each added EV; the peak loads are all shifted to 158 kW despite increasing peak load size.

Optimal operation percentage $\phi$ is high in all cases and stays at approximately 99%. Moreover, $\phi$ slightly increases when there are three or four EVs connected to the smart grid. Thus, the electrical substation operation is almost always within a beneficial interval and adding EVs even readily increases its performance.

Mean per half-hour standard deviation $\bar{\sigma}$ is higher in the controlled cases, when MPC is applied, compared to the uncontrolled case. $\bar{\sigma}$ also increases for each added EV. Namely, volatility rises in...
pursuance of peak load-shifting. In summary, for Scenario 1 over period October 2017, MPC manages to shift higher peak loads for each added EV while simultaneously maintaining beneficial electrical substation operation, however, at the price of increased volatility.

Table 6.2: Scenario 1 results, February 2018

<table>
<thead>
<tr>
<th></th>
<th>$P_{max}$ [kW]</th>
<th>$\phi$ [%]</th>
<th>$\bar{\sigma}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>336.5574</td>
<td>74.6377</td>
<td>6.8848</td>
</tr>
<tr>
<td>Controlled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 EVs</td>
<td>326.9809</td>
<td>77.8382</td>
<td>7.2853</td>
</tr>
<tr>
<td>1 EV</td>
<td>327.8893</td>
<td>77.7778</td>
<td>7.3361</td>
</tr>
<tr>
<td>2 EVs</td>
<td>330.3012</td>
<td>76.6908</td>
<td>7.4852</td>
</tr>
<tr>
<td>3 EVs</td>
<td>332.4623</td>
<td>76.2681</td>
<td>7.6381</td>
</tr>
<tr>
<td>4 EVs</td>
<td>332.6405</td>
<td>76.0870</td>
<td>7.7146</td>
</tr>
</tbody>
</table>

Table 6.2 presents the results of Scenario 1 over sample period February 2018 and shows that $P_{max}$ is lower when MPC is used. Furthermore, $P_{max}$ increases for each added EV. Thus, the max power at the electrical substation in Ramsjöåsen increases with increased number of EVs in the smart grid.

The optimal operation percentage $\phi$ is greater when MPC is utilised. Contrary to the results from Table 6.1, $\phi$ is generally rather low and further decreases for each added EV.

Mean per half-hour standard deviation $\bar{\sigma}$ is greater when MPC is applied and also increases as EVs are added to the smart grid.

The overall outcome of Scenario 1 over period October 2017 is better than over period February 2018. For Scenario 1 in February 2018, MPC manages to shift peak loads but not equalise $P_{max}$ while the number of EVs increase. Additionally, the electrical substation operates less within a beneficial interval in February 2018 and adding EVs decreases its performance. Nevertheless, peak loads are shifted at the price of higher volatility in both periods.

The results of Scenario 1 are graphed on the following page in Figures 6.1, 6.2 and 6.3. In the graphs, the results of the controlled case are illustrated together with its uncontrolled reference, which denotes the result of the uncontrolled standard case when MPC is not used.
Figure 6.1: Max power $P_{max}$, Scenario 1.

Figure 6.2: Optimal electrical substation operation percentage $\phi$, Scenario 1.

Figure 6.3: Mean per half-hour standard deviation $\bar{\sigma}$, Scenario 1.
6.2 Scenario 2

Scenario 2 investigates how peak load-shifting and optimal electrical substation operation are affected by EV charge speed and charge level. The results from Scenario 2 are summarised in Table 6.3 and Table 6.4.

Table 6.3: Scenario 2 results, October 2017

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$ [kW]</th>
<th>$\phi$ [%]</th>
<th>$\bar{\sigma}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>Standard case</td>
<td>163.2231</td>
<td>99.2778</td>
</tr>
<tr>
<td>Controlled</td>
<td>Low EV charge level</td>
<td>158.1918</td>
<td>99.3333</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>5.3206</td>
</tr>
<tr>
<td></td>
<td>Medium EV charge level</td>
<td>158.1919</td>
<td>99.3889</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>5.4143</td>
</tr>
<tr>
<td></td>
<td>High EV charge level</td>
<td>158.2690</td>
<td>99.4444</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>5.4644</td>
</tr>
<tr>
<td></td>
<td>Full EV charge level</td>
<td>159.3635</td>
<td>99.4444</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>5.8582</td>
</tr>
</tbody>
</table>

Table 6.3 shows that the outcomes of $P_{\text{max}}$ and $\phi$ are better when MPC is used. Moreover, $P_{\text{max}}$ and $\phi$ increase when EV charge speed and charge level rise. Hence, higher EV charge speed and charge level entail higher max power but also increased operating within a beneficial interval at the electrical substation in Ramsjöäsen.

Mean per half-hour standard deviation $\bar{\sigma}$ is greater when MPC is applied and also increases as EV charge speeds and charge levels increase.

Table 6.4: Scenario 2 results, February 2018

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$ [kW]</th>
<th>$\phi$ [%]</th>
<th>$\bar{\sigma}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>Standard case</td>
<td>336.5574</td>
<td>74.6377</td>
</tr>
<tr>
<td>Controlled</td>
<td>Low EV charge level</td>
<td>329.7966</td>
<td>76.8720</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>7.5540</td>
</tr>
<tr>
<td></td>
<td>Medium EV charge level</td>
<td>331.9757</td>
<td>76.6304</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>7.6924</td>
</tr>
<tr>
<td></td>
<td>High EV charge level</td>
<td>333.3510</td>
<td>76.2077</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>7.6914</td>
</tr>
<tr>
<td></td>
<td>Full EV charge level</td>
<td>333.5059</td>
<td>75.2415</td>
</tr>
<tr>
<td></td>
<td>and charge speed</td>
<td></td>
<td>7.7838</td>
</tr>
</tbody>
</table>

The behaviours of $P_{\text{max}}$ and $\bar{\sigma}$ indicated by Table 6.4 are similar to the behaviours of $P_{\text{max}}$ and $\bar{\sigma}$ indicated by Table 6.3. However, the increase in $P_{\text{max}}$ is greater in Table 6.4. Moreover, $\phi$ in Table 6.4 increases when MPC is utilised but contrary to the results from Table 6.3, $\phi$ is generally rather low and further decreases when EV charge speed and charge level increase.

As in Scenario 1, the overall outcome of Scenario 2 over period October 2017 is better than over period February 2018. Further, when comparing the results of Scenario 1 and Scenario 2 it is clear
that reducing the number of EVs facilitates more for the smart grid than reducing the EV charge level and charge speed.

The results of Scenario 2 are graphed on the following page in Figures 6.4, 6.5 and 6.6. In the graphs, the results of the controlled case are illustrated together with its uncontrolled reference, which denotes the result of the uncontrolled standard case when MPC is not used.
Figure 6.4: Max power $P_{\text{max}}$, Scenario 2.

Figure 6.5: Optimal electrical substation operation percentage $\phi$, Scenario 2.

Figure 6.6: Mean per half-hour standard deviation $\bar{\sigma}$, Scenario 2.
6.3 Scenario 3

Scenario 3 investigates how peak load-shifting and optimal electrical substation operation are affected by ES charge speed. The results from Scenario 3 are summarised in Table 6.5 and Table 6.6.

### Table 6.5: Scenario 3 results, October 2017

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$ [kW]</th>
<th>$\phi$ [%]</th>
<th>$\bar{\sigma}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard case</td>
<td>163.2231</td>
<td>99.2778</td>
<td>4.4682</td>
</tr>
<tr>
<td>Controlled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very slow ES charge speed</td>
<td>159.3039</td>
<td>99.3889</td>
<td>4.8991</td>
</tr>
<tr>
<td>Slow ES charge speed</td>
<td>158.8391</td>
<td>99.5000</td>
<td>5.0145</td>
</tr>
<tr>
<td>Medium ES charge speed</td>
<td>158.2690</td>
<td>99.4444</td>
<td>5.5133</td>
</tr>
<tr>
<td>Fast ES charge speed</td>
<td>157.9104</td>
<td>99.3333</td>
<td>6.2854</td>
</tr>
<tr>
<td>Very fast ES charge speed</td>
<td>156.7508</td>
<td>99.3333</td>
<td>7.5076</td>
</tr>
</tbody>
</table>

$P_{\text{max}}$ in Table 6.5 is reduced for higher ES charge speeds. It is essential to observe that in Scenario 3 a smart-grid component with facilitating purposes is adjusted as compared to Scenario 1 and Scenario 2 where aspects of the EVs, which burden the grid, are adjusted. Consequently, in Scenario 3 the magnitude of the peak loads does not vary depending on the sub-scenario. Hence, the conclusion that larger peak loads are shifted when ES charge speed increases can be drawn from Table 6.5.

$\phi$ increases until it reaches its peak when ES charge speed is "Slow" and decreases for higher charge speeds. Consequently, having a slower ES charge speed benefits optimal operation percentage of an electrical substation.

Similar to previous results, $\bar{\sigma}$ is larger when MPC is applied. However, in this scenario the rise in $\bar{\sigma}$ is more drastic; peak loads are shifted at a higher price of volatility.

### Table 6.6: Scenario 3 results, February 2018

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$ [kW]</th>
<th>$\phi$ [%]</th>
<th>$\bar{\sigma}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard case</td>
<td>336.5574</td>
<td>74.6377</td>
<td>6.8848</td>
</tr>
<tr>
<td>Controlled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very slow ES charge speed</td>
<td>332.0375</td>
<td>75.9662</td>
<td>7.4817</td>
</tr>
<tr>
<td>Slow ES charge speed</td>
<td>332.0785</td>
<td>76.2077</td>
<td>7.5509</td>
</tr>
<tr>
<td>Medium ES charge speed</td>
<td>332.6532</td>
<td>76.0870</td>
<td>7.7176</td>
</tr>
<tr>
<td>Fast ES charge speed</td>
<td>334.1522</td>
<td>75.8454</td>
<td>8.3297</td>
</tr>
<tr>
<td>Very fast ES charge speed</td>
<td>333.2003</td>
<td>75.7850</td>
<td>9.2015</td>
</tr>
</tbody>
</table>

29
$P_{\text{max}}$ in Table 6.6 increases as ES charge speed rises, contrary to $P_{\text{max}}$ in Table 6.5. Therefore, slower ES charge speeds entail larger peak load-shifting in February 2018 and the opposite is true in October 2017.

The outcomes of $\phi$ and $\bar{\sigma}$ are similar in Table 6.6 and Table 6.5; $\phi$ peaks when ES charge speed is "Slow" and the rise in $\bar{\sigma}$ is rather drastic.

The results of Scenario 3 are graphed on the following page in Figures 6.7, 6.8, and 6.9. In the graphs, the results of the controlled case are illustrated together with its uncontrolled reference, which denotes the result of the uncontrolled standard case when MPC is not used.
Figure 6.7: Max power $P_{\text{max}}$, Scenario 3.

Figure 6.8: Optimal electrical substation operation percentage $\phi$, Scenario 3.

Figure 6.9: Mean per half-hour standard deviation $\bar{\sigma}$, Scenario 3.
Chapter 7

Discussion

The results presented indicate that applying MPC on the smart grid in Ramsjöåsen reduces the max power and increases optimal operation percentage at the electrical substation. However, MPC also entails higher volatility for the electrical substation.

First, it is essential to observe that the max power $P_{\text{max}}$ is a global maximum over the entire sample period, i.e. it is the largest peak load at the electrical substation in Ramsjöåsen in either October 2017 or February 2018. Accordingly, the controlled $P_{\text{max}}$ may not occur at the same time-stamp as the uncontrolled $P_{\text{max}}$; the $P_{\text{max}}$’s do not necessarily represent the same peak load. Thus, when comparing $P_{\text{max}}$ in the controlled and uncontrolled case one cannot quantify the peak load-shifting. Namely, the comparison does not give knowledge about how much the peak loads are reduced by. The conclusion that can be drawn however, is that the maximum has been minimized. Hence, the largest peak load is reduced, which certainly is the most important peak to reduce.

Second, the way that the controlled and uncontrolled cases are defined further complicates the comparison of $P_{\text{max}}$ between the cases. The uncontrolled case does not utilise all smart-grid components but merely adds total consumption power, PV power (negative) and actual EV power (time series generated from charge profiles provided by Chargestorm) at each time $t$. Hence, no MPC control is applied and the sum of all power at time $t$ is

$$w_1 + w_2 + EV_{\text{actual}}^{\text{power}}.$$ 

So, in the uncontrolled case it is assumed that all PV power at time $t$ is converted without any loss and utilised instantaneously.

In contrast, the controlled case is the outcome of applying MPC and utilising all smart-grid components. Total power at time $t$ is the sum of power to all smart-grid components

$$x_7 = w_1 + w_2 + u_1 + u_2 + u_3 + u_4 + u_5 + u_6.$$ 

The calculation of the sum of power is case dependent ($t$ at $P_{\text{max}}$ will most likely be different for the two cases) and thus one cannot quantify the peak load-shifting by merely comparing the $P_{\text{max}}$’s presented in the results.

Third, it is also important to note that the objective function, over which the MPC optimizes, is a trade-off between peak load-shifting, EV charging, ES battery charging and operation of the Tune collective. Peak loads can only be shifted if it does not interfere with desired behaviour of the EVs,
ES battery and Tune collective. Thus, if the model cannot shift peak loads and simultaneously charge the EVs sufficiently, keep the ES charging within its beneficial interval and lessen Tune operation deviation, the outcome will be a trade-off between the previously mentioned. This is exemplified through the results of Scenario 1 and Scenario 3 in October 2017: The Scenario 3 results show that detrimental ES charging in October reduces $P_{\text{max}}$. Hence, $P_{\text{max}}$ in Scenario 1 probably remains at 158 kW (for all controlled sub-scenarios) and is not further reduced because that would require detrimental ES charging, which is not allowed in Scenario 1.

The results of the simulated scenarios show that the outcomes are worse in months with high peak loads (February 2018) compared to months with less exceptional peak loads (October 2017). The results of the scenarios in October 2017 have larger peak load-shifting, higher optimal operation percentage and lower volatility. This is probably because

1. The peak loads in October are generally much lower than in February.
2. There is much more solar radiation in October than in February.

Hence, more electricity can be stored in the ES battery and used subsequently to shift, the not so high, peak loads. The MPC will perform well in months with lower peak loads and higher solar radiation because the smart grid is not burdened. Moreover, the flexibility in heat pumps is more important in months like February since less solar power production entails less ES battery charging.

Scenario 1 clearly demonstrates how heavily EVs overload the smart grid. When comparing a burdened month like February with a month like October, there is a significant difference in how much peak loads the MPC manages to shift. With each added EV the pressure on the smart grid increases. During months like October it is possible to handle the additional EV load. However, peak load-shifting becomes difficult during months when power consumption is already high. Nevertheless, the MPC still manages to reduce the max power at the electrical substation in Ramsjöösen in all controlled sub-scenarios.

Scenario 2 demonstrates that decreasing the EV charge speed and charge level can alleviate the pressure on the smart grid. Furthermore, when comparing Scenario 2 with Scenario 1, it is clear that regulating the amount of EVs connected to the smart grid is more effective than regulating the charging speed and charge level of the EVs. The outcome of Scenario 1, especially in February 2018, is better than the outcome of Scenario 2; $P_{\text{max}}$'s are lower, $\phi$'s are higher and $\bar{\sigma}$'s are lower in Scenario 1.

Regarding Scenario 3 in February 2018, the MPC performs best for slower ES charge speeds; max power $P_{\text{max}}$ and volatility $\bar{\sigma}$ are at their lowest and optimal operation percentage $\phi$ is at its highest. However, $P_{\text{max}}$ in October 2017 decreases even though the ES charge speed rises. This is most likely due to the pressure on the smart grid being small in October, as explained above. Hence, high ES charging speeds are not useful when the smart grid is already heavily burdened.

Slower ES charge speeds are presumably better in February because it forces the MPC to plan more and prioritise which peak loads should be shifted. If the ES battery can be charged and discharged anyhow, the peak load-shifting will be more frequent but small. Slower charge speeds entail less frequent but larger peak load-shifts. Furthermore, slower ES charging speeds are also better for the battery lifetime.

Scenario 3 compared with Scenario 1 and Scenario 2, in February 2018, shows that the ES battery’s ability to alleviate the smart grid is small compared to the EV load. Namely, the load put on by the EVs is much greater than the ES’s ability to relieve the grid. Nevertheless, this result is expected
since, as mentioned, EVs actually burden the smart grid while the ES battery facilitates it. If what burdens the smart grid is removed then, obviously, the pressure on the grid will decrease.

All scenarios indicate that volatility rises in pursuance of peak load-shifting. The increase in volatility is particularly high in Scenario 3, when ES charging speed is regulated. Consequently, MPC shifts peak loads at the price of higher volatility. This increase in volatility might be related to the forested values used in the MPC. However, the impact of this increased volatility is unknown. Thus, how increased volatility affects the smart grid and the electrical substation in Ramsjöäsen needs further investigation.

7.1 Conclusion and Further Research

This thesis has shown that an MPC based smart-grid aggregator improves the performance of the smart grid in Ramsjöäsen; applying control on the system results in better performance, irrespective of how the smart grid is burdened. Further, this thesis indicates that an MPC based smart-grid aggregator improves Ramsjöäsen’s electrical substation operation. In summary, conclusions drawn from the results are that

- Applying MPC and control on the smart grid improves the outcome of $P_{max}$ and $\phi_{max}$; max power is reduced and optimal operation percentage is higher.
- Applying MPC and control on the smart grid increases $\bar{\sigma}$; MPC shifts peak loads at the price of higher volatility.
- Regulating the amount of EVs connected to the smart grid is more effective than regulating the charging speed and charge level of the EVs.
- The ES battery’s ability to alleviate the smart grid is small compared to the EV load.
- Slower ES charge speeds are favourable.

Proposals on how to maximize peak load-shifting are difficult to produce as problems regarding smart-grid efficiency are characterised by multiple conflicting perspectives. This is especially true when considering peak load-shifting on a macro level. It has been demonstrated how this thesis' micro scaled smart-grid solution is a trade-off between the different smart-grid components. When considering smart-grid efficiency on a macro scale however, one will have to take more and also completely distinct perspectives into account, e.g. the energy market or electricity pricing. What is most beneficial in one perspective will probably not be beneficial in the remaining perspectives.

However, this thesis supports future research by investigating on a micro level the necessary conditions to facilitate the more complex issues on a macro level. Consequently, this thesis serves as proof-of-concept and contributes with some theoretical basis for future peak load-shifting in smart grids and real-world implementations of an MPC based smart-grid aggregator.
Bibliography


Becker, A. Project manager at Ferroamp AB. Interview 9 April, 2018.


Lindborg, J. Project manager at Sustainable Innovation AB. Meeting 2 May, 2018.


Lindborg, J. and Ridenour, J. Project manager at Sustainable Innovation AB and thesis supervisor from Ngenic. Meeting 12 April, 2018b.


Appendix A

Complementary Mathematical Theory

A.1 Convexity

This thesis considers convex optimization problems as one can derive strong optimization conditions for such problems. An optimization problem

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad x \in F,
\end{align*}
\]

is convex if \( F \) is a convex set and \( f(x) \) is a convex function on \( F \) \cite{Sasane2012}. Convex optimization problems have the following properties: a stationary point of a convex function is necessarily a minimum point, any found local minimum point is also a global minimum point and first-order optimality condition is necessary and sufficient \cite{Griva2009}.

A subset \( C \subset \mathbb{R}^n \) is a convex set if for any two points \( x, y \) in \( C \), the line segment between \( x, y \) is also in \( C \). Thus, the subset \( C \subset \mathbb{R}^n \) is convex if

\[
(1 - t)x + ty \in C, \tag{A.1}
\]

for all \( x, y \in C \) and all \( t \in (0, 1) \).

Figure A.1: The left figure shows convex sets. The right figure shows non-convex sets \cite{Sasane2012}.
Assuming $C$ is a convex set, a function $f : C \rightarrow \mathbb{R}$ is a convex function if
\[
f((1-t)x + ty) \leq (1-t)f(x) + tf(y),
\] (A.2)
for all $x, y \in C$ and all $t \in (0,1)$; the line segment between $(x, f(x))$ and $(y, f(y))$ is always above the function graph of $f$ (Sasane and Svanberg, 2012).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{convex_function}
\caption{Convex function where $C = \mathbb{R}$ (Sasane and Svanberg, 2012).}
\end{figure}

\section*{A.2 Quadratic Programming}

\textit{Quadratic programming} (QP) is a specific kind of mathematical optimization problem where a quadratic function is optimized subject to linear constraints. A QP problem is presented by
\[
\begin{aligned}
\text{minimize} & \quad x^T Q x + 2q^T x + r \\
\text{subject to} & \quad a_i^T x \leq b_i, \quad i = 1, \ldots, m, \\
& \quad g_i^T x = h_i, \quad i = 1, \ldots, p,
\end{aligned}
\] (A.3)
where $Q \in \mathbb{R}^{n \times n}$ is symmetric, $q \in \mathbb{R}^n$ and $r$ is scalar. The linear constraints of (A.3) form a convex set, which is proved through utilisation of (A.1). Moreover, given (A.2) one can prove that quadratic functions are convex for positive semidefinite $Q$. Hence, (A.3) is a convex problem if $Q$ is positive semidefinite ($Q \geq 0$) (Sasane and Svanberg, 2012).

\section*{A.3 MPC Objective Function}

The objective of problem (3.1) is to minimize the total state cost and control cost, while penalising deviation of the states $x_t$ from the desired reference states $x_{r,t}$, i.e. at each time $t$ the objective is
\[
(x_t - x_{r,t})^T Q_1 (x_t - x_{r,t}) + u_t^T Q_2 u_t
\]

\footnote{The $Q_f$-term is not included in this demonstration, however, the reformulation of $(x_N - x_{r,N})^T Q_f (x_N - x_{r,N})$ is analogous to what is presented.}
Performing the matrix multiplications yields
\[ (x_t^T Q_1 - x_{r,t}^T Q_1)(x_t - x_{r,t}) + u_t^T Q_2 u_t = x_t^T Q_1 x_t - x_{r,t}^T Q_1 x_t - x_t^T Q_1 x_{r,t} + x_{r,t}^T Q_1 x_{r,t} + u_t^T Q_2 u_t. \]
The term \( x_{r,t}^T Q_1 x_{r,t} \) can be omitted as the problem is minimized with respect to \( x_t \), and thus the objective at time \( t \) is
\[ x_t^T Q_1 x_t - 2x_t^T Q_1 x_{r,t} + u_t^T Q_2 u_t. \]

### A.4 Reformulation of MPC problem

Problem (3.1) can be reformulated as a QP in the control inputs \( u_t \) only, which will be demonstrated in this section. Given the constraint
\[ x_{t+1} = A x_t + B u_t + D w_t, \]
future system states \( x_t \) can be predicted using the given initial state \( x(0) = x_0 \). The predictions \( x_t \) will be linear in \( x_0, u \) and \( w \). Thus, by letting \( X = (x_0, x_1, \ldots, x_N) \), \( X \) will be linear in \( x_0, U = (u_0, u_1, \ldots, u_{N-1}) \) and \( W = (w_0, w_1, \ldots, w_{N-1}) \).

\[ x_0 = \text{given}, \]
\[ x_1 = A x_0 + B u_0 + D w_0, \]
\[ x_2 = A x_1 + B u_1 + D u_1 = A^2 x_0 + A B u_0 + A D w_0 + B u_1 + D w_1, \]
\[ \vdots \]
\[ x_N = A^N x_0 + \sum_{t=0}^{N-1} [A^t B u_{(N-1-t)} + A^t D w_{(N-1-t)}]. \]

This system of equations can be expressed in matrix form
\[
\begin{bmatrix}
  x_0 \\
  x_1 \\
  x_2 \\
  \vdots \\
  x_N
\end{bmatrix}
= \begin{bmatrix}
  0 & 0 & \ldots & 0 \\
  B & 0 & \ldots & 0 \\
  A B & B & \ldots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  A^{N-1} B & A^{N-2} B & \ldots & B
\end{bmatrix}
\begin{bmatrix}
  u_0 \\
  u_1 \\
  u_2 \\
  \vdots \\
  u_{N-1}
\end{bmatrix}
+ \begin{bmatrix}
  0 & 0 & \ldots & 0 \\
  D & 0 & \ldots & 0 \\
  A D & D & \ldots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  A^{N-1} D & A^{N-2} D & \ldots & D
\end{bmatrix}
\begin{bmatrix}
  w_0 \\
  w_1 \\
  w_2 \\
  \vdots \\
  w_{N-1}
\end{bmatrix}
+ \begin{bmatrix}
  I \\
  A \\
  \vdots \\
  A^N
\end{bmatrix}
x_0,
\]
i.e.
\[ X = G_u U + G_w W + H x_0. \]

Further, the state and control constraints of (3.1) become
\[
\begin{bmatrix}
  C^x \\
  \vdots \\
  0 \\
  0 & \cdots & 0 \\
  0 & \cdots & 0 & C^f
\end{bmatrix}
X \leq \begin{bmatrix}
  c^x \\
  \vdots \\
  c^x \\
  c^x \\
  c^f
\end{bmatrix},
\]
\[
\begin{bmatrix}
  C^u \\
  \vdots \\
  0 \\
  0 & \cdots & 0 \\
  0 & \cdots & 0 & C^u
\end{bmatrix}
U \leq \begin{bmatrix}
  c^u \\
  \vdots \\
  c^u \\
  c^u \\
  c^u
\end{bmatrix}.
\]
Thus, 
\[ \bar{C}^x X \leq \bar{c}^x, \quad \bar{C}^u U \leq \bar{c}^u. \]

The objective function of (3.1) is reformulated analogously.
\[
\sum_{t=0}^{N-1} [x_t^T Q_1 x_t - 2x_t^T Q_1 x_{r,t} + u_t^T Q_2 u_t] + x_N^T Q_f x_N - 2x_N^T Q_f x_{r,N} = \\
= X^T \bar{Q}_1 X - 2X^T \bar{Q}_1 X_r + U^T \bar{Q}_2 U,
\]

where
\[
\bar{Q}_1 = \begin{bmatrix}
Q_1 & 0 & \cdots & 0 \\
0 & \ddots & 0 & \vdots \\
\vdots & 0 & Q_1 & 0 \\
0 & \cdots & 0 & Q_f
\end{bmatrix}, \quad \bar{Q}_2 = \begin{bmatrix}
Q_2 & 0 & \cdots & 0 \\
0 & \ddots & 0 & \vdots \\
\vdots & 0 & Q_2 & 0 \\
0 & \cdots & 0 & Q_2
\end{bmatrix}.
\]

Consequently, QP problem (A.4) in X and U is obtained.
\[
\begin{align*}
\text{minimize} & \quad X^T \bar{Q}_1 X - 2X^T \bar{Q}_1 X_r + U^T \bar{Q}_2 U \\
\text{subject to} & \quad X = G_u U + G_w W + Hx_0, \\
& \quad \bar{C}^x X \leq \bar{c}^x, \\
& \quad \bar{C}^u U \leq \bar{c}^u.
\end{align*}
\]

To obtain a QP in U only, the equality constraint of (A.4) is inserted into both the objective function and state inequality constraint of (A.4), resulting in QP (A.5).
\[
\begin{align*}
\text{minimize} & \quad U^T \bar{P} U + 2U^T \bar{q} \\
\text{subject to} & \quad \bar{C} U \leq \bar{c},
\end{align*}
\]

where
\[
\bar{P} = G_u^T \bar{Q}_1 G_u + \bar{Q}_2, \\
\bar{q} = G_u^T \bar{Q}_1 Hx_0 + G_u^T \bar{Q}_1 G_w W - G_u^T \bar{Q}_1 X_r, \\
\bar{\bar{C}} = \begin{bmatrix}
\bar{C}^x G_u \\
\bar{C}^u
\end{bmatrix}, \\
\bar{\bar{c}} = \begin{bmatrix}
\bar{c}^x - \bar{\bar{C}}^x Hx_0 - \bar{\bar{C}}^x G_w W \\
\bar{c}^u
\end{bmatrix}.
\]

Application of MPC on problem (A.5) with prediction length N and simulation length T is illus-
Data: $x_0$
Result: $U = (u_0(x_0), u_1(x_1), \ldots, u_{N-1}(x_{N-1}))$

let $t \leftarrow 0$
while $t < T$ do
  measure $x_t$
  solve finite-horizon MPC problem (3.1) via the QP (A.5);
  extract the obtained optimal control $U^* = (u^*_t(x_t), u^*_{t+1}(x_t), \ldots, u^*_{t+N-1}(x_t))$
  apply first element of $U^*$: $u_t = u^*_t(x_t)$
  let $t \leftarrow t + 1$
end

Algorithm 2: MPC algorithm

A.4.1 Convexity of Reformulated MPC

The QP problem (A.5) is convex if the symmetric matrix $\bar{P}$ is positive semidefinite; $\bar{P} \geq 0$. This puts certain requirements on (3.1), which will be presented in this section.

The sum of symmetric matrices is a symmetric matrix. Likewise, the sum of positive semidefinite matrices is a positive semidefinite matrix (Norman and Wolczuk 2012). Consequently, as $\bar{P} = G_u^T \bar{Q}_1 G_u + \bar{Q}_2$, one requires both $G_u^T \bar{Q}_1 G_u$ and $\bar{Q}_2$ to be symmetric and positive semidefinite.

First,

$$\bar{Q}_2 = \begin{bmatrix} Q_2 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & Q_2 & 0 \\ 0 & \cdots & 0 & Q_2 \end{bmatrix}$$

is obviously symmetric. Since $\bar{Q}_2$ is a diagonal matrix, $\bar{Q}_2 \geq 0$ if and only if (iff) its diagonal elements $Q_2 \geq 0$.

Second, $G_u^T \bar{Q}_1 G_u$ results in a symmetric matrix (can be proved by performing the matrix multiplication). Positive semidefiniteness of $G_u^T \bar{Q}_1 G_u$ is proved by utilising the following: a symmetric matrix $A \in \mathbb{R}^{n \times n}$ is positive semidefinite if $x^T Ax \geq 0$ for all non-zero $x \in \mathbb{R}^n$ (Norman and Wolczuk 2012) (Sasane and Svanberg 2012). So, for some non-zero vector $v \in \mathbb{R}^n$, let

$$v^T G_u^T \bar{Q}_1 G_u v = (G_u v)^T \bar{Q}_1 (G_u v) = (G_u v)^T \begin{bmatrix} Q_1 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & Q_1 & 0 \\ 0 & \cdots & 0 & Q_f \end{bmatrix} (G_u v).$$

Since $(G_u v)$ is a vector, $(G_u v)^T \bar{Q}_1 (G_u v) \geq 0$ when $\bar{Q}_1 \geq 0$. Additionally, $\bar{Q}_1$ being a diagonal matrix entails $\bar{Q}_1 \geq 0$ iff its diagonal elements $Q_1 \geq 0$, $Q_f \geq 0$.

In summary, if $\bar{P}$ is positive semidefinite and symmetric then (A.5) is a convex QP. This in turn requires $Q_1, Q_2$ and $Q_f$ in (3.1) to be positive semidefinite and symmetric. As $Q_1 \geq 0, Q_2 > 0, Q_f \geq$
0 are symmetric in (3.1), the convexity requirements of $\tilde{P}$ are already fulfilled\footnote{The reformulation of (A.4) to (A.5) does not have an effect on convexity since the reformulation entails looking at a subset of the convex set in (A.4).}

As regards to (3.1) this means that all costs, i.e. all elements in the Q-matrices, must be non-negative ($Q_1$) or positive ($Q_2$). Further, any cross-costs, i.e. non-diagonal elements in the Q-matrices, must be defined symmetrically; e.g. if the state of EV 1 is to penalise the state of EV 2, then the state of EV 2 must equally penalise the state of EV 1.