Interest Rate Smoothing versus Serially Correlated Errors in Taylor Rules: Testing the Tests

Peter Welz and Pär Österholm
INTEREST RATE SMOOTHING VERSUS SERIALLY CORRELATED ERRORS IN TAYLOR RULES:
TESTING THE TESTS

PETER WELZ AND PÅR ÖSTERHOLM
Interest Rate Smoothing versus Serially Correlated Errors in Taylor Rules: Testing the Tests∗

Peter Welz† and Pär Österholm‡

This version 31 March 2005

Abstract

This paper contributes to the recent debate about the estimated high partial adjustment coefficient in dynamic Taylor rules, commonly interpreted as deliberate interest rate smoothing on the part of the monetary authority. We argue that a high coefficient on the lagged interest rate term may be a consequence of an incorrectly specified central bank reaction function. Focusing on omitted variables, our Monte Carlo study first generates the well-known fact that all coefficients in the misspecified equation are biased in such cases. In particular, if relevant variables are left out from the estimated equation, a high partial adjustment coefficient is obtained even when it is in fact zero in the data generating process. Misspecification also leads to considerable size distortions in two tests that were recently proposed by English, Nelson, and Sack (2003) in order to distinguish between interest rate smoothing and serially correlated disturbances. Our results question the common interpretation of very slow partial adjustment as interest rate smoothing in estimated dynamic Taylor rules.

JEL classifications: C12, C15, E52

Keywords: Monetary policy, Taylor rule, Interest rate smoothing, Serially correlated error term, Omitted variables

∗We are grateful to Annika Alexius, Nils Gottfries, Rolf Larsson and seminar participants at Sveriges Riksbank and Uppsala University for valuable comments on this paper. Financial support from the Jan Wallander and Tom Hedelius foundation and Sparbankernas forskningsstiftelse is gratefully acknowledged.

†Department of Economics, Uppsala University, Box 513, 751 20 Uppsala, Sweden. e-mail: peter.welz@econ.uu.se

‡Department of Economics, Uppsala University, Box 513, 751 20 Uppsala, Sweden. e-mail: par.osterholm@nek.uu.se
1 Introduction

For nearly a decade, the Taylor (1993) rule has been unquestioned as a good tool for monetary policy evaluation.\footnote{For empirical analyses see for instance Clarida, Galí and Gertler (1998, 2000), Taylor (1999), Gerlach and Schnabel (2000), Huang, Margaritis, and Mayes (2001) and Orphanides (2001). For some theoretical work see for instance Levin, Wieland, and Williams (1999), Rudebusch (2002a) and Leitemo and Söderström (2004).} Notwithstanding this achievement, a number of recent writers have raised serious criticisms against the Taylor rule, Rudebusch (2002b) being among the most influential. Using term structure evidence, he shows that a Taylor rule with partial adjustment, which is interpreted as ‘interest rate smoothing’ by the central bank, implies much more interest rate predictability than can be found in the data. By contradicting a large part of the existing literature, Rudebusch also points to the implausibility of very slow partial adjustment, or equivalently, a high degree of interest rate smoothing in quarterly data, since this would imply that the central bank only closes a small fraction of the gap between the actual and the desired target interest rate each quarter\footnote{As we will discuss in the main text, a very common estimate in the literature suggests that the central bank closes half of the gap in about a year.}. He suggests that the estimated degree of interest rate smoothing found in most studies instead is due to serially correlated errors caused by ‘appropriate response[s] to special circumstances’ which are not captured by the variables in the Taylor rule.

Though almost observationally equivalent, interest rate smoothing and serially correlated errors lead apart from the economic interpretation to different statistical implications. English, Nelson, and Sack (2003) use this fact to develop two tests that distinguish between the two cases, and in applying their tests to U.S. data, they, as well as Castelnuovo (2003a, 2003b), find support for interest rate smoothing, thereby questioning Rudebusch’s claim.

The purpose of this paper is to further investigate the reasons for the finding of strong interest rate smoothing in estimated Taylor rules. Focusing on omitted variables as a likely cause, we use a Monte Carlo study that builds the data generating process on a New Keynesian macro model in order to investigate the bias in the coefficients of the
Taylor rule and the size properties of the English, Nelson, and Sack (2003) (ENS) tests when the estimated reaction function is misspecified. As expected\textsuperscript{3}, estimating a standard Taylor rule with inflation and output gap as only explanatory variables when the central bank’s true reaction function contains additional persistent variables results in severely biased coefficient estimates, and the ENS tests are found to massively overreject the null hypothesis of no interest rate smoothing at conventional significance levels.

The methodological setup in this paper takes its theoretical starting point in the critique put forward by Svensson (2003, 2004) that the Taylor rule is unlikely to be the solution to a central bank’s optimisation problem, in which the objective is to stabilise inflation and the output gap. If there are important state variables other than inflation and the output gap, the rule will not be optimal; instead, the number of response coefficients that need to be fixed in the central bank’s reaction function would have to be increased in accordance with the number of state variables in the economy. Put differently, inflation deviations from an inflation target and the output gap are unlikely to be sufficient statistics for the state of the economy and, hence, for the characterisation of central bank behaviour. The claim by Svensson also seems to have empirical relevance; for instance Österholm (2005) points out that cointegration is a necessary condition – both for consistent estimation of parameters and compatibility between the theoretical model and the data – given the highly persistent variables in the Taylor rule. Österholm concludes that the Taylor rule is likely to be misspecified, since a cointegrating relationship between nominal interest rates, inflation and the output gap finds very little support in that study.

As the results in this paper show that the recently developed tests by English, Nelson, and Sack (2003) seem unreliable in distinguishing between interest rate smoothing and serially correlated errors, we argue that there are strong reasons to question the traditional interpretation of a slow partial adjustment towards the target rate as a high degree of interest rate smoothing. Our results provide a credible explanation for the empirical

\textsuperscript{3}An early source is Grilliches (1961).
phenomenon of strong interest rate smoothing. They are in line with the findings in Rudebusch (2002b), and they support Svensson’s (2003, 2004) theoretical view that central banks do take a wide selection of economic variables into consideration in their decision making process.\(^4\)

This paper is organised as follows. Section two presents the Taylor rule and Section three the framework behind the ENS test. Section four describes the macroeconomic model and the data generating process (DGP) used in the Monte Carlo study. In Section five, the results of the Monte Carlo study using a number of different DGPs are discussed. Finally, Section six concludes with a brief discussion of the findings in this study.

2 The Taylor rule

To organise the discussion we first give an account of the Taylor rule and then provide a brief but necessarily incomplete overview of the theoretical and empirical literature on interest rate smoothing. For a more thorough exposition, see for instance Taylor (1999).

2.1 Basic specification and empirical evidence

The original formulation of the Taylor (1993) rule is given by

\[ i_t = r^* + \pi_t + f_\pi (\pi_t - \pi^*) + \phi_y y_t, \tag{1} \]

where \( i_t \) is the central bank policy rate, \( r^* \) is the equilibrium real interest rate, \( \pi_t \) the twelve month inflation rate, \( \pi^* \) the inflation target of the central bank and \( y_t \) the output gap. Based on calibration, Taylor found that a rule with the parameters set to \( r^* = \pi^* = 2 \) and \( f_\pi = \phi_y = 0.5 \), tracked the actual federal funds rate fairly well between 1987 and 1992.

\(^4\)This viewpoint also finds support among practitioners. As for instance Ben S. Bernanke (2004), Member of the Board of Governors of the US Federal Reserve System, remarked: ‘..., my forecast of controlled inflation is based on more than output gap arguments. Other factors likely to keep inflation at modest levels include continuing rapid gains in productivity, which have kept growth of unit labor costs at a very low level; unusually high price-cost margins in industry, which provide scope for firms to absorb future cost increases without raising prices; globalization and intensified competition in product markets; and the recent strengthening of the dollar.’
Note that the equilibrium real interest rate as well as the inflation target are assumed to be constant here. This fact will be taken up later on.

Adding an error term and collecting constants in the intercept, equation (1) can be reformulated as

\[ i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \varepsilon_t, \]

where \( \phi_0 \equiv r^* - (\phi_\pi - 1) \pi^* \) and \( \phi_\pi \equiv (1 + f_\pi) \). The rule in equation (2), or versions thereof allowing for forward looking behaviour, has generally been the starting point in the empirical literature.\(^5\)

In this literature it has also been shown that adding a lagged interest rate term such that

\[ i_t = (1 - \lambda) \left( \phi_0 + \phi_\pi \pi_t + \phi_y y_t \right) + \lambda i_{t-1} + \varepsilon_t, \]

where \( 0 \leq \lambda < 1 \), improves the empirical fit considerably. The lagged interest rate term is commonly interpreted as deliberate interest rate smoothing on the part of the monetary authority. Note also that equation (3) has a partial adjustment structure in which the term \( \tilde{i}_t \equiv \phi_0 + \phi_\pi \pi_t + \phi_y y_t \) is the target interest rate that depends on the state of the economy and that the central bank attempts to achieve over time.

As an example of a typical finding in the empirical literature, we present the results from least squares regressions of the static equation (2) and the dynamic equation (3) using quarterly U.S. data on the federal funds rate, CPI inflation and output gap from 1987Q1 to 1999Q4.\(^6\) Newey-West standard errors are reported in parentheses, the results for the constants have been neglected here for brevity.

---

\(^5\)There is a fairly extensive discussion in the literature regarding the timing of the explanatory variables in the Taylor rule. For instance McCallum and Nelson (1999) have argued that the central bank should react to lagged values of inflation and the output gap due to informational delays. Another suggestion has been to use forecasts of the regressors in order to capture the potentially forward looking behaviour of central banks. For a study in which several different approaches are taken to deal with this issue, see Orphanides (2001). The timing issue is not important in this study though. As the data is generated, some issues regarding informational delays can be dealt with by assumptions and normalisations. We will therefore adhere to Taylor’s original formulation regarding timing.

\(^6\)Data were supplied by the Bureau of Economic Analysis (CPI inflation), Federal Reserve Bank of St Louis (GDP and federal funds rate) and Congressional Budget Office (potential GDP).
The results for the estimated static Taylor rule are shown in (S). The estimated coefficients are not too far away from those suggested by Taylor (1993) but the diagnostic test statistics indicate the presence of autocorrelation as well as heteroskedasticity in the residuals.

\[ i_t = 1.15 \pi_t + 0.62 y_t \]  
\[ R^2_{adj} = 0.76 \quad \hat{\sigma}_\epsilon = 0.85 \quad DW = 0.47 \]
\[ AR(4) : \chi^2(4) = 27.33 \quad ARCH(4) : \chi^2(4) = 12.59 \]

The estimated dynamic Taylor rule (D) appears to fit the data better with a higher adjusted $R^2$ and a more than halved standard error relative to the static equation. The coefficient estimates are nevertheless markedly different. Despite inclusion of the lagged dependent variable, the autocorrelation tests still show serial correlation in the residuals, but the heteroskedasticity problem has vanished.

\[ i_t = 0.18 (1.01\pi_t + 1.04y_t) + 0.82 i_{t-1} \]  
\[ R^2_{adj} = 0.95 \quad \hat{\sigma}_\epsilon = 0.38 \quad DW = 0.79 \quad Durbin H = 4.74 \]
\[ AR(4) : \chi^2(4) = 19.49 \quad ARCH(4) : \chi^2(4) = 4.04 \]

Given the above results, it is surprising that so many writers then go on and take the estimated partial adjustment coefficient as indication of intended interest rate smoothing by the central bank, leaving aside the basic fact that all coefficients in this equation are inconsistently estimated because of the significant lagged dependent variable together with the presence of autocorrelated disturbances. For instance Clarida, Galí and Gertler (1998, 2000), Gerlach and Schnabel (2000) and Doménech, Ledo, and Taguas (2002) study dynamic Taylor rules over different sample periods across different countries. All these authors find smoothing parameters with large and significant values which they

\footnote{The restriction $\hat{\phi}_\pi = 1.5$ is rejected, while $\hat{\phi}_y = 0.5$ is not rejected at the 5% level.}
interpret as evidence for the hypothesis that central banks adjust the interest rate very gradually towards the target interest rate. This conclusion however seems questionable given the implications of this parameter for the partial adjustment mechanism. Note for instance that the conclusion from our estimation of (D) with quarterly data would be that it takes almost a year until the bank has closed half of the gap between the actual federal funds rate and the intended interest rate target, and that the value of 0.92 found by Clarida, Galí, and Gertler (1998) for the Federal Reserve Bank using monthly data from 1979 to 1994 implies that approximately half the intended adjustment has taken place after about nine months. Even though there seems to be agreement in the profession that central banks dislike aggressive movements of their instruments because such action is believed to unsettle financial markets, these adjustment processes appear to be implausibly slow. On a more general level Hendry (1995, p. 259) remarks that ‘long lags in partial adjustment models may be an artefact of that type of model’, and he further notes that the adjustment parameter often lies in the interval (0.8, 0.95) ‘regardless of application’. These econometric issues raise our doubts regarding the appropriateness of the dynamic Taylor rule in its common specification as for instance in equation (3).

2.2 Related literature

The observation that dynamic interest rate rules perform well empirically, combined with the interpretation of the lagged interest rate term as interest rate smoothing, has triggered a wealth of theoretical literature that investigates whether interest rate smoothing is optimal from a monetary policy perspective. The results are not clear cut and appear to be dependent on the structural macroeconomic model.

For instance Aoki (2003) shows that interest rate smoothing may be optimal in the presence of noisy indicator variables because it implies some policy cautiousness in the presence of uncertainty. Further, in a purely forward looking model Woodford (2003) establishes that under commitment to optimal policy, a lagged interest rate term in the monetary reaction function may be optimal because it induces history dependence that helps
to stabilise inflation expectations. Using larger models Levin, Wieland, and Williams (1999) claim that interest rate smoothing in the short-term interest rate may provide control over long-term rates, because expected sustained movements on the short-term interest rate have a greater impact on long-term interest rates.

In a more empirically oriented study, Goodhart (1997) investigates the interest rate setting behaviour of several central banks and finds that they tend to move the interest rate in small steps in the same direction between reversals. This fact does, however, not provide a direct justification for interest rate smoothing. Slow, stepwise movements in the interest rate could obviously be due to an explicit smoothing objective. They could on the other hand also be the appropriate reaction to the central bank’s perception of the slowly moving state of the economy.

Finally, in a follow-up study to Rudebusch (2002b), Söderlind, Söderström, and Vredin (2004) find that the inflation rate and the output gap are relatively easy to predict which should imply a good predictability of the nominal interest rate as well. However, using survey evidence the authors cannot support this fact, leaving them with the conclusion that the two sides of the Taylor rule do to match up in terms of predictability.

Optimality and theoretical models aside, the aim of most papers in the field has been to evaluate actual central bank behaviour; we next present the latest framework for this purpose.

3 Two tests for interest rate smoothing

In the standard empirical specification, the error term $\varepsilon_t$ in (2) is assumed to be serially uncorrelated. However, as mentioned by Rudebusch (2002b) and discussed in English, Nelson, and Sack (2003) an alternative explanation for estimating a significant coefficient on the lagged interest rate term may arise from serial correlation in the error term (Grilliches (1961)). Assuming that the autocorrelation is of first order, the interest rate
equation may be specified as
\[ i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + v_t, \quad v_t = \rho v_{t-1} + \varepsilon_t, \]  
(4)

where \( \varepsilon_t \) is assumed to be \( i.i.d. \) with mean zero. Subtracting \( i_{t-1} \) from both sides and letting again \( \bar{i}_t \equiv \phi_0 + \phi_\pi \pi_t + \phi_y y_t \) denote the target rate of the central bank, equations (3) and (4) can both be rewritten on the form
\[ \Delta i_t = \gamma_1 \Delta \bar{i}_t + \gamma_2 \left( \bar{i}_{t-1} - i_{t-1} \right) + \varepsilon_t. \]  
(5)

Under interest rate smoothing, \( \gamma_1 = \gamma_2 = 1 - \lambda \), but under serially correlated errors \( \gamma_1 = 1 \) and \( \gamma_2 = 1 - \rho \). English, Nelson, and Sack (2003) suggest a test for serial correlation versus interest rate smoothing that can be based on a non-linear least squares estimation of equation (5). Note, that the hypotheses are not nested; that is, rejecting the null hypothesis \( H_0 : \gamma_1 = 1 \) requires in addition rejection of the hypothesis that \( \gamma_1 = \gamma_2 \), if one wants to conclude that there is pure interest rate smoothing. From this point of view a likelihood ratio test might be preferable to a simple coefficient test as it is conducted by English, Nelson, and Sack (2003).

In order to obtain a nested test equation, the authors also extend the model to a more general form and allow for both interest rate smoothing and serially correlated errors in one specification, that is,
\[ i_t = (1 - \lambda) \bar{i}_t + \lambda i_{t-1} + v_t, \quad v_t = \rho v_{t-1} + \varepsilon_t. \]  
(6)

Like before, the model can be rewritten as
\[ \Delta i_t = (1 - \lambda) \Delta \bar{i}_t + (1 - \lambda)(1 - \rho)(\bar{i}_{t-1} - i_{t-1}) + \lambda \rho \Delta i_{t-1} + \varepsilon_t. \]  
(7)

We will base our Monte Carlo study in this paper on the two equations (5) and (7), since they have been used in empirical studies by English, Nelson, and Sack (2003) and Castelnovo (2003a, 2003b). These authors suggest that interest rate smoothing is present even after taking serially correlated errors and omitted variables into account. The question of spurious rejection of the null hypothesis of no interest rate smoothing in the above
models is addressed by English, Nelson, and Sack (2003). The authors point out that omitted variables may generate this phenomenon, but they consider measurement error, parameter instability of the response coefficients and changed levels of the target variables as likely culprits as well. Castelnuovo (2003b) tests for omitted variables and finds that the square of the output gap\footnote{This is taken to represent asymmetric preferences of the central bank; see for instance Surico (2002).}, the exchange rate and the M3 growth rate are all significant when individually included in the equation. However, the interest rate smoothing parameter is still found significant and the author concludes that this fact is not due to omitted variables.

Inspired by the claims of Svensson (2003) and Österholm (2005) that the Taylor rule is misspecified and supported by the term structure evidence in Rudebusch (2002b) and Söderlind, Söderström, and Vredin (2004), the finding of a high degree of interest rate smoothing in the empirical literature will be challenged in the next section. Castelnuovo (2003b) tests for omitted variables one at a time, but we claim that more than just one single variable is omitted from the central bank’s reaction function. We investigate the consequences of this misspecification type in a Monte Carlo study in the next following sections.

4 Model and data generating process

The model that will be used for the simulations in this paper is a standard New Keynesian macro model where the period is assumed to be one quarter. Based on Svensson (1997, 1999), the fundamental equations of the model are a Phillips curve relation

\[ \pi_t = \alpha_0 + \alpha_\pi \pi_{t-1} + \alpha_y y_{t-1} - \alpha_q q_{t-1} + \epsilon^\pi_t \]  

(8)

and an aggregate demand equation

\[ y_t = \beta_0 + \beta_y y_{t-1} - \beta_r (i_{t-1} - E_{t-1} [\pi_t]) + \beta_z z_{t-1} + \epsilon^y_t. \]  

(9)

The variables \( q_t \) and \( z_t \) do not carry specific economic meaning in this setup. The two variables should be interpreted more broadly as a catch-all for other variables that would
be included in a more elaborate and realistic model. For simplicity, these variables are therefore only modelled as first order autoregressive processes, that is

\[ z_t = \rho_z z_{t-1} + \varepsilon^z_t \]  
\[ q_t = \rho_q q_{t-1} + \varepsilon^q_t, \]

where the vector of disturbances \( (\varepsilon^\pi_t, \varepsilon^y_t, \varepsilon^q_t, \varepsilon^z_t) \)' is assumed to follow a multivariate normal distribution \( N(0, \Sigma) \) and \( \Sigma \) is not restricted to be diagonal.

The aim of the present study is not to develop a more realistic structural model of the economy. We merely attempt to show that the omission of relevant persistent variables, that central banks are likely to respond to, has implications for the conclusions about central bank behaviour drawn from quantitative analyses of simple Taylor rule specifications. Hence, throughout the paper these two variables will be treated as ‘unknown’.

Following Svensson (2003, 2004), it could however be argued that these variables represent important state variables such as the real exchange rate, the current account, terms of trade, foreign output, foreign interest rates or judgement.\(^9\) Another candidate can be found by noting that the intercept \( \phi_0 \) in equation (2) is a function of the equilibrium real interest rate and the inflation target. Even in simple New Keynesian models of the business cycle the equilibrium real interest rate is actually time varying, since it depends on the underlying exogenous (demand and supply) shocks. Furthermore, Woodford (2003) shows in a model with endogenous capital accumulation that the equilibrium real interest rate depends on the capital stock as well. For simplicity suppose that this interest rate follows an AR(1) process,

\[ r^*_t = \mu + \gamma_r r^*_{t-1} + \varepsilon^r_t. \]

Then equation (2) changes to

\[ i_t = \mu + \gamma_r r^*_{t-1} - (\phi_\pi - 1) \pi^* + \phi_y y_t + \phi_y y_t + \varepsilon^i_t + \varepsilon^r_t. \]

This is of course just one other way of motivating the case for omitted variable bias in estimated Taylor rules but it is appealing because it introduces a persistent variable that

\(^9\)For instance, Schächter and Stokman (1995) analyse an interest rate reaction function for the Bundesbank and find the following significant effects (signs in parentheses): inflation gap (+), money growth rate gap (+), indicator of business cycle sentiment (+), unemployment rate (-), D-Mark-depreciation against the US$ (+), current account balance (-), US-interest rate (-).
depends on underlying real factors and hence gives a good motivation for assuming a non-zero correlation structure between the AS/AD shocks and the shocks to $r_t^*$, or more generally, as in the present analysis, between the synthetic variables and output and inflation, respectively.\textsuperscript{10,11}

It would generally be more accurate to model our two synthetic variables as endogenous in a more realistic model of the economy. However, as we wish to keep the discussion general, we choose a simple approach which still allows us to make the methodological point. In addition, as becomes clear from the details in Appendix A, including an equation for the interest rate this model has a restricted first order vector autoregression representation in $\pi_t, y_t, q_t, z_t$ and $i_t$.

Monetary policy is assumed to minimise quadratic deviations of inflation from its target and output from its potential. As shown for instance by Svensson (1999), the solution to the central bank’s problem can then be expressed as the following instrument rule

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_q q_t + \phi_z z_t,$$

(13)

where the parameters $\phi_j$ depend non-linearly on the structural parameters in equations (8) and (9) as well as the parameters describing the preferences of the central bank.

In order to take into account the competing hypotheses, data will be generated from the structural model in equations (8) to (11) together with each of the three different specifications of the interest rate rule given in equations (14) to (16) below.

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_q q_t + \phi_z z_t + \epsilon_t^i$$

(14)

$$i_t = (1 - \lambda) (\phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_q q_t + \phi_z z_t) + \lambda i_{t-1} + \epsilon_t^i$$

(15)

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_q q_t + \phi_z z_t + v_t, \quad v_t = \rho v_{t-1} + \epsilon_t^i$$

(16)

Equation (14) is in the spirit of the original formulation suggested by Taylor. In equation (15) the central bank is applying interest rate smoothing but there are no serially

\textsuperscript{10}There is a growing empirical literature on the estimation of the unobserved time varying equilibrium real interest rate (Laubach and Williams (2003), Neiss and Nelson (2003)).

\textsuperscript{11}Note also that a similar reasoning holds for a time varying inflation target.
correlated shocks, and finally, in equation (16) there are serially correlated shocks but no interest rate smoothing. We further assume that monetary shocks are uncorrelated with the other shocks of the economy; see Appendix A for the details of the shock covariance matrix. Using these three specifications for the central bank’s reaction function, we will be able to investigate bias in the estimated coefficients of the Taylor rule and the size properties of the ENS tests.

For the coefficients on the known variables we choose values in equations (8) and (9) that are in line with those found in numerous studies, such as for instance the estimation results of Rudebusch and Svensson (2002). Consequently, the parametrisation pertains to quarterly data, and the coefficient values are given in Appendix A. Considering the parameters in the central bank’s reaction functions we set $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\phi_q = -0.8$, $\phi_z = 0.2$ and $\phi_0$ such that the steady state annualised interest rate is equal to 4%. Note that for this model, the values $\phi_j$ are not in line with optimal monetary policy. This could be motivated by for instance deliberately less aggressive policy on the part of the central bank or uncertainty about the underlying macroeconomic model. Such a ‘rule of thumb’ modelling of central bank behaviour in empirical structural models has been used by many authors.

5 Simulations and results

Using the parameterised model in equations (8) to (11) together with one of the three interest rate rules in equations (14) to (16), we simulate this economy for 160 periods. Thereafter we estimate the equations (3), (5) and (7) and perform the tests suggested

\[\begin{align*}
\text{12} & \text{Using the objective function assumed in Svensson (1999) and setting the discount factor to 0.99 and a weight of 0.5 on the output gap (i.e. the central bank only attempts to minimise deviations of inflation and output from target), the parametrisation of our model would imply } \phi_\pi = 12.86, \; \phi_y = 10.93, \; \phi_q = -0.23 \\
& \text{and } \phi_z = 1.1.
\end{align*}\]

\[\begin{align*}
\text{13} & \text{In order to ensure independence of initial values we draw 1000 observations and discard the first 840. 160 periods correspond to 40 years of data in our set-up. The effects of sample size has been investigated by using only 80 observations in each sample. These results – which are available upon request – show no qualitative differences between the two sample sizes.}
\end{align*}\]
by English, Nelson, and Sack (2003). In order to investigate the bias of the estimated coefficients and the properties of the tests in the presence of misspecification, the equations will also be estimated with $z_t$ and $q_t$ left out one at a time and both at the same time.

We perform 1000 Monte Carlo replications in which the models are estimated and conduct Wald tests on $\gamma_1$, $\lambda$ and $\rho$ in equations (5) and (7).

5.1 The benchmark specification

Table 1 in Appendix B.1 reports the results from the Monte Carlo analysis when data generation is based on the central bank’s reaction function with white noise errors and without interest rate smoothing. The upper part of the tables provides the results on the linear equation (3), the middle part on the NLLS estimations of equations (5) and (7), and the bottom part reports the empirical size of the tests. As should be expected, the results from the ‘standard’ specification in equation (3) indicate that the true coefficient values are recovered in the estimation when all variables are included in the estimation. On the contrary, when one or both of $q_t$ or $z_t$ are omitted from the interest rate equation, the estimated coefficients are strongly biased. Notably, $\lambda$ is estimated to be 0.62 when its true value is zero and both $\phi_\pi$ and $\phi_y$ experience a substantial upward bias. For instance, when $q_t$ is omitted $\phi_\pi$ is biased up to 1.9 from its true value of 1.5 while on average $\phi_y = 1.23$ compared to the true value of 0.5.

Turning to the estimations using equations (5) and (7) we find that also in this case variable omission leads to considerable bias in some of the coefficient estimates. The bias is plausibly weaker when $z_t$ is left out and stronger when $q_t$ and $z_t$ are omitted at the same time. In general, because both $q_t$ and $z_t$ are correlated with output and inflation, and because they are relevant explanatory variables in our setup, omitting them from the interest rate equation renders the remaining estimated coefficients biased and inconsistent.

---

14Instead of estimating equation (3) with non-linear least squares, the equivalent model $i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \lambda i_{t-1} + \epsilon_t'$ was estimated and the coefficients were thereafter transformed to the long-run values in order to be comparable to the other estimated coefficients in the paper.
Considering the ENS tests, the null hypothesis is $\gamma_1 = 1$ in equation (5) and $\lambda = 0$ and $\rho = 0$, respectively, in equation (7). The tests have the correct size when all variables are included in the equations, but there are notable size distortions when one or two variables are left out. This is most accentuated for the test on $\gamma_1$ where leaving out variable $q_t$ leads to a size of 74.2%. In the test that allows for interest rate smoothing and autocorrelation correction simultaneously, the effect is weaker but the empirical size is still between 2.5 and 4.5 times the nominal.

Next, in Table 2 the results for data generated from the model with interest rate smoothing are reported. Since $\gamma_1 = 1$ and $\lambda = 0$ are rejected with a frequency very close to 100% we do not report all figures in detail. It is clear that both tests have excellent power properties. However, we find again considerable bias in all coefficients when variables are omitted from equation (3). Omitting $q_t$ leads for instance to an estimated value of $\lambda$ that is 75% larger than its true value. For equations (5) and (7) there is also a bias when variables are omitted, though it is typically fairly small for $\lambda$, especially using equation (7). Regarding the response coefficients on the other hand, $\phi_y$ is severely biased in both equations and $\phi_\pi$ is biased in equation (5). In the more general ENS test equation, that allows for serially correlated errors and smoothing simultaneously, we find the explanation for these biased estimates in the case of omitted variables. The serial correlation coefficient for the error process, $\rho$, is large and significantly different from zero in all cases of omitted variables which is shown by the 100% rejection rates of the null hypothesis that $\rho$ is equal to zero when $q_t$ or both variables are omitted.

Finally, Table 3 reports on the estimation results with data that was generated with serially correlated errors but without interest rate smoothing. Regarding coefficient bias, the story is repeating itself from the first DGP. When $q_t$ or both variables are omitted, the bias is severe for the response coefficients regardless of specification. The partial adjustment coefficient $\lambda$ is also strongly biased upward for equations (3) and (5). In the ENS tests, the null hypotheses are now that $\gamma_1$ is equal to unity in equation (5) and that $\lambda$ is equal to zero in equation (7). Again the size of the tests is biased upward in the case
of omitted variables leading to the conclusion that there is interest rate smoothing when in fact there is none.

There are several reasons for the obtained results. As stated above, omitting the two relevant variables $q_t$ and $z_t$ from the test equations has two effects: the errors will be serially correlated, and the estimated coefficients will be biased and inconsistent due to the contemporaneous correlation between $q_t$ and $z_t$ and output and inflation, respectively.

But why do the tests that are designed to allow for serially correlated errors not work? In the benchmark case we calibrate the model such that the AR(1)-processes for $z_t$ and $q_t$ have the same persistence but enter the interest rate equation with different coefficients. As a result, and because we have also assumed that the two processes are uncorrelated, their sum is again an AR(1)-process with the same persistence. Now, consider first the case in which the true interest equation is assumed to have white noise errors. Then, omitting either or both of $q_t$ and $z_t$ introduces serial correlation of first order into the disturbances. But, as the data is generated, the equation already features white noise errors, so that the actual disturbances follow an ARMA(1,1) process which results from the sum of the AR(1) and the white noise process. However, both test equations, (5) and (7), assume a correction for autocorrelation of first order, which in this case is not appropriate leaving the disturbances non-spherical.

Turning to the case where the interest equation with autocorrelated errors is used in the data generating process, the failure of the tests has a similar explanation. Now, omitting one or both of $q_t$ and $z_t$ again introduces serial correlation of first order into the disturbances, but by construction this persistence differs from the one assumed for the error process in the test equations. As a result two AR(1)-processes with different persistence are added implying that the actual disturbances follow an ARMA(2,1)-process. Again both tests do not allow for this error structure, and this leads to overrejection of the null hypothesis of no interest rate smoothing and biased coefficient estimates.

\[\text{For a simple proof of this, see for instance Hamilton (1994).}\]
5.2 Sensitivity analysis

In addition to the above arguments, there is not only feedback to output and inflation from \( z_t \) and \( q_t \) but the shocks of these variables are also contemporaneously correlated with the shocks to output and inflation. A sensitivity analysis, with the results given in column 2 of Table 4 in Appendix B.2, shows that removing these correlations weakens – but does not remove – the bias in the test results. We can conclude that the contemporaneous correlation of the shocks is important for the bias in the tests but it is not the only reason. Furthermore, whilst attractive in theory, a zero correlation between the underlying shocks does not seem to be reasonable in practice.

There are several other ways the model parameters could be changed and we report only two further results. First, the feedback to output and inflation from \( q_t \) and \( z_t \) contributes to some extent to the failure of the tests. Omitting these two variables from the aggregate supply equation (8) and aggregate demand equation (9) leads in fact to slightly smaller size distortions relative to the benchmark case as shown in column 3 of Table 4. Hence, the exact way how \( q_t \) and \( z_t \) enter the structural equations does not seem to steer our main conclusions as long as shocks to them are correlated with inflation and the output gap. If we assume in addition that there is zero correlation between the disturbances of the omitted variables and the other variables in the system, we find that the simple test performs reasonably well as shown in column 4 of Table 4. Serious size distortions can still be found in the more general test for testing the hypothesis \( \lambda = 0 \) on the DGP with white noise errors in the interest rate equation. For all practical purposes, this exercise seems fairly unrealistic though, since including \( q_t \) and \( z_t \) in the interest rate equation in these cases has a strange economic interpretation as the central bank has no reason to react to them on the basis of the model. However, from an econometric point of view this exercise indicates that the simple test detects omission of the synthetic variables as autocorrelated errors whereas the more general test is still biased. Hence, implicitly it provides information on how the aforementioned ARMA-structure in the errors influences the nominal size of the tests.
To sum up, the main outcome is that both tests feature considerable size distortions when relevant variables are left out from the interest rate equation. On a more general level, the assumed first-order autocorrelation structure of the disturbances, which these tests are based on, implies restrictions that may not be valid when applied to an autocorrelation structure that results from omission of relevant variables. In fact, as is well known, assuming an AR(1) disturbance structure imposes a common factor restriction that should be tested beforehand. The imposition of this common factor restriction has already been criticised by Hendry and Mizon (1978), and Mizon (1995) shows that autoregressive least squares estimation can yield inconsistent estimates even when the errors have an AR(1) structure.

6 Discussion and conclusion

The present study suggests that a high degree of interest rate smoothing, in an otherwise standard Taylor rule with responses of the nominal interest rate to deviations of inflation from a constant inflation target and deviations of output from its natural level, is likely to result from econometric misspecification of the estimated equation. Our results indicate that the tests suggested by English, Nelson, and Sack (2003) may not be able to distinguish between interest rate smoothing and serially correlated disturbances in Taylor-type rules, when in fact the rule is misspecified. In this paper the misspecification pertains to omitted variables because that may be the most obvious case. The omission of relevant variables leads to biased and inconsistent coefficient estimates and is highly likely to induce a disturbance structure that the tests are not designed for.

Variable omission does not seem an unlikely cause of misspecification in estimated central bank reaction functions that only include the inflation and the output gap. The practical decision making process in central banks appears to depend on a wide range of economic indicators, such as for instance monetary aggregates, the exchange rate, the current account and financial market variables to name but a few (see also the citation of Bernanke in footnote 4). The information content of this broad set of indicators may
not be sufficiently well approximated by just inflation and the output gap.\textsuperscript{16} Moreover, as Svensson (2003, 2004) discusses formally and as one can infer from press conferences at central banks, judgement may play an important role in the decision making process. How this judgement can be accounted for in a statistically sensible way is, however, beyond the scope of this paper.

The results of this paper question the standard conclusion that the large and significantly estimated coefficient on the lagged interest rate should be interpreted as an intended high degree of interest rate smoothing. We are able to show that this ‘smoothing’ or partial adjustment coefficient may not be very informative about the true degree of interest rate smoothing because it may hide omitted variable bias due to misspecification of the estimated equation and its disturbance structure. This finding does not only raise further doubts regarding the conclusions in the literature trying to evaluate central bank performance and preferences using Taylor rules, it also offers a credible explanation to the inconsistencies between the Taylor rule and the data that have recently been brought to researchers’ attention.

Finally, at a more general level, we would like to relate our study to macroeconomic modelling methodology; as mentioned by English, Nelson, and Sack (2003), serially correlated error terms signal that something systematic has been left out of the estimated equation. The authors name one very important implication of this: as long as the omitted variables that generate the serially correlated error term are orthogonal to the regressors in the equation, modelling them as autoregressive process is a valid approach. However, the authors appear to overlook the practical importance of this statement. In macroeconomics there are extremely few variables that could credibly be claimed to be orthogonal to the regressors. Therefore it will generally be the case – as shown in this paper – that the estimated coefficients are biased when the error terms are serially correlated. This should make most researchers question macroeconometric analyses that attempt to model

\textsuperscript{16}See for instance Goodhart and Hofmann (2002) for an empirical study that shows how the omission of various asset price variables leads to considerable changes in the remaining coefficients in an estimated Taylor-type rule.
serially correlated errors statistically. A more favourable avenue may be to model these terms by economic variables as already suggested by Grilliches (1961) and Hendry (1995).
References


BERNANKE, B. S. (2004): “Gradualism,” Speech at an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington.


Appendix

A.1 The structural model

The structural model is given in equations (8) to (11). We further append three different specifications of a Taylor rule as outlined in Section 3. Based on the paper by Rudebusch and Svensson (2002) we also assume the following additional parameter values: $\alpha_\pi = 0.92$, $\alpha_y = 0.15$, $\beta_y = 0.90$, $\beta_r = 0.1$, and in addition we set $\alpha_q = 0.23$, $\beta_z = 0.11$, $\rho_q = 0.95$, $\rho_z = 0.95$ and the smoothing parameter $\lambda = 0.4$. We further assume a steady state (annualised) inflation rate of 2% ($\pi = 2$) and steady state (annualised) interest rate rate of 4%, which implies $\phi_0 = 1$. It follows that $\alpha_0 = \frac{\pi}{1-\alpha_\pi}$ and $\beta_0 = \beta_r [\phi_0 + \pi (\phi_\pi - \alpha_\pi)]$.

\[
\Omega = \begin{bmatrix}
0.15 & -0.0121 & 0 & 0.0758 & -0.0379 \\
0.20 & 0 & -0.0991 & 0.0550 \\
0.05 & 0 & 0 \\
0.2294 & 0 & 0.1725 \\
\end{bmatrix},
\]

that is, we assume that monetary shocks are uncorrelated with shocks to other variables. The upper-left $3 \times 3$ part of this matrix is based on a trivariate VAR(2) using U.S. data on $\pi_t$, $y_t$ and $i_t$ from 1987Q1 to 1999Q4, where we have set the covariance between the interest rate and output and inflation, respectively, to zero, since they were estimated with values very close to zero. The stochastics of the variables $q_t$ and $z_t$ are however unknown and are therefore chosen to ensure that $\Omega$ is positive definite at the same time as the unconditional first two moments of $\pi_t$, $y_t$ and $i_t$ in the real data are approximately replicated.

A.2 Partial adjustment and white noise model

Using the interest rate specification given in equation (15) we can write the model in matrix form as follows

\[
M x_t = C + A x_{t-1} + \varepsilon_t
\]
where \( x_t = (\pi_t \ y_t \ i_t \ q_t \ z_t)' \), \( \varepsilon_t = (\varepsilon_t^\pi \ \varepsilon_t^y \ \varepsilon_t^i \ \varepsilon_t^q \ \varepsilon_t^z)' \) and the system matrices are

\[
M = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
-\phi_x(1 - \lambda) & -\phi_y(1 - \lambda) & 1 & -\phi_q(1 - \lambda) & -\phi_z(1 - \lambda) \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
\alpha_0 \\
\beta_0 \\
(1 - \lambda)\phi_0 \\
0 \\
0
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
\alpha_\pi & \alpha_y & 0 & -\alpha_q & 0 \\
\alpha_\pi \beta_r & \beta_y + \alpha_y \beta_r & -\beta_r & -\alpha_q \beta_r & \beta_z \\
0 & 0 & \lambda & 0 & 0 \\
0 & 0 & 0 & \rho_q & 0 \\
0 & 0 & 0 & 0 & \rho_z
\end{bmatrix}
\]

Then data is generated from \( x_t = M^{-1}C + M^{-1}A x_{t-1} + M^{-1}\varepsilon_t \).

For the white noise model, we set \( \lambda = 0 \) in the above specification.

### A.3 Serial correlation model

Now the equation for the interest rate is as in (16). Hence the VAR will have six equations as we treat the serially correlated error as an additional state variable

\[
\tilde{M} \tilde{x}_t = \tilde{C} + \tilde{A} \tilde{x}_{t-1} + R \varepsilon_t
\]

where \( \tilde{x}_t = (\pi_t \ y_t \ i_t \ q_t \ z_t \ v_t)' \), \( \varepsilon_t = (\varepsilon_t^\pi \ \varepsilon_t^y \ \varepsilon_t^i \ \varepsilon_t^q \ \varepsilon_t^z)' \) and the system matrices are

\[
\tilde{M} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
-\phi_\pi & -\phi_y & 1 & -\phi_q & -\phi_z & -1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\tilde{C} = \begin{bmatrix}
\alpha_0 \\
\beta_0 \\
(1 - \lambda)\phi_0 \\
0 \\
0
\end{bmatrix}
\]

\[
\tilde{A} = \begin{bmatrix}
\alpha_\pi & \alpha_y & 0 & -\alpha_q & 0 & 0 \\
\alpha_\pi \beta_r & \beta_y + \alpha_y \beta_r & -\beta_r & -\alpha_q \beta_r & \beta_z & 0 \\
0 & 0 & 0 & \rho_q & 0 & 0 \\
0 & 0 & 0 & \rho_z & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_q & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_z
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

Then data is generated from \( \tilde{x}_t = \tilde{M}^{-1}\tilde{C} + \tilde{M}^{-1}\tilde{A} \tilde{x}_{t-1} + \tilde{M}^{-1}R \varepsilon_t \)
# Appendix

## B.1 The benchmark specification

Table 1: Results from data with white noise errors

<table>
<thead>
<tr>
<th>Equation</th>
<th>All variables</th>
<th>$q$ omitted</th>
<th>$z$ omitted</th>
<th>$q, z$ omitted</th>
<th>true</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>1.00</td>
<td>0.21</td>
<td>0.94</td>
<td>0.23</td>
<td>1.00</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.50</td>
<td>1.90</td>
<td>1.53</td>
<td>1.88</td>
<td>1.50</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.50</td>
<td>1.23</td>
<td>0.62</td>
<td>1.16</td>
<td>0.50</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-0.80</td>
<td>-0.81</td>
<td>-0.80</td>
<td>-0.80</td>
<td>-0.80</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>0.20</td>
<td>-0.13</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$\phi_5$</td>
<td>-0.00</td>
<td>0.62</td>
<td>0.10</td>
<td>0.62</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### NLLS estimation of test equations

| $\phi_0$ | 1.00 | 0.17 | 0.93 | 0.17 | 1.00 |
| $\phi_1$ | 1.50 | 1.92 | 1.53 | 1.92 | 1.50 |
| $\phi_2$ | 0.50 | 1.43 | 0.61 | 1.43 | 0.50 |
| $\phi_3$ | -0.80 | -0.80 | -0.80 | -0.80 | -0.80 |
| $\phi_4$ | 0.20 | 0.00 | 0.20 | 0.20 | 0.20 |
| $\phi_5$ | -0.00 | 0.02 | 0.03 | 0.02 | 0.00 |
| $\phi_6$ | 1.00 | 0.68 | 0.94 | 0.68 | 1.00 |
| $\phi_7$ | 1.03 | 0.08 | 0.68 | 0.08 | 1.00 |

### Test size

- Size in % $H_0 : \gamma_1 = 1$: 5.30, 35.30, 75.80, 5.00
- Size in % $H_0 : \lambda = 0$: 5.30, 35.30, 75.80, 5.00
- Size in % $H_0 : \rho = 0$: 7.60, 100.00, 78.70, 5.00

Notes: 1000 Monte Carlo replications. Nominal size of tests is five percent.

Equation (3): $i_t = \lambda i_{t-1} + (1 - \lambda)\tilde{i}_t + \varepsilon_t^i$. Long-run coefficients calculated after OLS estimation.

Equation (5): $\Delta i_t = \gamma_1 \Delta \tilde{i}_t + \gamma_2 \left( \tilde{i}_{t-1} - i_{t-1} \right) + \varepsilon_t^i$

Equation (7): $\Delta i_t = (1 - \lambda) \Delta \tilde{i}_t + (1 - \lambda) (1 - \rho) \left( \tilde{i}_{t-1} - i_{t-1} \right) + \lambda \rho \Delta i_{t-1} + \varepsilon_t^i$
Table 2: Results from data with white noise errors and partial adjustment

<table>
<thead>
<tr>
<th>Equation</th>
<th>all variables</th>
<th>q omitted</th>
<th>z omitted</th>
<th>q, z omitted</th>
<th>true</th>
</tr>
</thead>
</table>

**OLS estimation of linear interest rate equation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation (3)</th>
<th>Equation (3)</th>
<th>Equation (3)</th>
<th>Equation (3)</th>
<th>Equation (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>1.00</td>
<td>0.19</td>
<td>0.95</td>
<td>0.23</td>
<td>1.00</td>
</tr>
<tr>
<td>$\phi_{pi}$</td>
<td>1.50</td>
<td>1.91</td>
<td>1.52</td>
<td>1.89</td>
<td>1.50</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.50</td>
<td>1.28</td>
<td>0.63</td>
<td>1.18</td>
<td>0.50</td>
</tr>
<tr>
<td>$\phi_q$</td>
<td>-0.80</td>
<td>-0.83</td>
<td>-0.80</td>
<td>-0.80</td>
<td>-0.80</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.20</td>
<td>-0.17</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.39</td>
<td>0.71</td>
<td>0.46</td>
<td>0.71</td>
<td>0.40</td>
</tr>
</tbody>
</table>

**NLLS estimation of test equations**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>1.00</td>
<td>-0.01</td>
<td>0.94</td>
<td>-0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\phi_{pi}$</td>
<td>1.50</td>
<td>2.00</td>
<td>1.53</td>
<td>2.00</td>
<td>1.50</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.49</td>
<td>1.36</td>
<td>0.64</td>
<td>1.35</td>
<td>0.50</td>
</tr>
<tr>
<td>$\phi_q$</td>
<td>-0.80</td>
<td>-0.83</td>
<td>-0.80</td>
<td>-0.80</td>
<td>-0.80</td>
</tr>
<tr>
<td>$\phi_z$</td>
<td>0.20</td>
<td>-0.03</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.40</td>
<td>0.42</td>
<td>0.45</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.03</td>
<td>0.81</td>
<td>0.07</td>
<td>0.82</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.60</td>
<td>0.45</td>
<td>0.57</td>
<td>0.46</td>
<td>0.60</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.62</td>
<td>0.13</td>
<td>0.49</td>
<td>0.13</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Test size**

<table>
<thead>
<tr>
<th>Test</th>
<th>Size in % $H_0: \rho = 0$</th>
<th>$7.60$</th>
<th>$100.00$</th>
<th>$18.40$</th>
<th>$100.00$</th>
<th>$5.00$</th>
</tr>
</thead>
</table>

Notes: 1000 Monte Carlo replications. Nominal size of tests is five percent.

Equation (3): $i_t = \lambda i_{t-1} + (1 - \lambda)\tilde{i}_t + \varepsilon_t^i$. Long-run coefficients calculated after OLS estimation.

Equation (5): $\Delta i_t = \gamma_1 \Delta \tilde{i}_t + \gamma_2 \left( \tilde{i}_{t-1} - i_{t-1} \right) + \varepsilon_t^i$

Equation (7): $\Delta i_t = (1 - \lambda) \Delta \tilde{i}_t + (1 - \lambda)(1 - \rho)(\tilde{i}_{t-1} - i_{t-1}) + \lambda \rho \Delta i_{t-1} + \varepsilon_t^i$
Table 3: Results from data with first order serial correlation errors

<table>
<thead>
<tr>
<th>equation</th>
<th>OLS estimation of linear interest rate equation</th>
<th>NLLS estimation of test equations</th>
<th>Test size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all variables</td>
<td>q omitted</td>
<td>z omitted</td>
</tr>
<tr>
<td>( \phi_0 )</td>
<td>( \phi_{0L} )</td>
<td>( \phi_{0N} )</td>
<td>( \phi_{0N} )</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>1.00</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>1.50</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>0.50</td>
<td>1.28</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
<td>( \phi_{\pi L} )</td>
<td>( \phi_{\pi N} )</td>
<td>( \phi_{\pi N} )</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>-0.80</td>
<td>-0.83</td>
</tr>
<tr>
<td>( \phi_y )</td>
<td>( \phi_{y L} )</td>
<td>( \phi_{y N} )</td>
<td>( \phi_{y N} )</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>0.20</td>
<td>-0.17</td>
</tr>
<tr>
<td>( \phi_q )</td>
<td>( \phi_{q L} )</td>
<td>( \phi_{q N} )</td>
<td>( \phi_{q N} )</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>0.39</td>
<td>0.71</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( \lambda_{L} )</td>
<td>( \lambda_{N} )</td>
<td>( \lambda_{N} )</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>1.00</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>1.50</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>0.50</td>
<td>1.45</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>( \gamma_{1 L} )</td>
<td>( \gamma_{1 N} )</td>
<td>( \gamma_{1 N} )</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>( \gamma_{2 L} )</td>
<td>( \gamma_{2 N} )</td>
<td>( \gamma_{2 N} )</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>1.00</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Test size
- Size in % \( H_0 : \gamma_1 = 1 \) | \( 8.00 \) | \( 72.60 \) | \( 34.50 \) | \( 73.10 \) | \( 5.00 \)
- Size in % \( H_0 : \lambda = 0 \) | \( 6.30 \) | \( 58.80 \) | \( 19.30 \) | \( 60.20 \) | \( 5.00 \)

Notes: 1000 Monte Carlo replications. Nominal size of tests is five percent.

Equation (3): \( i_t = \lambda i_{t-1} + (1 - \lambda)\hat{i}_t + \varepsilon_t \). Long-run coefficients calculated after OLS estimation.

Equation (5): \( \Delta i_t = \gamma_1 \Delta \hat{i} + \gamma_2 (\hat{i}_{t-1} - i_{t-1}) + \varepsilon_t \)

Equation (7): \( \Delta i_t = (1 - \lambda)\Delta \hat{i} + (1 - \lambda)(1 - \rho)(\hat{i}_{t-1} - i_{t-1}) + \lambda \rho \Delta i_{t-1} + \varepsilon_t \)
### B.2 Sensitivity analysis

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>White noise errors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H_0: \gamma_1 = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>5.30</td>
<td>5.10</td>
<td>5.60</td>
<td>5.10</td>
</tr>
<tr>
<td>(q) omitted</td>
<td>74.20</td>
<td>11.80</td>
<td>47.20</td>
<td>6.30</td>
</tr>
<tr>
<td>(z) omitted</td>
<td>35.30</td>
<td>12.10</td>
<td>18.20</td>
<td>8.40</td>
</tr>
<tr>
<td>(q, z) omitted</td>
<td>75.80</td>
<td>13.90</td>
<td>46.80</td>
<td>5.60</td>
</tr>
<tr>
<td>(H_0: \lambda = 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>5.30</td>
<td>5.10</td>
<td>5.80</td>
<td>5.20</td>
</tr>
<tr>
<td>(q) omitted</td>
<td>12.50</td>
<td>18.00</td>
<td>8.20</td>
<td>23.40</td>
</tr>
<tr>
<td>(z) omitted</td>
<td>22.20</td>
<td>10.20</td>
<td>13.40</td>
<td>14.70</td>
</tr>
<tr>
<td>(q, z) omitted</td>
<td>13.00</td>
<td>18.30</td>
<td>8.70</td>
<td>23.60</td>
</tr>
<tr>
<td>(H_0: \rho = 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>7.60</td>
<td>7.70</td>
<td>7.20</td>
<td>7.40</td>
</tr>
<tr>
<td>(q) omitted</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>(z) omitted</td>
<td>78.70</td>
<td>91.20</td>
<td>91.50</td>
<td>94.00</td>
</tr>
<tr>
<td>(q, z) omitted</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td><strong>Partial adjustment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H_0: \rho = 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>7.60</td>
<td>7.80</td>
<td>7.50</td>
<td>7.40</td>
</tr>
<tr>
<td>(q) omitted</td>
<td>100.00</td>
<td>100.00</td>
<td>99.10</td>
<td>100.00</td>
</tr>
<tr>
<td>(z) omitted</td>
<td>18.40</td>
<td>39.80</td>
<td>39.70</td>
<td>55.60</td>
</tr>
<tr>
<td>(q, z) omitted</td>
<td>100.00</td>
<td>100.00</td>
<td>99.80</td>
<td>100.00</td>
</tr>
<tr>
<td><strong>Serial correlation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H_0: \gamma_1 = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>8.00</td>
<td>9.40</td>
<td>6.70</td>
<td>7.60</td>
</tr>
<tr>
<td>(q) omitted</td>
<td>72.60</td>
<td>11.00</td>
<td>59.60</td>
<td>7.30</td>
</tr>
<tr>
<td>(z) omitted</td>
<td>34.50</td>
<td>11.20</td>
<td>17.80</td>
<td>6.60</td>
</tr>
<tr>
<td>(q, z) omitted</td>
<td>73.10</td>
<td>11.60</td>
<td>57.50</td>
<td>7.50</td>
</tr>
<tr>
<td>(H_0: \lambda = 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.20</td>
</tr>
<tr>
<td>(q) omitted</td>
<td>58.80</td>
<td>8.30</td>
<td>29.80</td>
<td>5.80</td>
</tr>
<tr>
<td>(z) omitted</td>
<td>19.30</td>
<td>6.90</td>
<td>9.70</td>
<td>4.90</td>
</tr>
<tr>
<td>(q, z) omitted</td>
<td>60.20</td>
<td>8.30</td>
<td>29.40</td>
<td>5.20</td>
</tr>
</tbody>
</table>

Notes: All values are percentages

(1): Benchmark specification
(2): Sensitivity analysis 1: Blockdiagonal \(\Omega\)
(3): Sensitivity analysis 2: \(q_t\) and \(z_t\) omitted from AS/AD, i.e. \(\alpha_q = \beta_z = 0\)
(4): Sensitivity analysis 3: Blockdiagonal \(\Omega\) and \(\alpha_q = \beta_z = 0\)


2004:9 Sören Blomquist and Vidar Christiansen, Taxation and Heterogeneous Preferences. 29 pp.


* A list of papers in this series from earlier years will be sent on request by the department.


2005:11 Martin Ågren, Myopic Loss Aversion, the Equity Premium Puzzle, and GARCH. 34 pp.


See also working papers published by the Office of Labour Market Policy Evaluation
http://www.ifau.se/

ISSN 0284-2904