A Pion Collector Based on Superconducting Solenoids

A Feasibility Study for the ESSnuSB

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Abstract
The European Spallation Source Neutrino Super-Beam project (ESSnuSB) plans to use the powerful proton beam from the ESS linac to produce a neutrino super beam in a dedicated target. The super beam must consist of either muon neutrinos or muon antineutrinos and be directed towards an underground detector 540 km from the source. A neutrino horn based on a magnetic toroid is normally employed for collecting the pions that emerge from the target and soon after emit neutrinos as they decay. However, the horn cannot accept the nominal pulse from the linac due to ohmic heating from the high current driving the structure. As an alternative, we have studied the use of solenoids for pion collection. The solenoids, that would be made superconducting for continuous operation, are however blind to the pion charge wherefore a charge separation stage is needed. This report describes our efforts of combining solenoid and dipole fields in order to collect pions from the target and separate the pion charges before they have time to decay. The study indicates that focusing with solenoids is possible, but that simultaneous charge separation can only be achieved with limited efficiency and acceptance in terms of pion divergence and momentum. Further studies are required for full understanding of the limitations.
1 Introduction

As the European Spallation Source (ESS) [1] is under construction in Lund, Sweden, there are ongoing activities aiming at exploiting the full physics potential of the ESS. The main objective is of course neutron production for neutron scattering experiments, and that relies on the acceleration of a 62.5 mA proton beam distributed over 2.86 ms bunch trains at 14 Hz, to an energy of 2.0 GeV [2]. This world-unique beam power of 5 MW offers opportunities to study the physics beyond the material science primarily targeted. For example, there is a plan to study neutron-antineutron oscillations and neutron decay in a beamline dedicated to fundamental physics [3].

Another project based on the ESS linac is the ESS Neutrino Super Beam (ESSnuSB) study [4]. ESSnuSB intends to use the unprecedented beam power to produce neutrinos, in a second target station, and detect some of these neutrinos in an underground mine far from the neutrino production. Through this scheme one means to measure neutrino oscillations at the second oscillation maximum, with the goal of discovering the long-sought CP violation in the leptonic sector. Since neutrinos are elusive particles that rarely interact with matter it is essential to have an intense proton beam at the generation stage, and a large detector volume at the detection location. By doubling the duty factor of the linac from 4% to 8%, possibly by simply doubling the pulse repetition rate, one could have 5 MW beam on the neutron target and 5 MW on the neutrino target.

To generate a neutrino flux at the far detector of sufficient intensity requires efficient collection of charged pions generated in the target, which later decay and form the neutrino super-beam. The magnetic structure normally employed for this task, called a magnetic horn, was invented by S. van der Meer [5] and consists of a toroidal magnet where the particles that are produced at a large angle are forced back into the forward direction by the magnetic field. The particles must cross the walls of the structure in order to reach the field region, which is generated by a current of the order of 350 kA flowing through the walls. This enormous current results in significant heating of the conductor, heating which can only be sustained for a very short time. The nominal ESS pulse duration of 2.86 ms is two orders of magnitude too long and need to be reduced before it reaches the horn. Energy deposition from particles that traverse the horn surface makes it particularly difficult to make a superconducting horn, which would enable DC operation. Instead, the proton pulse that arrives at the horn must be shortened. The plan at ESSnuSB is to compress the proton pulse in one or several accumulator rings located at the end of the linac. Using the ring the 2.86 ms pulse can be reduced to 1.32 µs [6].

Unfortunately, the need for an accumulator ring brings more necessary changes to the linac. The high intensity of the ESS beam poses a challenge for the injection into the ring, which can be helped by charge exchange. This means that, instead of a proton beam, an $H^{-}$ beam is injected and stripped from the two
electrons just after the injection point. This, however, means that the linac must accelerate $H^-$ in addition to the nominal proton beam. In addition, space charge effects in the ring may limit the maximum intensity in the accumulator ring to half or a fourth of the nominal ESS pulse. If so, the $H^-$ beam pulse must be restructured into four shorter pulses spaced at 70Hz, in between the nominal proton pulses. Furthermore, the acceleration of two different beam particles is a big challenge from a beam optics point of view, on top of the technical issues brought by the required second injection. Together, these changes to the ESS linac means a larger risk to the neutron production together with a higher cost for the project. The ultimate way of avoiding these issues would be to find a pion collector that directly accepts the full 2.86ms pulse from the ESS linac.

2  Physics and Detection

When protons from the ESS linac penetrate the neutrino target, nuclear processes occur that result in the emission of secondary particles, charged pions among others. A positively charged pion decays into an anti-muon and a muon neutrino, as shown at the top of Fig. 1 (a). The muon neutrino then travels through the surface of the earth and further on to the far detector, where it will be detected, at some probability. Along the way some of these muon neutrinos will oscillate, i.e. will change flavor and be identified as electron neutrinos, when they reach the detector. If the total flux of muon neutrinos at the target is known, the fraction of oscillated particles can be determined. A comparison will be made of the spectrum of oscillated neutrinos starting from positive pions and of the spectrum from negative pions, Fig. 1(c). This is equivalent to comparing the oscillation probabilities for particle and anti-particle and is thus a handle on CP violation in the leptonic sector.

The detector foreseen for the ESSnuSB project is an adapted design of the MEMPHYS detector [7, 8] developed for the EUROnu project [9]. It consists of half a Megaton volume of clear water, surrounded by photomultipliers. When a neutrino interacts with nuclear particles in the detector, secondary leptons are emitted. As the speed of these leptons is higher than the speed of light in the detector medium, meaning water, a shock wave of light, called Cherenkov radiation, is emitted. This light propagates through the crystal-clear water as a ring and is picked up by the photomultipliers. A rather diffuse light burst is emitted when an electron neutrino interacts in the detector, compared to the well defined ring of light detected after a muon neutrino interaction. Thus the detector can sense the neutrino flavor. In order to distinguish a neutrino from an anti-neutrino, however, a magnetic field through the detector would be needed. In a magnetic field, the charged secondary particles will follow trajectories of which the curvature depends on the mass-to-charge ratio. Such a detector, e.g. one based on magnetized liquid argon, has been foreseen for similar long baseline
\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \quad (a) \]
\[ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad (b) \]
\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (c) \]
\[ \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad (d) \]

Figure 1: Full decay chain of the pions: The \( \pi^+ \) meson decay emits an anti-muon and a muon neutrino \( (a) \). The anti-muon decays further and emits a positron, an electron neutrino and an muon anti-neutrino \( (b) \). The \( \pi^- \) meson emits a muon and a muon anti-neutrino \( (c) \). The muon decays further and emits an electron, an electron anti-neutrino and a muon neutrino \( (d) \).

experiments, such as the LBNE project \([10, 11]\), but is both more complex and expensive - if even feasible in the case of ESSnuSB - to realize and maintain. To this end, ESSnuSB plans for a water detector, blind to lepton charge, implying that it must be ensured already at the production stage that the super-beam consists purely of either muon neutrinos or muon anti-neutrinos. The neutrino selection will take place through focusing of \( \pi^+ \) while defocusing or discarding the \( \pi^- \), and vice versa. Figure 2 shows an overview of the project, from the proton beam on the target, via pion to muon or electron neutrino in the detector. Figure 1 shows the decay chain of the \( \pi^- \) meson. Shown at the top is the emitted muon neutrino that is used to form a super-beam. In \( \pi^- \) mode, all muon anti-neutrinos coming from the decay of a \( \pi^+ \) will contaminate the experiment. The same goes for the neutrinos emerging from the further decay of the daughter muon, which must be absorbed before it decays.

With the detector placed 540 km from the neutrino source, less than 50 events per year are expected, a number which depends on the actual value of the CP violating phase. In Ref. \([12]\) the performance of the ESSnuSB experiment is treated in detail.

\section{Requirements from the Experiment}

The small number of events expected at the detector makes the experiment sensitive to both systematic and statistical errors. The pion collector plays a key role in reducing these errors and it needs to fulfill a number of requirements that will be explained here.
3.1 Focusing efficiency

With the pion collector we wish to redirect a highly divergent group of pions towards the detector location, in large that means point-to-parallel focusing. However, irrespective of the level of focusing of the pions there will be an additional smearing of the neutrino distribution coming from the kinematics of the pion decay. The smearing effect can be calculated, thus providing us with the accuracy of focusing that the collector should provide. To this end, we consider the angular distribution of the decay in the center-of-mass frame, together with the Lorentz boost due to the pion momentum.

Table 1: The pion and muon mass and mean life time in the rest frame.

<table>
<thead>
<tr>
<th>particle</th>
<th>property</th>
<th>symbol</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm$</td>
<td>mass</td>
<td>$m_\pi$</td>
<td>139.57</td>
<td>MeV/$c^2$</td>
</tr>
<tr>
<td>$\pi^\pm$</td>
<td>life time</td>
<td>$\tau_\pi$</td>
<td>26</td>
<td>ns</td>
</tr>
<tr>
<td>$\mu^\pm$</td>
<td>mass</td>
<td>$m_\mu$</td>
<td>105.66</td>
<td>MeV/$c^2$</td>
</tr>
<tr>
<td>$\mu^\pm$</td>
<td>life time</td>
<td>$\tau_\mu$</td>
<td>2200</td>
<td>ns</td>
</tr>
</tbody>
</table>

In the rest frame of the pion, i.e. the center of mass (cms) frame, the neutrino and the muon are emitted back to back. That means, assuming that the neutrino mass is negligible, that their momenta, $p_\nu$ and $p_\mu$, are equal in value but opposite in direction, and uniquely defined as [13]

$$p_\nu = p_\mu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}c \approx 30 \text{ MeV}/c$$

(1)

where $c$ is the speed of light, $m_\pi$ the pion mass and $m_\mu$ the mass of the muon, both at rest, the values of which are listed in Table 1. The distribution in direction, or emission angle $\theta$ with respect to the traveling direction of the pion, is
uniform in the cms frame. In the boosted frame, the lab frame, there is a weighted probability that the neutrino will be emitted in a certain direction. This probability distribution $\rho(\theta)$ depends on the energy of the decaying pion and looks as following:

$$\rho(\theta) = \frac{1}{2} \sin \theta \frac{1 - \beta^2}{(1 - \beta \cos \theta)^2} \quad (2)$$

with $\beta = \nu c = \sqrt{1 - 1/\gamma^2}$ being the relativistic pion speed. The higher the energy of the pion, the more boosted in the forward direction is the emission of the secondary particles, as is shown in Fig. 3 for three pion momenta corresponding to $\gamma = 2, 4$ and $6$. By looking at how the peak of the density distribution moves as the pion momentum increases we can extract the most probable emission angle as a function of the pion momentum. Figure 4 shows this relation, where we denote the most probable emission angle as $\langle \theta \rangle$. At pion momentum $p_\pi = 600\text{ MeV/c}$ we obtain $\langle \theta \rangle = 0.13\text{ rad} \approx 7^\circ$. The lower bound (lower dashed line), at which the density is half of the maximum value, is $0.04\text{ rad}$ and the upper bound (upper dashed line) $0.32\text{ mrad}$. Since the neutrino emission angle cannot be reduced through focusing we conclude that the angle $\langle \theta \rangle$ sets the limit to which accuracy we need to focus pions.

![Figure 3: The probability density $\rho(\theta)$ of the neutrino emission as a function of emission angle $\theta$ in the lab frame for different pion momenta.](image-url)
Figure 4: The most probable neutrino emission angle $\langle \theta \rangle$ as a function of the pion momentum $p_\pi$ in the lab frame. The dashed lines correspond to the left and right bounds where the probability density is reduced to half of its maximum value, see Fig. 3.

3.2 Chromatic aberration

The emission angle of the neutrino has a direct correlation with the neutrino energy in the lab frame, given by the kinematic relation

$$p_\nu = \frac{1}{2} \frac{m_\pi^2 c^4 - m_\mu^2 c^4}{E_\pi - p_\pi c \cos \theta}$$

(3)

where $E_\pi = \sqrt{p_\pi^2 c^2 + m_\pi^2 c^4}$ and $\theta$ is the angle that the neutrino makes with the pion trajectory in the lab frame. Figure 5 shows the distribution in Eq. 3 for three specific cases where the pion energy corresponds to $\gamma = 2$, $\gamma = 4$ and $\gamma = 6$. To obtain the final neutrino distribution in energy and direction we must convolute the equivalent pion distribution with the relation in Eq. 3, a calculation I will omit here.

3.3 Contamination

The long baseline experiment has one ultimate goal: to measure the neutrino oscillation ratio of particle/antiparticle by measuring the spectra of muon neutrinos and electron neutrinos. It is of utmost importance that the neutrino super-beam consists purely of muon neutrinos (> 99% [14]) in order not to contaminate the experiment. The two main sources of contamination becomes clear from looking at the decay chains of $\pi^\pm$ in Fig. 1. The first source originates from the fact that the detector is blind to charge, i.e. that a neutrino and an anti-neutrino
Figure 5: The correlation between neutrino momentum and emission angle with respect to the pion trajectory in the lab frame, shown for three pion energies ($\gamma$).

give rise to identical signals. Having pions of the undesired sign in the neutrino beam means that we are looking at particle and antiparticle at the same time. In the target, $\pi^+$ are produced at a higher rate than the $\pi^-$, approximately with a ratio of 1.66 [15], see Table 2 where the number of particles produced per proton on target are listed for the four most abundant particle species. Nevertheless, there is enough pions of the wrong sign to perturb the measurement. The collector must therefore have the vital functionality of focusing one pion charge while diverging the other, something that is intrinsic to the van der Meer horn.

Table 2: Number of particles produced per proton on target (p.o.t.). These numbers have been obtained through a FLUKA simulation.

<table>
<thead>
<tr>
<th>particle</th>
<th>$\pi^+$</th>
<th>$\pi^-$</th>
<th>$\mu^+$</th>
<th>$\mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#/p.o.t.</td>
<td>0.29</td>
<td>0.17</td>
<td>$3 \cdot 10^{-4}$</td>
<td>$7 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

The second source of contamination is the further decay of the muon antimuon emitted together with the muon neutrino. An electron neutrino is emitted when the anti-muon decays as shown in Fig. 1. Such neutrinos would pollute the oscillation measurement by falsely indicating a higher oscillation ratio. Apart from eliminating the pions of the wrong sign, the focusing system must be designed in such a way that the number of pions that decay is maximized with respect to the number of muons that have time to decay. The total energy and the life time of pions and muons, given in Tab. 1, will thus influence the maximum length allowed for the focusing system and the length of the decay tunnel. We need to both focus pions of one charge and dispose the pions of the other charge before the pions start decaying. In addition, we need a decay tunnel which allows for a maximum number of pions to decay while a minimum amount of the muons
decay before they are absorbed at the end of the tunnel.

We use the numbers from Tab. 1 to estimate the distance traveled by a particle before decay. If the total energy of a relativistic particle is \( E \) and its rest mass \( m_0 \) we get a relativistic Lorentz factor \( \gamma = E/m_0c^2 \) which helps us to calculate the mean life time of the particle in the lab frame \( \tau = \gamma \tau_0 \) where \( \tau_0 \) is the mean life time in the rest frame. Using the Lorentz factor we can also calculate the speed \( v \) of the particle since \( v = \beta/c \) and \( \beta = \sqrt{1 - 1/\gamma^2} \). Because of the high speed, the particle will experience length contraction. Inversely, the lab frame will observe time dilation. The end result, in the lab frame, is that the particle will reach further than expected before it decays. We can calculate the mean distance \( d \) traveled before decay, seen from the lab frame:

\[
d = v\tau = c\sqrt{1 - 1/\gamma^2} \cdot \gamma \tau_0 = c\tau_0 \sqrt{\gamma^2 - 1}.
\]

Expressed in momentum \( p \) and rest mass \( m_0 \) we get

\[
d = c\tau_0 \frac{pc}{m_0c^2}.
\]

As an example we look at a pion with total energy \( E = 500 \text{ MeV} \). That means that \( \gamma \approx 3.6 \) and \( v = 0.96c \). The mean distance traveled by the pion before decay is then \( d_{\text{lab}} = 27 \text{ m} \). Compare this to the distance calculated directly from the life time in the rest frame \( d_0 = 7.5 \text{ m} \).

We can also use the half life to estimate the distance from the source at which 1% of the pions have decayed. For \( \gamma = 3.6 \) this is only 0.37 m, which gives the order of the maximum length of the charge separation system.

Let us now consider a pion and a muon with the same momentum. The ratio of distance traveled by the two particles is then given by

\[
\frac{d_\pi}{d_\mu} = \frac{\tau_\pi}{\tau_\mu} \frac{m_\mu c^2}{m_\pi c^2} \approx 0.009
\]

The lifetime of the two particles thus work to our advantage, since at the same momentum the muon travels almost 100 times longer before it decays. The contamination from muon decay is thus suppressed by shortening the decay tunnel behind the collector. However, it needs to be long enough for the pions to decay so that the neutrino intensity is maximized. The optimum length which balances these factors has been estimated to 25 m [15]. In this length about 50% of the pions have decayed but very few of the muons. A longer tunnel does not significantly increase the signal level but clearly increases the contamination level.

To conclude, we have identified the functions to be fulfilled by the pion collector and following decay channel as follows:

1. The pions must be focused towards the detector to an accuracy such that the final divergence is dominated by the pion decay kinematics. In this case it means a maximum divergence around a few tens of milliradians.
2. The pions must be charge-separated at the source.

3. The decay of pions must be negligible before the charge separation becomes effective. This requirement limits the length of the charge separation system to of the order of a meter.

4. The length of the decay tunnel must optimize the neutrino beam signal-to-noise ratio.

4 The Source

The target design intended for the ESSnuSB is an adaptation of a design made through the EUROnu project [16]. It consists of a cylindrical canister containing a packed bed of titanium spheres, cooled with cold high-pressure helium gas. The target cylinder, 78 cm long and 3 cm diameter, is placed in a field free region inside the neutrino horn. The pion distribution expected from this target is shown in Fig. 6 and was obtained through a FLUKA simulation [17, 18] using a 2 GeV proton beam. Here, the distribution in pion momentum and exit angle with respect to the proton beam direction, is displayed. The distribution in horizontal position and horizontal momentum is shown in Figs. 7, which illustrates some of the challenges of the focusing system: Both the wide angular spread and the momentum distribution call for a large acceptance system.

![Figure 6: Pion distribution generated with FLUKA. A 2 GeV proton beam was used in the simulation. The intensity (color) scale is logarithmic.](image)

In Fig. 8 we look at the distribution in detail. Figure 8(a) shows a histogram of the transverse exit position of the pions, expressed in \( r = \sqrt{x^2 + y^2} \), the distance
Figure 7: A histogram of the horizontal position $x$ and momentum $p_x$ of all pions when they exit the target, generated with FLUKA. The intensity (color) scale is logarithmic.

from the axis and the center of the target. Almost all particles - 98.9% - exit the target at $r = 1.5$ cm, meaning through the side wall of the cylinder. Only 0.4% exit through the front end cap and 0.75% are backscattered. In Fig. 8(b) we look at the angle $\theta$ that the exiting particle trajectory makes with the longitudinal axis. The distribution in angle is very wide and peaks at 0.5 rad with a full width at half maximum (fwhm) of over 1 rad. If we look at a more narrow momentum span, here 600 MeV/c±60 MeV/c, the width is decreased to 0.7 rad.

As illustrated here, the pions emerging from the target differ from a conventional beam in more than one sense: 1) It has a very wide momentum distribution, which leads to chromatic effects in both the pion collection and the neutrino emission; 2) The angular distribution is equally wide and challenging to handle; 3) The ”beam” consists of mainly two particle species, one of which needs to be discarded. While the van der Meer Horn can handle these three challenges with a reasonable efficiency, the challenge is to identify a collector based on a solenoid that could do the same. In the sections that follow that will be treated.

5 A Solenoid Collector: Background

With a superconducting solenoid instead of a normal-conducting horn for pion collection, the ESSnuSB project will profit from a substantial reduction in cost and complexity. The pion collector could then be powered continuously instead of in pulsed mode, which means that the nominal ESS proton pulse can be accepted directly from the linac. It also means that neither the accumulator ring for pulse
Figure 8: A closer look at the pion distribution reveals that the vast majority of the pions exit the target through the side of the cylinder and not through the end cap (a). The pions that exit with a direction in the forward hemisphere are distributed in angle $\theta$ with respect to the forward direction as shown in (b). If we zoom in on the pions with momentum 600 MeV $\pm$10% we see that the distribution becomes more narrow but still with a peak at nearly 0.5 rad, i.e. 28°.

compression nor the H$^-$ operation of the linac would be needed.

The advantages of a solenoid collector has been recognised by other superbeam projects, by studies for future muon colliders and associated neutrino factories, and as part of other systems which rely on the focusing of secondary particles from a target, such as positron sources. This study continuous the research and development done for other projects. A short survey of previous studies will be presented here.

With the prospect of a muon collider and an associated neutrino factory at Brookhaven National Laboratory (BNL) [19] efforts were made to assess the performance of a solenoid collection system in the early 21st century. Like other neutrino factory projects, the goal of the pion collection was to match the daughter muons to an accelerator acceptance. The neutrinos then result from the muon decay in straight sections of the muon accelerator. The collector under study included a tapered solenoid and a liquid mercury jet. Experimental tests of the liquid mercury jet target were performed at BNL, the results of which are discussed in Ref. [19]. In addition, a focusing system based on solenoids tapering from a maximum field of 20 T to the minimum 1.25 T was designed [20, 21]. This system, connote as adiabatic device, is based on a similar focusing scheme designed by Chehab for a positron source at LAL [22] in 1983.

The adiabatic device was first introduced by Helm at SLAC already in 1962 [23] and has been used mainly for positron sources. The advantage of the tapered scheme to a simple solenoid is the increased acceptance of transverse momentum. The continuous decrease in magnetic field is accompanied by an increase of the bore radius in order to transform a small, highly divergent source to a large but
parallel pion distribution. In Ref. [24] from 2001 Chehab makes a theoretical comparison of the adiabatic device, a simple solenoid, and a third system called the quarter-wave transformer (QWT). The latter consists of two solenoids: a short solenoid with strong magnetic field followed by a long solenoid with weak field. The QWT, which is also commonly used at positron sources, can thus be considered as an intermediate step between the single solenoid and the continuously tapered adiabatic device.

The work on a liquid mercury jet surrounded by a high field solenoid collector was continued by Kirk et al. The study led to a proof-of-principle experiment in 2008, MERIT, of a pulsed superconducting 15 T solenoid [25], intended for a 4 MW proton beam from the hypothetical Super AGS proton driver, an upgrade of the Alternating Gradient Synchrotron (AGS) at BNL. The design field of 15 T was reached in 2006. Similarly, CERN has been involved in a study for a neutrino factory as a step towards a muon collider. There, B. Autin and Ph. Royer investigated the possibilities of using a superconducting solenoid instead of the van der Meer horn [26], also around the turn of the century, 2000. No further results were reported until very recently, when Hansen published a thesis on the topic [27]. Hansen has been working on the optimization of a pion collector for a neutrino factory. Similarly to the previous CERN study, the system is based on a liquid mercury jet, and a strongly tapered magnetic field.

Diwan et al. have also studied the use of solenoid focusing for a neutrino super-beam, detailed in Ref. [28] from 2001. Diwan, Kahn and Palmer came as far as to, in Ref. [29] from 1999, compare the focusing efficiency of a solenoid system with a van der Meer horn, with the conclusion that the solenoid performs even better than the horn in the energy range of $0.45 < E_{\nu} < 1.28$. Note, though, that this investigation was made for a proton beam energy of 28.3 GeV, i.e. ten times higher than ESSuSB. In this case, the secondary pions are less divergent and therefore "easier" to collect. In Ref. [30] from 2006, Kahn also points out that they were unable to maintain the focusing of the pions as they left the magnetic field. While for a neutrino factory this is less of an issue, since it is enough to match the pion distribution with to the accelerator acceptance, at a limited distance from the target, it implies a significant loss in collection efficiency for super-beam projects.

In addition, muon collider and neutrino factory studies are not compelled to early transverse pion charge separation but can perform transverse or longitudinal charge separation of the muons later down stream, see e.g. Chehab in Ref. [24]. For the super-beam case, Kahn suggests inserting a dipole in between solenoids in the suggested tapered system, though that scheme was never tested [31] and the study was not pursued further.

In conclusion, there are several previous studies showing promising results, but several important steps are missing in proving the suitability of a solenoid collector. Sufficient collection efficiency was shown by Diwan, Kahn and Palmer, given certain beam conditions. The most important piece missing is the charge
selection scheme, the performance of which is essential for the CP violation experiment with a non-magnetized detector. In the remainder of this report I will discuss the focusing properties of solenoid systems, and attempts to add a charge selection step, for the specific beam conditions of the ESSnuSB.

6 The solenoidal field

The solenoid magnet is characterized by a magnetic field solely in the longitudinal direction, which makes it an axisymmetric system. An ideal solenoid has a constant, homogeneous field $B_z$ inside the magnet and no field outside the magnet. In reality, the solenoid is constructed from current loops around the axis, which generates the magnetic field at the center of the loops. We can calculate the magnetic field on the axis induced by electric current loops surrounding the axis by applying Biot-Savard’s law, which will not be shown here. The field strength is maximum at the center of the magnet and decreases on the sides, and stretches longitudinally outside the current windings as so called end field, see Fig. 9. For many applications, however, we can assume the ideal hard-edge model shown in the same figure. Here, the integral from one field-free region to the field-free region on the other side is the same in the two cases. By field-free we have assumed that the residual field is a factor $10^{-4}$ weaker than the maximum field.

![Figure 9: The magnetic field on axis along the axis for a hard-edge model solenoid compared with a solenoid where the end field is taken into account.](image)

To the first order, the field inside the magnet is homogeneous, independent of the radial position. For accurate tracking of a divergent beam, such as the pion beam from the ESS neutrino target, higher order corrections to the field are important. In this report, we content ourselves with the first-order approximation.

Now, we want to look at the effect of the solenoid on particles of charge $e$. As a first approach we consider particles that stay close to the central axis so that the
paraxial approximation can be applied. We can then treat particle propagation through a solenoid, or any other magnet, as optical rays passing through lenses. We let a so-called transfer matrix represent each magnetic element and combine the matrices to form a series of accelerator components. In this way we can study the action of one or several magnets on a particle or particle distribution.

In Ref. [32] Royer gives a thorough description of the solenoid magnetic field, including the transfer matrix representing a solenoid in the hard-edge model. The solenoid transfer matrix can be constructed as a product of three matrices. The first matrix

\[ M_1 = M_{\text{edge}}(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 1 & 0 \\ -\alpha & 0 & 0 & 1 \end{pmatrix} \]  

(7)

represents the entrance edge of the solenoid, where the particle is given an azimuthal kick \( \alpha = eB/2p \) due to the change in magnetic field in the longitudinal direction. Here, \( B \) is the longitudinal magnetic field on axis, \( e \) is the elementary charge and \( p \) the momentum of the particle. Inside the magnetic field region the particle follows a helical trajectory with a radius \( r_0 = p_\perp/eB \) that increases with the transverse momentum \( p_\perp \). The rotation frequency is called the Larmor frequency and is given by \( \omega = eB/2\gamma m \). The number of turns \( n_L \) depends of course on the length of the magnet and on the longitudinal momentum as \( n_L = eBL/(2\pi p) \). The matrix

\[ M_2 = M_{\text{rot}} = \begin{pmatrix} 1 & \frac{L}{\gamma} \sin \theta & 0 & \frac{L}{\gamma}(1 - \cos \theta) \\ 0 & \cos \theta & 0 & \sin \theta \\ 0 & -\frac{L}{\gamma}(1 - \cos \theta) & 1 & \frac{L}{\gamma} \sin \theta \\ 0 & -\sin \theta & 0 & \cos \theta \end{pmatrix} \]  

(8)

describes this rotation, where \( \theta = eBL/p \) is the completed angle. At the exit edge, again the particle receives a kick, now in the opposite direction compared to at the entrance, thus eliminating the azimuthal momentum given there.

\[ M_3 = M_{\text{edge}}(-\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\alpha & 0 \\ 0 & 0 & 1 & 0 \\ -\alpha & 0 & 0 & 1 \end{pmatrix} \]  

(9)

The total effect of the solenoid on the beam is then given by the matrix \( M_{\text{sol}} \) which is a combination of the matrices above. We then have

\[ M_{\text{sol}} = M_3M_2M_1 = \begin{pmatrix} C^2 & CS/\alpha & CS & S^2/\alpha \\ -CS\alpha & C^2 & -S^2\alpha & CS \\ -CS & -S^2/\alpha & C^2 & CS\alpha \\ S^2\alpha & -CS & -CS\alpha & C^2 \end{pmatrix} \]  

(10)
with \( C = \cos(\theta/2) \), \( S = \sin(\theta/2) \), \( \theta = 2L\alpha \) and \( \alpha = eB/2p \).

We can of course also consider only a part of the solenoid. For example, if the particle source is placed inside the solenoid there will be no "entrance kick". Such a situation can be expressed by the matrix \( M_{\text{half}} = M_{\text{edge}}(-\alpha)M_{\text{rot}}(L') \) where \( L' \) is the effective length from the source to the exit edge of the solenoid.

With the transfer matrix we can easily propagate a single particle or an ensemble of particles through a solenoid. The main disadvantage of the transfer matrix approach is that end fields are not taken into account. If end fields play a significant role we must turn to Reiser’s detailed treatment of solenoids and other axisymmetrical systems in Ref. [33]. Reiser derives an equation that describes the evolution of the beam radius \( r \), or beam envelope, in such a system. The paraxial ray envelope equation, (eq. 3.49 in Ref. [33]) reads

\[
 r'' + \frac{\gamma'}{\gamma\beta^2} r' + \frac{\gamma''}{2\gamma\beta^2} r + \left( \frac{qB}{2mc\gamma} \right)^2 r - \left( \frac{p_\theta}{mc\gamma} \right)^2 \frac{1}{r^2} = 0
\]

but can be simplified significantly when there is no acceleration or deceleration, i.e. when \( \gamma' = \gamma'' = 0 \). In Eq. (11) \( p_\theta = \gamma mr^2\theta + eA_\theta r \) is the canonical momentum where \( A_\theta \) depends on the field at the source point.

In the coming section we will use Eq. (11) to study the focusing action of solenoids.

### 6.1 Solenoid focusing

The focal length of a solenoid can be calculated either from the transfer matrix or from Reiser’s Eq. 3.144 in Ref. [33], which takes the end fields into account. Let us consider a magnet with field strength \( B(z) \) on the axis. A particle of charge \( e \), mass \( m \) and relativistic speed and Lorentz factor \( \beta \) and \( \gamma \) will see a focal length of

\[
 \frac{1}{f} = \left( \frac{e}{2mc\beta\gamma} \right)^2 \int_{z_1}^{z_2} B(z)^2 dz
\]

(12)

For a hard-edge model where \( B(z) = 0 \) outside the magnet and \( B(z) = B_0 \) inside the magnet of length \( L \) we obtain a simple expression for the focal length:

\[
 \frac{1}{f} = \left( \frac{eB_0}{2mc\beta\gamma} \right)^2 L = \left( \frac{eB_0}{2p} \right)^2 L
\]

(13)

where \( p \) is the momentum of the particle.

To get a feeling for these numbers we calculate the focal strength based on the same beam parameters as before: \( E = 500 \text{ MeV} / c \) which gives \( p = 480 \text{ MeV} / c \) for pions. For a moderate magnet field strength of \( B_0 = 2 \text{ T} \) inside a magnet of length \( L = 1 \text{ m} \) and \( B = 0 \) outside. That leaves us with a focal length \( f = 2.6 \text{ m} \). If we increase the field strength to \( 10 \text{ T} \) we reduce the focal length \( f = 10 \text{ cm} \).
That means that for the strong fields that have been foreseen so far for solenoid collectors the focal length is much smaller than the actual magnet length, which in this case is equal to one Larmor wavelength.

The particular focusing we want to achieve is to transform the very divergent pion "beam" into an almost parallel beam. Because of the large distances we can consider it as point-to-parallel focusing. In Ref. [34] K. McDonald uses a hard-edge model of a solenoid to demonstrate that a solenoid can focus particles of any energy if the source is placed outside of the magnetic field. The same conclusion can be drawn either through the use of the transfer matrix, or by numerically solving the paraxial ray equation, Eq. (11). See Fig. 10 for an example.

\[
P_n = \frac{eB_0 L}{(2n + 1)\pi c} \quad n = 0, 1, 2, \ldots
\]

where \( L \) is the length of the magnet with constant field strength \( B_0 \) on axis along the axis. As usual, \( e \) is the elementary charge and \( c \) is the speed of light. However straightforward it is to analytically arrive to the same conclusion, it is challenging to achieve this discrete focusing in practice. As soon as we consider the end fields along the axis of the magnet the relation does no longer hold. We illustrate this with an example in Fig. 11 where we show the beam envelope evolution from the center of a solenoid to a few meters behind it. The magnetic field is optimized for reducing the final divergence of a beam with momentum \( P_0 \). The divergence

![Figure 10: If the source is placed well outside the magnet, where the end field is very weak, a simple solenoid is enough to achieve point-to-parallel focusing, assuming paraxial ray conditions.](image-url)
of higher order momenta $P_1$ and $P_2$ is not equally minimized, as stated in Eq. 14, though reduced compared to the initial divergence shown as a reference with the dashed line.

![Graph showing magnetic field and target position](image)

Figure 11: Target inside a single solenoid optimized for quasi point-to-parallel focusing at the momentum $P_0$. Point-to-parallel focusing is not reached for the higher-order momenta stated in Eq. 14.

The explanation to the fact that the envelope grows as soon as the magnetic field strength decreases is to be found in the first assumptions of the envelope equation, that the canonical momentum in the azimuthal direction $p_\theta$ is a constant of motion in an axisymmetrical system. This, called Busch’s theorem, can be expressed as

$$p_\theta = m\gamma r^2 \frac{d\theta}{dt} + \frac{eB_z r^2}{2} = \text{constant}. \quad (15)$$

If the source of radius $r_0$ is placed inside the magnet where the field is $B_0$, at which point $d\theta/dt = 0$, we can calculate this constant of motion to be $p_\theta = eB_0 r_0^2/2$. After the magnet, as the magnetic field $B_z$ goes to zero and the azimuthal angle remains unchanged, i.e. $d\theta/dt = 0$ again, we get that the radius $r$ inevitably grows. This implies that, in an analytical sense, it is impossible to focus a beam using one solenoid if the beam particles are born inside the magnetic field.

A solution to the fundamental limitation stated by Bush’s theorem is to let the envelope grow slowly by adding a weak magnetic field behind the first stronger solenoid. This combination of a short but strong solenoid followed by a long but weak solenoid, sometimes called the quarter-wave transformer [24], transforms a small but highly divergent beam into a large, quasi-parallel beam. An example is shown in Fig. 12, where we have recovered the point-to-parallel focusing from Fig. 10 but with the target placed inside the first magnet. These results, however, hold only for a beam with a narrow energy distribution. To increase the energy
acceptance we turn to the so-called *adiabatic device* [22]. Instead of two solenoids, we use a continuously decreasing magnetic field accompanied with an increase of the bore radius, so that the highly divergent beam is allowed to slowly expand while losing transverse momentum. The tapering of the field $B$ and the bore radius $R$ along the longitudinal coordinate $z$ is described by

$$B(z) = \frac{B_0}{1 + az} \quad R(z) = R_0\sqrt{1 + az}$$

(16)

where $B_0$ and $R_0$ are the field and aperture, respectively, at the beginning of the magnet, where the target is located. The parameter $a$ affects the focusing and also the acceptance in transverse momentum [29]. An example of such focusing is shown in Fig. 13. Figures 10, 12 and 13 all show the same beam parameters: relativistic Lorentz factor $\gamma = 4$, initial radius $r_0 = 10$ mm, and initial divergence $r'_0 = 100$ mrad.

![Graph of magnetic field and target position](image)

**Figure 12:** Quarter wave transformer: A strong but short solenoid followed by a long weak solenoid. With this setup it is possible to achieve at least quasi point-to-parallel focusing.

Now, we look at the focusing potential for varying particle momentum. In Fig. 14 we have used the same transverse beam parameters and the same simple solenoid as in Fig. 10 and we look at the envelope evolution for particles with Lorentz factors $\gamma = 3, 4$ and $5$. We see strong divergence after the solenoid for the non-matched energies. The same case is shown for the quarter-wave transformer in Fig. 15, though the focusing is smoother and the energy acceptance is improved. Even slightly better is the adiabatic device in Fig. 16.

Here, each system as been optimized to match specific conditions but will accept small deviations from the same. The results are not necessarily in line with the final acceptance of these three systems. To establish that extensive studies, moving away from the paraxial approximation, would be required.
Figure 13: The adiabatic device: A tapered magnetic field gives the best energy acceptance for quasi point-to-parallel focusing.

Figure 14: Changing the particle momentum means that we lose the focusing.

6.2 Charge separation

The charge selection capability of the pion focusing scheme is indispensable. Since a solenoid focuses both positive and negative particles in a similar manner, we add a dipole field to the system. As a first approach, we ignore the focusing and consider the charge selection only.

We start from the angular distribution of $\pi^+$ and $\pi^-$ in the horizontal plane, $x$, displayed in Fig. 17. All pions traveling in the backward direction have been discarded, which accounts for the abrupt drop at $\pm \pi/2$. With these strong tails, it is clear that the separation must be substantial. As a first crude estimation of the degree of separation needed we apply a hypothetical dipole field in the
Figure 15: Also the quarter-wave transformer works best as a narrow-band collector and has been used as such in positron sources [35].

Figure 16: With the tapered field the momentum acceptance is increased but is far from complete.

vertical direction so that each particle is deflected in the horizontal plane. The
deflection angle, $\phi(p) = \phi_0 p_0 / p$, depends on the particle momentum $p$, where
$\phi_0$ is the deflection angle for particles with momentum $p_0 = 400\text{ MeV}/c$. The
direction of deflection depends on the pion charge. Behind the dipole field, the
horizontal angular distribution of particles form two partially overlapping peaks.
An example, where $\phi_0 = 1\text{ rad}$ has been used, is shown in Fig. 18. Now we
assume that we collect pions with a certain acceptance $\pm \delta$ around the central
deflection angle. By comparing the number of pions of each charge collected
within this acceptance we can calculate the contamination levels. In Fig. 18, the
acceptance is $\delta = 0.2\text{ rad}$ and is marked with gray bands around the central angle
$\phi_0 = 1\text{ rad}$. We observe that because of the large momentum spread and the
nonlinear dispersion the peaks are not located at the central deflection angle.

\[ \text{Figure 17: The horizontal angular distribution of the } \pi^+ \text{ and the } \pi^- \text{ mesons, where all the charged pions traveling in the backward hemisphere have been discarded. There are 1.66 times more } \pi^+ \text{ than } \pi^- \text{ produced.} \]

By varying the deflection angle, or separation angle \( \phi_0 \) we can identify the separation level needed to reach \(<1\%\) contamination. Figure 19 indicates that for \( \delta = 0.2 \text{rad}\) this occurs at around \(0.9 \text{rad}\). At this separation angle, however, the density peak is far from the acceptance band, which means a reduced efficiency. Although, it is possible to shift the acceptance band to around the peak it would increase the contamination. In order to simultaneously keep the contamination low and the collection efficiency high we must reduce the angular spread before we attempt to separate the charges. In practice, this means that we need to first focus and then apply a dipole field. In the next section we will look at a design of such a scheme.

7 Simultaneous focusing and charge separation

In the previous chapter we looked at the focusing capabilities of solenoid systems. In addition, we concluded that it is necessary to collect the pions before we add a dipole field for charge separation. Here, we try a scheme that combines the focusing and charge separating functions. A requirement for the system is that the absorber/disperser is located maximum around \(1-2 \text{m}\) from the target. If the distance is larger the contamination is likely to be too high.

Even though we have pointed out the insufficiency of the paraxial ray approximation, we use it here as a first step. Through that we aim at identifying limitations to what can be achieved with simple optics. For computational ease,
Figure 18: The horizontal angular distribution of the pions after passing through a dipole field corresponding to a $\phi_0 = \pm 1$ rad central deflection angle. The gray bands mark a $\pm \delta = \pm 2$ rad acceptance around $\phi_0$ where pions are collected and counted.

Figure 19: Contamination as a function of separation angle $\phi_0$. The acceptance is here set to $\pm 0.2$ rad.

we use transfer matrices to propagate pion through our system of solenoid and dipole fields, and limit ourselves to considering hard-edge magnets.

Let us start with studying the quarter wave transformer from Fig. 12. This solenoid doublet transforms the small divergent beam to a large quasi-parallel beam. In the drift between the two solenoids we place a dipole field that gives a horizontal kick in a direction defined by the pion charge. We then let the kicked beam pass through the second solenoid before we look at the distribution of the positive and negative pions, respectively. At this point the separation between
the charges should be sufficient for complete removal of one of them.

In order to reduce the distance from the source to the charge separation point we double the strength of the QWT in Fig. 12 and reduce the length of each element by a factor two. We use a beam with mean momentum $p_0 = 550\text{ MeV}/c$, initial rms size of 1 mm and divergence 50 mrad. By adding a $\varphi_0 = \pm 0.5\text{ rad}$ kick between the solenoids we obtain the beam distribution in Fig. 20, here with a monochromatic beam. The charges are nicely separated behind the QWT, shown in Fig. 20(b), and the divergence effectively reduced in Fig. 20(a).

Figure 20: A stronger version of the QWT from Fig. 12 represented by transfer matrices and with a horizontal kick applied between the two solenoids. The resulting distribution in horizontal phase space is shown in (a) and the cross section in (b).

When an rms momentum spread of $\sigma_p/p_0 = 0.1$ is included, the situation changes dramatically. We now obtain the distributions in Fig. 21, where the color of each marker indicates the momentum $p$ of the macroparticle, given by the colorbar in MeV/c. Note that the kick applied to each macroparticle depends on its momentum as $\varphi = \varphi_0 p_0 / p$. The dispersion leads to a distortion of the beam distribution so that the final divergence is larger than in Fig. 20(a). The charge separation is also less clean than in Fig. 20(b).

The overlap between the two pion charges caused by the momentum spread cannot be overcome by applying a stronger kick since that only increases the dispersion. It would result in a larger and more divergent final beam but not solve the contamination issue. With this simple scheme and this rather well-behaved beam we get about 0.3% contamination at the source also for a kick twice as strong, 1 mrad. Since the real pion distribution is both more divergent and has a wider momentum distribution we turn next to a more elegant combination of solenoid and dipole fields.

Let us consider the series of components sketched out in Fig. 22. We begin with a target inside a solenoid which is responsible for the initial collection. This
Figure 21: The horizontal phase space distribution (a) and the transverse cross section (b) behind the second solenoid after an rms momentum spread of $\sigma_p/p_0 = 0.1$ has been included. The colorbar indicates momentum in MeV/c.

Figure 22: A block diagram of the full system, from initial collection, through charge separation to final focusing.

solenoid should be rather short and with a relatively strong field in order to maximize the number of pions collected. Following the first solenoid we place another solenoid with a dipole field superimposed. This is the charge separation stage, where we choose to combine a solenoid and a dipole in order for the pions to stay focused. After the charge separation stage there should be maximum separation of the two charges. This implies that the length of the separation magnet should be adapted to the particle momentum so that the particle performs half a Larmor rotation in the magnet. After this section we place an absorber or possibly a disperser. Here, we make sure that only one charge goes through. The dispersion generated by the dipole field needs to be canceled for the selected charge. This is done by adding at least one more dipole, together with more solenoids to maintain the beam. Finally, when the dispersion is canceled, we place a final focusing system, probably of the tapered kind. The last stage should be optimized so that the angular divergence is reduced below the kinematic limit explained in section 3.1.
Having a dipole field superimposed with a solenoid field is equivalent to having a tilted solenoid, from a field direction point of view. Instead of rotating the solenoid field components we handle the tilt by rotating the coordinate system.

A first attempt to split the $\pi^+$ and $\pi^-$ beams is shown in Fig. 23. Figure 23(a) shows the phase space coordinates and Fig. 23(a) the transverse position coordinates of pions at the location of the absorption. For this case, the same beam parameters as in Fig. 21 have been used. The solenoid strengths and lengths have thus been optimized for a beam with 1 mm rms beam size, 50 mrad divergence and average momentum $p_0 = 550$ MeV, corresponding to roughly $\gamma = 4$. The dipole field component chosen corresponds to a deflecting angle of 0.74 rad whereas the tilt of the coordinate system is 0.5 rad. A deflection angle of 0.5 rad gave insufficient charge separation, of the same order as the system in Fig. 21. After the field adjustments were made an rms momentum spread of $\sigma_p = 0.1$ was added.

We observe that the degree of separation has indeed improved compared to the case shown in Fig. 21. Also the divergence at the point of the absorber is smaller. Further reduction is possible by adding solenoids, thus forming a final focusing stage.

![Figure 23: The horizontal phase space distribution (a) and the transverse cross section (b) behind the second solenoid which has a dipole component added to the vertical plane. The same beam parameters as in Fig. 21 have been used but the final divergence is smaller and the contamination is removed and. The colorbar indicates momentum in MeV/c.](image)

To summarize, we have been able to collect pions from a small but divergent source and turned them into a larger but less divergent pion beam. In addition we have separated these charges sufficiently well for us to remove the undesired pion charge. We have done so successfully for a Gaussian pion distribution in transverse phase space and in momentum. An rms divergence of roughly 0.1 rad can be tolerated, and an rms momentum spread of 10% around the peak of 400-
600 MeV/c. With large divergence or larger momentum spread the separation is insufficient and consequently the contamination tolerance of the experiment is surpassed.

For further advancing this study, it is necessary to abandon the transfer matrix formalism and turn to full three dimensional particle tracking. Based on such simulations, in e.g. FLUKA, the conclusion can be drawn on the efficiency of the focusing and charge selection of a pion collector based on solenoids.

8 Conclusion and Outlook

We have investigated the feasibility of using superconducting solenoids in combination with magnetic dipole fields to produce a pure, well collimated neutrino super-beam. A superconducting solenoid has strong advantages compared to a van der Meer horn since it can be operated in DC mode and therefore has a longer lifetime can be expected. This also implies that it can accept the full length pulse from the ESS linac, thus making the accumulator ring and subsequent $H^-$ operation of the linac superfluous. It also has disadvantages, such as strong requirements on shielding of the superconducting coil, and the "blindness" to charge when looking at the focusing properties. A note on shielding is found in Appendix A.

This study has focused on the feasibility of using a superconducting solenoid for the ESSnuSB. We have investigated the focusing capabilities of different systems including solenoids, as well as looked at possible charge separation schemes. This preliminary study indicates that the solenoid systems perform well in terms of focusing but that charge selection is a challenge. Together, the two functions put a limit on the momentum and divergence acceptance which does not match the pion distribution emerging from the target.

We have shown that a combination of solenoids and dipoles can be used to focus and separate the charges of pions with an rms divergence of roughly 0.1 rad and rms momentum spread of 10%. Larger momentum spreads may lead to contamination.

This study assumes approximations that are not applicable for larger beams and larger momentum distributions. Continued investigations should include full three-dimensional tracking rather than computations based on the paraxial ray approximation. As an alternative to transverse charge separation, longitudinal separation using RF cavities could be considered. The latter would be beneficial for a future muon collider.
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A Shielding of superconducting coil

The coil of the superconducting solenoid must be protected from pions that elude the focusing efforts, through internal shielding. The shielding will occupy the space in the magnet bore so that higher current is needed to obtain the desired magnetic field in the center. There is of course a limit to what amount of shielding is practically feasible and we want to early identify the shielding needs.

To this end, I made an estimate of how much lead or concrete it takes to absorb the shower from a pion with momentum 1 GeV/c. A first number was obtained through analytical calculations using a continuous slowing-down approximation, which arrived at the result of roughly 1 m for lead. A Monte Carlo simulation with FLUKA gave a more optimistic result: 0.4 m of lead or 1 m of concrete to stop 90% of the pion energy.

For details, see Ref. [36].