Geometrical projectile shapes effect on hypervelocity impact

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Abstract

All spacecraft confronts threat of space debris and meteoroids. Spherical projectiles are common shape used to study impact of space debris on spacecraft structures or shielding system. However, real space debris which is a threat to spacecraft is not likely to be spherical. There is a need to study the influence of projectile shape as non-spherical shapes may cause greater damage to spacecraft than spherical projectiles with same impact conditions.

The aim of this research is to investigate the effect of projectile shapes focusing on impact velocity in hypervelocity range by numerical simulations. The geometrical shapes of projectiles that were simulated are sphere, cylinder and cube. Projectiles and targets are made of aluminium, 2024. All projectiles have equivalent mass hence equivalent impact energy The impact velocities are 3 km/s, 5 km/s, 7 km/s and 9 km/s. Smoothed particle hydrodynamics (SPH) is applied.

The shapes of debris clouds, velocities of debris clouds, residual velocity of projectiles, dimension of target hole after impact, impact induced stress and target failure modes are investigated and compared between different projectile shapes. Shapes of debris clouds generated by impact of cylindrical and cubic projectiles have spike-like in frontal area of the debris clouds, which do not exist in that of spherical projectile. Velocities of debris clouds generated by the impacts of cylindrical and cubic projectiles are higher than that of spherical projectile in hypervelocity impact range, i.e. higher than 5 km/s.

The conventional spherical projectile is not the most dangerous case of space debris. With equivalent mass, a cylindrical projectile, with length-to-diameter ratio equal to 1, is more lethal than a spherical projectile. A cubic projectile is more lethal than a spherical projectile and cylindrical projectile, with length-to-diameter ratio equal to 1 and with an equivalent mass.
I would like to thank Prof. Rade Vignjevic for his advices, recommendations, guidance and supervising me through this work.

I would also like to thank Dr. James Campbell who always gives me suggestions and has been very helpful and to Mr. Nenad Djordjevic and Dr. Kevin Hughes.

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Thanks especially to my beloved parents, sister and brother who always support me in every step I take and everywhere I am.
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<tr>
<td>A</td>
<td>Area</td>
</tr>
<tr>
<td>E</td>
<td>Internal energy</td>
</tr>
<tr>
<td>E₀</td>
<td>Initial internal energy</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>T*</td>
<td>Homogenous temperature</td>
</tr>
<tr>
<td>V</td>
<td>Volume</td>
</tr>
<tr>
<td>Vₛ</td>
<td>Specific volume</td>
</tr>
<tr>
<td>U</td>
<td>Shock velocity</td>
</tr>
<tr>
<td>uₚ</td>
<td>Particle velocity</td>
</tr>
<tr>
<td>v₀</td>
<td>Initial velocity</td>
</tr>
<tr>
<td>c</td>
<td>Wave velocity</td>
</tr>
<tr>
<td>ρ</td>
<td>Material density</td>
</tr>
<tr>
<td>C₀</td>
<td>Speed of sound in the material</td>
</tr>
<tr>
<td>W</td>
<td>Kernel function</td>
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<tr>
<td>H</td>
<td>Smoothing length</td>
</tr>
<tr>
<td>σ</td>
<td>Stress</td>
</tr>
<tr>
<td>σ*</td>
<td>Ratio of pressure to effective stress</td>
</tr>
<tr>
<td>σᵧ</td>
<td>Yield stress</td>
</tr>
<tr>
<td>ε</td>
<td>Strain</td>
</tr>
<tr>
<td>ε*</td>
<td>Non-dimensional strain rate</td>
</tr>
<tr>
<td>εᵖ</td>
<td>Effective plastic strain</td>
</tr>
<tr>
<td>εᵣ</td>
<td>Strain at fracture</td>
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## Abbreviations

<table>
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<th>Description</th>
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<tr>
<td>SPH</td>
<td>Smoothed Particle Hydrodynamics</td>
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<tr>
<td>MCM</td>
<td>Meshless Continuum Mechanic</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary Earth Orbit</td>
</tr>
<tr>
<td>HEO</td>
<td>Highly Elliptical Orbit</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Admin</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>UN</td>
<td>United Nations</td>
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1. Introduction

Geometrical shape projectile effects on hypervelocity impact are applicable to impact of space debris to spacecraft. The space debris threat is explained to aid in the understanding of the confronting threat of spacecraft. Spacecraft shielding systems and the effect of space debris in general are described. The objectives, methodologies and outline of the research are described in this chapter.

1.1 Space Debris Threat

It has been more than 50 years since the beginning of space era, starting at the first Sputnik launch in 1957. Mankind continues launching and delivering spacecraft, be it rockets, satellites, shuttles or probes into the space. Numerous parts of spacecraft are abandoned in the space. Their sizes may be as small as aluminium oxide particles from solid rocket exhausts, or fasteners, to non-functioning satellite or launch vehicle upper stages (Lacoste, 2009). These objects that are abandoned near the Earth are still orbiting around the Earth and become space debris. All spacecraft will be confronted collision of space debris and meteoroids during their function lifetime (Wertz et al., 1999). The collision may cause minor damage to major damage that can leads to the loss of mission and spacecraft. The level of damage depends on the size, impact velocity, impact angle, space debris density, and so on. The debris as small as 1 cm may cause serious damage to spacecraft while particle sizes 0.1 cm may give rise to surface erosion in the long term (Tribble, 2003 and UN, 1999).

1.2 Spacecraft Shielding System

Spacecraft shielding system is a passive mean to protect spacecraft against space debris and meteoroids. Shielding system is effective to protect spacecraft against objects smaller than 1 cm in diameter (UN, 1999). The simple and practical shielding system is Whipple bumper shield that consists of a single thin plate called bumper placed at a short distance ahead of a primary structure of the spacecraft, pressure or rear wall (Chi, 2008). The concept is to fragment and/or vaporise the projectile through the impact with the bumper, typically metallic or composite thin plates. Intensity of impulse is reduced before the debris cloud of the projectile and bumper hits spacecraft structure and leads to
the loss of mission and/or spacecraft. The performance of shields is dependent on distance between the bumper and the primary structure wall, the material of the shield, the thickness of the shield and so on.

Spacecraft shielding systems have been continuously developed, varying both the number of bumpers and the materials used. The main concept is still to fragment and/or vaporise the projectile. The resulting debris cloud expands and hits the next layer over a larger area, dispersing the energy of the projectile and the impulsive load (Campbell 1998). The characteristics of the debris cloud indicate the ability of the bumper of fragmentation and vaporisation of the projectiles. In designing spacecraft shielding system, it is crucial to comprehend debris cloud of the impact between of space debris and the bumper.

1.3 Effect of Space Debris Shapes

Spherical projectiles have been the conventional shape used in most of the experiments and researches. Morrison and Elfer (1988) have found cylinders with a length to diameter ratio of 1 are much less penetrating than equivalent mass spheres at low impact velocities of 3 km/s but the ranking reversed at more than 6 km/s (Elfer 1996). Elfer has stated that this was because the cylinder shatters more easily than the sphere at 3 km/s. At more than 6 km/s, in normal impacts the cylinder creates a debris cloud with a spike of which the tip is the most lethal. The tip appears to be a spall fragment from the bumper, normal to the flat of the cylinder that can penetrate the rear wall.

Rolsten Hunt and Wellnitz (1964) found that significant differences in the impact pattern of explosively-launched disc with different orientation in their studies of principles of meteoroid protection. Friend, Murphy, and Gough (1969) used polycarbonate cylindrical and conical and cup-shaped projectile in their studies of debris cloud pressure distribution on secondary surface. Morrison (1972) proved that in particular cases a cylindrical shape is more dangerous than spherical projectiles.

Piekutowski (1987, 1990) stated that the cylindrical projectiles are more efficient penetrators of double-sheet structures than spherical projectiles of equivalent mass, and the incline cylindrical projectile causes more severe damage to the structure.
Buyuk (2008) has studied ellipsoidal projectiles and compared the results of oblate and prolate with sphere. He concluded that it is not necessarily correct that the ideal spherical projectiles are the most dangerous threat, and also presented that the most dangerous case of ellipsoidal projectiles orientation is not the case with the longest or shortest side of the ellipsoidal projectiles parallel or perpendicular to the impact direction.

1.4 Research Objectives

Radar and ground-based studies of orbital space debris has discovered that orbital space debris is not only composed of spheres. Their sizes and shapes are varied (Williamsen, 2008). Beside conventional studies in spherical projectiles, studies are required for non-spherical projectiles with different shapes in order to take them into account when designing spacecraft shielding. In this research, the influence of projectiles with different geometrical shapes is to be investigated. Numerical simulations are applied due to their advantages of inexpensiveness, less man-power, resource and time consuming in comparison to actual experiments.

The initial objective of this research is to investigate the effect projectile shape by plotting ballistic limit curves for each projectile. The first dual-wall structure was constructed. Its output file was 10 GB which was too large. Plotting a ballistic limit curve, many simulations are required. In order to observe characteristics of debris clouds in different projectile shapes, many curves are required.

Study of debris cloud produced by projectile impact on the bumper can be used for development of the method for approximating the pressure loading imposed on the rear wall or spacecraft structure. A spacecraft designer can estimate the response of a structure by using the approximate pressure-loading history (Piekutowski, 1990).

The main objective of this research is to investigate the influence of projectile shapes on spacecraft shielding systems in the hypervelocity impact range. The extended objectives of this research are the following:

- Simulate the impact of different shapes of projectiles on thin plate targets using smoothed particle hydrodynamics methods.
Investigate the influence of different geometrical projectile shapes on debris cloud characteristics and the fragmentation of projectiles with equivalent mass impacting on thin plate target in hypervelocity impact.

1.5 Research Methodologies

The methodologies of the research are:

(i) Literature review for comprehension of the existing problems of space debris and its threat, its general background for defining scope, objective, methodologies and further simulation set up for this research.

(ii) Study the associated physical impact phenomena based on space debris impacts on spacecraft shields or spacecraft.

(iii) Study the methods used in the impact of projectiles in this range of velocity for numerical simulation i.e. smoothed particle hydrodynamics methods.

(iv) Set up a simulation based on available experimental data to validate numerical simulation. Validating numerical simulation by comparisons of characteristics of debris cloud generated by numerical simulation and actual experiments. Accuracy of numerical simulation is calculated.

(v) Simulate aluminium spherical, cubic and cylindrical projectiles impacts on single thin aluminium plates. All projectiles are made of aluminium and have equivalent mass. Observe characteristics of debris cloud generated by each geometrical projectile.

(vi) Compare the characteristics of debris cloud generated by different projectile shapes and finally conclude with suggested future work.

Properties of debris cloud to be observed:

- Debris cloud shape and size
- Debris cloud velocity profile and debris cloud velocities at particular points
- Target hole dimension and failure mode
- Impact induced stress
1.6 Research Outline

Chapter 1. *Introduction*, explains the space debris threat, the basic spacecraft shielding system, geometrical shapes of space debris, defines the problem and scope, objectives, methodologies and outline of this research.

Chapter 2. *Orbital Space Debris and Protection*, describes the introduction to orbital space debris, the space debris environment and activities on this issue, the spacecraft shielding system and debris cloud characteristics.


Chapter 4. *Smoothed Particle Hydrodynamics Method*, describes the introduction to SPH and numerical simulations, spatial discretizations: Eulerian, Lagrangian, artificial viscosity, SPH formularisation, and Implementation in MCM (Meshless Continuum Mechanic Code).

Chapter 5. *Simulation Validation*, presents simulation features, MCM modelling and the results of spherical and cylindrical projectiles impact on single plates imitating actual experiments.

Chapter 6. *Geometrical shape projectiles impact simulations*, presents model descriptions, measured parameters, modelling and results of spherical, cylindrical and cubic projectile impact on single thin plate target.

Chapter 7. *Comparisons of geometrical shape projectile impact*, presents comparisons of debris cloud shapes and velocities, target hole dimensions, target failure modes, and induced stresses generated by different shape of projectiles impact from chapter 6.

Chapter 8. *Conclusion*, concludes geometrical shape effects on hypervelocity impact and future work recommendation.

Chapter 9, *References*, provides references of this research.

Appendix A, *MCM codes*, presents an example of MCM codes
2. Orbital Space Debris and Protection

In this chapter space debris and protection against it are explained. The studies in space debris are divided into three main topics, which are measurements, modelling & risk assessment, and mitigation. The characteristic length of space debris is explained. The spacecraft shielding system is introduced and its development is explained. The ballistic limit curve is described.

2.1 Introduction to Orbital Space Debris

Orbital space debris, also called space debris, space junk or space waste, is the man-made objects which is no longer used (UN, 1999) generally refer to it as the results of space missions, no longer serving any functions anymore. This has drawn attention to many authorities and organisations around the world that are aware of the growth of space debris and its threat to spacecraft. Space debris is orbiting around the Earth with very high velocity.

![Figure 2.1. Trackable objects in orbit around the Earth (ESA, 2008)](image)

In Figure 2.1, the illustration of the trackable objects in orbit around the Earth by European Space Agency is shown. Space debris population is densest in Low-Earth Orbit (LEO) and Geostationary Orbit (GEO), which are the orbits that are used for Earth observation and communication. The growth of space debris population in LEO is
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approximately 5 percent per year. With the effort of all nations and many organisations in the last few decades, it is expected that the growth of the space debris population will decrease and the amount space debris will be mitigated. This mitigating effort can be achieved by de-orbiting to the atmospheric drag, placing spacecraft in disposal or graveyard orbits, and so on (NASA, 2005).

![Space Debris Objects > 1mm](image)

**Figure 2.2. Spatial density of space debris by altitude according to ESA MASTER-2001 (Wiki: Michael Oswald, 2004).**

The elaboration of space debris density is graphed in Figure 2.2. The impact velocities of space debris are dependent on the orbiting altitude (Committee on Space Shuttle Meteoroids/Debris Risk Management, 1997). Space debris in LEO is travelling with higher velocity than debris in GEO. Thus the impact velocity of space debris in LEO, approximately between 10-15 km/s, is higher than that in GEO which is approximately 0.5 km/s. To visualise this velocity range, one can compare with objects on Earth such as bullet. The rifle bullet velocity is between 1.0 and 1.5 km/s (Pereyra M. 1999). Therefore, impact velocity in LEO is roughly 10 times higher that rifle bullets velocity. According to NASA (2009), 1-centimeter of aluminium travelling at this range of velocity may cause damage as harmful as 400-lb safe travelling at 60 mph. Figure 2.3 shows the growth of space debris population in the last four decades.
2.2 Study in Space Debris

Space debris has been studied in three main aspects according to United Nation conference in New York, USA, 1999. The space debris problem has been investigated and divided in three main categories (UN, 1999) which are space debris measurement, space debris modelling & risk analysis, and space debris mitigation:

2.2.1. Space Debris Measurement

Space debris is tracked and monitored. Tracking can be performed using either ground-based systems or space-based systems. Generally, ground-based systems are capable to track space debris of size 10 cm or larger and space-based systems are able to track space debris of size 1 cm or larger (Tribble, 2003).

Ground-based measurement systems have two main categories of measurement, which are radar measurement and optical measurement. The radar measurement can be performed in all weather and in both day and night time but it is limited to small objects in long range (UN, 1999) whereas optical measurement needs to be performed while the space debris is sunlit and the background sky is dark. Therefore, the observation in LEO can be performed in limited periods of the day while in HEO (High-Earth orbit) it can be performed at all times of the day. According to ESA, 2009, (Lacoste, 2009) radar measurements are suitable for the LEO regime below 2000 km. Optical measurements are suitable for GEO and HEO observations.
Space-based measurement systems have two categories. The first is retrieved surfaces and impact detectors, approached by analysing the exposed spacecraft surface to the space environment after its return to Earth. Examples of well-known projects of this type of measurement are Long-Duration Exposure Facility (LDEF), European Retrievable Carrier (EURECA), etc. The second category uses sensors to measure impact fluxes and the meteoroid and space debris population. Space-base measurement has higher resolution and performance than ground-based systems, with the trade-off of higher cost. Examples of this space-based measurement are DEBIE (Debris In-Orbit Evaluator) launched in 2001 and DEBIE II launched in 2008, Infra-red Astronomical Satellite (IRAS) in 1983, MSX spacecraft in 1996 and Geostationary Orbit Impact Detector (GORID) in 1996.

Cataloguing is where a record is taken of orbital space debris population characteristics that have been derived from measurement or record. The United States Space Command catalogue and the space object catalogue of Russian Federation are space debris catalogues that are frequently updated. ESA has the Database Information System Characterising Objects in Space (DISCOS), which is based on those two catalogues belonging to the United States and the Russian Federation.

### 2.2.2. Space Debris Modelling and Risk Assessment

Models of space debris environment rely upon historical data and are limited by the sparse amount of data available to validate the derived characteristics. Space debris models have been continuously developed by various organisations, such as EVOLVE by the NASA Johnson Space Center and MASTER by ESA.

Space debris risk assessment is divided into collision risk assessment and re-entry risk assessment. Collision risk assessment is utilised to enhance safety of space operation, designing spacecraft with pre-flight risk assessment within acceptable levels. It is also used to design the type and location of space debris shielding to protect the crew and critical subsystems and components. Re-entry risk assessment is the assessment of the chance of uncontrolled re-entry from the Earth orbit.
2.2.3. Space Debris Mitigation

Mitigation consists of two main concepts, which are not to make the existing problem worse, and for a reduction of the rate of increase of space debris over time; to strive to protect the commercially valuable low Earth and geostationary orbits through protection strategies (ESA, 2009).

Reduction in the rate of increase of space debris has three approaches. First, avoidance of debris generated under normal operation resulting making the existing problem worse. Second, prevention of on-orbit break-ups is by preventing on-orbit explosion and collision. Third, de-orbiting and re-orbiting of space objects.

![Figure 2.4. Space debris sizes and protection methods.](image)

Protection of spacecraft can be achieved by both shielding and collision avoidance. Space debris shielding is quite effective against small particles. Particles smaller than 0.1 cm, can cause erosion in the long term but no serious immediate damage (Tribble 2003 & Vedder, 1976). Figure 2.4 shows protection of spacecraft against space debris with respect to space debris size. Typical space debris shields can protect spacecraft structures against particles 0.1 to 1 cm in size. All objects 1 to 10 cm in size cannot currently be blocked by shielding technology nor be tracked for collision avoidance by operational means. Nonetheless, risk analysis has very much concern in the range of space debris size that spacecraft shielding cannot protect against. The risk of being impacted by space debris size 5 mm is 0.005 time per ten-day mission (Committee on Space Shuttle Meteoroids/Debris Risk Management, 1997). Therefore, risk analysis is the only available solution for space debris of sizes between 1 to 10 cm. Space debris sizes larger than 10 cm are trackable by ground-based systems. In addition, protection
against objects 1 to 10 cm in size can be achieved through special features in designing of spacecraft systems, such as redundant systems, frangible structures, pressure vessel isolation capabilities, etc.

### 2.3 Characteristic Length of Space Debris

To define different shapes of space debris for radar detection, characteristic length is a parameter characterized size of space debris developed by NASA (Hill et al, 2008)). Characteristic length is defined as:

\[ L = \frac{X + Y + Z}{3} \]

Where X, Y and Z are the maximum orthogonal projections of object

It has been used for the detectable radar cross-section. The characteristic length of sphere is its diameter. For a cube, the characteristic length is its dimensions in length, width and thickness. If spherical and cubic space debris are made of identical material, then they possess equivalent mass and volume. The cubic space debris has a smaller characteristic length and less detectable than spherical space debris.

### 2.4 Spacecraft Shielding System

Beside man-made space debris, meteoroids which are natural objects in the solar system (Wiki, 2009) may collide with the spacecraft and consequently cause damage to the spacecraft. The meteoroid threat has been a concern since the beginning of human spaceflight when the space debris has not yet accumulated and become an important issue of space hazards. Nevertheless, collision of meteoroids is less harmful to the spacecraft in comparison with the collision of space debris, due to the fact that the density of the meteoroids is much less than space debris even though impact velocity of meteoroids may be higher (Jolly, 1997). Hence, the key to design spacecraft in order to withstand both space debris and meteoroids is to consider the impact of space debris to spacecraft or its shielding system.

#### 2.4.1. Shielding System Development

The first spacecraft shielding against meteoroids and space debris was introduced by Fred Whipple in 1947, which is still being used until present day (Jolly, 1997). The Whipple bumper shield has one outer bumper placed at the short distance ahead of a
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primary structural system. The concept is to fragment and/or vaporise the projectile through the impact with the bumper, typically metallic or composite thin plates. The main concept is to fragment or vaporise the projectile. The resulting the debris cloud expands and hits the next layer over a larger area, dispersing the energy of the projectile and the impulsive load. The Whipple bumper shocks the projectile and creates a debris cloud containing smaller, less lethal, bumper and projectile fragments. The full force of debris cloud is diluted over a larger area on the spacecraft rear wall.

![Figure 2.5. Whipple bumper shield after impact of projectile (left) and rear wall of a Whipple shield impacted fragmentation of projectile and bumper (right) (Hyde, HITF, 2009)](image)

Spacecraft shielding has been continuously developed. Various materials are adopted to apply in spacecraft shielding system, for examples, aluminium, Nextel™, Kevlar®, aluminium mesh, aluminium honeycomb sandwiches, etc. The basic idea of fragmenting and vaporising a projectile is also applied to other shielding systems that have been developed after Whipple bumper shield, such as stuffed Whipple shield, mesh double bumper shield and multi-shock shield (Schonberg, 1999).

![Figure 2.6. Stuffed Whipple shield after impact of a projectile (left) and mesh double bumper shield after impact of a projectile (right) (Hyde, HITF, 2009)](image)
2.4.2. Ballistic Limit Curves

The Ballistic Limit Curve, BLC, is a practical diagram used to describe the protective performance of a shield. Debris impact velocities are plotted against the size of debris, with a threshold for penetration and no penetration. The region below the graph is the no-penetration area. The region above the graph is penetration area. The criteria of penetration and no-penetration are generally predefined in spacecraft requirements and depend on the critical component being protected. It can be developed either by ballistic limit equations or experimental data.

![Ballistic Limit Curves Diagram](image)

**Figure 2.7. Generic Single-Wall and Double-wall Ballistic Limit Curves (Schonberg, 2008).**

Generic single-wall and double-wall ballistic limit curves are represented in Figure 2.7. Ballistic limit curves are typically the threshold between rear-wall perforation and no perforation. In double-wall, as shown in Figure 2.7 that equivalent projectile diameter may cause penetrate in ballistic range while in higher velocity it may not penetrate through the rear-wall. This is due to projectiles at low velocity do not fragment and break up as in higher velocity in shatter range. The fragmentation of projectiles mitigates impulse density reaching the rear-wall. In the double wall, the curve is similar with single wall at the beginning and rises up in transition region where the effect of shock wave in the solid takes place (Schonberg, 2008 and Hayashida, 1991). Further increase velocity, melting and vaporising effect are concerned with higher intensity of shock.
propagate into the projectile and target, consequently some energy is left causing change of state (Piekutowski, 1990).

### 2.5 Debris Cloud Characteristics

Piekutowski (1996) has described the debris cloud produced by spherical projectiles as composing of three major features, which are ejecta, external bubble of debris and internal structure. An ejecta veil is ejected from the impact side of bumper and almost entirely composes of bumper. An expanding bubble of bumper debris forms at the rear side of the bumper. The internal feature consists of projectile debris located inside and at the front of the external bubble of bumper debris.

![Debris Cloud Structure](image.jpg)

*Figure 2.8. Debris cloud structure produced by spherical projectile. (Piekutowski, 1996).*

The velocities of debris cloud after impact is widely represented in terms of normalised velocities which are the ratio of local velocities of debris cloud to impact velocity of projectile.
Debris produced by cylindrical projectiles has structures as shown in Figure 2.9. It consists of the main debris cloud, inner cone and front cone. Piekutowski, 1987, performed experiments using copper as a bumper material and an aluminium cylindrical projectile. He found that the front cone of debris cloud consists of bumper material.

The population of space debris has continuously grown. Spacecraft are threatened by collision of space debris. The spacecraft shielding system has been developed due to the fact that all spacecraft confronts collisions of space debris and micrometeoroids. Studies on space debris have been performed in three main categories, which are space debris measurement, modelling & risk analysis and mitigation. Characteristics of debris clouds are different for different projectile shapes.

The following steps, after understanding the problems and the main features concerned with the topic, is to study the associated physical impact phenomena that will be presented in the next chapter.
3. Impact Phenomena

The brief explanations of impact phenomena, impact physics, hypervelocity impact, and failure modes are presented. Comprehension of impact phenomena is crucial for understanding and performing simulation of hypervelocity impact. This chapter starts with classification of impact phenomena. The impact physics section explains the mechanism, properties and characteristics of impact. Hypervelocity impacts, which are in the interest of this research, are described. Finally, target failure modes are explained.

3.1 Classification of Impact Phenomena

Impact Phenomena can be classified as elastic and inelastic impacts. Elastic impact is where there is no loss of translational kinetic energy, i.e. no translational kinetic energy is dissipated into heat nor converted to neither rotation kinetic, vibration nor material deformation energy (Vignjevic, 2008). The inelastic impact is, on the other hand, where translational kinetic energy exists. In this research, it is focused on inelastic impact. The impact of deformable bodies influenced by stress wave propagation and inertia effects is called impact dynamics.

Impact dynamics has two main features that are different from the classical mechanics of deformable bodies, which are the influence of stress wave propagation in the material and the importance of inertia effects that have to be taken into account in all of the governing equations based on the fundamental laws of mechanics and physics. Generally, impacts are transient phenomena where steady states conditions never take place.

3.2 Impact Physics

Impact physics of hypervelocity projectiles impacting on a thin plate is the interest of this research. The concerned mechanism and phenomena of such impact are discussed in this section. The basic stress wave and its propagation mechanism are described. Stress induced by impact is explained.
### 3.2.1. Stress Wave

Stress waves are stresses that transfer within the material bodies from impacting surfaces inward to material bodies. The collision generates stresses in the material of the bodies that must deform in order to bear these stresses. The stresses and deformations do not immediately spread through all of material bodies. Compression forces particles closer together, which takes time and requires relative motion. Stresses and deformations are transmitted from the impacting surface to all parts of the bodies in the form of waves.

Stress wave mechanism is illustrated in Figure 3.1, one-dimensional compressive stress wave propagation. Assume that an unstressed bar is exposed suddenly to a pressure that remains constant thereafter. The bar is assumed to be composed of a large number of very thin layers of particles parallel to the impact plane. Considering one-dimension wave propagation, numbers within the particles represent layer numbers of particles. Immediately after impact the pressure is supported entirely by inertia of the first particle, which is particle at the surface. The first particle is accelerated. The spring between first and second particle is compressed as the first particle moves toward second particle. Compressive stress from the spring accelerates second particle. The second particle moves and the spring between second and third particle is compressed. Compressive stresses build and accelerate consecutive particles forming a stress wave.

![Figure 3.1. One-dimensional compressive stress wave propagation (Lepage, 2003)](image)

Stress waves reflect when they encounter boundaries. Similar to general waves, stress waves reflect in opposite phase when encountering free boundary. The phase of reflecting waves remain unchanged when encountering fixed boundary. In compressive stress waves encountering a free boundary, the reflecting waves are tensional stress...
waves. On the other hand, compressive stress waves reflected as compressive stress waves when encountering a fixed boundary (Zukas, 1982).

Considering the propagation of a uni-axial stress wave in a bar for small strains and displacements from non-deformed shape, free body diagram is shown in Figure 3.2.

Applying Newton’s second law to the element dx:

\[ (\sigma + \frac{\partial \sigma}{\partial x} dx - \sigma) A = \rho dxA \frac{\partial v}{\partial t} \]  

(3.1)

Equation (3.1) is simplified and can be written:

\[ \frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \]  

(3.2)

The continuity condition derives from the definitions of strain and velocity:

\[ \varepsilon = \frac{\partial u}{\partial x} \quad \text{and} \quad v = \frac{\partial u}{\partial t} \]  

(3.3a) and (3.3b)

The Equation (3.3a) and (3.3b) can be written:

\[ \frac{\partial \varepsilon}{\partial x} = \frac{\partial^2 u}{\partial x^2} \quad \text{and} \quad \frac{\partial v}{\partial t} = \frac{\partial^2 u}{\partial t^2} \]  

(3.4a) and (3.4b)
Impact Phenomena

Left-hand side of Equation (3.2) can be written:

$$\frac{\partial \sigma}{\partial x} = \frac{\partial \sigma}{\partial \varepsilon} \cdot \frac{\partial \varepsilon}{\partial x} = \frac{\partial \sigma}{\partial \varepsilon} \cdot \frac{\partial^2 u}{\partial x^2}$$  \hspace{1cm} (3.5)

Substituting Equation (3.4) and (3.5), stress wave equation is obtained:

$$\frac{\varepsilon^2}{\alpha^2} = c^2 \frac{\varepsilon^2}{\alpha^2}$$  \hspace{1cm} (3.6)

Where \( c = \frac{\sqrt{S}}{\sqrt{\rho}} \) is the wave velocity and \( S \) is slope of stress-strain curve.

If stress applied to material is lower than yield strength, stress waves generated are in elastic region called elastic stress wave. In the elastic region, \( S \) is the Young’s modulus of material. In the plastic region where applied stress is more than yield strength of material, plastic stress waves are generated. Usually, the slope of stress-strain curve in plastic region is lower than in elastic region. Therefore, the stress wave velocity in plastic region is lower than one in elastic region. Nevertheless, some cases exist where the stress-strain curve in plastic region concaves up. The plastic stress wave velocity is higher than one of elastic stress wave, and consequently there is a single plastic shock wave in the material (Vignjevic, 2008).

### 3.2.2. Impact Induced Stress

Once impact occurs, stress and strain are induced in material, starting from the impact surface and transmit inward material bodies. Stress and strain are induced through the material bodies. Induced stress and strain in material by impact can be estimated by applying fundamental conservation laws and wave propagation concepts.

To analyse impact induced stress and strain, the event of a moving rigid wall impacting a stationary bar are considered. This analysis is the same as a moving bar impacting a stationary wall. First, considering the rigid wall travels at initial velocity, \( v_0 \) and bar is at rest. At the time \( dt \) after impact, the rod will be deformed with distance \( v_0 \ dt \). The original surface is deformed to plane A. The impact induced strain up to plane A at time \( dt \). Disturbance in the bar at time \( dt \) reaches plane B, the material of the bar behind plane B is undisturbed and has no velocity and no stress induced. The distance from original surface of the bar to plane B is \( c \ dt \) where \( c \) is the wave velocity. The impact induced stress up to plane B at time \( dt \).
Illustration of impact between rigid wall and bar is shown in Figure 3.3. Applying law of conservation of momentum to analyse impact between rigid wall and bar, impulse transmitted to the rod is \( \sigma A \, dt \) where A is cross section of the bar. The change of momentum is the product of mass and velocity of all particles under disturbance between original surface of the bar to plane B which is \( \rho A \, c \, dt \, v_0 \). The force impulse is equal to change of momentum:

\[
\sigma A \, dt = \rho A \, c \, dt \, v_0 \tag{3.7}
\]

Equation (3.7) can be simplified to obtain impact-induced stress:

\[
\sigma = \rho c v_0 \tag{3.8}
\]

This impact-induced stress in Equation (3.8) is derived for the case that the rod is initially at rest and unstressed. The impact of a moving projectile on stationary target is the interest of this research. The projectile and target are made of the identical material and therefore have the identical properties, i.e. identical material density and stress wave velocity. If the velocities in the material are measured with respect to the impact interface, the interface can be considered as a rigid surface to projectile and target. At the interface, stress magnitude in the projectile is equivalent to one in the target:

\[
\sigma_1 = \sigma_{2t} \tag{3.9}
\]
Where subscripts 1 and 2 refer to projectile and target, respectively.

Equation (3.9) can be written:

\[ \rho_1 c_1 v_1 = \rho_2 c_2 v_2 \]  

(3.10)

The particle velocity in the projectile, \( v_1 \) and particle velocity in the target, \( v_2 \) are relative to the interface. Therefore, the sum of particle velocities after impact is equal to initial velocity of projectile before impact and it can be written as:

\[ v_0 - v_1 = v_2 \]  

(3.11)

Substitute Equation (3.11) to Equation (3.10). Material density and wave velocity in material are the same since both the projectile and the target are made of the same material. Hence the impact is symmetrical and magnitudes of particle velocity in projectile and in target are equal. The relationship between magnitudes of velocities can be written as:

\[ v_1 = v_2 = \frac{v_0}{2} \]  

(3.12)

The impact-induced stress can be expressed as a function of the material properties and impact velocity:

\[ \sigma = \rho c \frac{v_0}{2} \]  

(3.13)

Equation (3.13) is used to predict the impact-induced stress.

### 3.2.3. Shock wave

Shock wave is a type of propagating disturbance carrying energy through the medium. It is crucial to comprehend the basic idea of the shock wave. It leads to shock in solids, which is an important feature to study in hypervelocity impact. Shock waves are an integral part of dynamic behaviour of materials under high pressure. Shock waves form discontinuity in the medium between in front of and behind the shock wave as shown in Figure 3.4 (Campbell, 1994).
Properties of the medium are abruptly changed between in front and behind the shock wave, such as pressure, velocity, and density. Relationships between these properties have been developed to define the relationship between in front of and behind the shock. These equations are known as Rankine-Hugoniot relations. These relate the pressure, $P$, internal energy, $E$, specific volume, $V$, and density, $\rho$, in term of the shock velocity, $U$, and the particle velocity behind the shock, $u_p$, where the medium properties in front of the shock relate to undisturbed properties, i.e. particles at rest. The zero subscripts refer to properties of material in front of the shock.

The relationship of material properties obtained from applying conservation of mass, in a coordinate system in which the material in front of the shock is stationary:

$$\rho_0 U = \rho (U - u_p)$$  \hspace{1cm} (3.14)

Applying the law of conservation of momentum; that is, the difference in momentum is equal to the impulse per unit cross-section area. After simplification, the equation can be written:

$$P - P_0 = \rho_0 U u_p$$  \hspace{1cm} (3.15)

The conservation of energy can be obtained by setting up an equalisation between the difference of work done by pressure in front of and behind the shock and the total energy, kinetic plus internal, between in front of and behind the shock:

$$Pu_p = \frac{1}{2} \rho_0 U u_p^2 + \rho_0 U (E - E_0)$$  \hspace{1cm} (3.16)
Substitute \( u_p \) and \( \rho_0 U \) from laws of conservation of mass and momentum Equation in (3.14) and (3.15) and replace \( 1/\rho \) by \( V \), then simplifying. The Hugoniot energy equation is obtained:

\[
E - E_0 = \frac{1}{2} (P + P_0)(V_0 - V)
\]  

These equations involve five variables, i.e. \( \rho \), \( \sigma \), \( E \), \( u \) and \( U \). Hence there are two independent variables. The Hugoniot curve is a material property that is the locus of attainable shock states and is equivalent to a stress-strain curve if uni-axial stress is used as the fourth equation. Therefore knowledge of one of any variables, with application of these four equations, is sufficient to determine other variables.

The simplest form of Hugoniot curve is a diagram where shock velocity is a function of particle velocity. This can be obtained from the impact of two identical flat plates. The particle velocity behind the shock is half the impact velocity. The relationship between shock velocity and particle velocity has been found to be linear in many materials:

\[
U = C_0 + S_1 u_p
\]  

Where \( S_1 \) is experimentally determined material constant and \( C_0 \) is the speed of sound in the unstressed material.

\[\text{Figure 3.5. Example of Hugoniot Curve, representing material behaviour under a uniaxial strain state (Campbell, 1994)}\]

Shock waves will be not be generated if the impulsive load is not large enough. An elastic or elastic-plastic wave will be generated and propagate through the material.
Figure 3.5 shows an example of Hugoniot curve. At low pressure, the curve is linear and the wave travels through the material as an elastic wave. The point where the linear relationship of Hugoniot curve ends is the Hugoniot Yield Stress or Hugoniot Elastic Limit.

### 3.3 Hypervelocity Impact

Hypervelocity impact is significant for studying space debris impacting on spacecraft, and its further application to spacecraft protection design. Spacecraft encounter space debris and micrometeoroids, most of them travelling in the hypervelocity region. Hypervelocity impact usually occurs in the region where impact velocity is higher than speed of sound in the collision materials (Zukas, 1982). It refers to velocities so high that the strength of materials upon impact is very small compared to inertial stresses. For aluminium, hypervelocity impact is of order of 5 km/s and higher.

Hypervelocity impact creates a crater which results from a solid target melting and vaporising. Targets and projectiles behave like fluid and crater parameters depend more on projectile and target densities and impact energy. Hence, the principal of fluid mechanics may be used at least for starting of hypervelocity impact analysis.

Hypervelocity impact can be classified into 3 categories based on target thickness, which are semi-infinite target, intermediate thickness target and thin target.

An impact into semi-infinite target is one that has no stress wave reflection effect on the pulverisation of the projectile and target from the rear side of the target. Shock waves are generated after the projectile impacts on the target where peak stresses are largely higher than material strength. Projectile impact causes a crater in the target. The crater shape depends on the shape of projectile and materials present. The chucky projectile, such as sphere or cube, will create a hemispherical crater. The longer projectile will create a deeper crater.

An impact into intermediate thickness target concerns the effect of stress wave reflection on the pulverisation of the projectile and target from the rear side of the target. When primary compressive stress shock waves detach from the expanding crater surface and reach rear side of the wall which is a free surface, the waves reflect as tensional stress waves back. The tensional stress waves are generated to satisfy the condition of stress at
any free surface remains zero at all time. They travel backward into the target in the opposite direction of compressive stress waves. The instantaneous pressure at any point in the target is the algebraic sum of forward running compressive stress wave and rearward running tensile stress wave. When the pressure at any point exceeds ultimate dynamic tensile-yield strength, the target at that point will fracture. The spallation or spall of rear surface of the target is formed with depth approximately one third of target thickness. The crater growth begins at the front surface of the target and continues until approximately two thirds of the target thickness where its floor meets the uppermost spall plane resulting in target perforation. This situation is known as ‘ballistic limit conditions’.

![Wave propagation and perforation mechanism in intermediate thickness target](image)

*Figure 3.6. Wave propagation and perforation mechanism in intermediate thickness target (Zukas, 1982).*

An impact into thin targets has similar results to impacts into intermediate thickness targets. There is an effect of stress wave reflection on the pulverisation. The energetic projectile penetrates through the thin target plate with only small fraction of projectile kinetic energy to form perforation in the target. The shock waves propagate forward into the target and backward into the projectile until they confront the free surface of the target or projectile. The shock wave in the target generally reflects as a tensile stress wave back into target when encountering a free surface of the rear surface of the target. Meanwhile, a primary shock wave in the projectile may encounter a side and rear surface of projectile and reflects as tensile stress wave back into the projectile.

Thin plates are used as bumpers to protect spacecraft from space debris. Fragmentation, melting and vaporisation of the projectile and target reduce the intensity of impulse of impact before impacting to spacecraft. This concept is applied to spacecraft shielding system.

Figure 3.7 depicts the mechanism of projectile impact into a dual-wall structure. Projectile fragmentation, melting and vaporisation of the projectile and the bumper
reduce the intensity of impulse of projectile impact before striking to spacecraft. Vaporised cloud and particles of bumper and projectile disperse over a large area of the rear wall or spacecraft wall depending on standoff distance, etc. Individual craters in the rear wall may be generated. Spalling or spallation may occur as the rear wall can be considered as an intermediate thickness target or thin target. The criteria of failure of rear wall are different depending on each predefine failure of the mission.

![Figure 3.7. Normal Impact of a Dual-Wall Structure. (Schonberg, 1991).](image)

### 3.4 Target Failure Mode

Failure can occur in various modes. It depends on each mission and component critical criterion whether a target is identified as failure. Failure occurs in different modes depending on several characteristics of both the projectile and target, such as impact velocity and obliquity, geometry of projectile and target and material properties. A target may fail in one failure mode or in combination of many failure modes. Figure 3.8 depicts target failure modes that mostly occur in projectile impact on the thin to intermediate thickness target.

Fracture occurs due to the initial compression stress wave in a material with low density. Radial fracture behind initial wave in a brittle target of its strength is lower in tension.
than in compression. Spall failure occurs due to the reflection of initial compression wave as tension causing a tensile material failure. Plugging is formed due to the formation of shear bands. Frontal and rearwards petaling is the result of bending moment, as the target is pushed. Fragmentation generally occurs in case of brittle targets. Hole in ductile material is enlarged (Corbett, 1996 and Lepage, 2003).

![Figure 3.8. Failure modes of various perforation mechanisms (Backman, 1978).](image)

Impact phenomena in hypervelocity impacts are concerned with stress wave propagation and shock waves in solids. Target failure modes are different for each material of impact, geometry of problem, impact velocities and obliquity. The next step is to apply physical phenomena into simulation. The development of the method used for simulation is explained in the following chapter.
4. Smoothed Particle Hydrodynamics Method

To perform a simulation properly, comprehension of the numerical simulation technique is required. Smoothed Particle Hydrodynamics (SPH) method is implemented to the simulations in this research. This chapter presents the basic description of SPH and a short history concerned with its application in hypervelocity impact simulation. The procedures of numerical simulation are provided. Important features of the method are explained, i.e. spatial description, time integration, artificial viscosity, and formularisation. Finally the implementation in MCM is explained.

4.1 Introduction to SPH and Numerical Simulation

Smoothed Particle Hydrodynamics (SPH) is a gridless Lagrangian method. The mesh-free property is the main difference between SPH and classical finite element method. It was developed for simulations of extreme deformation and high pressure in which classical finite element the mesh tangling exists. It uses hydrodynamics computer codes or hydrocodes, which are used to simulate dynamic response of material and structure to impulse and impact with large deformation (Hallquist, 2006 and Hyde, 2008). The solution is obtained by applying physical continuum equations. Computational grids are generated in both space and time. SPH divides the fluid into a set of discrete elements. The discrete elements are referred as particles. These particles have a spatial distance, so-called smoothing length. Properties of particles are smoothed by kernel function. Physical quantity of any particle can be obtained by summing the relevant properties of all particles that are located within smoothing length.

Smoothed Particle Hydrodynamics was first introduced by Lucy, Gingold and Monaghan in 1977, originally to simulate problems in astrophysical phenomena. Later, it has been widely applied in continuum solid and fluid mechanics such as fluid dynamics to simulate ocean waves, aerodynamics and gas dynamics. Libersky and Petschek (1990) applied SPH to elastic-plastic solid by adding of strength effects to simulate hypervelocity impact. An axisymmetric model of SPH was introduced in 1993 (Petschek and Libersky 1993, Joshson, 1993) by ignoring the hoop stresses at axes and representing a smoothed torus of material about the axes of symmetry. The three-
The procedure of conducting a numerical simulation is described in the diagram in Figure 4.1. It begins with observation of physical phenomena, followed by establishing a mathematical model with some simplifications and necessary assumptions. The mathematical model can be expressed in ordinary differential equations (ODE) or partial differential equations (PDE), as governing equations. The boundary condition (BC) and initial condition are required to specify properly in space and/or time. Geometries of problems are divided into discrete components in order to numerically solve the problems. The technique of discretization is different for each method. Coding and implementation of numerical simulation for each problem are applied. The accuracy of numerical simulation is dependent on mesh cell size, initial and boundary condition and others applied earlier (Liu et al, 2003).

![Diagram of numerical simulation procedure](image)

**Figure 4.1. Procedure of conducting numerical simulation (GR Liu & MB Liu, 2003)**

### 4.2 Spatial Discretization

Geometries or materials in finite element techniques are divided into small interconnected sub-regions or elements in order to determine the solution with differential equations. The spatial discretization or mesh of space is basically divided into two categories, which are Eulerian method and Lagrangian method:
4.2.1. Eulerian Method

In an Eulerian mesh, the mesh or grid is fixed and coincident with spatial points as material flows through it, as shown in Figure 4.2a. All the mesh nodes, and cell boundaries, remain fixed in the mesh at all time. Mass, momentum and energy flow through the cell boundaries. The new properties of each cell are calculated from the quantities of flow into and out of the cell. Only current properties are available. The time history of particular particles cannot be tracked. Eulerian mesh has to be applied in all working space even there is no material in such cell. This requires more CPU time than Lagrangian mesh.

4.2.2. Lagrangian Method

In Lagrangian mesh, the mesh or grid is coincident with a material particular point. The mesh or grid moves along with material at all time as shown in Figure 4.2b. The element deforms with the material. Therefore, each particular point of material can be tracked. The calculation in Lagrangian mesh can be performed faster due to the fact that calculation is performed only where material exists.

Figure 4.2a and 4.2b. 2-D Eulerian mesh and Lagrangian mesh, left and right respectively (Zukas, 1990)
Smoothed Particle Hydrodynamics Method

Asra Monthienthong

It is suitable for problems with high distortion. However, the distortion of the mesh and mesh tangling may occur in extremely high deformation. Elements which experience high distortion may cause instabilities and abnormal simulation termination.

4.2.3. Alternative Method

To avoid the disadvantages of Eulerian and Lagrangian method, there have been researches to develop a method which would combine the advantages of both. Arbitrary Lagrangian Eulerian method is a method which its nodes can be programmed to move arbitrarily. Therefore, the advantages of both Eulerian and Lagrangian can be exploited. Usually the nodes on the boundaries are moved to remain on the boundaries, while the interior nodes are moved to minimise mesh distortion. The element erosion method, which deletes the heavily distorted elements from the material, has been added to conventional Lagrangian method. The SPH method adds the meshless property to conventional Lagrangian method as described earlier.

4.3 Artificial Viscosity

Numerical simulations in hypervelocity impact are always concerned with shock waves. Discontinuity between regions in front of and behind the shock is a problem causing unstable oscillations of flow parameters behind the shock. This problem is solved by adding the artificial viscosity to in term, $q$, to pressure to mitigate the discontinuities in the regions between in front of and behind the shock. This artificial viscous term, $q$, damps out the oscillation behind the shock (Hiermaier, 1997).

The viscous term, $q$, is usually expressed in term of quadratic function which is $\varepsilon_{kk}$, the trace of the strain tensor (Hallquist, 1998)

\[
q = \rho_l \left( C_0 \epsilon_{kk}^2 - C_1 c \epsilon_{kk} \right) \quad \text{for} \quad \dot{\epsilon}_{kk} < 0
\]

\[
q = 0 \quad \text{for} \quad \dot{\epsilon}_{kk} \geq 0
\]  

Where $C_0$ and $C_1$ are the quadratic and linear viscosity coefficients which has default value of 1.5 and 0.06 respectively

$c$ is the local speed of sound, $l$ is the characteristic length
4.4 SPH Formalisation

Properties of materials in the Smoothed Particle Hydrodynamics method are smoothed by the interpolation formula and kernel function instead of obtain by divided grids. The function $f(x)$ at the position $x$ is an integration of function $f$ of neighbouring positions $x'$ times an interpolating kernel function $W(x - x', h)$:

$$\langle f(x) \rangle = \int f(x') W(x - x', h) dx'$$  \hspace{1cm} (4.3)

Where $h$ is a smoothing length

$\langle f(x) \rangle$ is a kernel approximation of function $f(x)$

The brackets $( )$ referred to kernel approximation.

The consistency of the kernel function is necessary. Therefore, the kernel function is required to have a compact support, which is zero everywhere but in the finite domain within the range of smoothing length, $2h$ as in Equation (4.4). The smoothing length and set of particles within it are depicted in Figure 4.3. The kernel function is required to be normalised as in Equation (4.5).

$$W(x - x', h) = 0 \text{ for } |x - x'| \geq 2h$$  \hspace{1cm} (4.4)

$$\int W(x - x') dx' = 1$$  \hspace{1cm} (4.5)

Figure 4.3 Smoothed particle lengths and set of particle within (Colebourn, 2000).
Consistency requirements were formulated by Lucy (1977) to ensure the kernel function is a Dirac function if \( h \) is zero or approaching to zero and the kernel function \( f(x) \) is equal to \( f(x') \) if smoothing length is zero:

\[
\lim_{h \to 0} W(x - x', h) = \delta(x - x', h) \tag{4.6}
\]

\[
\lim_{h \to 0} \langle f(x) \rangle = f(x) \tag{4.7}
\]

If there are \( n \) particles for \( f(x) \) integration, therefore the Equation (4.3) is the summation:

\[
\langle f(x) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x'^j) W(x - x'^j, h) \tag{4.8}
\]

Where \( \frac{m_j}{\rho_j} \) is the volume associated with the point or particle \( j \).

\[
\langle f(x'^j) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x'^j) W(x'^j - x'^j, h) \tag{4.9}
\]

### 4.2.1. Conservation equations

The equations of conservation of mass, momentum and energy in Lagrangian framework are applied as following:

\[
\frac{d\rho}{dt} = -\rho \frac{\partial v_\alpha}{\partial x_\beta} \tag{4.10}
\]

\[
\frac{dv_\alpha}{dt} = -\frac{1}{\rho} \frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} \text{ or } \frac{dv_\alpha}{dt} = \frac{\partial}{\partial x_\beta} \left( \frac{\sigma_{\alpha\beta}}{\rho} \right) + \frac{\sigma_{\alpha\beta}}{\rho^2} \frac{\partial \rho}{\partial x_\beta} \tag{4.11}
\]

\[
\frac{dE}{dt} = \frac{\sigma_{\alpha\beta}}{\rho} \frac{\partial v_\alpha}{\partial x_\beta} \text{ or } \frac{dE}{dt} = \frac{\partial}{\partial x_\beta} \left( \frac{\sigma_{\alpha\beta} v_\alpha}{\rho^2} \right) - \frac{\sigma_{\alpha\beta} v_\alpha}{\rho^2} \frac{\partial \rho}{\partial x_\beta} \tag{4.12}
\]

Where \( \alpha \) and \( \beta \) denote component of variables.

Where the derivative of space with respect to time is velocity:

\[
v_\alpha = \frac{dx_\alpha}{dt} \tag{4.13}
\]
Applying kernel estimation into equations of conservation in Equation (4.10) to (4.12):

\[
\frac{d\rho}{dt} = - \int W \rho \frac{\partial v_a}{\partial x_a} dx' \tag{4.14}
\]

\[
\frac{dv_a}{dt} = \int W \frac{\partial}{\partial x_a} \left( \frac{\rho v_a}{\rho} \right) dx' + \int W \frac{\rho v_a}{\rho^2} \frac{\partial \rho}{\partial x_a} dx' \tag{4.15}
\]

\[
\frac{dE}{dt} = \int W \frac{\sigma_{ab}}{\rho} \frac{\partial (p \nu_a)}{\partial x_b} dx' - \int W \frac{\rho v_a}{\rho^2} \frac{\partial \rho}{\partial x_b} dx' \tag{4.16}
\]

All the equations from Equation (4.14) to (4.16) can be written in:

\[
\int W f(x') \frac{\partial g(x')}{\partial x'} dx' \tag{4.17}
\]

Where \( g(x') \) represents different function in each equation from Equation (4.14) to (4.16).

Taylor series about \( x' = x \) is:

\[
\int W f(x') \frac{\partial g(x)}{\partial x} dx' = \int \left\{ f(x) \frac{\partial g(x)}{\partial x} + (x - x') \frac{d}{dx} \left( f(x) \frac{\partial g(x)}{\partial x} \right) + \ldots \right\} W dx' \tag{4.18}
\]

\( W \) is an even function, therefore the odd powers of \( x - x' \) disappear. The second and higher order terms are ignored. The overall order of the method is:

\[
\int W f(x') \frac{\partial g(x)}{\partial x'} dx' = \left( f(x) \frac{\partial g(x)}{\partial x} \right)_{x = x'} \tag{4.19}
\]

Substituting \( \frac{\partial g(x')}{\partial x'} \) for \( \frac{\partial g(x)}{\partial x} \), the follow relation is obtained:

\[
\left( f(x) \frac{\partial g(x)}{\partial x} \right)_{x = x'} = f(x) \int W \frac{\partial g(x)}{\partial x'} dx' \tag{4.20}
\]

Applying relation in Equation (4.20) to Equation (4.14), (4.15) and (4.16):
\[
\langle \frac{d\rho}{dt} \rangle = -\rho \int W \frac{\partial v'_a}{\partial x'_\beta} dx'
\]

(4.21)

\[
\langle \frac{dv_a}{dt} \rangle = \int W \frac{\partial}{\partial x'_\beta} \left( \frac{\sigma_{a\beta}}{\rho} \right) dx' + \frac{\sigma_{a\beta}}{\rho^2} \int W \frac{\partial \rho'}{\partial x'_\beta} dx'
\]

(4.22)

\[
\langle \frac{dE}{dt} \rangle = \frac{\sigma_{a\beta}}{\rho^2} \int W \frac{\partial (\rho' v'_a)}{\partial x'_\beta} dx' + \frac{\sigma_{a\beta} v'_a}{\rho^2} \int W \frac{\partial \rho'}{\partial x'_\beta} dx'
\]

(4.23)

All the Equation from (4.21) to (4.23) contains kernel estimations of spatial derivation as follow in Equation (4.24):

\[
\langle \frac{\partial f(x)}{\partial x_a} \rangle = \int W \frac{\partial f(x'\rangle}{\partial x'_a} dx'
\]

(4.24)

Applying integration by parts:

\[
\langle \frac{\partial f(x)}{\partial x_a} \rangle = Wf(x) - \int W \frac{\partial f(x)}{\partial x_a} dx'
\]

(4.25)

The multiplication of the kernel function and function \(f\) can be written as:

\[
Wf(x) = \int \frac{\partial (Wf(x))}{\partial x_a} dx'
\]

(4.26)

Using Green’s theorem:

\[
\int \frac{\partial (Wf(x))}{\partial x_a} dx' = \int_S W f(x) n_t dS
\]

(4.27)

Therefore, the surface integration is zero, if the region is not within smoothing length or if variables are absent on the boundary of the body. If none of aforementioned conditions exists, modification is to be made to account for boundary conditions.

Derivative of function \(f\) in Equations (4.21) to (4.23) is generally substituted by derivative of kernel function:
\[ \int W \frac{\partial f(x)}{\partial x_a} dx' = - \int f(x) \frac{\partial W}{\partial x_a} dx' \] (4.28)

Therefore, Equation (4.21) to (4.23) can be written as:

\[ \left( \frac{d\rho}{dt} \right) = \rho \int v'_a \frac{\partial W}{\partial x_a} dx' \] (4.29)

\[ \left( \frac{dv_a}{dt} \right) = - \int \frac{\sigma_{\alpha\beta}}{\rho} \frac{\partial W}{\partial x'_\beta} dx' - \frac{\sigma_{\alpha\alpha}}{\rho^2} \int \rho \frac{\partial W}{\partial x_a} dx' \] (4.30)

\[ \left( \frac{dE}{dt} \right) = - \frac{\sigma_{\alpha\beta}}{\rho^2} \int \rho v'_a \frac{\partial W}{\partial x'_\beta} dx' + \frac{\sigma_{\alpha\beta} v_a}{\rho^2} \int \rho v'_a \frac{\partial W}{\partial x'_\beta} dx' \] (4.31)

Converting the continuous volume integral to summation of discrete points:

\[ \frac{d\rho^i}{dt} = \rho^i \sum_{j=1}^{N} m^j \left( v^i_j - v^i_\beta \right) \frac{\partial W^{ij}}{\partial x'_{i\beta}} \] (4.32)

\[ \frac{dv^i_a}{dt} = - \sum_{j=1}^{N} m^j \left( \frac{\sigma_{\alpha\beta}^j}{\rho^j} - \frac{\sigma_{\alpha\alpha}^j}{\rho^j} \right) \frac{\partial W^{ij}}{\partial x'_{i\beta}} \] (4.33)

\[ \frac{dE^i}{dt} = - \frac{\sigma_{\alpha\beta}^i}{\rho^i} \sum_{j=1}^{N} m^j \left( v^i_j - v^i_\beta \right) \frac{\partial W^{ij}}{\partial x'_{i\beta}} \] (4.34)

where \[ W^{ij} = W(x^i - x^j, h) \]

The Equations (4.32), (4.33) and (4.34) are the basic equations of conservation, which are used in the SPH method.

### 4.2.2. Kernel Function

Kernel functions which are frequently used are Guassian and B-spline functions. B-spline, proposed by Monaghan (1983), is explained as it is applied in this research.
\[ W(v, h) = \frac{C}{h^D} \begin{cases} 
\left( 1 - \frac{3}{2}v^2 + \frac{3}{4}v^3 \right) & , \quad v < 1 \\
\frac{1}{4}(2 - v)^3 & , \quad 1 \leq v \leq 2 \\
0 & , \quad \text{otherwise} 
\end{cases} \quad (4.35) \]

Where \( v = \frac{|x-x'|}{h} \)

D is the number of dimension of the simulation

C is the scaling factor that depends on number of dimension and ensures the consistency condition in Equation (4.4) and (4.5).

C values are:

\[ C = \begin{cases} 
\frac{2}{3} & , \quad D = 1 \\
\frac{10}{\pi} & , \quad D = 2 \\
\frac{1}{\pi} & , \quad D = 3 
\end{cases} \quad (4.36) \]

### 4.2.3. Smoothing Length

If the smoothing length remains constant, in large deformation where particles may separate from each other, neighbouring particles may not be enough for the calculation to establish smoothness. In compressive large deformation, if there are too many neighbouring particles within the smoothing length, too many particles are taken to calculate the properties of each local function. Therefore, the smoothing length is required to vary in large deformation. The idea is to keep number of neighbouring particle constant (Benz, 1990). The equation of variable smoothing length is:

\[ h = h_0 \left( \frac{\rho_0}{\rho} \right)^{\frac{1}{\pi}} \quad (4.37) \]

Where \( h_0 \) and \( \rho_0 \) are constant and \( n \) is number of dimension.
4.5 Implementation in MCM

Meshless Continuum Mechanics, MCM, code was developed by Cranfield University. MCM is used in this research to simulate spherical, cubic and cylindrical projectiles in the hypervelocity region with the application of smoothed particle hydrodynamics method. The model is described in an input file. It includes the model configuration, i.e. nodal coordination, equation of state, projectile and target mass, material type, initial velocity, smoothing length, etc. An example of input file is in appendix A. The input files are submitted to run in high performance computing systems of Cranfield University. After tasks’ computations are complete, the output files are obtained and illustrated in LS-PrePost®.

The Johnson-Cook strength model in MCM is an elastic-plastic hydrodynamics model. This model is suitable for high rate deformation of materials. The application is also applied in ballistic penetration and impact simulations. The formulation is described. The constants for many materials are given by Johnson and Cook (1983).

The yield strength is determined as (Lin, 2004)

\[
\sigma_y = [A + B(\bar{\varepsilon}^p)^n][1 + c \ln \dot{\varepsilon}^*][1 - (T^*)^m]
\]  

(4.38)

Where \( \bar{\varepsilon}^p = \int_0^t d\bar{\varepsilon}^p \) is the effective plastic strain

\( \dot{\varepsilon}^* \) is non-dimensional strain rate

\( T^* \) is homogenous temperature, it is a ratio of current temperature to melting temperature.

Temperature change is computed adiabatically since transient dynamic occurs in a short time interval and the actual heat transfer is negligible. Parameters A, B, c, m and n are experimentally determined by a curve-fitting procedure. The spall model is set to be pressure limit model.

The strain at fracture in Johnson-Cook model is given by (Lin, 2004):

\[
\varepsilon_f = [D_1 + D_2 \exp(D_3 \sigma^*)][1 + D_4 \ln(\dot{\varepsilon}^*)][1 + D_5 T^*]
\]  

(4.39)
Where $\sigma^*$ is the ratio of pressure to effective stress:

$$D = \sum \frac{\Delta \rho}{\varepsilon_f}$$

Parameters of aluminum 2024-T3 applied in MCM is summarised in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$ (g/cm$^3$)</td>
<td>2.77</td>
</tr>
<tr>
<td>Shear modulus, $G$ (GPa)</td>
<td>28.6</td>
</tr>
<tr>
<td>$A$ (MPa)</td>
<td>265</td>
</tr>
<tr>
<td>$B$ (MPa)</td>
<td>426</td>
</tr>
<tr>
<td>Strain hardening exponent, $n$</td>
<td>0.34</td>
</tr>
<tr>
<td>Strain rate dependence coefficient, $c$</td>
<td>0.015</td>
</tr>
<tr>
<td>Temperature dependence exponent, $m$</td>
<td>1</td>
</tr>
<tr>
<td>Melt temperature, $T_m$ (K)</td>
<td>775</td>
</tr>
<tr>
<td>Room temperature, $T_r$ (K)</td>
<td>293</td>
</tr>
<tr>
<td>Reference strain rate, $\dot{\varepsilon}_0$</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Specific heat, $c_v$ (J·kg$^{-1}$·K$^{-1}$)</td>
<td>875</td>
</tr>
<tr>
<td>Pressure cutoff, $\rho_{off}$, or failure stress, $\sigma_m$ (MPa)</td>
<td>-1200</td>
</tr>
</tbody>
</table>

An equation of state (EOS) describes the hydrostatics behaviour of stress and strain. A Hugoniot curve is required to develop an equation of state. It is the relationship between pressure, internal energy and relative volume. The equation of state used in this research is the Gruneisen equation of state with cubic shock velocity-particle velocity defines pressure. (Hallquist, 2006)

For compression:

$$p = \frac{\rho_0 c^2 \mu}{1+(1-\frac{\gamma_0}{2})\mu\frac{a}{2}\mu^2} + (\gamma_0 + \alpha \mu)E$$  \hspace{1cm} (4.40)
For expansion:

\[ p = \rho_0 C^2 \mu + (\gamma_0 + a \mu)E \quad (4.41) \]

Where:
- \( E \) is the internal energy per initial volume
- \( C \) is the intercept of \( \mu_s - \mu_p \) curve
- \( \gamma_0 \) is the Gruneisen coefficient
- \( a \) is the first order volume correction to \( \gamma_0 \)
- \( S_1, S_2, S_3 \) are the first, second and third slope coefficients of the shock velocity-particle velocity curve.
- \( \mu = \frac{1}{\nu} - 1 \)

**Table 4.2. Material properties of aluminium 2024-T3 (Matweb, 2009 and Campbell, 1994).**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity curve intercept, ( C )</td>
<td>0.524</td>
</tr>
<tr>
<td>First slope coefficient, ( S_1 )</td>
<td>1.400</td>
</tr>
<tr>
<td>Gruneisen coefficient, ( \gamma_0 )</td>
<td>1.970</td>
</tr>
<tr>
<td>First order volume correction coefficient, ( a )</td>
<td>0.480</td>
</tr>
</tbody>
</table>

The second slope coefficient, \( S_2 \), and third slope coefficient, \( S_3 \), are zero. Initial internal energy and initial relative volume are set to zero.

SPH is a gridless Lagrangian method. Particles represent the materials in the problem. Properties of particles are smoothed by the properties of neighbouring particles within the smoothing length. SPH is applicable to hypervelocity impact. Formularisation of SPH has been developed. The Johnson-Cook material model and Gruneisen equation of states are implemented. The following step after the study of the used method is to set up simulations according to available experiments, in order to validate assumptions and applied parameters. Simulation validation is presented in the next chapter.
5. Simulation Validation

This chapter presents simulation validation. Prior to simulations of the different shapes of projectiles, the simulation set up, assumptions and used parameters need to be validated. This is in order to determine reliability of the simulation and to attest that the assumptions, used parameters and set up are done properly. In this chapter the validations of the simulation of spherical and cylindrical projectiles are explained. The accuracy of the simulation is presented.

5.1 Validation and Resources

In order to determine the accuracy and reliability of the simulation, the validation of simulation set up, assumptions and used parameters needs to be performed. The shape of a debris cloud and its velocity at particular points are compared with experimental data. The shape of a spherical projectile’s impact on a thin plate target is observed and compared between experiment and simulation. The simulation was set up according to experiments of Piekutowski (1996).

Morrison (1972) has compared the debris cloud of spherical and cylindrical projectiles. The validation of cylindrical projectiles is performed according to his experiment. Nonetheless, exact parameters of the experiment such as the time after impact of measurement and taking picture are not given. Therefore, only the trends of the debris cloud are observed and compared with simulation results. There are studies in cylindrical projectile shapes but most of experimental data available are in terms of damage on a dual-wall structure and ballistic limit curves. Cubic projectile impacts have not been validated.

5.2 Validation of Spherical Projectile

Validation of spherical projectile impacts on thin plated has been performed by repeating two experiments of Piekutowski (1996). The results display the shape of the debris cloud and local velocity at particular points of debris cloud. There is no available data to validate the target deformation and induced stress. Therefore, in this validation the shape of debris cloud and local velocity at particular point are observed and compared between simulations and experiments.
5.2.1. Model Configurations

Configurations of experiments which will be imitated are shown in Table 5.1. Both target and projectiles are made of aluminium 2017-T4. All projectiles have diameter, D, of 9.35 mm. The thicknesses of the targets are 2.19 mm and 3.96 mm with the ratios of target-thickness-to-projectile-diameter of 0.233 and 0.424 for case 1 and 2 respectively. The initial velocities or projectiles are 6.64, and 6.68 for case 1 and 2 respectively. The shapes and velocities of the debris cloud were measured at time after impact, τ, of 6.6 μs and 8.4 μs for case 1 and 2 respectively.

The simulation is set according to experimental set up. Width and length of the plates are 25.0 m. The Johnson-cook material model and Gruniesen equation of state are applied. Only one quarter is modelled as it has less cost in term of CPU time and memory. Inter-particle distance of projectile is 0.4 mm for both cases. In thin plate targets, inter-particle distances are 0.36 mm and 0.39 mm for case 1 and 2 respectively.

Table 5.1. Spherical projectile impact on thin plate target configuration description for validation.

<table>
<thead>
<tr>
<th>Case</th>
<th>D (mm)</th>
<th>t(mm)</th>
<th>t/D</th>
<th>v(km/s)</th>
<th>τ(μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.35</td>
<td>2.19</td>
<td>0.233</td>
<td>6.64</td>
<td>6.6</td>
</tr>
<tr>
<td>2</td>
<td>9.35</td>
<td>3.96</td>
<td>0.424</td>
<td>6.68</td>
<td>8.4</td>
</tr>
</tbody>
</table>

5.2.2. Comparisons between experiments and simulations

Distance between impact surface of thin plate target and frontal area of debris cloud is measured at specific time which is 6.6 μs and 8.4 μs after impact for case 1 and 2 respectively. The results are shown in Table 5.2.
Table 5.2. Comparisons between experimental data and simulating data.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th></th>
<th></th>
<th>Case 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Simulation</td>
<td>Error (%)</td>
<td>Experiment</td>
<td>Simulation</td>
<td>Error (%)</td>
</tr>
<tr>
<td><strong>Shape</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impact surface to debris cloud front</td>
<td>4.83</td>
<td>4.50</td>
<td>6.83</td>
<td>4.36</td>
<td>4.41</td>
<td>1.15</td>
</tr>
<tr>
<td>Widest part of debris cloud</td>
<td>4.44</td>
<td>4.17</td>
<td>6.08</td>
<td>3.49</td>
<td>3.55</td>
<td>1.72</td>
</tr>
<tr>
<td>Impact surface to inner feature</td>
<td>2.67</td>
<td>2.73</td>
<td>2.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Normalised velocity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>At frontal part of debris cloud</td>
<td>0.894</td>
<td>0.934</td>
<td>4.51</td>
<td>0.793</td>
<td>0.8165</td>
<td>2.96</td>
</tr>
</tbody>
</table>

The average accuracy of debris cloud shape is 3.41 percent. The average accuracy of debris cloud velocities is 3.74 percent. Velocities and shape of debris cloud should be performed at all points but in these experiments only debris cloud velocities and shape at some points are measured in experiments. Nonetheless, more measurements should have been performed to have more reliability of accuracy, which leads to reliability of simulation.

Figure 5.1a and 5.1b. Comparisons of experimental (Piekutowski, 1996) and simulating debris cloud shape of case 1
Pictures to compare between the debris clouds of experiments and simulation in case 1 are shown in Figure 5.1. It is good agreement between the debris cloud shapes from experiment and simulation. The external bubble is composed of bumper martial as shown in red colour. Projectile particles are in the internal feature as shown by the blue colour. However, if projectile and thin plate targets consist of more particles, a better view of debris clouds may be obtained. The separation and features of the debris cloud will be clearer and more distinguishable.

5.3 Validation of Cylindrical Projectile

Morrison (1972) has investigated effect of cylindrical projectiles and compared with spherical projectiles with equivalent masses. The projectiles are both made of aluminium, impacting on an aluminium double-sheet structure at a velocity near 7 km/s. Figure 5.2a shows the impact of a cylindrical projectile, with a length to diameter ratio of cylinder equal to one, onto a thin plate target.

![Figure 5.2a and 5.2b. Pictures of debris clouds of cylindrical projectile impact on thin plate target from experiment (Morrison, 1972) and simulation.](image)

The spike-like debris cloud of a cylindrical projectile is observed which is distinctly different from one of spherical projectile. A picture from a simulation of cylindrical projectile with length to diameter ratio equal to one is illustrated in Figure 5.2b to compare with experimental debris cloud. It is shown that the spike-like shape of debris
cloud frontal area is observed. The configurations of experiments and simulations are not identical. Only the trend and probable shape of debris cloud is compared. The spike-like part or front cone in the simulation is composed of bumper material. It agrees with experiments of Piekutowski, as explained in Chapter 2.

The simulation set up has shown good agreement with experimental data. Nonetheless, more validation should be performed. Further experimental data is required to validate both spherical and non-spherical projectile cases. None of induced stress and target deformation, which are to be investigated in the next chapter, has been validated due to the unavailability of experimental data. The following step after simulation validation is to simulate spherical, cylindrical and cubic projectiles impacting on thin plate targets, which will be presented in the next chapter.
6. Geometrical Projectile Shapes Impact Simulation

This chapter presents the simulations used to study the effect of projectile shape. Simulations are performed for spherical, cylindrical and cubic projectiles at varying impact velocities. In this chapter, models description is explained. MCM code is described along with procedures of the MCM application. The measured parameters are explained. Finally, the modelling and results of impacts of spherical, cylindrical and cubic projectiles on thin plate targets are presented.

6.1 Model description

Targets and projectiles in all simulations are made of aluminium 2024, as this material has been widely used in hypervelocity impact experiment (Piekutowski, 1996, Elfer, 1996 and Morrison 1972, and so on). An identical target is used in all simulations in order to remove influence of target on simulation results. Each projectile shape is simulated with 4 initial projectile velocities which are 3, 5, 7, and 9 km/s toward target in direction normal to the target. The simulations of projectile impact velocity of 7 km/s are illustrated in this chapter.

In order to compare only the influence of the projectile shape, the dimensions of the projectiles are chosen to keep the mass constant, and hence for a given initial velocity, the impact energy remains constant. A spherical projectile of diameter 5.0 mm is taken as the reference case, giving a mass of 0.1813 g. A cylindrical projectile of equal mass has height and diameter of 4.37 mm. Likewise the cubic projectile is modelled with width, length and thickness 4.03 mm.

The target plate thickness is 2.20 mm as a bumper of space debris shielding is typically between 1.27-2.50 mm (Destefani et al, 2006 and Clough, 1970) and the rear wall thickness is between 3.20-4.80 mm (Christiansen et al, 1995). The plate is modelled as a 25mm by 25 mm square. This represents a compromise as modelling a large target is expensive in CPU time and memory, while if the target plate is too small edges are close enough to influence the results. Nonetheless, previous experiments with spherical projectiles with diameter in the order of 5 mm generate holes on the target smaller than 19 mm in diameter, depending on impact velocity, plate thickness, etc. (Elfer 1996 and
Therefore, a target with width and length 25.00 mm is modelled as the target for all simulations.

The interval between state plots is 0.05 μs. Simulation duration is between at the impact and 30 μs after impact.

As the models are symmetrical in the planes parallel to projectile initial velocity, symmetry boundary conditions are used to reduce the CPU time and memory and only one quarter of the problem is actually represented. All models use an initial cubic packing for the SPH particles with the interval between particles is defined by an inter-particle distance. Following the requirements for the symmetry planes, no particle lies on a plane of symmetry and the nearest particles to the axes are allocated in half of the inter-particle distance from axes.

6.2 Measured parameters

Properties and characteristics of debris clouds and targets are observed in different projectile shapes and initial velocities. Selected parameters to be measured are described including the reasons of those parameters’ observations:

6.2.1. Debris Cloud Shapes

The shape of debris cloud generated by hypervelocity impact depends on various parameters such as target thickness, projectile diameter and impact velocity.

6.2.2. Debris Cloud Velocities

Debris cloud velocities are measured from the velocity profiles after impact. The velocity of projectiles and the target indicates the space debris shielding performance in mitigating the impulsive force prior to striking on the spacecraft wall. The velocities of ejecta bouncing backward are not of interest in this research.

6.2.3. Selected Nodal Velocities

Selected nodal velocities are measured to track their velocities. It is difficult to track particular parts of material in actual experiments but it can be observed in numerical
Geometrical Projectile Shapes Impact Simulation

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Additionally, nodal velocities implied duration of transition phenomena of impact and the starts of stability condition which takes place afterward.

6.2.4. Stress waves

Stresses in the impact direction are measured. Morrison (1972) has given the reasons which cylindrical projectiles are more penetrating than spherical projectiles. He mentioned that the shock wave generated in a thin plate or target by spherical projectile impact is a hemispherical shape whereas the shock wave generated by a cylindrical projectile impacting by its flat face is planar shape. Therefore, stress waves propagate in the target and projectile are observed.

Applying Equation (3.13), \( \sigma = \rho \frac{v_0^2}{2} \), developed from two rigid body impacting with identical material. The density of aluminium is 2770 kg/m\(^3\), speed of sound in aluminium is 4877 m/s (Matweb, 2009), and initial projectile velocity is 7 km/s. The expected stress is 47.2 GPa.

6.2.5. Target failure

Failure modes of the target are observed and the dimensions of the hole in the target after the impact are observed to investigate the effect of different projectile shape on target.

6.3 Spherical Projectile Impact

The model consists of a spherical projectile 5.00 mm in diameter and a plate 25.00-mm in width and length and 2.20 mm thickness (25.00 mm x 25.00 mm x 2.20 mm). The model is shown in Figure 6.1a and Figure 6.1b. There are 24 particles on 5-mm diameter of spherical projectile. Inter-particle distance is 0.20 mm for both of projectile and target. The distance between the most top and the bottom particle is 4.80 mm plus two halves of both diameter each side. Therefore, a spherical projectiles size of 5.00 mm is generated. 2.20-mm target thickness consists of 10 particles and 62 particles lie on one quarter of 25-mm width and length of the target. The spherical projectile and the target are represented by 40475 particles. The initail projectile velocity is in negative z-direction where positive z-direction points upward. Figure 6.1a is full considered configuration. Figure 6.1b is the actual model constructed and analysed in MCM.
Cross-section views are shown for better views of projectile and target fragmentation at 0.5 μs, 1 μs and 2 μs after impact in Figure 6.2a, 6.2b, and 6.2c respectively. The particles of the projectile are represented in blue and those of target are in red. 0.5 μs after impact the projectile and target are deformed. Some particles of both projectile and target bounce backward. At 2 μs after impact, the spherical projectile is fragmented completely. Most of the fragmentation continues travel forward along with material of the target, which travels ahead of fragmentation of the projectile.
A set of seven nodes within the projectile and the target have been selected to track the velocities. Figure 6.3. Nodal vertical velocities for the selected nodes are plotted in Figure 6.4. All selected particles are ones nearest to xz plane, i.e. half inter-particle distance from xz plane.

![Cross-section of model](image)

*Figure 6.3. Cross-section of model illustrated the nodes and their numbers for reference in plot Figure 6.4.*

![Nodal velocities](image)

*Figure 6.4. Nodal velocities in negative z direction of spherical projectile initial velocity of 7 km/s corresponded to nodes in Figure 6.3.*
The node numbers displayed in Figure 6.3 correspond to the curve labels in Figure 6.4. Node 1 and node 2, belonging to the projectile, change z velocity from an initial 7 km/s down to 4.5 km/s and 5.2 km/s respectively; whereas node 3, belonging to the side of the projectile, continuously decelerates in z-direction to the velocity approaching zero. Node 4, located in middle of target prior to impact, accelerates to more than 5 km/s within 0.5 μs and travels in velocity of 5.5 km/s after 2 μs. Node 5 and node 6 accelerate in the direction of impact and opposite direction respectively before stabilising to a constant velocity approaching zero. Node 7 of target has zero velocity at all times before and after impact since it is situated far from impact area.

Deleting particles belonging to projectile, target damage is observed. The failure mode can be observed in Figure 6.5a. It is a petal failure mode. The dimension of hole in the target is 13.1 mm in average of all directions.

The velocity profile of the resulting debris cloud is shown in Figure 6.6b for 6 μs after the impact. At this time, the maximum velocity in the z-direction is 5.55 km/s, normalised velocity 0.79, at the central frontal area of space debris. The majority of the projectile material is in the frontal region of the debris cloud.
At 0.1 μs after impact, the maximum stress in the impact direction is 456.5 GPa. It is only a few particles that have a stress of this value. The expected stress is 10 times lower. This may cause by the noise in solution. The actual value should be in the light blue range as majority of particles are in light blue. Stress is between 46.5 GPa and 137.0 GPa. At 0.2 μs after impact, the majority of particles have stress between 54.8 GPa and 82.2 GPa. Stress waves propagate to more than half of the sphere at 0.2 μs after impact where the deformation of the target is approaching to half thickness of the target. The shock wave that propagates through the target is in hemispheric shape. At 0.3 μs after impact, the majority of particles have stress between 24.4 GPa and 76.6 GPa. Stresses in the vertical direction at 0.1 μs, 0.2 μs and 0.3 μs after impact are shown in Figure 6.7a, 6.7b and 6.7c.

Figure 6.6a and 6.6b. Debris cloud and its velocity profiles at 6 μs after impact of spherical projectile with initial velocity of 7 km/s.

Figure 6.7a, 6.7b and 6.7c. Stress in z-direction at 0.1 μs, 0.2 μs and 0.3 μs after impact respectively of spherical projectile projectile with initial velocity of 7 km/s, unit in 100 GPa.
6.4 Cylindrical Projectile Impact

The model consists of a cylindrical projectile of size 4.37 mm in diameter and length and a plate 25 mm in width and length and 2.20 mm thickness (25.00 mm x 25.00 mm x 2.20 mm) as shown in Figure 6.8a and 6.8b. There are 21 particles along the axis of the cylinder and 10 particles along the radius. Inter-particle distances are equal to 0.2185 mm in the cylindrical projectile and 0.2000 mm in the target. 2.20-mm target thickness consists of 10 particles and 62 particles lie on one quarter of 25-mm width and length of the target. The cylindrical projectile and target are represented by 40099 particles. The initial projectile velocity is in the negative z-direction where positive z-direction points upward as shown in Figure 6.8a and 6.8b. Figure 6.8a is full considered configuration. Figure 6.8b is the actual model constructed and analysed in MCM.

Cross-section views of impact are shown for better views of the projectile and the target fragmentation at 0.5 μs, 1 μs and 2 μs after impact in Figure 6.9a, 6.9b, and 6.9c respectively.

The particles of the projectile are represented in blue and those of the target are in red. After impact, the projectile and target are deformed. Some material from both projectile and target form the back-splash. At 1 μs after impact, the spike-like shape in the front side of the debris cloud is formed. The spike-like part of the debris cloud consists of particles that belong to target. The majority of the projectile material is in the frontal region of the debris cloud but not in the spike-like part of debris cloud. This agrees with experiment observation of Piekutowski (1987) that the spike-like part, so-called cone, is composed of bumper material. Separation of the most frontal layer of the target arises.
There is no such separation of the material layer observed in actual experiments. Increasing resolution would be one possible approach.

A set of nine nodes within the projectile and target have been selected to track the velocities. Figure 6.10. Nodal vertical velocities for the selected nodes are plotted in Figure 6.11. All selected particles are ones nearest to xz plane, i.e. half inter-particle distance from xz plane. The node numbers displayed in Figure 6.10 correspond to the curve labels in Figure 6.11. Node 1 and node 2, belonging to central part of cylindrical projectile, change z velocity from an initial 7 km/s down to 4.2 km/s and 4.5 km/s within 2 μs after impact respectively, whereas node 3, belonging to the side of the cylindrical projectile, continuously decelerates in z-direction from an initial 7 km/s down to 1.5 km/s. Node 4, located in the mid-front part of the projectile prior to impact decelerates to a velocity of 5 km/s. Node 5 bounces back to velocity of 2.5 km/s in opposite direction of initial projectile velocity. Node 6 and node 7 which belong to bumper central area of impact accelerate immediately after impact. At 2.5 μs after impact, the particles travel at constant velocities of 5.3 km/s and 6.3 km/s respectively. Node 8 slowly bounces back and node 9 is immobile with no velocity at all time since it is far from impact area.
Figure 6.10. Cross-section of model illustrated the nodes and their numbers for reference in plot Figure 6.11.

Figure 6.11. Nodal velocities in negative z direction of cylindrical projectile initial velocity of 7 km/s corresponded to nodes in Figure 6.10.

Deleting particles belonging to projectile, target damage is observed. The failure mode can be observed in Figure 6.12a. It is a petal failure mode. The dimension of hole in the target is 12.9 mm in average of all directions.
Figure 6.12a and 6.12b. Target failure and target hole dimension after impact of cylindrical projectile with initial velocity of 7 km/s.

The velocity profile of the resulting debris cloud is shown in Figure 6.13b for 6 μs after the impact. At this time the maximum velocity in the z-direction is 6.5 km/s, normalised velocity is 0.93, at the central frontal area of the space debris. Piekutowski, 1987, has studied debris clouds produced by cylindrical projectiles. He found that the spike-like part travels as a coherent unit which agree with the simulation result but in his experiments it was found that the spike-like part has velocity of 5 percents higher than impact velocity whereas in the simulation, it is 7 percents less.

Figure 6.13a and 6.13b. Debris cloud and its velocity profiles at 6 μs after impact of cylindrical projectile with initial velocity of 7 km/s.
Stresses in vertical direction at 0.1 μs, 0.2 μs and 0.3 μs after impact are shown in Figure 6.14. At 0.1 μs after impact the maximum stress in the impact direction is 299.1 GPa. It is only a few particles that have stress of this value. The expected stress is 6 times lower. This may cause by the noise in solution. The actual value should be in the light blue and light green range as the majority of particles are in light blue where the stress is between 59.8 GPa to 119.7 GPa. At 0.2 μs after impact the majority of particles have stress between 49.5 GPa to 83.6 GPa. The outer part of stress propagation is a curvature whereas the inner part is planar. At 0.3 μs after impact, the majority of particles have stress between 24.5 GPa and 63.7 GPa.

### 6.5 Cubic Projectile Impact

The model consists of a cubic projectile sized 4.03 mm (4.03 mm x 4.03 mm x 4.03 mm), and a plate 25.00 mm in width and length and 2.20 mm thickness (25.00 mm x 25.00 mm x 2.20 mm) as shown in Figure 6.15a and Figure 6.15b. One quarter model of the projectile consists of 20 particles in height and 10 particles in width. The inter-particle distance is 0.2105 mm in the projectile and 0.2000 mm in the target. The target plate is identical to the spherical and cylindrical cases, with 10 particles through the thickness and 62 particles in one quarter of the whole width. The cubic projectile and target are represented by 40440 particles. The initial projectile velocity is in the negative z-direction where positive z-direction points upward in Figure 6.15a and Figure 6.15b.
Cross-section views of impact are shown for better views of projectile and target fragmentation at 0.5 μs, 1 μs and 2 μs after impact in Figure 6.16a, 6.16b, and 6.16c respectively. The particles of the projectile are represented in blue and those of target are in red. After impact the projectile and target deformed. Some material from both projectile and target form the back-splash. At 1 μs after impact, the spike-like part of debris cloud consisting of particles which belong to the target.

**Figure 6.15a and 6.15b.** Cubic projectile and thin plate target configuration and cross-section of model with SPH particles, respectively.

**Figure 6.16a, 6.16b and 6.16c.** Cross-section view of one quarter model of cubic projectile with impact velocity of 7 km/s at 0.5 μs, 1μs and 2 μs after impact in left, middle and right respectively.
A set of nine nodes within the projectile and the target have been selected to track the velocities, Figure 6.17. Nodal vertical velocities for the selected nodes are plotted in Figure 6.18. All selected nodes are ones nearest to xz plane, i.e. half inter-particle distance from xz plane. The node numbers displayed in Figure 6.17 correspond to the curve labels in Figure 6.18.

![Cross-section of model illustrating nodes and their numbers for reference in plot Figure 6.18.](image)

*Figure 6.17. Cross-section of model illustrated the nodes and their numbers for reference in plot Figure 6.18.*

![Nodal velocities in negative z direction of cubic projectile initial velocity of 7 km/s corresponded to nodes in Figure 6.17.](image)

*Figure 6.18. Nodal velocities in negative z direction of cubic projectile initial velocity of 7 km/s corresponded to nodes in Figure 6.17.*
Node 1, node 2 and node 4, belonging to central part of cubic, change z velocity from an initial 7 km/s down to 3.8 km/s for node 1 and to 5.0 km/s for both node 2 and node 4 whereas node 3, belonging to side of cubic projectile, decelerates and its velocity in the z direction approaches zero. Node 5 bounces back to velocity 4.2 km/s in the opposite direction of initial projectile velocity. Node 6, belonging to target, accelerates to an equivalent velocity of node 4 which belongs to the projectile. Node 7, belonging to opposite side of impact surface of target, accelerates to 6.5 km/s. Node 8 gradually move in opposite direction of impact before its velocity approaches zero. Velocity of node 9 fluctuates slightly near zero.

![Image](image1.png)

**Figure 6.19a and 6.19b. Target failure and target hole dimension after impact of cubic projectile with initial velocity of 7 km/s.**

Deleting particles belonging to projectile, target damage is observed. The failure mode can be observed in Figure 6.19a. It is a petal failure mode. The dimension of hole in the target is 12.6 mm in average of all directions.

![Image](image2.png)

**Figure 6.20a and 6.20b. Debris cloud and its velocity profiles at 6 μs after impact of cylindrical projectile with initial velocity of 7 km/s.**
The velocity profile of the resulting debris cloud is shown in Figure 6.20b for 6 μs after the impact. At this time, the maximum velocity in the z-direction is 6.5 km/s at the central frontal area of space debris. Similar to the cylindrical projectile, the majority of the projectile material is in the frontal region of the debris cloud but not in the spike-like part of debris cloud.

![Figure 6.20b](image)

**Figure 6.20b.** Stress in z-direction at 0.1 μs, 0.2 μs and 0.3 μs after impact of cylindrical projectile with initial velocity of 7 km/s, unit in 100 GPa.

At 0.1 μs after impact, the maximum stress in impact direction is 270.5 GPa. It is only few particles that have stress of this value. The expected stress is 5.5 times lower. This may be caused by the noise in solution. The actual value should be in the light blue and light green range as the majority of particles are in light blue. Stress is between 54.1 GPa to 108.2 GPa. At 0.2 μs after impact, the majority of particles have stress between 47.8 GPa to 80.7 GPa. At 0.3 μs after impact, the majority of particles have stress between 18.1 GPa and 47.8 GPa. The outer part of stress propagation is a curve whereas inner part is planar. Stress in vertical direction at 0.1 μs, 0.2 μs and 0.3 μs after impact are shown in Figure 6.21.

![Stress in vertical direction](image)

**Figure 6.21.** Stress in z-direction at 0.1 μs, 0.2 μs and 0.3 μs after impact of cubic projectile with initial velocity of 7 km/s, unit in 100 GPa.

In this chapter, the simulations were performed with spherical, cylindrical and cubic projectiles. Shapes and velocities of debris cloud of each projectile are observed and their properties were collected. Induced stress waves in targets are measured and their shapes are observed for each projectile. Target deformation data is collected. The following step is to compare the collected data of the different projectile shapes with initial projectile velocity of 3 km/s, 5 km/s, 7 km/s and 9 km/s. The comparisons of properties and parameters of different shapes of projectile are to be performed and presented in the next chapter.
7. Comparisons of Geometrical Projectile Shapes Impact

In this chapter, comparisons of results of spherical, cylindrical and cubic projectile impact on the single thin plate target are presented. Debris cloud shapes are illustrated. Debris cloud velocities’ profiles are displayed. Normalized velocities are collected and plotted. Target failure modes are compared along with the dimension of the hole on target and target displacements. Induced stresses are observed.

7.1 Debris Cloud Shapes

Side views of the impact are illustrated in Figure 7.1. Debris cloud of spherical, cubic and cylindrical projectiles are compared at impact point, 3 μs and 6 μs after impact with impact velocity 7 km/s. Sizes of debris clouds are measured. \( L_0 \) is the length of debris cloud measured from target surface of opposite to impact surface to the frontal area. \( L \) is length of debris cloud excluding the spike part. The widest diameter of debris cloud, \( D \), is measured. Table 7.1 shows the length and the diameter of each debris cloud at 6 μs after impact of initial velocity of 7 km/s.

*Table 7.1. Length and diameter of debris cloud at 6 μs after impact of initial velocity of 7 km/s.*

<table>
<thead>
<tr>
<th>Shape</th>
<th>( L_0 ) (mm)</th>
<th>( L ) (mm)</th>
<th>( D ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>30.4</td>
<td>30.4</td>
<td>26.0</td>
</tr>
<tr>
<td>Cylinder</td>
<td>36.8</td>
<td>29.6</td>
<td>28.4</td>
</tr>
<tr>
<td>Cube</td>
<td>37.4</td>
<td>28.0</td>
<td>27.0</td>
</tr>
</tbody>
</table>

Debris cloud of spherical projectiles is narrowest. The widest debris cloud is one of cylindrical projectile impact. The length including spike of debris cloud of cubic projectile is highest where one of spherical projectile is lowest. The lengths of debris clouds excluding the spike are slightly different. Compared to the spherical projectile with equivalent mass, the debris cloud generated from cubic and cylindrical projectiles’ impact are larger in diameter. The spike-like parts of debris cloud from cubic and
cylindrical projectiles continue grow with elapse time. The spike-like part of cubic is wider. It may be cause by the larger impact surface of the cubic projectile. Further investigation is required as the different is not significant.

Figure 7.1. Comparison of debris cloud generated by spherical, cubic and cylinder projectiles (top, middle, and bottom) impact on thin plate at impact, 3 μs and 6 μs after impact with impact velocity 7 km/s.

7.2 Debris Cloud Velocities

The velocities of debris clouds after impact are observed. Velocities in the impact direction distributions of debris cloud at 6 μs for each projectile shape are illustrated in Figure 7.2. Maximum velocities in negative z-direction are in the middle frontal area of
Comparisons of Geometrical Projectile Shapes Impact

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debris cloud. The maximum velocity varies with projectile shapes. Therefore in these illustrations the identical velocity colour scale for different projectiles might not represent equal velocity.

![Figure 7.2. Comparison of velocity in impact direction of debris cloud generated by spherical, cubic and cylinder projectiles impact on thin plate at impact 6 μs after impact with impact velocity 7 km/s, left, middle, and right respectively.](image)

The maximum velocities of all projectile shapes are normalized as shown in Table 7.2. Simulating each projectile shape with impact velocity 3, 5, 7 and 9 km/s, the normalized maximum axial velocities of all cases are obtained and plotted in Figure 7.3.

**Table 7.2. Maximum axial velocities and normalised maximum axial velocities of 6 μs after impact with impact velocity 7 km/s.**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Maximum Axial</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>5.54611</td>
<td>0.79230</td>
</tr>
<tr>
<td>Cube</td>
<td>6.50901</td>
<td>0.94144</td>
</tr>
<tr>
<td>Cylinder</td>
<td>6.35286</td>
<td>0.90755</td>
</tr>
</tbody>
</table>

The debris cloud resulting from cubic and cylindrical projectiles, which impact the bumper with a flat surface, produce different debris cloud features. The cubic projectile gives a higher normalised axial velocity after impact than the cylindrical projectile in the 3 – 9 km/s impact velocity range. The debris cloud from a spherical projectile has a lower normalised velocity than cubic and cylindrical projectiles in most cases of impact velocities range of 3 – 9 km/s. The normalised maximum velocity of the spherical projectile is only slightly higher than ones of the cylindrical projectile at 5 km/s, and
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lower than the cubic projectiles in all cases. The cubic and cylindrical projectiles have higher velocities and normalised velocities in impact velocity cases higher than 5 km/s, which is in hypervelocity range of aluminium.

![Figure 7.3. Normalised maximum axial velocities at 6 μs after impact for each projectile shape versus impact velocities.](image)

Both cylindrical and cubic projectiles impact on target with their flat face, therefore the shocks propagates in targets are in planar shape, comparatively to shock propagation in targets hit by spherical projectiles, which is a hemispherical shape. The reason of the debris cloud of cubic projectiles has a higher velocity than cylindrical projectile may be the larger frontal area of cubic projectile. Therefore, the larger shock is generated and propagates through the target surface as the impact of flat surface of the cubic projectile has an area of 16.24 mm$^2$ and the impact of the flat surface of the cylindrical projectile has an area of 14.99 mm$^2$.

### 7.3 Dimension of target hole

All the targets fail in petal failure mode. The dimensions of the hole in the targets are slightly different. Measured dimensions of the holes in the targets are 13.1 mm, 12.9mm, 12.6 mm for spherical, cylindrical and cubic projectiles respectively. The difference is too low to verify the effect of geometrical projectile shape on target hole. Further study needs to be performed.
Comparisons of Geometrical Projectile Shapes Impact

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Figure 7.4a, 7.4b, 7.4c. Comparison of target of spherical, cylindrical and cubic projectiles in left, middle, and right respectively at 9 μs after impact with impact velocity of 7 km/s.

7.4 Induced Stress

Shape of stress in the target impacted by the spherical projectile is hemispherical whereas in the cylindrical and cubic projectiles are planar in the middle as shown in Figure 7.5a, 7.5b and 7.5c. There are noises in solution, causing maximum stress values to be much higher than expected ones. The magnitude of stresses in targets impacted by cylindrical and cubic projectiles are lower than one of the spherical projectile as shown in Table 7.3. Nevertheless, more validation is required.

Figure 7.5a, 7.5b and 7.5c. Stress in z-direction at 0.2 μs after impact of spherical, cylindrical and cubic projectile, left, middle and right respectively with initial velocity of 7 km/s.
Table 7.3. Majority of stress in z-direction at 0.2 μs and 0.3 μs after impact of spherical, cylindrical and cubic projectile, left, middle and right respectively with initial velocity of 7 km/s.

<table>
<thead>
<tr>
<th>Time after impact</th>
<th>Sphere (GPa)</th>
<th>Cylinder (GPa)</th>
<th>Cube (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 μs</td>
<td>54.8-82.2</td>
<td>49.5-83.6</td>
<td>47.8-80.7</td>
</tr>
<tr>
<td>0.3 μs</td>
<td>24.4-76.6</td>
<td>24.5-63.7</td>
<td>18.1-47.8</td>
</tr>
</tbody>
</table>

Comparisons of four properties resulting from the impacts have been performed. The debris cloud of the cylindrical and cubic projectiles has spike-like parts in the fronts while one of spherical projectile does not. Normalised velocities of cubic projectiles are highest in all cases except in the case of impact velocity of 3 km/s of which cylindrical projectile is higher. Target damage from each projectile is only slightly different from each other. Stress in the z-direction is highest in the spherical projectile. The conclusion is presented in the following chapter.
8. Conclusion

In this research, numerical simulations were performed for the impact of spherical, cylindrical and cubic projectiles on a thin plate target to investigate the effect of projectile shape focusing on impact velocity in hypervelocity range. The smoothed particle hydrodynamics method was applied. Projectiles and targets were made of aluminium, 2024. All projectiles have equivalent mass and hence equivalent impact energy.

The shape of debris cloud, velocities of debris cloud, residual velocity of projectiles, dimension of target hole after impact, impact induced stress and target failure modes are investigated and compared. Validation of debris cloud shapes and velocities of debris cloud were performed with the spherical projectile. Basic validation of debris cloud shape was performed with the cylindrical projectile. Induced stresses were not validated. Further validations in both spherical and non-spherical projectile are required.

8.1 Geometrical projectile shapes effect

The geometrical projectile shapes effect on debris cloud shape, debris cloud velocity, residual projectile velocity, and induced stresses have been studied.

Shapes of debris clouds generated by impact of cylindrical and cubic projectiles have a spike-like in frontal area of debris clouds, which does not exist in one of spherical projectile. Both cylindrical and cubic projectiles impact on target with their flat face, therefore the shocks propagates in targets are in planar shape, comparatively to shock propagation in targets impacted by spherical projectiles, which is a hemispherical shape as concluded by Morrison. This may be the reason for spike formed in frontal area of projectiles.

Velocities of debris clouds generated by the impacts of cylindrical and cubic projectiles are higher than that of spherical projectile in the impact velocity range of 7-9 km/s. At the impact velocity range of 5 km/s, that of spherical projectile is approximately equivalent to that of cylindrical projectile. The cubic projectile has highest velocity of debris clouds in all case of velocity range 3-9 km/s. Both cylindrical and cubic projectiles impact on the target with their flat surface. The reason that debris cloud of
cubic projectiles has a higher velocity than cylindrical projectile may be the larger frontal area of cubic projectile. Therefore, the larger shock is generated and propagates through the target.

The conventional spherical projectile is not the most dangerous case of space debris. Space debris is not usually found to be in spherical shape. A cylindrical projectile, with length-to-diameter ratio equal to 1, is more lethal than a spherical projectile with an equivalent mass in hypervelocity impact range as it has higher velocity and normalised velocity. The impact of cubic projectile with its face is more lethal than that of spherical projectile and cylindrical projectile, with length-to-diameter ratio equal to 1, with an equivalent mass.

8.2 Future work

There are various works to be developed in the field of projectile shapes effect hypervelocity impact. The suggested future works are:

- Quantitatively analyse debris cloud characteristics produced by various projectile shapes to investigate its influence on debris cloud characteristics including the relationship of debris cloud and damage in the witness plate.
- Investigate the change of material state and shock propagation in the debris cloud and their effect on damage of witness plate with different projectile shapes.
- Simulate in a dual-wall structure to obtain ballistic limit curves for each projectile shape and to investigate damage in the witness plate.
- Compare different projectile shapes with controlled characteristic length.
- Investigate geometrical shapes of projectiles such as flake, cone and pyramids.
- Investigate the influence of obliquity and orientation for each shape.

Other future work is to validate velocities profiles of cylindrical and cubic projectile by experiments and to validate induced stress by impact.
9. References


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Pereyra M. (1999). *Speed of Bullet*. available at: 


References


Appendix A. MCM codes

An example of MCM codes for impact of spherical projectile on thin plate target are presented in this appendix. The full code of a simulation has 700 pages. Therefore, the part that is repeating generation of 40475 nodes is deleted. The units are cm and gram.

Spherical projectile

```
sphere14 file for version 2 format

* Comment line

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* Control card 2: Time control

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* Control card 3: Output file control

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* Control card 4: Input and initialization options

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* Control card 5: Analysis options
```
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* 0 1 0 1 *

* Control card 6: Interpolation options *

2 1 40 *

* Control card 7: Blank at this time *

*

* Material model *

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Gruneisen EOS
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Appendix A

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