Two-Dimensional Compressible Solar Wind Flow Past an Open Terrestrial Magnetopause

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Sammanfattning


Vid växelverkan mellan jordens magnetfält och det interplanetära magnetfältet (IMF) så kan det, under vissa förutsättningar uppstå magnetisk reconnection. Förutsättningarna för magnetisk reconnection studeras och själva återkopplingenantas ske norr om ekvatorslinjen.

Två olika punkter undersöks, två respektive fyra jordradier norr om den subsolära punkten. Det visar sig att större fluktationer i plasmats variabler ju närmare man kommer stagnationspunkten.
Abstract

In this thesis a two dimensional compressible flow of the shocked solar wind plasma in the magnetosheath region along the magnetopause is studied. A curvilinear coordinate system is chosen to study the behavior of the velocity and the magnetic field at different positions close to a reconnection point along the magnetopause. Magnetohydrodynamic (MHD) equations are used to analyze the plasma flow. A model by Henri Cabannes (1951) is used to describe the shape of the bow shock and the magnetopause.

The interaction of the Earth’s magnetic field with an interplanetary magnetic field (IMF) can under some circumstances result in the phenomenon of magnetic reconnection. The conditions for magnetic reconnection are considered. Magnetic reconnection is assumed to occur north of the subsolar point. Two points are considered, two respectively four earth radii north of the subsolar point. It is seen that in the transition through the magnetopause there are larger fluctuations in velocity and magnetic field distributions closer to the subsolar point.
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1. Introduction

Mankind has always been interested in phenomenon that take place in outer space and one of them is the so called northern light, which is visible to the bare eye. Research and observations due to the fascination of the sun and the whole space has been done throughout the century. Many models and ideas about solar-terrestrial relationships have been done but it was in the early 1930s space physic flourished. Chapman and Ferraro came with a model of the solar wind plasma i.e a stream of electrons and ions leaving the sunspots streaming out in space, and they developed the idea of the interaction between the solar wind and the earth’s magnetic field leading to the concept of the Earth’s magnetosphere.

In the 1940s Alvé developed the theory of magnetohydrodynamics, or MHD, which combine magnetic field (magneto), fluids (hydro) and movement (dynamics). In MHD the plasma is treated as an conducting fluid.

In ideal MHD the fluid is assumed to be a perfect conductor and have little or no resistivity, but in reality there are no perfect conductors. That means that MHD equations are only valid as long as you can ignore the diffusion. When two magnetic domains comes close together the current sheet that separates them can collapse and cause a phenomena called magnetic reconnection.

Magnetic reconnection was first discussed by Sweet (1958) and Parker (1957). In the late 1950s and early 1960s the existence of the solar wind plasma was proved experimentally with high technology equipment such as satellites and rockets and since then the interest and knowledge for space has increased rapidly.

The magnetic field of the Earth reminds of a field from a bar magnet, the field lines originate in the south pole and end up in the north pole.

![Figure 1.0.1. The field lines of a bar magnet (to the left) and the Earth (to the right). Note that magnetic north pole corresponds to the geographic south pole (and vice versa) on the Earth](image)

The magnetic field around the Earth is believed to originate from the center of the planet but how it is created is not completely understood. In the center of the planet there is a core of iron that could create the magnetic field but the problem is that the core is too hot and therefore not magnetic, but the fact remains, there is a magnetic field around the planet. One probable explanation is the Earth’s rotation around it’s own axis. It is known that magnetic fields are produced by motion of electrical charges and it’s believed that electrical currents created by a rotation of the liquid outer core (constituent of iron and
nickel) coupled with convective effects could be the explanation of the Earth’s magnetic field. This process is called the dynamo effect.

The magnetic field creates a protecting “bubble”, called the magnetosphere and on the outer limit, where the earth’s magnetic field meets the solar winds magnetic field, there is a discontinuity called the magnetopause.

The magnetosphere is of great importance to life on Earth as it stops the solar wind from ripping away our atmosphere and also stops the plasma from burning the crust. If the magnetic field would disappear, all life on Earth will vanish. When the magnetopause prevents the solar wind to enter the magnetosphere a bow shock is created further out and the solar wind properties are altered after penetrating the bow shock. The plasma continue to stream against the magnetosphere in the region between the magnetopause and the bow shock, this region is called the magnetosheath.

Some high energetic particles can penetrate the magnetopause and be trapped in Earth’s magnetic field. As the particle moves along the magnetic field lines, they enter the atmosphere near the north- and south poles (in the so called polar cusps) where the field lines disappear into the planet.

Figure 1.0.2. The picture shows, amongst other things, the bow shock, magnetosheath, magnetopause, magnetosphere and the polar cusps.

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\[1\] The particles collide with molecules in the air which then will get a higher energy, when they fall back to the ground state they will give out light in different colors. That’s what we call Northern light (southern light at the south pole).
2. Theory

The solar wind is super sonic until it hits the bow shock, then the velocity drops down to subsonic. This is not the only property that changes after the bow shock, the pressure, density and magnetic field are also altered in the magnetosheath. The shocked solar wind continue to flow through the magnetosheath until it arrives to the magnetosphere.

The plasma can penetrate the magnetopause and enter the magnetosphere at certain locations under special conditions. One such process is called magnetic reconnection and the conditions fot his process to occur are described later. An overview of the plasma flow in the magnetosheath is seen in figure(1.0.2)

2.1. The Solar Wind

The solar corona, with a temperature of order $10^6 K$, withstands a much higher gas pressure then the interstellar space which results that an ionized plasma gets ejected from the sun (solar wind). The difference in gas pressure gives the solar wind enough energy to escape the solar gravity. The velocity of these free electrons and ions can vary between $250 \text{ km/s}$ and $900 \text{ km/s}$ depending on solar activities.

The high density in the corona gives a high collision rate but after the plasma is ejected the rate decreases and the solar wind gradually becomes collisionless. In a collisionless plasma, the conductivity becomes large and Ohms law

$$j = \sigma (E + u \times B),$$

is reduced to

$$E + u \times B = 0.$$  \hfill (2.1.2)

The high conductivity and a non-zero velocity ($u \neq 0$) gives that the magnetic field from the corona will be frozen in the solar wind and follow the plasma flow through the interplanetary space.

2.1.1. Frozen In Theorem

The frozen in condition means that motion along the field lines doesn’t change the field and motions crossing the field lines carry the field with them. If we visualize a certain amount of the solar wind plasma with a magnetic field $B_1$ moving through a cross section, $A_1$, the magnetic flux becomes $\Phi_1 = B_1 A_1$.

What the frozen in theorem says is that if the same amount of plasma later moves through a different cross section, $A_2$, it will have a different magnetic field, $B_2$, but the same magnetic flux (i.e $B_2 A_2 = B_1 A_1$).

To validate the concept of the frozen in theorem we use the induction equation,

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \frac{1}{\mu_0 \sigma} \nabla^2 B.$$  \hfill (2.1.3)
Introducing the dimensionless magnetic Reynolds number $R_m$\textsuperscript{1} the second part (diffusion term) on the right hand side disappears due to the high conductivity of the plasma i.e. $R_m \gg 1$. Equation (2.1.3) becomes

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B).$$

(2.1.4)

We consider the rate of change of the magnetic flux $\Phi$ through a moving contour

$$\frac{d\Phi}{dt} = \int \int_S \frac{\partial B}{\partial t} \cdot ds + \oint_C B \cdot (v \times dl).$$

(2.1.5)

Using some vector algebra the second part of the right hand side can be written as

$$\oint_C B \cdot (v \times dl) = -\int \int_S \nabla \times (u \times B) \cdot ds.$$ \hspace{1cm} (2.1.6)

Combining (2.1.5) with (2.1.6) give us

$$\frac{d\Phi}{dt} = \int \int_S \left( \frac{\partial B}{\partial t} - \nabla \times (u \times B) \right) \cdot ds.$$ \hspace{1cm} (2.1.7)

Since equation (2.1.4) holds we must have

$$\frac{d\Phi}{dt} = 0,$$ \hspace{1cm} (2.1.8)

which means that the magnetic field lines are frozen in the plasma and moving along with it.

\textbf{2.2. Magnetic Reconnection}

It was in the 1940s the first idea of magnetic reconnection were first discussed and since then the progress has been huge. Today magnetic reconnection is one of the main research areas in plasma physics. The magnetic field in the plasma flow contains stored magnetic energy which is released and transferred to the charged particles in the solar wind at reconnection. This phenomenon makes it possible for the solar winds constituents to penetrate the magnetopause and enter the magnetosphere with an energy boost. Magnetic reconnection is possible when two magnetic field of opposite polarisation get close together, then the field lines of the magnetic fields breaks down and ties together in a new topology. They will form a “slingshot” that fires jets of particles in opposite direction\textsuperscript{2}.

\textsuperscript{1} The magnetic Reynolds number is a dimensionless quantity depending on magnetic properties. $R_m = \mu \sigma U L$, where $U$ is a characteristic velocity and $L$ is a characteristic length, $\sigma$ is the conductivity and $\mu$ is permeability.

\textsuperscript{2} What really happen is that the field has rejoined in a lower energy state and the extra energy has been transferred to the plasma flow.
The plasma flow ejected from the Sun has an embedded magnetic field, the Interplanetary Magnetic Field (IMF), with a magnetic Reynolds number much larger than unity. This means that the solar wind plasma is a highly conducting medium and as we discussed earlier diffusion of the magnetic field lines through the plasma can not occur and the frozen in concept is valid. When the IMF is directed southwards and approaches the Earth’s magnetic field we have two magnetic fields with opposite polarities. Because the Earth’s magnetic field is like a dipole magnet with the magnetic field lines pointing from the geographical south to north (figure3.1.2), the first condition for reconnection is fulfilled.

Another condition is that the velocity cannot be too high if reconnection should be possible, the plasma needs to be slow enough to be subalfvénic, i.e

$$M_A = \frac{U}{v_A} < 1.$$  \hfill (2.2.1)

$U$ is the velocity along the magnetopause and $v_A$ is the Alfvén velocity that depends on the magnetic field,

$$v_A = \sqrt{\frac{B^2}{\mu_0 \rho}}.$$  \hfill (2.2.2)

$B$ is the magnetic field, $\rho$ the density and $\mu_0$ the permeability.

Magnetic reconnection can take place as long as these conditions are satisfied. The frozen in condition of the solar wind breaks down and the magnetic flux is no longer preserved. The two different magnetic fields lines may diffuse, allowing the interplanetary and terrestrial field lines to connect.

The magnetic field is increasing when the plasma moves along the magnetopause and so is the velocity. Eventually the velocity will become too high to allow reconnection, despite that the density is decreasing and that the magnetic field is increasing.
3. Analysis

As a first step a more general boundary layer analysis (BLA) at the reconnection point is considered. Then we introduce a model for the flow inside the magnetosheath which will give more accurate boundary conditions. Before we establish how the plasma flows inside the magnetosheath an analysis of the crossing from the outside bow shock to the magnetosheath is considered. This gives conditions that can be inserted into the BLA and will give solutions that describes the velocity and magnetic field in the vicinity of the reconnection point.

3.1. Boundary Layer Analysis

In our analysis we use a curve-linear coordinate system that moves along the magnetopause with the $z$-coordinate in the tangent direction and the $x$-coordinate in the normal direction (see fig.3.1.1)

$$r(z)$$

Figure 3.1.1. Coordinate system. The number $r(z)$ is the “height” of the reconnection point from the sub solar point.

3.1.1. The equations and the dimensionless variables

The plasma flow movement is described by Navier-Stokes Equation with an extra body force due to the magnetic field

$$\rho(u \cdot \nabla)u = \mu \nabla^2 u - \nabla P + (B \cdot \nabla)B. \quad (3.1.1)$$

The magnetic field is described by the induction equation (2.1.3) and stationary magnetic field is assumed (i.e. $\frac{\partial B}{\partial t} = 0$) so

$$\nabla \times (u \times B) + \frac{1}{\mu_0 \sigma} \nabla^2 B = 0. \quad (3.1.2)$$

The flow is considered to be two dimensional

$$u = (u_x, 0, u_z) \quad (3.1.3)$$

$$B = (B_x, 0, B_z). \quad (3.1.4)$$
and the following dimensionless variables are introduced

\[
\frac{u}{v_A} \rightarrow u_x \\
\frac{u}{v_A} \rightarrow u_z \\
\frac{B_x}{B_0} \rightarrow B_x \\
\frac{B_z}{B_0} \rightarrow B_z \\
\frac{E_z}{v_A B_0} \rightarrow E_z \\
\frac{x}{R_E} \rightarrow x \\
\frac{z}{R_E} \rightarrow z.
\]

(3.1.5)

Here \(B_0\) is the \(z\)-component of the magnetic field, \(R_E\) is earth radius and \(v_A\) is the Alfvén speed. Equation (3.1.2) and equation (3.1.1) becomes (see Schlichting [10])

\[
\frac{1}{R_m} \frac{\partial B_x}{\partial x} + u_x B_z - B_x u_z + E_y = 0
\]

(3.1.6)

\[
\rho u_x \frac{\partial u_x}{\partial x} + \rho u_z \frac{\partial u_z}{\partial z} = \frac{\partial P}{\partial x} + B_x \frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} + R \frac{\partial^2 u_x}{\partial x^2}
\]

(3.1.7)

\[
\rho u_x \frac{\partial u_z}{\partial x} + \rho u_z \frac{\partial u_z}{\partial z} = \frac{\partial P}{\partial z} + B_x \frac{\partial B_x}{\partial x} + B_z \frac{\partial B_z}{\partial z} + R \frac{\partial^2 u_z}{\partial x^2}
\]

(3.1.8)

One of Maxwells equations and the stationary continuity equation for compressible flow are also needed

\[
\nabla \cdot \mathbf{B} = 0
\]

(3.1.9)

\[
\nabla \cdot (\rho u) = 0.
\]

(3.1.10)

In parabolic coordinates the equations (3.1.9)-(3.1.10) implies that

\[
\frac{\partial B_x}{\partial x} + \frac{1}{r(z)} \frac{\partial}{\partial z} (r(z) B_z) = 0
\]

(3.1.11)

\[
\frac{\partial (\rho u_x)}{\partial x} + \frac{1}{r(z)} \frac{\partial}{\partial z} (\rho r(z) u_z) = 0.
\]

(3.1.12)

### 3.1.2. The DeHoffmann-Teller Frame

To exclude the electric field (i.e. \(E_y\)) the DeHoffmann-Teller frame of reference is used (HTF, [5]). The velocity component \(u_z\) is divided into a DeHoffmann-Teller velocity \((U_{HT})\) and the velocity that the plasma is moving with respect to the HTF

\[
u^*_z = u_z + U_{HT}.
\]

(3.1.13)

note that \(u^*_z\) is the “original” velocity and \(u_z\) and \(U_{HT}\) is the new velocity components.

Introducing a fast variable \(\xi\)

\[
\xi = R^\frac{1}{2} x
\]

(3.1.14)

the derivative becomes

\[
\frac{\partial}{\partial x} = R^\frac{1}{2} \frac{\partial}{\partial \xi}.
\]

(3.1.15)
The Induction equation (3.1.2) and the Navier-Stokes equation (3.1.1) with the DeHoffmann-Teller velocity and the variable substitution then become

\[
\frac{R^2}{R_m} \frac{\partial B_z}{\partial \xi} + u_\xi B_z - B_\xi (u_z + U_{HT}) + E_y = 0 \quad (3.1.16)
\]

\[
R^2 \rho u_\xi \frac{\partial u_\xi}{\partial \xi} + \rho (u_z + U_{HT}) \frac{\partial u_\xi}{\partial \xi} = -R^2 \frac{\partial \rho}{\partial \xi} + R^2 B_\xi \frac{\partial B_\xi}{\partial \xi} + \frac{B_\xi}{R} \frac{\partial^2 u_\xi}{\partial \xi^2}
\]

\[
R^2 \rho u_\xi \frac{\partial u_\xi}{\partial \xi} + \rho (u_z + U_{HT}) \left( \frac{\partial u_\xi}{\partial \xi} + \frac{\partial U_{HT}}{\partial \xi} \right) = \frac{-\partial P}{\partial \xi} + R^2 B_\xi \frac{\partial B_\xi}{\partial \xi} + B_\xi \frac{\partial B_\xi}{\partial \xi} + \frac{\partial^2 u_\xi}{\partial \xi^2} \quad (3.1.17)
\]

The equations (3.1.11) - (3.1.12) is altered to

\[
R^2 \rho u_\xi \frac{\partial u_\xi}{\partial \xi} + \frac{1}{r(z)} \frac{\partial}{\partial z} (r(z) B_z) = 0 \quad (3.1.19)
\]

and

\[
R^2 \rho u_\xi \frac{\partial u_\xi}{\partial \xi} + R^2 u_\xi \frac{\partial \rho}{\partial \xi} + \frac{1}{r(z)} \frac{\partial}{\partial z} (\rho r(z)(u_z + U_{HT})) = 0 \quad (3.1.20)
\]

We have chosen \( U_{HT} = U_{HT}(z) \) such that \( E_y + B_\xi U_{HT} = 0 \).

To get a more solvable set of equations a perturbation expansion for large \( R \) and \( R_m \) is considered. As in ordinary boundary layer theory in fluid mechanics we choose the following expansions

\[
u_\xi = u_\xi^{(0)} + R^{-\frac{1}{2}} u_\xi^{(1)} + ...
\]

\[
\quad u_z = U_{HT} + u_z^{(0)} + R^{-\frac{1}{2}} u_z^{(1)} + ...
\]

\[
\quad B_\xi = B_\xi^{(0)} + R^{-\frac{1}{2}} B_\xi^{(1)} + ...
\]

\[
\quad B_z = B_z^{(0)} + R^{-\frac{1}{2}} B_z^{(1)} + ...
\]

\[
\quad \rho(\xi, z) = \rho_0(z) + R^{-\frac{1}{2}} \rho_1(\xi, z).
\]

Here the Reynolds number \( (R) \) is assumed to be of the same order as Reynolds magnetic number \( (R_m) \). The perturbation expansion (3.1.21) in equation (3.1.16)-(3.1.20) gives to leading order \( O \left( R^2 \right) \)

\[
\rho_0 u_\xi^{(0)} \frac{\partial u_\xi^{(0)}}{\partial \xi} = B_\xi^{(0)} \frac{\partial B_\xi^{(0)}}{\partial \xi} \quad (3.1.22)
\]

Maxwell’s equation and the continuity equation of order \( O \left( R^0 \right) \) are

\[
\frac{\partial B_\xi^{(0)}}{\partial \xi} = 0 \Rightarrow B_\xi^{(0)} = B_\xi^{(0)}(z) \quad (3.1.23)
\]

\[
\frac{\partial u_\xi^{(0)}}{\partial \xi} = 0 \Rightarrow u_\xi^{(0)} = u_\xi^{(0)}(z) \quad (3.1.24)
\]

which means that \( u_\xi^{(0)} \) and \( B_\xi^{(0)} \) only depends on \( z \). This fact and the induction equation of order \( O \left( R^0 \right) \) gives

\[
B_z^{(0)} = \frac{B_\xi^{(0)} u_\xi^{(0)}}{u_\xi^{(0)}} \quad (3.1.25)
\]

and used in equation (3.1.22) it results in
The derivative is different from zero which gives
\[ B_{\xi}^{(0)} = \pm u_{\xi}^{(0)} \sqrt{\rho_0} \] (3.1.27)
\[ B_{\xi}^{(1)} = \pm u_{\xi}^{(0)} \sqrt{\rho_0} \] (3.1.28)
where the plus- and minus sign correspond to solutions to the north- and south of the reconnection point respectively. When equation (3.1.23) and equation (3.1.24) is used in equation (3.1.17) of order \( O(R^2) \) it is clear that
\[ \frac{\partial P}{\partial \xi} = 0. \] (3.1.29)
This is an important property, it means that the modified pressure
\[ P = p + \frac{B^2}{2\mu_0} \] (3.1.30)
is constant through the transition layer.

3.1.3. Solving the equations

The solution

The Navier-Stokes equation in the z-direction of order \( O(R^0) \) is
\[
\rho_0 u_\xi^{(0)} \frac{\partial u_\xi^{(0)}}{\partial \xi} + \rho_0 u_\xi^{(0)} \frac{\partial u_\xi^{(0)}}{\partial z} + \rho_1 u_{\xi}^{(0)} \frac{\partial u_{\xi}^{(0)}}{\partial \xi} + 
\rho_0 u_{\xi}^{(0)} \frac{\partial \rho_0^{(0)}}{\partial z} + \rho_0 U_{HT} \frac{\partial u_{\xi}^{(0)}}{\partial z} + \rho_0 U_{HT} \frac{\partial B_{\xi}^{(0)}}{\partial z} = 0.
\]
Differentiate the induction equation with respect to \( \xi \) of order \( O(R^0) \)
\[
\sqrt{\rho_0} \frac{\partial^2 u_{\xi}^{(0)}}{\partial \xi^2} - u_{\xi}^{(0)} \frac{\partial \rho_0^{(0)}}{\partial \xi} = u_{\xi}^{(0)} \frac{\partial B_{\xi}^{(1)}}{\partial \xi} - u_{\xi}^{(1)} \frac{\partial \rho_0^{(0)}}{\partial \xi} + u_{\xi}^{(0)} \sqrt{\rho_0} \frac{\partial B_{\xi}^{(1)}}{\partial \xi} + U_{HT} \frac{\partial B_{\xi}^{(1)}}{\partial \xi} = 0.
\]
A combination of equation (3.1.19) and equation (3.1.20) gives us the relation
\[
u_{\xi}^{(1)} \rho_0 + u_{\xi}^{(0)} \rho_1 - B_{\xi}^{(1)} \sqrt{\rho_0} = -\frac{1}{2} \frac{\partial \rho_0}{\partial z} \int_{-\infty}^{\xi} u_{\xi}^{(0)} d\xi - \frac{1}{r(z)} \frac{d}{dz} (r(z) \rho_0 U_{HT})
\]
which together with equation (3.1.31) and equation (3.1.32) gives the final equation,
We introduce the following variables substitution

$$\eta = \xi g(z)$$

$$Z = z$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial Z} + \xi g'(z) \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial \xi} = g(z) \frac{\partial}{\partial \eta}$$

$$\frac{\partial^2}{\partial \xi^2} = g^2(z) \frac{\partial^2}{\partial \eta^2}$$

$$\frac{\partial \tilde{u}^{(0)}}{\partial z} = \frac{\partial \tilde{u}^{(0)}}{\partial Z} + \eta \frac{q'(z)}{g(z)} \frac{\partial \tilde{u}^{(0)}}{\partial \eta}$$

and define \( u_z^{(0)} \) as

$$u_z^{(0)} = \frac{\partial f}{\partial \eta} = \frac{\partial}{\partial \eta} f(Z, \eta).$$

These conditions applied in equation (3.1.34) gives

$$-\rho_0 U_{HT} \frac{\partial U_{HT}}{\partial z} - \frac{dP}{dz} + \left( \frac{\partial^2 f}{\partial \eta^2} \right) \frac{d\rho_0}{dz} + \left( 1 + \rho_0 \frac{R}{\rho_0} \right) g^2 \frac{\partial^3 f}{\partial \eta^3}$$

$$+ \frac{\partial^3 f}{\partial \eta^3} \left( \frac{\partial \rho_0}{\partial \eta} \int_{-\infty}^{\eta} \frac{d\rho_0}{dz} \eta \frac{d\rho_0}{dz} (r(z) \rho_0 U_{HT}) \right)$$

$$-\rho_0 U_{HT} \left[ \left( \frac{\partial^2 f}{\partial \eta^2} + \eta \frac{\partial^2 f}{\partial \eta^2} \right) \right] + U_{HT} \frac{\partial f}{\partial \eta} \left( \frac{1}{\rho_0} \frac{\partial}{\partial \eta} (r(z) \rho_0) - \frac{\partial}{\partial \eta} \right)$$

$$- \rho_0 \frac{U_{HT}}{r(z)} \left( \frac{\partial^2 f}{\partial \eta^2} \left( r(z) \frac{\partial f}{\partial \eta} \right) + \eta \frac{\partial^2 f}{\partial \eta^2} \left( r(z) \frac{\partial f}{\partial \eta} \right) \right) \right) = 0.$$ (3.1.37)

Further \( f \) is expanded in \( f_0 \) and \( f_1 \)

$$f(Z, \eta) = f_0(\eta) + Z f_1(\eta)$$

so the equation evolves to

$$-\rho_0 U_{HT} \frac{\partial^2 f_0}{\partial z^2} - \frac{dP}{dz} + \left( 1 + \rho_0 \frac{R}{\rho_0} \right) g^2 (f_0'' + Z f_1'')$$

$$+ (f_0'' + Z f_1'') \left( \frac{\partial \rho_0}{\partial \eta} \int_{-\infty}^{\eta} \frac{d\rho_0}{dz} (f_0' + Z f_1') \eta \frac{d\rho_0}{dz} (r(z) \rho_0 U_{HT}) \right)$$

$$- \rho_0 \frac{U_{HT}}{r(z)} \left( f_0' \frac{\partial}{\partial \eta} (r(z)) + Z f_1' \frac{\partial}{\partial \eta} (r(z)) \right)$$

$$+ (f_0'' + Z f_1'') \frac{d\rho_0}{dz} - 2 \rho_0 U_{HT} \left[ \left( f_1'' + \eta \frac{\partial f_1'}{\partial \eta} (f_0'' + Z f_1'') \right) \right]$$

$$+ U_{HT} \left( f_0'' + Z f_1'' \right) \left( \frac{1}{\rho_0} \frac{\partial}{\partial \eta} (r(z) \rho_0) - \frac{\partial}{\partial \eta} \right) = 0.$$ (3.1.39)

By putting \( Z = 0 \) in equation (3.1.39) and taking the homogeneous part of the differential equation, we find

$$f_0'''(\eta) - \frac{2 \rho_0 (0) U_{HT}(0) \frac{d}{d \eta} \eta f_0''(\eta)}{1 + \rho_0 (0) \frac{R}{\rho_0}} = 0.$$ (3.1.40)

Knowing that \( f_0 \) only depends on \( \eta \) the term in front of \( f_0'' \) must be a constant, which is chosen to be equal to 2.
Solving the homogeneous equation for \( f \)

\[
\frac{-2\rho_0(0)U_{HT}(0)g'}{(1 + \rho_0(0)\frac{\rho}{R_m})g^3} = 2
\]

This gives a differential equation where the solution is the inverse transition layer thickness \( g \)

\[
g = \pm \sqrt{\frac{\rho_0(0)U_{HT}(0)}{2 (1 + \rho_0(0)\frac{\rho}{R_m})Z}}
\]

where the plus and minus sign corresponds to the location of the reconnection point. The transition layer thickness is

\[
\frac{1}{g} = \delta(Z) = \pm \sqrt{\frac{2 (1 + \rho_0(0)\frac{\rho}{R_m})Z}{\rho_0(0)U_{HT}(0)}}
\]

At the reconnection point \( Z = 0 \) the transition layer thickness \( \delta(Z) \) is assumed to be equal to zero.

The coordinate system is chosen and moves along the magnetopause so that the reconnection point is always at \( Z = 0 \) and the interesting area is in the vicinity of that point. In order to see the behavior a Taylor expansion of \( \rho_0(Z) \), \( U_{HT}(Z) \) and \( r(Z) \) at \( Z = 0 \) is considered

\[
\rho_0(Z) = \rho_0(0) + Z\rho'_0(0)
\]

\[
U_{HT}(Z) = U_{HT}(0) + ZU'_{HT}(0)
\]

\[
r(Z) = r(0) + Zr'(0).
\]

The Taylor expansion gives the final touch to the equation before the solutions can be obtained. It becomes a very long equation that will be solved for different orders to get a numerical solution for the magnetic field. The assumptions and expansion has transformed equation(3.1.34) to

\[
\begin{align*}
(\rho_0(0) + Z\rho'_0(0))(U_{HT}(0) + ZU'_{HT}(0))U'_{HT}(0) \\
-\frac{4\rho}{dz} + \left(1 + (\rho_0(0) + Z\rho'_0(0))\frac{\rho}{R_m}\right)g^2 (f''_0 + Zf''_1) + \\
(f''_0 + Zf''_1) \frac{\rho}{R_m} \int f'_{\xi}(\xi g) + Zf'_{1}(\xi g) d\xi \\
- (f''_0 + Zf''_1) \frac{2\eta}{\rho} \frac{\rho}{R_m} [r(z)(\rho_0(0) + Z\rho'_0(0))(U_{HT}(0) + ZU'_{HT}(0))] \\
- (\rho_0(0) + Z\rho'_0(0))(U_{HT}(0) + ZU'_{HT}(0)) \left\{2f_1 + 2\eta\frac{\rho}{R_m} (f''_0 + Zf''_1)\right\} \\
- (\rho_0(0) + Z\rho'_0(0))(U_{HT}(0) + ZU'_{HT}(0)) \frac{1}{\rho(0) r(z)} \int (r(z)(\rho_0(0) + Z\rho'_0(0))) \\
+ (U_{HT}(0) + ZU'_{HT}(0))(f''_0 + Zf''_1) \frac{1}{\rho(0)} \left(\frac{\rho}{R_m} (r(z)(\rho_0(0) + Z\rho'_0(0)) \right) \\
+ (f''_0 + Zf''_1) \frac{\rho}{R_m} (\rho_0(0) - (U_{HT}(0) + ZU'_{HT}(0))(f''_0 + Zf''_1) \frac{1}{2}\rho_0(0) = 0.
\end{align*}
\]

Solving the homogeneous equation for \( f''_0(\eta) \) by using equation(3.1.47) of order \( O(\frac{1}{R}) \)

\[
\rho_0(0)U_{HT}(0)\eta f''_0(\eta) + \frac{1}{2}\rho_0(0)U_{HT}(0)f''_0(\eta) = 0 \Rightarrow f''_0(\eta) + 2\eta f''_0(\eta) = 0
\]
which gives the solution for $u_z^{(0)}$

$$u_z^{(0)} = f_0'(\eta) = C_A + C_B erf(\eta). \quad (3.1.49)$$

By choosing the constants $C_A = 0$ and $C_B = 1$ the above solution of $u_z^{(0)}$ becomes an error function.

When we now have the solution of $f_0'(\eta)$, it is straightforward to obtain a differential equation for $f_1'$ from equation (3.1.47) of order $O (R^0)$

$$\left(f_1'' + 2\eta f_1' - 4f_1\right) = 2U''_{HT}(0) + 2 \left(\frac{\rho'(z)}{\rho_0(z)} + \frac{2}{2} \left(\frac{\rho'_0(0)}{\rho_0(0)} + \frac{U''_{HT}(0)}{U_{HT}(0)}\right)\right) \eta f_0''$$

$$+ \frac{2}{\rho_0(0)U_{HT}(0)} \frac{dP}{d\zeta} - \frac{\rho'_0(0)}{\rho_0(0)} f_0' - \frac{2\rho'_0(0)}{\rho_0(0)U_{HT}(0)} (f_0')^2 - \frac{\rho'_0(0)}{1 + \rho_0(0) / \rho_m} f_0''.' \quad (3.1.50)$$

The next step is to identify the constants in front of the different $f_0$ terms, they are

$$C_1 = -2\frac{\rho'_0(0)}{\rho_0(0)}$$

$$C_2 = \left[4\frac{\rho'(z)}{\rho_0(z)} + 5 \left(\frac{\rho'_0(0)}{\rho_0(0)} + \frac{U''_{HT}(0)}{U_{HT}(0)}\right)\right]$$

$$C_3 = -\frac{\rho'_0(0)}{1 + \rho_0(0) / \rho_m})$$

$$C_4 = -2\frac{\rho'_0(0)}{\rho_0(0)U_{HT}(0)}.$$

To get the numerical values of these constants, so that equation (3.1.50) can be solved, a model of the flow between the bow shock and magnetopause is needed which is presented in section (3.2).

For now a more general solution of $f_1'$ is obtained with all the constants set to unity and the terms not including some sort of derivative of $f_0$ is set to zero (fig 3.1.2).

![Figure 3.1.2](image-url)

Figure 3.1.2. The figure shows the total particular solution of $f_1'$ with the constants set to unity. The figure shows that there will be a change during the transition.

### 3.2. Magnetosheath model

A right-handed coordinate system with the origin at the sub solar point on the bow shock and with the $x$-axis pointing towards the Earth in the solar wind flow direction. This coordinate system is chosen to agree with the model H. Cabannes made for the shape of the bow shock [3].

The velocity, pressure and density outside the bow shock is considered to be constant in this model for the magnetosheath flux (i.e. velocity $u_1 = q_1 \hat{x}$, pressure $p_1$ and the density $\rho_1$).
3.2.1. Before and after the bow shock

To calculate how the parameters inside and outside the bow shock relate to each other, the continuity equation (3.2.1), the energy equation (3.2.2), momentum equation (3.2.3) and the perfect gas relation (3.2.4), are considered.

In this case the bow shock has a parabolic shape (fig1.0.2) (Oblique shock wave) which means that the normal (\( \hat{n} \))- and tangential (\( \hat{t} \)) components on the in- and outside are preferred in the calculations. The shape of the bow shock is given by equation (3.2.5) ([3]). So we have

\[
\rho_1 u_{1n} = \rho_2 u_{2n} \quad (3.2.1)
\]
\[
h_1 + \frac{1}{2} u_1 \cdot u_1 = h_2 + \frac{1}{2} u_2 \cdot u_2 \quad (3.2.2)
\]
\[
- [p\hat{n}]_1^2 = [\hat{n} \cdot (\rho u)]_1^2 \quad (3.2.3)
\]
\[
h = C_p T = \frac{\gamma p}{(\gamma - 1)p} \quad (3.2.4)
\]
\[
x = x(r) = \frac{r^2}{2R} + \sum_{i=2}^{\infty} \lambda_i \frac{r^{2i}}{2iR^{2i-1}} \quad (3.2.5)
\]

where \( \rho \) is the density, \( p \) is the pressure, \( u \) total velocity (the total velocity contains a normal and tangential part \( u = u_n \hat{n} + u_t \hat{t} \)) and \( h \) is the specific enthalpy. Setting the distance between the bow shock and the magnetopause to \( L \) the shape of the magnetopause can be obtained

\[
x_{mp} = x_{mp}(r) = L + \frac{r^2}{2R_{mp}} + \sum_{i=2}^{\infty} \sigma_i \frac{r^{2i}}{2iR_{mp}^{2i-1}}. \quad (3.2.6)
\]

\( R_{mp} \) is the radius of curvature of the magnetopause. It is important to note that this radius is different from the bow shocks radius of curvature, \( R \).

The reconnection point is altered by taking different \( Z \) values, so a relation between \( r \) and \( Z \) needed. Since \( Z \) is the length along the magnetopause, parabolic globally and ordinary Cartesian locally, the relation becomes

\[
Z = \int_0^r \sqrt{1 + x'(r)^2}dr. \quad (3.2.7)
\]

Here \( x(r) \) is the same as equation (3.2.6).

Equation (3.2.5) gives the normal (3.2.8) an tangent (3.2.9) to the bow shock

\[
\hat{n} = \frac{\hat{x} - x'(r)\hat{r}}{\sqrt{1 + (x'(r))^2}} \quad (3.2.8)
\]
\[
\hat{t} = \frac{x'(r)\hat{x} + \hat{r}}{\sqrt{1 + (x'(r))^2}}. \quad (3.2.9)
\]

The first step is to eliminate \( h \) by using equation (3.2.4) in equation (3.2.2) and so we get a new energy equation

\[
2 \frac{\gamma}{(\gamma - 1) p_2} + u_{n1}^2 = 2 \frac{\gamma}{(\gamma - 1) p_2} + u_{n2}^2. \quad (3.2.10)
\]

The equations (3.2.1),(3.2.3) and (3.2.10) gives a relation between the inside and outside pressure

\[
\frac{p_2}{p_1} = 1 + 2 \frac{\gamma}{(\gamma + 1)} \left( \frac{p_1 V_1^2}{\gamma p_1} - 1 \right) \quad (3.2.11)
\]
and if we use the Mach number (normal direction)
\[ M_n^2 = \frac{\rho V_n^2}{\gamma p} = \frac{M^2}{\left(1 + (x'(r))^2\right)^{1/2}} \] (3.2.12)
the relation becomes
\[ \frac{p_2}{p_1} = 1 + 2\frac{\gamma}{(\gamma + 1)} (M_{1n}^2 - 1). \] (3.2.13)

In a similar manner the ratio for the density can be obtained
\[ \frac{\rho_1}{\rho_2} = \left(\frac{\gamma - 1}{\gamma + 1}\right) + \frac{2}{M_{2n}^2(\gamma + 1)}. \] (3.2.14)

In the shocked solar wind plasma the Mach number will be different due to variation of the solar wind properties. From equation (3.2.13) the relation between the Mach number before and after the bow shock is derived,
\[ M_2^2 = \frac{1 + (\gamma - 1) M_1^2}{2\gamma M_2^2 - \gamma - 1}. \] (3.2.15)

The velocity after the bow shock is chosen to be of the form \( u_2 = u_x \hat{x} + u_r \hat{r} \) and as earlier the velocity before the bow shock is \( u_1 = q_1 \hat{x} \) which together with (3.2.8) and (3.2.9) gives the normal and tangential components
\[
\begin{align*}
\mathbf{u}_1 \cdot \hat{n} &= u_{1n} = q_1 \frac{q_1}{\sqrt{1 + (x'(r))^2}}, \\
\mathbf{u}_2 \cdot \hat{n} &= u_{2n} = \frac{u_x - u_x x'(r)}{\sqrt{1 + (x'(r))^2}}, \\
\mathbf{u}_1 \cdot \hat{t} &= u_{1t} = \frac{q_1 x'(r)}{\sqrt{1 + (x'(r))^2}}, \\
\mathbf{u}_2 \cdot \hat{t} &= u_{2t} = \frac{u_x x'(r) + u_r}{\sqrt{1 + (x'(r))^2}}.
\end{align*}
\] (3.2.16)

Using equation (3.2.1) in equation (3.2.3) makes it clear that the tangential velocity is the same on both side of the bow shock. Relations of the velocity outside and the velocity inside in \( \hat{r} \) direction is derived using this fact
\[ u_{1t} = u_{2t} \Rightarrow u_r = (q_1 - u_x) x'(r). \] (3.2.17)

A ratio between the velocity inside and outside in \( \hat{x} \) direction follows from equation (3.2.1) and equation (3.2.14)
\[
\frac{u_x}{q_1} = \frac{2}{(\gamma + 1)} \left\{ \frac{x'(r)}{1 + x'(r)^2} + \frac{1}{M^2} + \frac{\gamma - 1}{2} \right\}. \] (3.2.18)

### 3.2.2. In the Magnetosheath and at the Magnetopause

The numerical values of the calculated constants in section (3.1) is given from the properties of the plasma flow along the magnetopause. A stream function (\( \Psi \)) is needed to obtain the knowledge how the properties alters in the magnetosheath. The velocity can be expressed in derivatives of the stream function
\[
\frac{\partial \Psi}{\partial r} = Ar \rho u_x, \quad \frac{\partial \Psi}{\partial x} = -Ar \rho u_r, \] (3.2.19)
here \( A \) is a constant. The model used for the stream function is[3]
\[
\Psi(x, r) = \sum_{i=1}^{\infty} r^{2i} \psi_{2i}(x) = \sum_{i=1}^{\infty} \sum_{n=0}^{\infty} r^{2i} x^n \psi_{2i}^{(n)}(0). \] (3.2.20)
When the coefficients $\psi_{2i}^{(n)}(0)$ is calculated, the constant $A$ is chosen so that $\psi_2(0)^2 = 1$. To determine the stream function we also need the models for the velocity, density and pressure

$$u_x(x, r) = \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} r^{2i} n! U_{2i}^{(n)}(0),$$

$$u_r(x, r) = \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} r^{2i} n! U_{2i+1}^{(n)}(0),$$

$$p(x, r) = \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} r^{2i} n! P_{2i}^{(n)}(0),$$

$$\rho(x, r) = \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} r^{2i} n! \alpha_{2i}^{(n)}(0).$$

The interesting positions lie along the magnetopause ($x = x_{mp}(r)$) and the velocity along the magnetopause can be derived from equation (3.2.19)

$$u_x = \frac{1}{Ar\rho} \left. \frac{\partial \Psi}{\partial r} \right|_{x=x_{mp}(r)}$$

$$u_r = -\frac{1}{Ar\rho} \left. \frac{\partial \Psi}{\partial x} \right|_{x=x_{mp}(r)}$$

To determine all the coefficients in the expressions (eq(3.2.21)-eq(3.2.24)) the relations at the bow shock are used. But that will not be enough, a description of the motion after the bow shock is also needed

$$\rho \left( u_x \frac{\partial u_x}{\partial x} + u_r \frac{\partial u_r}{\partial r} + \frac{\partial p}{\partial x} \right) = 0$$

$$\rho \left( u_x \frac{\partial u_x}{\partial x} + u_r \frac{\partial u_r}{\partial r} + \frac{\partial p}{\partial r} \right) = 0$$

$$\frac{\partial}{\partial x} \left( r \rho u_x \right) + \frac{\partial}{\partial r} \left( r \rho u_r \right) = 0$$

$$u_x \frac{\partial}{\partial x} \left( p \rho^{-\gamma} \right) + u_r \frac{\partial}{\partial r} \left( p \rho^{-\gamma} \right) = 0$$

Using the expression (eq(3.2.21-3.2.24)) in the equations describing the alteration during the transition of the bow shock (eq(3.2.13-3.2.14) and eq(3.2.18-3.2.17)) some coefficients will automatically show themself. Adding the description of the flow (eq(3.2.27-3.2.30)) will help us to find the other coefficients (for the calculations see Appendix A).

The pressure, velocity and density (eq(3.2.13)-(3.2.18)) inside of the bow shock is calculated in a straightforward way when the outer quantities are known.

The expressions are now obtained and will be expressed with origin at the stagnation point and with a parabolic coordinate system where the $Z$-axis will follow the magnetopause and the other axis will point in the normal direction of the magnetopause. The fact that the model describing the properties of the solar wind is expressed with the coordinate system at the sub solar point force us to find a relation between the $r$-coordinate of that system and the $Z$-coordinate of the Magnetopause system. This is done by calculating the length of the Magnetopause using equation (3.2.7).

The constants are defined at the reconnection site and therefore we need to have the origin of our coordinate system there. In order to set the origin at the reconnection site a Taylor expansion is done at the reconnection point, by putting

$$Z^* = Z - Z_{rec} = 0.$$

the constants can be obtained. $Z^*$ is the distance from the reconnection point, $Z$ is the distance from the stagnation point and $Z_{rec}$ is the distance to the reconnection point from the stagnation point.
3.2.3. Introducing the magnetic field to obtain DeHoffmann-Teller Velocity

One flaw of the model used in subsection(3.2.2) and the relations obtained in subsection(3.2.1) is that it doesn’t include the magnetic field. To derive the DeHoffmann-Teller velocity \((U_{HT})\), needed in our constants, the magnetic field in the solar wind has to be included. First we state that the magnetic field in the normal direction \(\hat{\mathbf{n}}\) outside the bow shock is the same as precisely inside \((\mathbf{B}_1 = \mathbf{B}_2)\).

In order for the reconnection to take place, it has been stated earlier that the flow inside the bow becomes subalfvénic

\[
M_A = \frac{U}{v_A} < 1. \quad (3.2.32)
\]

\(U\) is the flow velocity inside the bow shock and \(v_A\) is the Alfvén velocity defined as

\[
v_A = \sqrt{\frac{B^2}{\mu_0 \rho}}. \quad (3.2.33)
\]

In order to check that the subalfvénic condition is fulfilled a normalization of the magnetic field is needed. It is important that it is normalized with the same scale as the velocity. The normalized Alfvén Mach number becomes

\[
M_A = \frac{\sqrt{\mu_0 U}}{\mathbf{B}} M_A^\infty \quad (3.2.34)
\]

with

\[
M_A^\infty = \frac{q_1}{\sqrt{\mu_0 \rho_1}} \quad (3.2.35)
\]

\(M_A^\infty\) is the Alfvén Mach number outside the bow shock, \(B_{imf}\) is the interplanetary magnetic field and as before \(q_1\) is the solar wind velocity outside the bow shock and \(\rho_1\) is the density outside.

The magnetic field, \(\mathbf{B}\), in the solar wind satisfy Maxwells equation (i.e eq(3.1.9)). This immediately allows us to define the magnetic field as

\[
\mathbf{B} = \nabla \times \mathbf{A}, \quad (3.2.36)
\]

since the divergence of a curl automatically becomes zero. \(\mathbf{A}\) is called the vector potential and is chosen to have the same value precisely after the bow shock (BS)

\[
A_{BS}^{out} = A_{BS}^{in}. \quad (3.2.37)
\]

A model for the vector potential is needed in order to calculate the magnetic field. Our model has the form,

\[
A(x, r) = \sum_{i=1}^{\infty} r^{2i} A_{2i}(x) = \sum_{i=1}^{\infty} r^{2i} x^n a_{2i}^{(n)}. \quad (3.2.38)
\]

To find the coefficients we need the equation for the current density

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \quad (3.2.39)
\]

The magnetic field is expressed as the vector potential (eq(3.2.36)) and the fact that the current density is zero \((\mathbf{J} = 0)\) are used to modify equation(3.2.39) so the coefficients in equation(3.2.38) can be calculated. Equation(3.2.39) becomes

\[
\frac{r}{\partial r} \left( \frac{1}{r} \frac{\partial A(x, r)}{\partial r} \right) + \frac{\partial^2 A(x, r)}{\partial x^2} = 0. \quad (3.2.40)
\]

For the complete calculation of the coefficients see Appendix B.
When the vector potential is calculated its trivial to find the magnetic field and then the equations (3.1.21) and (3.1.28) can be used to derive the DeHoffmann-Teller velocity

\[ U_{HT} = u_z - \frac{B_z(0)}{\sqrt{\rho_0}}. \]  

(3.2.41)

Applying equation (3.1.28), the DeHoffmann-Teller velocity (eq(3.2.41)) becomes

\[ U_{HT} = u_z + \frac{B_z(0)}{\sqrt{\rho_0}}. \]  

(3.2.42)

When \( \eta \to \infty \) then \( u_z^0 \to \frac{B_0^0}{\rho} \) and used for \( Z = 0 \) the constants in equation (3.1.49) get new values, \( C_A = 0 \) and \( C_B = \frac{B_0(0)}{\sqrt{\rho_0}} \). Using the expressions and the same Taylor expansion as in the previous section the values of the DeHoffmann-Teller velocity can be obtained.

It’s important to note that \( u_z \) is the velocity along the magnetopause and is derived by taking the dot-product between the total velocity vector \( (u_x \hat{x} + u_r \hat{r}) \) and the tangent of the magnetopause (equation (3.2.9)), but is also important to remember that the magnitude of \( u_z \) is the same as the magnitude of the velocity vector

\[ |u_z| = \sqrt{(u_x^2 + u_r^2)}. \]  

(3.2.43)

The expressions needed to calculate the constants are now derived. By choosing a reconnection point that satisfies the condition for reconnection (see section (2.2)) and using the Maple code in appendix A and appendix B the constants numerical values are obtained.
4. Results

The change of the velocity and magnetic field in the transition from the magnetosheath to the magnetosphere is analyzed. The properties of the plasma will change depending on where on the magnetopause the transition occur. Therefore different reconnection points are considered with a restriction that magnetic reconnection is allowed (i.e subalfvénic equation(3.2.32)). To obtain results the following parameters in the solar wind outside the bow shock are considered,

\[ q_1 = 400 \text{ km/s} \]
\[ n = 5 \text{ cm}^{-3} \]
\[ B_{\text{imf}} = 22 \text{nT}. \]  

Here \( q_1 \) is the velocity, \( n \) is the particle density and \( B_{\text{imf}} \) is the interplanetary magnetic field that is frozen in the solar wind (from the Sun). These parameters corresponds to the Mach number,

\[ M_\infty = 8. \]

The Alfvén Mach number outside (equation(3.2.35))becomes

\[ M_\infty^A = 1.8637 \]

in this case. That means that reconnection is only possible at a distance less then seven earth radii from the sub solar line along the magnetopause. Solar wind flow with Mach number

\[ M_\infty = 6 \]

is also considered, it will have the same parameters as equation(4.0.1) except the velocity. In this case the outer flow will have a velocity of

\[ q_1 = 300 \text{ km/s}. \]

When the velocity outside is altered and the Mach number is changed also the Alfvén Mach is changed, in this case to

\[ M_\infty^A = 1.3979. \]

As stated earlier magnetic reconnection is a way for the charged particles in the solar wind plasma to leave the magnetosheath and enter the magnetosphere. The magnetic field and velocity will be altered during the transition from the outside (\( \eta \to \infty \)) to the inside (\( \eta \to -\infty \)). The location of the reconnection is chosen to be at two earth radii and at four earth radii.

4.1. Reconnection site at 2 earth radii

The first reconnection point that is considered is positioned at two earth radii from the sub solar point. A difference between going north and going south along the magnetopause is expected due to change of the DeHoffmann-Teller velocity, \( U_{HT} \). The two cases of \( M = 8 \) and \( M = 6 \) (different speed in the unshocked solar wind) are analyzed. The plots of the magnetic field are scaled with \( \sqrt{\rho_0} \), there \( \rho_0 \) is the density at the reconnection point, this is important to remember when comparing the magnetic fields for different Mach numbers.
4.1.1. Magnetic field

In figure(4.1.1) we see how the magnetic field (with $M = 6$) evolve as we travel north (left) and south (right) of the reconnection point along the magnetopause.

![Figure 4.1.1](image1.png)

Figure 4.1.1. It is shown how the magnetic field behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 6$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green. To the left is the northern case and to the right we’ll see what happens in the southern case.

When increasing the speed of the solar wind to $M = 8$ the magnetic field shows a similar development in the transition through the magnetopause as seen in figure(4.1.2).

![Figure 4.1.2](image2.png)

Figure 4.1.2. It is shown how the magnetic field behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 8$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green. To the left there is the northern case and to the right there is the southern case.

Figure(4.1.1) and figure(4.1.2) appear to be identical but there is a slight difference, the magnetic fields have a larger spread during the transition with the lower Mach number ($M = 6$).
4.1.2. Velocity

It is stated that the plasma velocity is zero at the stagnation point and then increasing with $Z$ along the magnetopause. When reconnection occur the velocity will naturally change due to the plasma entering of the magnetosphere.

![Figure 4.1.3](image)

Figure 4.1.3. It is shown how the plasma velocity behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 6$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green.

As seen in figure(4.1.3) the velocity at the magnetosheath side of the magnetopause doesn’t change much in the northern case but on the inside they will have gained a different velocity boost. The difference when the solar wind has a higher initial speed outside the bow shock ($M = 8$) gives a similar development at reconnection.

![Figure 4.1.4](image)

Figure 4.1.4. It is shown how the plasma velocity behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 8$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green.

4.2. Reconnection site at 4 earth radii

It is also considered that we could have a reconnection point at a distance of four earth radii from the sub-solar point. The plasma’s magnetic field and velocity during transition is suspected to be different from the reconnection site at two earth radii due to different initial condition. The velocity along
the magnetopause will be higher when $Z$ is higher. The magnetic field at the magnetopause in the magnetosheath will also be different.

### 4.2.1. Magnetic field

The development of the magnetic field for $M = 6$ is seen in figure(4.2.1). Note that the difference between the northern and southern case is significantly smaller than the difference at two earth radii.

![Figure 4.2.1](image1)

Figure 4.2.1. It is shown how the magnetic field behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 6$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green. Northern case is to the left and the southern case is to the right.

With the Mach number increased to $M = 8$ the magnetic field shows a smaller spread compared to $M = 6$, just as it did at the reconnection point at two earth radii. The magnetic field once again shows a similar behavior for the northern and southern case, see figure(4.2.2).

![Figure 4.2.2](image2)

Figure 4.2.2. It is shown how the magnetic field behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 8$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green. The figure shows the northern (left) and the southern case (right).

### 4.2.2. Velocity

The velocity development at $M = 6$ during the transition is considered in figure(4.2.3), going north and south of the reconnection point.
Figure 4.2.3. It is shown how the plasma velocity behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 6$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green.

If the Mach number is considered to be $M = 8$, we obtain a similar plot but with less variation at the boundary. The variation is not big but exists.

Figure 4.2.4. It is shown how the plasma velocity behaves under the transition from the magnetosheath to the magnetosphere for Mach number $M = 8$ with different $Z$ values. $Z = 0$ corresponds to the reconnection point (black), the line corresponding to $Z = 0.25$ earth radii north(south) is red, $Z = 0.5$ is blue and $Z = 0.75$ is green.

The conclusion of the velocity at the site located at four earthradii is that the spread is bigger to the north, because to velocity on the magnetopause increases more rapidly there.

4.3. Summary

The magnetic field of the plasma changes direction and the velocity changes during the transition as expected. Due to the energy transfer from the magnetic field to the plasma during the transition the particles will receive a speed boost in a direction from the reconnection site, which is seen in the plots.

The difference between Mach 8 and Mach 6 is less than expected, but an interesting result is that the plasmaflow seems to have a higher velocity at the reconnection point for Mach 6. The reason is that due to higher speed outside the bow shock the density will be larger in the Magnetosheath for Mach 8 then for Mach 6. The fact that the magnetic field will be the same will cause the velocity to be lower for Mach 8.
The velocity will have different fluctuations during transition depending if it is south or north of the reconnection site. The reason is the difference in initial condition of the plasma flow, especially the DeHoffmann Teller velocity will behave different.

The magnetic field has a much bigger variation at the reconnection site at two earth radii, the reason for this is that the closer to the stagnation point you get the more rapid is the variation of the physical variables.
Bibliography

A. Appendix Maple Code 1

Here you find the maple code used to obtain the expressions for the stream function, pressure and density. It is much code but the mathematics is straightforward. It is only a matter of using solving systems of equation.

```maple
restart: unprotect(gamma):

> Pressure

>p(x,r):=A[0,0]+A[0,1]*x+A[0,2]*x^2+A[0,3]*x^3+A[0,4]*x^4+r^2*A[2,0]+A[2,1]*x+A[2,2]*x^2+r^4*A[4,0];
gamma:=5/3;m:=subs(gamma=5/3,M=6,1/(2/(gamma+1)/M^2+(gamma-1)/(gamma+1))-1);

> Density

>rho(x,r):=alpha[0,0]+alpha[0,1]*x+alpha[0,2]*x^2+alpha[0,3]*x^3+alpha[0,4]*x^4+r^2*alpha[2,0]+alpha[2,1]*x+alpha[2,2]*x^2+r^4*alpha[4,0];

Velocity, u(x,r) = ux and v(x,r) = ur.

> u(x,r):=u[0,0]+u[0,1]*x+u[0,2]*x^2+u[0,3]*x^3+u[0,4]*x^4+r^2*u[2,0]+u[2,1]*x+u[2,2]*x^2+r^4*u[4,0];
> v(x,r):=r*(u[1,0]+u[1,1]*x+u[1,2]*x^2+u[1,3]*x^3)+r^3*u[3,0];

Identify Coefficients.

Velocity at bow shock

> eq1:=subs(x=r^2/2,u(x,r))-(1-mu^2)*r^2/(1+r^2)-1/(m+1);
> eq1:=convert(taylor(eq1,r=0,5),polynom);
> eq11:=coeff(eq1,r,0);
> u[0,0]:=1/(m+1);
> mu:=sqrt(abs((gamma-1)/(gamma+1)));
> eq2:=subs(x=r^2/2,v(x,r)-(1-(u[0,0]+u[0,1]*x+u[0,2]*x^2+u[0,3]*x^3+u[0,4]*x^4+r^2*u[2,0]+u[2,1]*x+u[2,2]*x^2+r^4*u[4,0]))*r);
> eq21:=coeff(eq2,r,1);
> u[1,0]:=simplify(1-1/(m+1));

Density at BS

> eq3:=convert(taylor(subs(x=r^2/2,rho(x,r))-1/(1/(m+1)+(1/(m+1)-mu^2)*r^2),r=0,5),polynom);
> eq31:=coeff(eq3,r,0);
> alpha[0,0]:=m+1;

Pressure at BS

> eq4:=subs(x=r^2/2,p(x,r))-(1-mu^4)*(m+1)/(1-mu^2*m-mu^2)/(1+r^2)+mu^2;
> eq4:=convert(taylor(eq4,r=0,5),polynom);
> eq41:=coeff(eq4,r,0);
> A[0,0]:=factor((1-mu^4)*(m+1)/(1-mu^2*m-mu^2)-mu^2);
> u(x,r):=u[0,0]+u[0,1]*x+u[0,2]*x^2+u[0,3]*x^3+u[0,4]*x^4+r^2*u[2,0]+u[2,1]*x+u[2,2]*x^2+u[2,3]*x^3+u[2,4]*x^4;
> v(x,r):=v[0,0]+v[0,1]*x+v[0,2]*x^2+v[0,3]*x^3+v[0,4]*x^4+r^2*v[2,0]+v[2,1]*x+v[2,2]*x^2+v[2,3]*x^3+v[2,4]*x^4;
> rho(x,r):=rho[0,0]+rho[0,1]*x+rho[0,2]*x^2+rho[0,3]*x^3+rho[0,4]*x^4+r^2*rho[2,0]+rho[2,1]*x+rho[2,2]*x^2+rho[2,3]*x^3+rho[2,4]*x^4;
> p(x,r):=p[0,0]+p[0,1]*x+p[0,2]*x^2+p[0,3]*x^3+p[0,4]*x^4+r^2*p[2,0]+p[2,1]*x+p[2,2]*x^2+r^4*p[4,0];

Equations of motion

25
\[
\text{deq1 := } \text{Rho}(x,r) \cdot (U(x,r) \cdot \text{diff}(U(x,r),x) + V(x,r) \cdot \text{diff}(U(x,r),r)) + \frac{1}{\gamma} \cdot \frac{1}{M^2} \cdot \text{diff}(P(x,r),x); \\
\text{deq11 := } \text{coeff(deq1,r,0);} \\
\text{deq12 := } \text{expand(coeff(deq1,r,2));} \\
\text{deq2 := } \text{Rho}(x,r) \cdot (U(x,r) \cdot \text{diff}(V(x,r),x) + V(x,r) \cdot \text{diff}(V(x,r),r)) + \frac{1}{\gamma} \cdot \frac{1}{M^2} \cdot \text{diff}(P(x,r),r); \\
\text{deq21 := } \text{coeff(deq2,r,1);} \\
\text{deq22 := } \text{coeff(deq2,r,3);} \\
\text{deq3 := } \text{diff(Rho(x,r) \cdot r \cdot U(x,r),x)} + \text{diff(Rho(x,r) \cdot r \cdot V(x,r),r);} \\
\text{deq31 := } \text{coeff(deq3,r,1);} \\
\text{deq32 := } \text{coeff(deq3,r,3);} \\
\text{Using adiabatic} \\
\text{deq4 := } U(x,r) \cdot \text{diff}(P(x,r) \cdot \text{Rho}(x,r)^{-\gamma},x) + V(x,r) \cdot \text{diff}(P(x,r) \cdot \text{Rho}(x,r)^{-\gamma},r); \\
\text{deq41 := } \text{simplify(coeff(deq4,r,0)/U[0](x)/alpha[0](x)^{-\gamma});} \\
\text{deq42 := } \text{expand(coeff(deq4,r,2));} \\
\text{deq11;} \\
\text{deq31;} \\
\text{deq41;} \\
\text{System of equations to calculate more Coeff.} \\
\text{sys := deq11=0,deq31=0,deq41=0;} \\
\text{Order:=3;} \\
\text{Solving} \\
\text{dsolve(sys union A[0](0)=(-m-1+mu^2)/(-1+mu^2*m+mu^2),alpha[0](0)=m+1,U[0](0)=1/(m+1),alpha[0](x),U[0](x),A[0](x),type=series);} \\
\text{Giving values to coefficients.} \\
\text{Ap[0](x):=subs(M=6,U[1](0)=m/(m+1),convert(series(179/4+(-895/6*U[1](0))/(-179/4)*M^2+13/48)*x+3580/13*M^2*(-44293784*U[1](0)^2+726024*D(U[1])(0)+13*M^2*459980752)*D(U[1])(0)+1787136*U[1](0)^2*M^2-2197*D(U[1])(0)*M^2-2148+13*M^2-3)*x^2+O(x^3),x,3),polynom));} \\
\text{alphap[0](x):=subs(M=6,U[1](0)=m/(m+1),convert(series(48/13+(-96/13*U[1](0))/(-179/4)*M^2+13/48)*x+2304/169*M^2*(-59980752*D(U[1])(0)+726024*D(U[1])(0)*M^2-2197*D(U[1])(0)*M^2+16224*U[1](0)^2*M^2-2148+13*M^2-3)*x^2+O(x^3),x,3),polynom));} \\
\text{Up[0](x):=subs(M=6,U[1](0)=m/(m+1),convert(series(13/48+(-2148*12416U[1](0)+169*2M^2+832*U[1](0)^2*M^2-4613904*D(U[1])(0)+5584*D(U[1])(0)*M^2-2-169*D(U[1])(0)*M^2-4)*x^2+O(x^3),x,3),polynom));} \\
\text{deq21;Order:=2;} \\
\text{dsolve(deq21,U[1](x),type=series);} \\
\text{Up[1](x):=subs(M=6,U[1](0)=u[1,0],U[0](0)=1/(m+1),alpha[0](0)=alpha[0,0],convert(series(U[1](0)+(alpha[0](0)*U[1](0)^2+6/5*1/M^2*A[2](0))/alpha[0](0)/U[0](0))*x+O(x^2),x,2),polynom));} \\
\text{subs(x=0,diff(Up[1](x),x));} \\
\text{eq4:} \\
\text{eq42 := } \text{coeff(eq4,r,2);} \\
\text{A[0,1] := } \text{subs(x=0,diff(Ap[0](x),x));} \\
\text{solve(eq42=0,A[2,0]);} \\
\text{A[2,0] := } -1615/16; \\
\text{u[1,1] := } \text{subs(A[2](0)=A[2,0],x=0,diff(Up[1](x),x));} \\
\text{Up[1](x) := } \text{subs(A[2](0)=A[2,0],Up[1](x));} \\
\text{Ap[0](x) := } \text{subs(D(U[1])(0)=u[1,1],Ap[0](x));} \\
\text{Up[0](x) := } \text{subs(D(U[1])(0)=u[1,1],Up[0](x));} \\
\text{alphap[0](x) := } \text{subs(D(U[1])(0)=u[1,1],alphap[0](x));} \\
\text{u[0,1] := } \text{subs(x=0,diff(Up[0](x),x));} \\
\text{eq12 := } \text{coeff(eq1,r,2);} \\
\text{solve(eq12=0,u[2,0]);} \\
\text{u[2,0] := } -323/192; \\
\text{u[1,1] ;} \\
\text{eq22 := } \text{coeff(eq2,r,3);} \\
\text{solve(eq22=0,u[3,0]);} \\
\text{u[3,0] := } -1207/832; \\
\text{eq32 := } \text{coeff(eq3,r,2);} \\
\text{alpha[0,1] := } \text{subs(x=0,diff(alphap[0](x),x));}
> solve(eq32=0,alpha[2,0]);
> alpha[2,0]:=-516/169;
> deq12:=subs(M=6,D(U[1])(0)=u[1,1],U[0](x)=Up[0](x),alpha[0](x)=alphp[0](x),A[0](x)=Ap[0](x),U[1](x)=Up[1](x),deq12);
> deq22:=subs(M=6,D(U[1])(0)=u[1,1],U[0](x)=Up[0](x),alpha[0](x)=alphp[0](x),A[0](x)=Ap[0](x),U[1](x)=Up[1](x),deq22);
> deq32:=subs(M=6,D(U[1])(0)=u[1,1],U[0](x)=Up[0](x),alpha[0](x)=alphp[0](x),A[0](x)=Ap[0](x),U[1](x)=Up[1](x),deq32);
> deq42:=subs(M=6,D(U[1])(0)=u[1,1],U[0](x)=Up[0](x),alpha[0](x)=alphp[0](x),A[0](x)=Ap[0](x),U[1](x)=Up[1](x),deq42);
>
> sys2:=deq12=0,deq32=0,deq42=0;Order:=3;
> A[2,0];alpha[2,0];u[2,0];
> dsolve(sys2 union A[2](0)=A[2,0],alpha[2](0)=alpha[2,0],U[2](0)=u[2,0],alpha[2](x),U[2](x),A[2](x),type=series);

> Ap[2](x):=subs(M=6,simplify(convert(series(-1615/169 +
82944/76895*(284765/3145728*48^(1/3)*13^(2/3)+1966315/63700992*48^(1/3)*13^(2/3)*(4127/676 +192/13*U[3](0)))*48^(2/3)*13^(1/3)*x+(-17519074771/5299840-1586346/3185*U[3](0)+
1074/7*D(U[3](0))(0))*x^2+O(x^3),x,3),polynom)));
> Up[2](x):=simplify(subs(M=6,convert(series(323/192*(-13839426764800*340282608640/3*48^(2/3)*13^(1/3)*M^2+142918101560/3*48^(2/3)*13^(1/3)*x+(300794589/8153600+32757/2450*U[3](0)+
179/70*D(U[3](0))(0))*x^2+O(x^3),x,3),polynom)));
> alphap[2](x):=subs(M=6,simplify(convert(series(-516/169 +
82944/76895*(284765/3145728*48^(1/3)*13^(2/3)+1966315/63700992*48^(1/3)*13^(2/3)*(4127/676 +192/13*U[3](0)))*48^(2/3)*13^(1/3)*x+(-17519074771/5299840-1586346/3185*U[3](0)+
1074/7*D(U[3](0))(0))*x^2+O(x^3),x,3),polynom)));
>
> Order:=2;
> dsolve(deq22=0,U[3](x),type=series);
> Up[3](x):=subs(U[3](0)=u[3,0],U[2](0)=u[2,0],alpha[2](0)=alpha[2,0],U[3](0)+(-1749/338*U[2](0)
-140/13*U[3](0)+15*A[4](0)-4199/4608*alpha[2](0))*x);323/192 +
> eq43:=coeff(eq4,r,4); > A[2,1]:=subs(x=0,U[3](0)=u[3,0],diff(Ap[2](x),x));
> A[0,2]:=subs(x=0,diff(A[0](0),x,x)/2);
> solve(eq43=0,A[0,4]);
>
> A[4,0]:=-3257407/11648; > Up[3](x):=subs(A[4](0)=A[4,0],Up[3](x));
> u[3,1]:=coeff > Ap[2](x):=subs(U[3](0)=u[3,0],D(U[3])(0)=u[3,1],Ap[2](x));
> alphap[2](x):=subs(U[3](0)=u[3,0],D(U[3])(0)=u[3,1],alphap[2](x));
> Up[2](x):=subs(U[3](0)=u[3,0],D(U[3])(0)=u[3,1],Up[2](x));
> deq11:
> A[0,0];alpha[0,0];u[0,0];
> dsolve; 
> deq31;
> deq41;
> sys3:=deq11=0,deq31=0,deq41=0;
>
> Order:=5;
> dsolve(sys3 union A[0](0)=(-m+1+mu^2)/(-1+mu^2*(m+mu^2),alpha[0](0)=m+1,
U[0](0)=1/(m+1),alpha[0](x),U[0](x),A[0](x),type=series);
> Order:=4;dsolve(deq21,U[1](x),type=series);
> A[2,1]:=subs(x=0,diff(Ap[2](x),x));A[2,2]:=subs(x=0,diff(Ap[2](x),x,x)/2);
> u[0,1];u[0,2]:=subs(x=0,diff(Up[0](x),x,x)/2);alphap[2,2]:=subs(x=0,diff(alphap[2](x),x,x)/2);

> Ub[1](x):=subs(U[1](0)=u[1,0],alpha[0,0]=alpha[0,0],M=6,'@@'(D,2)(alpha[0](0))=2*alpha[0,2],
A[2](0)=A[2,0],D(A[2])(0)=A[2,1],'@@'(D,2)(U[0](0))=2*u[2,0],U[0](0)=u[0,0],D(U[0])(0)=u[0,1],
D(alpha[0])(0)=alphap[0,0],D(U[0])(0)=2*A[2,2],series(U[1](0)
+(-115*6*alpha[0](0)*U[1](0)+2*M^2+6*A[2](0)/M^2)*alpha[0](0)/U[0](0))*x^2 +1/10*(10*U[1](0)+3*alpha[0](0)*M^2+2*12*U[1](0)*alpha[0](0)*A[2](0))}
-6*U[0](0)*D(A[2])(0)*alpha[0](0)+6*U[0](0)*(D(alpha[0])(0)*A[2](0)
+5*D(U[0](0))*alpha[0](0)*2*U[1](0)*M*2
+6*D(U[0](0))*alpha[0](0)*A[2](0))*alpha[0](0)*2*M*2*U[0](0)*2*x*2
-4*alpha[0](0)*D(A[2])(0)*alpha[0](0)*3*M*4*U[0](0)*U[1](0)*2
-30*alpha[0](0)*D(A[2])(0)*alpha[0](0)*2*M*2*U[0](0)*A[2](0)
-30*alpha[0](0)*D(A[2])(0)*alpha[0](0)*M*2*U[0](0)*2*A[2](0)
+30*alpha[0](0)*D(A[2])(0)*alpha[0](0)*2*M*2*U[0](0)*2*A[2](0)
+150*alpha[0](0)*3*M*4*U[1](0)*0*U[1](0)*4*M*4
+240*alpha[0](0)*2*M*2*U[0](0)*60*U[1](0)*M*2*U[0](0)*D(A[2])(0)*alpha[0](0)*2
+60*alpha[0](0)*U[1](0)*M*2*D(U[0](0)*alpha[0](0)*A[2](0)
+150*D(U[0](0)*alpha[0](0)*3*M*4*U[1](0)*U[1](0)*3+180*D(U[0](0)*alpha[0](0)*2*M*2*U[1](0)*A[2](0)
+72*alpha[0](0)*A[2](0)*2-60*D(alpha[0](0)*M^2*U[0](0)*2*D(A[2])(0)*alpha[0](0)
+60*D(U[0](0)*alpha[0](0)*2*M*2*U[0](0)*2*A[2](0)*alpha[0](0)*2*M*2*U[0](0)*D(A[2])(0)
+60*D(U[0](0)*alpha[0](0)*2*M*2*U[0](0)*D(alpha[0](0)*alpha[0](0)*A[2](0)
+50*alpha[0](0)*2*M*2*U[0](0)*3*M*4*U[1](0)*0*2
+60*D(U[0](0)*alpha[0](0)*2*M*2*U[0](0)*D(alpha[0](0)*alpha[0](0)*A[2](0)

> u[1,2]:=coeff(U[1](x),x,2);
> u[1,3]:=coeff(U[1](x),x,3);
> alpha[0,2]:=1/2*subs(x=0,diff(alphap[0](x),x,x));
eq3;
> alpha[2,1]:=subs(x=0,diff(alphap[2](x),x));
> solve(eq3=0,alpha[4,0]);
> alpha[4,0]:=2092731/153790;
> Ab[0](x):=subs(M=6,U[1](0)=u[1,0],D(U[1])(0)=u[1,1],'
'@@'(D,2)(U[1])(0)=2*u[1,2],
'@@'(D,3)(U[1])(0)=6*u[1,3],series(179/4*(-7160*U[1](0)*M^2/(-2148+13*M^2)*x
+3580/13*M^2*(442934784*U[1](0)*2+27040*U[1](0)*2-59980752*D(U[1])(0))
+726024*D(U[1](0)*0*2-2197*D(U[1](0)*M^2/(-2148+13*M^2)*x^2
+(-3580/39*M^2*(-613097571910208*U[1](0)*D(U[1])(0)+456683479695360*U[1](0)*3*M^2
-59848727968*U[1](0)*4*D(U[1])(0)+4530397960*U[1](0)*0*6*D(U[1])(0)
-130709280*U[1](0)*5*M^2/(-2148+13*M^2)*x^3
+74211065450496*U[1](0)*0*2*D(U[1](0)))*(-2148+13*M^2)*5*x^3
+895/507*M^2/(-741397862400*M^4*D(U[1])(0)*0*U[1](0)*2
+73501043466240*D(U[1](0)*0*M^4/(-2148+13*M^2)*x^4
-164535311228626542264320*D(U[1](0)*0*U[1](0)*2*M^2
+12575114514630180864*D(U[1](0)*0*M^6*U[1](0)*0*2
-1159407721207037952*U[1](0)*0*M^6/(-2148+13*M^2)*x^5
+723617772952328*U[1](0)*0*M^8/(-2148+13*M^2)*x^6
-26542034373920*U[1](0)*0*M^10/(-2148+13*M^2)*x^7
+13769801500451029988*U[1](0)*0*M^4/(-2148+13*M^2)*x^8
-1186961366511916941312*U[1](0)*0*M^2/(-2148+13*M^2)*x^9
-8092471024883937705984*D(U[1](0)*0*2*M^2+103268111238375906816*D(U[1](0)*0*2*M^4
-8695557909052758464*D(U[1](0)*0*2*M^6+5427133296938496*D(U[1](0)*0*2*M^8
-19906532605440*D(U[1](0)*0*2*M^10+643727879587233792*U[1](0)*0*4*M^4
-165993998981508847616*alpha[3](D)(U[1](0)+40159050880*U[1](0)*M^4/12*D(U[1])(0)*0*U[1](0)*2
-34004975063334912*D(U[1](0)*0*M^8*U[1](0)*2+1520816128000*M^12*U[1](0)*0*4
+30119288160*M^12*D(U[1])(0)*2+202280998435232667042240*U[1](0)*0*4*M^2
+177160740715639794032*U[1](0)*0*M^4+602771475643813282176*alpha[3](D)(U[1](0)*0*M^2
+9121045697580752640*alpha[3](D)(U[1](0)*0*M^4+7359521667811120*alpha[3](D)(U[1](0)*0*M^6
-334056500285040*alpha[3](D)(U[1](0)*0*M^8+8087028887096*alpha[3](D)(U[1](0)*0*M^10
-815730721*alpha[3](D)(U[1](0)*0*M^12+1196119544954880*M^10*U[1](0)*0*4
+161027325634151448576*U[1](0)*0*M^4+6+49032071183206918421888*U[1](0)*0*alpha[3](D)(U[1](0)
+3677405338740588836146*D(U[1](0)*0*2)/(-2148+13*M^2)*7*x^4+O(x^5,x,5));
\[\begin{align*} &-169*D(U[1](0)*M^4)/(-2148+13*M^2)^3)*x^2+716/13*(276745431575808*\@*(D,2)(U[1])(0)\] 
&-74211065450496*U[1](0)*M^2*D(U[1])(0)+748559784960*U[1](0)*M^4*D(U[1])(0) 
&-90607792*U[1](0)*M^6*D(U[1])(0)+18267391878144*U[1](0)*3*M^2+15856214400*U[1](0)*3*M^6 
&-6699610057392*\@*(D,2)(U[1])(0)*M^2+60820482528*\@*(D,2)(U[1])(0)*M^4 
&-245396112*\@*(D,2)(U[1])(0)*M^6+7370434805760*U[1](0)*3*M^4+8998912*U[1](0)*3*M^8 
&+371293*\@*(D,2)(U[1])(0)*M^8 \end{align*}\]

\[\begin{align*} &+(-179/13*(-9124896768*M^12*D(U[1])(0)*U[1](0)^2-1583099377344*D(U[1])(0)*M^10*U[1](0)^2) 
&+170993835037963264*U[1](0)*M^6*U[1](0)^2 
&+78037058158166016*U[1](0)*M^6*\@*(D,2)(U[1])(0) 
&+204140565853184*U[1](0)*M^8*\@*(D,2)(U[1])(0) 
&+10131096759127179264*U[1](0)*M^10*\@*(D,2)(U[1])(0) 
&+45653697563057328512*U[1](0)*M^4*\@*(D,2)(U[1])(0) 
&+432402731726305296384*D(U[1])(0)*M^2 
&-75983225693484448*D(U[1])(0)*2*M^4+58527793618264512*D(U[1])(0)*2*M^6 
&-151807924388988*D(U[1])(0)*2*M^8-153127173888*D(U[1])(0)*2*M^10 
&+50897274594656256*U[1](0)*4*M^4-8127687653729346834432*\@*(D,3)(U[1])(0) 
&+123566104*U[1](0)*M^12*\@*(D,2)(U[1])(0)*1494724178608128*D(U[1])(0)*M^8*U[1](0)*2 
&+74879414*U[1](0)*M^12*\@*(D,2)(U[1])(0)*4+296747328*M^12*U[1](0)^2*2 
&+904049155017796039848*U[1](0)*M^4+463706356855737515552*\@*(D,3)(U[1])(0)*M^2 
&+701549669044673280*\@*(D,3)(U[1])(0)*M^4+5661170517306240*\@*(D,3)(U[1])(0)*M^6 
&-2569665386808*\@*(D,3)(U[1])(0)*M^8+62209714392*\@*(D,3)(U[1])(0)*M^10 
&-627485174*\@*(D,3)(U[1])(0)*M^12+43298445066240*M^10*U[1](0) 
&+1732620587285202112*U[1](0)*M^4/(-2148+13*M^2)*7)*x^4+O(x^5),x,5); 
\end{align*}\]
\[ 367740533874051883816416*D(U[1])(0)^2/(-2148+13*M^2)*x^4+O(x^5), x, 5) \]

\[
\text{psi}[4](x) := \text{convert}\left(\frac{1}{2}\text{expand} (\text{alphab}[0](x)*\text{Ub}[0](x)), \text{polynom}\right);
\]

\[
\text{psi}[2](x) := \text{convert}\left(\text{taylor} (\text{expand} (\text{alphab}[0](x)*\text{Ub}[0](x)), x, 5), \text{polynom}\right);
\]

\[
\text{psi}[6](x) := \text{convert}\left(\frac{1}{3}\text{expand} (\text{alphab}[0](x)*u[4,0]+\text{alphap}[2](x)*\text{Up}[2](x) +\text{alpha}[4,0]*\text{Ub}[0](x)), x, 1), \text{polynom}\right);
\]

The stream function.

\[
\text{Psi}(x, r) := \frac{1}{2}r^2(\text{psi}[2](x)+r^2\text{psi}[4](x)+r^4\text{psi}[6](x));
\]

\[
\text{fsolve}(1-70/13*x-1557/169*x^2-10009/17745*x^3-1845480*x^4=0, x);
\]

Curvature radius for the magnetopause

\[
Rmp := -1/2\text{diff}(\text{psi}[2](x), x)/\text{psi}[4](x);
\]

\[
h := \text{subs}(x=0.1434, Rmp);
\]

Checking the streamlines

\[
\text{contourplot} (\text{Psi}(x, r), x=0..0.4, r=-0.2..0.2, \text{contours}=100);
\]

Checking the streamlines

\[
\text{contourplot} (\text{fsolve}(1/2-160299871/3690960*H^4-35/13*H-1557/338*H^2-10009/35490*H^3=0, H), H);
\]

Shape of MP and BS

\[
Xmp(r) := H + X0*r^2 + X1*r^4 + X2*r^6; Xbs(r) := r^2/2;
\]

Final density

\[
\rho_1(x, r) := \alpha[0,0] + \alpha[0,1]*x + \alpha[0,2]*x^2 + \alpha[0,3]*x^3 + \alpha[0,4]*x^4 + r^2*(\alpha[2,0] + \alpha[2,1]*x + \alpha[2,2]*x^2) + r^4*\alpha[4,0];
\]
> rho2(r) := convert(taylor(subs(x=Xmp(r),rho1(x,r)),r=0..8),polynom);
> plot(rho2(r),r=0..0.4);
> #(4.08-3.95)/3.95;
> u1(x,r);
> Psi(x,r);
B. Appendix Maple Code 2

This is the maple code for calculating the constants needed to get numerical solutions. It also include the derivation of the vector potential (i.e magnetic field). The expression obtained in Appendix A is used here as well.

> restart:

Vector potential

> A(x,r):=r^2/2*(1+A1*x+A2*x^2+A3*x^3+A4*x^4+A5*x^5+A6*x^6
+ r^2*(B1+B2*x+B3*x^2+B4*x^3+B5*x^4)+r^4*(C1+C2*x+C3*x^2)
+ r^6*(D1+D2*x+D3*x^2)+r^8*(E1+E2*x)+r^10*F1);

No Current

> Aekv(x,r):=expand(r*diff(1/r*diff(A(x,r),r),r)+diff(A(x,r),x,x));

Identify coefficients

> e1:=coeff(coeff(Aekv(x,r),r,2),x,0);
> e2:=coeff(coeff(Aekv(x,r),r,2),x,1);
> e3:=coeff(coeff(Aekv(x,r),r,2),x,2);
> e4:=coeff(coeff(Aekv(x,r),r,2),x,3);
> e5:=coeff(coeff(Aekv(x,r),r,2),x,4);
> e6:=coeff(coeff(Aekv(x,r),r,2),x,5);
> e7:=coeff(coeff(Aekv(x,r),r,2),x,6);

> solve(e1=0,e2=0,e3=0,e4=0,e5=0,B1,B2,B3,B4,B5);

> Aekv1(x,r):=subs(B5 = -15/4*A6, B4 = -5/2*A5, B3 = -3/2*A4,
B2 = -3/4*A3, B1 = -1/4*A2,Aekv(x,r));

> A(x,r):=subs(B5 = -15/4*A6, B4 = -5/2*A5,
B3 = -3/2*A4, B2 = -3/4*A3, B1 = -1/4*A2,A(x,r));

> d1:=coeff(coeff(Aekv1(x,r),r,4),x,0);
> d2:=coeff(coeff(Aekv1(x,r),r,4),x,1);
> d3:=coeff(coeff(Aekv1(x,r),r,4),x,2);
> d4:=coeff(coeff(Aekv1(x,r),r,4),x,3);
> d5:=coeff(coeff(Aekv1(x,r),r,4),x,4);

> solve(d1=0,d2=0,d3=0,C1,C2,C3);

> Aekv2(x,r):=subs(C3 = 15/8*A6, C2 = 5/8*A5, C1 = 1/8*A4,Aekv1(x,r));

> A(x,r):=subs(C3 = 15/8*A6, C2 = 5/8*A5, C1 = 1/8*A4,A(x,r));

> f1:=coeff(coeff(Aekv2(x,r),r,6),x,0);
> f2:=coeff(coeff(Aekv2(x,r),r,6),x,1);
> f3:=coeff(coeff(Aekv2(x,r),r,6),x,2);
> f4:=coeff(coeff(Aekv2(x,r),r,6),x,3);

> solve(f1=0,f2=0,f3=0,D1,D2,D3);

> Aekv3(x,r):=subs(D2 = 0, D3 = 0, D1 = -5/64*A6,Aekv2(x,r));

> A(x,r):=subs(D2 = 0, D3 = 0, D1 = -5/64*A6,A(x,r));

> g1:=coeff(coeff(Aekv3(x,r),r,8),x,0);
> g2:=coeff(coeff(Aekv3(x,r),r,8),x,1);
> g3:=coeff(coeff(Aekv3(x,r),r,8),x,2);

> Aekv4(x,r):=subs(E1=0,E2=0,Aekv3(x,r));

> A(x,r):=subs(E1=0,E2=0,A(x,r));

> h1:=coeff(coeff(Aekv4(x,r),r,10),x,0);
> h2:=coeff(coeff(Aekv4(x,r),r,10),x,1);

> Xmp(r) := .1434+.6634936733*r^2+.7320336693*r^4+1.487498296*r^6;
\( Xbs(r) := r^2 / 2; \)
\( bs(r) := \text{subs}(x = Xbs(r), A(x, r)); \)
\( eq1 := \text{coeff}(bs(r), r, 2); \)
\( eq2 := \text{coeff}(bs(r), r, 4); \)
\( eq3 := \text{coeff}(bs(r), r, 6); \)
\( eq4 := \text{coeff}(bs(r), r, 8); \)
\( \text{solve}(eq1 = 0, eq2 = 0, eq3 = 0, eq4 = 0, A0, A2, A3, A4); \)
\( A(x, r) := \text{subs}(A0 = \text{MAG}, A2 = 2^*A1, A3 = 3/2^*A1-5/64^*A6+5/16^*A5, A4 = 1/2^*A1-15/64^*A6+15/16^*A5, A(x, r)); \)
\( mp(r) := \text{expand}(\text{subs}(x = Xmp(r), A(x, r))); \)
\( Ek1 := \text{coeff}(mp(r), r, 2); \)
\( Ek2 := \text{coeff}(mp(r), r, 4); \)
\( Ek3 := \text{coeff}(mp(r), r, 6); \)
\( \text{solve}(Ek1 = 0, Ek2 = 0, Ek3 = 0, A1, A5, A6); \)
\( A(x, r) := \text{subs}(A1 = -4.959728172, A5 = -42.17655231, A6 = 11.42664865, A(x, r)); \)
\( \text{expand}(A(x, r)); \)
\( \text{with(plots)}: \text{contourplot}(A(x, r), x = 0..0.4, r = -.2..0.2, \text{contours} = 100); \)
\( A(x, r) \)\( \text{magnetic field} \)
\( Bx := 1/r^*\text{diff}(A(x, r), r); \)
\( Br := -1/r^*\text{diff}(A(x, r), x); \)
\( B := Bx + Br \) \( \# \text{vektor} \)
\( Bx := \text{subs}(x = Xmp(r), Bx); \)
\( Br := \text{subs}(x = Xmp(r), Br); \)
\( Bz := -\sqrt{Bx^2 + Br^2}; \)
\( Bz1 := (-Bx^*(\text{diff}(Xmp(r), r)) + Br)/\sqrt{1+(\text{diff}(Xmp(r), r))^2}; \)
\( \text{length along the magnetopause} \)
\( xprim := -\text{diff}(Xmp(r), r); \)
\( # rprim := \sqrt{1/(2*x)}; \)
\( R := 10; \)
\( \text{Integrand} := \text{sqrt}(1+xprim^2); \)
\( \text{taylor}(\text{Integrand}, r = 0, 10); \)
\( \text{Integrand2} := \text{series}(1+.8804477095*r^2+3.49803582*r^4+13.05052560*r^6+8.525252146*r^8, r, 10); \)
\( \text{Rekv1} := \text{subs}(r = r(Z)); \)
\( \text{Rekv2} := \text{convert(Rekv1, \text{polynom})}; \)
\( \text{Rekv3} := \text{coeff(Rekv2, Z, 1)}; \)
\( \text{Rekv4} := \text{coeff(Rekv2, Z, 2)}; \)
\( \text{Rekv5} := \text{coeff(Rekv2, Z, 3)}; \)
\( \text{Rekv6} := \text{coeff(Rekv2, Z, 4)}; \)
\( \text{Rekv7} := \text{coeff(Rekv2, Z, 5)}; \)
\( \text{Rekv8} := \text{coeff(Rekv2, Z, 6)}; \)
\( \text{Rekv9} := \text{coeff(Rekv2, Z, 7)}; \)
\( \text{Rekv10} := \text{coeff(Rekv2, Z, 8)}; \)
\( \text{Rekv11} := \text{coeff(Rekv2, Z, 9)}; \)
\( \text{Rekv12} := \text{coeff(Rekv2, Z, 10)}; \)
\( \text{Rekv13} := \text{coeff(Rekv2, Z, 11)}; \)
\( \text{Rekv14} := \text{coeff(Rekv2, Z, 12)}; \)
\( \text{Rkoeff} := \text{solve} \{ \text{Rekv1} = 1, \text{Rekv2} = 0, \text{Rekv3} = 0, \text{Rekv4} = 0, \text{Rekv5} = 0, \text{Rekv6} = 0, \text{Rekv7} = 0, \text{Rekv8} = 0, \text{Rekv9} = 0, \text{Rekv10} = 0, \text{Rekv11} = 0, \text{Rekv12} = 0, \text{Rekv13} = 0, \text{Rekv14} \}; \)
\( \text{plot} \{ \text{subs}(\text{Rkoeff}, r(Z)), Z = 0..0.6 \}; \)
\( \text{r_av_z} := \text{subs}(r1 = 0, r2 = 0, r3 = 0, r4 = 0, r(Z)); \)
\( \text{r_av_z} := \text{subs}(Z = Z/13, \text{r_av_z}); \)
\( \text{RPRIM_av_z} := \text{diff}(\text{r_av_z}, Z); \)
\( \text{for i from 1 to 10 by 1 do} \)
\( \text{r_varde}[i] := \text{subs}(Z = i/26, \text{r_av_z}); \)
\( \text{evalf}(i/26); \)
end do;
> # r_varde:=subs(Z=2/13,r_av_z);  
> RPRIM:=diff(r_av_z > ,Z);  
> rprim:=subs(Z=z,RPRIM);  
> plot(z,r=0..1);  
> plot([Xmp(r),Xbs(r)],r=-1..1,x=0..1);  

Stream function

> Psi8(x,r):=1/2*r^2*(1-567/104*x-1521979/171520*x^2+8467901962333/1631381604000*x^3  
-35140037795/8320*x^4+r^2*2^4/(35/13+126959/8320*x+5852954713369/1055122432000*x^2)-3569/676*x^4);  
> Psi(x,r):=1/2*r^2*(1-70/13*x-1557/169*x^2-10009/17745*x^3-160299871/184580*x^4  
+r^2*2^4/(35/13+2563/169*x+8687359/153790*x^2)-3569/676*x^4);  

Density in the magnetosheath  
> rho1_8(x,r):=256/67+729/130*x-2258857413/90584000*x^2  
+24851009605268721/237909817600000*x^3  
-787295583437321321286993/92834314106060800000000*x^4  
+r^2*2^4*(-516/169-1176186/76895*x-351214713/1999270*x^2)+2092731/153790*x^4;  
> rho1(x,r):=48/13+72/13*x-4014/169*x^2+255275109/2691325*x^3  
-7527058114199796423000*x^4+r^2*2^4*(-516/169-1176186/76895*x-351214713/1999270*x^2)  
+2092731/153790*x^4;  

Density magnetopause  
> rho2(r):=convert(taylor(subs(x=Xmp(r),rho1(x,r)),r=0,8),polynom);  
> rho2z:=subs(r=r_av_z2,rho2(r));  
> drho2z:=diff(rho2z,Z):  

magnetic field expressed in z  
> Bz_z:=subs(r=r_av_z2,Bz);  

velocity from the stream function  
> ux(x,r):=1/r/rho1(x,r)*diff(Psi(x,r),r);#x:=Xmp(r):Uxmp:=ux(x,r);x:='x':  
> ur(x,r):=-1/r/rho1(x,r)*diff(Psi(x,r),x);#x:=Xmp(r) :Urmp:=ur(x,r);x:='x':  
> ux(x,r):=subs(x=Xmp(r),ux(x,r)):/2/sqrt(rho2z):  
> ur(x,r):=subs(x=Xmp(r),ur(x,r)):/2/sqrt(rho2z):  
> Uz:=sqrt((ux(x,r))^2+(ur(x,r))^2)/2;  
> Uz_av_z:=subs(r=r_av_z2,Uz):  
> dUz_z:=diff(Uz_av_z,Z):  
> #dUz:=subs(x=Xmp(r),dUz):  

Alfvén M  
> MAinf:=1.39794: # hast ute 300->1.39794 och 400->1.86392  
> MA:=evalf(Uz*sqrt(rho2(r)/Bz^2))*MAinf:  
> MA_z:=evalf(Uz_av_z2*Bz_z^2)/MAinf:  
> #r_varde:=.7678829693e-1;#0.9970210538e-1;  
> for i from 1 to 8 by 1 do  
> subs(r=r_varde[i],Alfven_MACH[i]=MA):  
> subs(Z=i/2,Alfven_MACHz[i]=MA_z):  
> end do;  
> for i from 1 to 8 by 1 do  
> #earth_rad[i]=subs(r=r_varde[i],[mag[i]=Bz,hast[i]=Uz,DeH[i]=U[HT],diffDeH[i]=dU[HT],  
> DiffHast[i]=dUz,dens[i]=rho2(r),altMag=Bz1]);
janne:=subs(Z=i/2,{mag[i]=Bz_z,hast[i]=Uz_av_z,DeH[i]=U[HT],DiffDeH[i]=dU[HT],
DiffHast[i]=dUz_z,dens[i]=rho2z,alfmach=evalf(MAinf*Uz_av_z*z*sqrt(rho2z/Bz_z^2))});

end do;
> mag2[2]:=subs(r=r_varde[2],Bz);dens2[2]:=subs(r=r_varde[2],rho2(r));
Alfven velocity
> VAinf:=1/MAinf; #km/s
> MUFF:=Uz/V[A];
> V[A]:=(abs(Bz)/sqrt(rho2(r)))*VAinf;
> for i from 1 to 7 by 1 do
>     AlfvenHast[i]=evalf(subs(r=r_varde[i],V[A]));
>     janne=evalf(subs(r=r_varde[i],MUFF));
> end do:
> AlfvenHast2[2]:=evalf(subs(r=r_varde[2],V[A]));

All Constants.
> for i from 1 to 8 by 1 do
    Z:=i/2;
    #earth_radii[i]=subs(r=r_varde[i],{mag[i]=Bz/abs(mag2[2]),hast[i]=Uz/AlfvenHast2[2],
    DeH[i]=U[HT]/AlfvenHast2[2],diffDeH[i]=dU[HT]/AlfvenHast2[2],DiffHast[i]=dUz/AlfvenHast2[2],
    dens[i]=rho2(r)/dens2[2],diffdens[i]=d(rho2(r))/dens2[2]}):
    #r:=r_varde[i]:
    #mag[i]:=subs(r=r_varde[i],Bz/abs(mag2[2]));
    #DeH[i]:=subs(r=r_varde[i],U[HT]/AlfvenHast2[2]):
    #hast[i]:=subs(r=r_varde[i],Uz/AlfvenHast2[2]):
    #DiffHast[i]:=subs(r=r_varde[i],dUz/AlfvenHast2[2]):
    #dens[i]:=subs(r=r_varde[i],rho2(r)/dens2[2]):
    #diffdens[i]:=subs(r=r_varde[i],d(rho2(r))/dens2[2]):
    mag[i/2]:=Bz_z:
    hast[i/2]:=Uz_av_z:
    DeH[i/2]:=U[HT]:
    diffDeH[i/2]:=dU[HT]:
    DeH_S[i/2]:=U[HT]:
    diffDeH_S[i/2]:=dU[HT]:
    DiffHast[i/2]:=dUz_z:
    dens[i/2]:=rho2z:
    #Z:=Z:
    #r(kvot[i]=subs(Z=i/2,r(kvot));
    #r(kvot)_z:=RPRIM/r_av_z:
    #r(kvot)_z2:=RPRIM_av_z/r_av_z2:
    #r(kvot)_z:=subs(Z=Z/13,r(kvot)_z):
    #r(kvot2)=subs(r=r_varde[2],r(kvot));
    #r(kvot)_z:=subs(Z=i/2,r(kvot)_z):
    #r(kvot2)_z:=subs(Z=i/2,r(kvot2)_z):
    end do;

end do;
> for i from 1 to 8 do
    Dens_kvot[i/2]=diffdens[i/2]/dens[i/2]:
end do;

r-ratio.
> r(kvot)_z:=RPRIM/r_av_z:
> r(kvot)_z2:=RPRIM_av_z/r_av_z2:
> r(kvot)_z:=subs(Z=Z/13,r(kvot)_z):
> #r(kvot2)=subs(r=r_varde[2],r(kvot));
> for i from 1 to 8 by 1 do
> #r(kvot)_z:=subs(Z=i/2,r(kvot)_z):
> r(kvot2)_z:=subs(Z=i/2,r(kvot2)_z):
> end do;

pressure
> P1_8(x,r):=319/4+14499/128*x-19905784679/44595200*x^2+164570212912*x
-350612735/151424*x^2+13257407/11648*r^4;
> P1(x,r):=179/4+895/8*x-176315/416*x^2+r^2*(-1615/16-749521/2912*x
-350612735/151424*x^2+13257407/11648*r^4);
> Pmp:=subs(x=Xmp(r),P1(x,r));
> Pmp:=subs(r=r_av_z,Pmp):Pmp_z:=subs(Z=Z/13,Pmp);
> Pprim := (diff(Pmp, Z));  
Pprim_z := diff(Pmp_z, Z);
> Pprim := subs(Z = z, Pprim);  
Pmp := subs(Z = z, Pmp);
> Pprim2 := abs(subs(r = r_varde[2], Pprim));  
Pmp2 := subs(r = r_varde[2], Pmp);
> plot([Pmp, Pprim], r = 0 .. 0.3, color = [red, black]);
> plot([Pprim_z, Pmp_z], Z = 0 .. 0.5);
> # Pprim := (diff(Pmp, r) * rprim);

pressure gradient

> for i from 1 to 8 by 1 do
> #subs(r = r_varde[i], dP/dz = Pprim/Pmp2, Tryck = Pmp/Pmp2):
> earth_radii[i/2] = subs(Z = i/2, {dP/dz = Pprim_z, Tryck = Pmp_z});
> end do;

> # plot(dU[HT], U[HT], r = 0 .. 0.3, color = [red, blue]);
> # U[HT] := subs(r = r_av_z, U[HT]);
> # dU[HT] := diff(U[HT], Z);
> plot(dU[HTu], U[HTu], Z = 0 .. 4, color = [red, blue]); plot(dU[HTs], U[HTs], Z = 0 .. 4, color = [red, blue]);
> taylor(drho2z/rho2z, Z = 1, 8);  
taylor(Bz_z, Z = 2, 8);  
taylor(Uz_av_z, Z = 2, 5);
> subs(Z = 1.5, 0.1931733964 + 0.1171368437*(Z-2)+0.2043838350e-1*(Z-2)^2
+ 0.8574014855e-2*(Z-2)^3+2.970748788e-2*(Z-2)^4);
> # subs(Z = 1.5, 0.9011165985e-1 + 0.9371475980e-1*(Z-1)+0.5817192650e-2*(Z-1)^2
+ 0.2663901966e-2*(Z-1)^3+6.116816145e-3*(Z-1)^4);

Constants for decided reconnection points. 1i corresponds to 1/2 Earth radii.

> for i from 4 to 8 by 4 do
> for Z = i/2 do
> Rkvoten = Rkvoten_z[i/2];
> MagField = mag[i/2];  
> Velocity = hast[i/2];  
> DeHoffmann = DeH[i/2];
> diff_DeHoff = diffDeH[i/2];
> Diff_Velocity = DiffHast[i/2];
> Density = dens[i/2];
> Alfven_Mach = alfmach[i/2];
> Diff_density = diffdens[i/2];
> dens_kvot[i/2] = dens[i/2]/dens[i/2];
> (dP/dz)[i/2] = subs(Z = i/2, Pprim_z);
> DeHoffmann_S = DeH_S[i/2];
> diff_DeHoff_S = diffDeH_S[i/2];
> end do;

# vekpot = A(x, r), ryck = P1(x, r), dens = rho2z;
C. Appendix Matlab code

C.1. The m-files for the equation

C.1.1. Equation for \( f_0'(\eta) \)

function g=Fhollprim(x,g)
g=[g(2); g(3); -2*x*g(3)];

C.1.2. Equation for \( f_1'(\eta) \)

function f=exjobbSenaste61(x,f)
Zr=1;
rhokvot=-0.03834;
HT=0.28021;
HTprim=0.287184;
HTkvot=HTprim/HT;
rho=3.8807;
tryck=-2.3303;
rvkvot=0.99646;
%constants
C1=-2*rhokvot;
C2=(4*rkvot+5*(rhokvot+HTkvot));
C3=0.5*C1;
C4=C1/HT;
P=tryck/(rho*HT);
Uprim=HTprim;
f=[f(2); f(3);
-2*x*f(3)+4*f(2)+C1*erf(x)+C3*4/sqrt(pi)*x.*exp(-x.^2)+
C2*(4/sqrt(pi))*x.*exp(-x.^2)+C4*erf(x).^2+Uprim+P];

C.2. The m-file that plots the velocity

The m-files that plot the velocity and magnetic field are similar, the difference is the plot output. A combination of the functions corresponds to the magnetic field, \( f_0'(\eta)+Z f_1'(\eta) \) (note that the magnetic field is scaled with \( \sqrt{\rho_0} \).

Z=1.0;
Zr=1; %reconnection point
%hast=0.09011;
rhokvot=-0.03834;
HT=0.28021;
HTprim=0.28718;
HTkvot=HTprim/HT;
rkvot=0.99646;
tryck=-2.3303;
rho=3.8807;
B0=-0.37448/sqrt(rho);
B1=-0.37395/sqrt(rho);
MagField=B0+(Z-Zr)*B1;
C1 = -2*rhokvot;
C2 = (4*tkvot + 5* (rhokvot + HTkvot));
C3 = 0.5*C1;
C4 = C1/HT;
Uprim = HTprim;
P = tryck/(rho*HT);
B = MagField
x0 = 0.00; xs = 4.00; xspan = x0:0.1:xs;
F10 = 0;
F20 = -(P + Uprim)/4-C4*0.0908;
F30 = -C1*0.282-0.282*C3-C2*0.282;
F0 = [F10,F20,F30];
G10 = 0;
G20 = -(Z-Zr)*randfel-(Z-Zr)*HTprim;
G30 = 1.128*B0;
G0 = [G10,G20,G30];
[x,f] = ode45('exjobbSenaste61',xspan,F0);
[x,g] = ode45('F nollprim',xspan,G0);
[mf,nf] = size(f);
randfel = (f(mf,2)-hast);
[xb,fb] = ode45('exjobbSenaste61',xspan,F0);
[xb,gb] = ode45('F nollprim',xspan,G0);
[mfb,nfb] = size(fb);
randfelb = (fb(mfb,2)-0);
plot(x,g(:,2)+(Z-Zr)*f(:,2)+HT+(Z-Zr)*HTprim,'k',xb,gb(:,2)+(Z-Zr)*fb(:,2)+HT+(Z-Zr)*HTprim,'k')