I. INTRODUCTION

Excessive release of phosphorus from the effluent of on-site sanitation facilities to the environment is one source of aquatic pollution. The primary effect of excess phosphorus in the aquatic environment is eutrophication. In Sweden, there are about one million private on-site wastewater treatment facilities, half of which are considered unacceptable by the Swedish Environmental Protection Agency [1]. According to Ejhed et al. [2], the most frequent on-site wastewater treatment systems in Sweden are: infiltration systems (40%), septic tanks (24%), collecting tanks (18%) and sand filter beds (13%). Nevertheless, present on-site sanitation systems have often insufficient phosphorus removal efficiency. Recent guidelines issued by Swedish EPA [3] recommended total phosphorus reduction of 70-90% in wastewater in small scale treatment facilities. Using reactive filter materials as compact filter systems following a biological pretreatment can be a convenient option for upgrading the present on-site sewage treatment facilities. An advantage of using reactive filter materials is that they can possibly be used as a soil amendment in agriculture after being enriched with phosphorus [4, 5, 6].

Phosphorus removal capacity of such filter materials has been the main focus of studies so far. Some studies were done on natural and industrial by-products, e.g. wollastonite [7] and slags from steel production [8], respectively. In addition to natural and industrial by-products, some studies have focused on commercial filter materials such as Polonite® [4], Filtralite P® [9], and Filtra P® [10]. Filtra P® was shown to be a promising material for the removal of phosphate (PO$_4$-P) from domestic wastewater with removal efficiency of more than 95% [10]. Nevertheless, Filtra P® has been only briefly investigated, i.e., [10, 11]. These studies were focused on the PO$_4$-P removal mechanism and the filtration efficiency of this material. No studies, however, have attempted to model the PO$_4$-P transport and retention in saturated Filtra P® columns which is likely to be valuable for the estimation of the longevity of this filter material.

Several researchers have modeled the transport and retention of phosphorus in natural and artificial filters. For instance, Notodarmojo et al. [12] presented a model for one-dimensional transport of phosphorus in soils and groundwater. Van der Zee and Gjaltema [13] and Van der Zee et al. [14] developed a PO$_4$-P transport model assuming one-dimensional steady state flow in soil columns. McGechan and Lewis [15] reviewed literature on phosphorus sorption by soils with the intention of selecting equations and parameter values for use in a soil phosphorus dynamics model. Lee et al. [16, 17] launched the first study on the phosphorus transport in saturated slag columns. In all these studies, the general one-dimensional convection-dispersion equation for saturated flow in porous media [18] was applied to model the transport and retention of phosphorus in columns filled with different materials. The only variation between the studies is the approximated phosphorus removal mechanism and thus the removal term in the equations. Rubin [19] listed six different classes of chemical reactions that can take place in solute transport and subsequently investigated the relation between the mathematical nature of the problem formulation and the chemical nature of reactions. PO$_4$-P removal mechanism of Filtra P® was investigated by [10, 11] who state that calcium-phosphate precipitation is important for removal of PO$_4$-P from wastewater.

One way of determining how much solute can be removed by the specific sorbent is batch experiments. The procedure is that results from the experiment are plotted on a
graph, called an isotherm that shows the solute concentration in equilibrium versus the amount sorbed onto the solid. The most common nonlinear sorption isotherms are the Langmuir and Freundlich isotherms. The application of these isotherms and their limitations are discussed widely in literature e.g. [20]. In the present study, sorption is referred to a process that ranges from adsorption to precipitation reactions which was defined by McBride [21], a definition convenient to use when dealing with reactive materials such as Filtra P for which the chemical processes governing the interaction are not fully known [20].

The main objective of this work was to investigate the feasibility of modeling PO$_4$-P transport and retention in fixed-bed columns filled with Filtra P by means of the convection-dispersion equation and a Langmuir isotherm. The experimental data used in this study came from [22] and [23]. The latter was used to obtain the Langmuir isotherm. The breakthrough curves obtained by the model were compared with the results from fixed-bed column experiments.

II. MATERIAL AND METHODS

A. Filtra P

Filtra P is a commercial filter material produced by heating up a mixture of limestone, gypsum, and iron. This product is categorized as an alkaline sorbent due to its high content of calcium hydroxide [10]. Table 1 shows the properties of this filter material. This material is delivered in the size of either 500 kg or 1000 kg per household for phosphorus treatment with the functioning time of minimum 1 year (www.nordkalk.com). Samples of Filtra P were obtained from Nordkalk Oyj Abp (Pargas, Finland).

B. Batch experiment

Batch equilibrium sorption experiments were carried out in 0.5 L Erlenmeyer flasks, containing 500 dm$^3$ of the PO$_4$-P solution (initial concentrations in the range of 3-1000 mg dm$^{-3}$), into which 25.0 g of Filtra P was added. The samples were equilibrated with PO$_4$-P solutions using KH$_2$PO$_4$ as a PO$_4$-P source. The flasks were placed on a shaking plate vibrating at 100 rpm for 0.5, 1, 2, 4, 8, 24, 48 hours for concentrations between 3-100 mg dm$^{-3}$ and for 48 hours for concentrations between 300-1000 mg dm$^{-3}$. The experiments were carried out at room temperature (ca. 20$^\circ$C). After shaking, the supernatant was directly filtrated through 0.45 µm filters, acidified and stored at 5$^\circ$C. The PO$_4$-P was analysed with the ICP technique without prior digestion since the sample was assumed to be totally dissolved during the procedure. In the supernatants, the pH was measured using a WTW pH330 pH-meter and a WTW SenTix41 pH-electrode.

<table>
<thead>
<tr>
<th>Element, g kg$^{-1}$ TS</th>
<th>TABLE I. PROPERTIES OF FILTRA P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca</td>
<td>355.21</td>
</tr>
<tr>
<td>Si</td>
<td>11.27</td>
</tr>
<tr>
<td>Al</td>
<td>4.57</td>
</tr>
<tr>
<td>Fe</td>
<td>3.24</td>
</tr>
<tr>
<td>Mg</td>
<td>2.22</td>
</tr>
<tr>
<td>K</td>
<td>1.44</td>
</tr>
<tr>
<td>Na</td>
<td>0.69</td>
</tr>
<tr>
<td>Ti</td>
<td>0.26</td>
</tr>
<tr>
<td>P</td>
<td>0.25</td>
</tr>
<tr>
<td>Mn</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Density, kg m$^{-3}$

1000

Uniformity coefficient, $Ca = d_{max}/d_{min}$

1.4

Effective grain size, $d_{min}$(mm)

4.6

The equilibrium concentration of PO$_4$-P in the solid phase $q_e$ (g kg$^{-1}$), was calculated from the initial concentration $C_0$ (mg dm$^{-3}$), and the equilibrium concentration $C_e$ (mg dm$^{-3}$), in each flask, using the following equation:

$$q_e = \frac{V(C_0 - C_e)}{m}$$  (1)

where $V$ is the volume of solution and $m$ is the mass of Filtra P.

The equilibrium data obtained from batch experiments were fitted using Langmuir isotherm model, which may be expressed as:

$$q_e = \frac{q_{max}bC_e}{(1 + bC_e)}.$$  (2)

In (2) $q_e$ is the solid phase equilibrium PO$_4$-P concentration (g kg$^{-1}$), $C_e$ is the liquid phase PO$_4$-P concentration in equilibrium (mg dm$^{-3}$), $q_{max}$ is the maximum uptake (g kg$^{-1}$) and $b$ is a constant related to energy of adsorption (dm$^{-1}$ mg$^{-1}$). The results of the batch experiments from [23] are shown in Fig. 1.

When fitting the Langmuir isotherm (2) to the experimental equilibrium data for the PO$_4$-P in the equilibrium concentrations $C_e = 0.22.93$ mg dm$^{-3}$ the following values on $q_{max}$ and $b$ were obtained, 4.30 g kg$^{-1}$ and 0.84 dm$^{-3}$ mg$^{-1}$, respectively, see Fig. 1b. The Langmuir
Figure 1. Batch experiment results. Where duplicates were performed, standard deviations are shown as error bars. Deviations for pH values were negligible.

The area above the breakthrough curve is proportional to the total solute removed from the feed stream upon the complete saturation of the bed [24]. Therefore, if the entire bed comes to equilibrium with the feed, the amount of PO₄-P removed from the solution, \( q_e \), may be calculated as (e.g. Izquierdo et al., [25]):

\[
q_e = \frac{C_0 V_p}{m} \int_0^N (1 - \frac{C}{C_0}) dN
\]

where \( q_e \) is the amount of PO₄-P removed (g of PO₄-P per kg of Filtra P), \( C_0 \) and \( C \) are PO₄-P concentration at the column influent and effluent (mg dm⁻³) respectively, \( V_p \) is the bed pore volume (dm³), \( m \) is the mass of Filtra P in the column (g), and \( N \) is the number of pore volumes.

The integral represented by (3) was solved numerically in Matlab.

D. Mass transport model

In order to simulate the PO₄-P breakthrough curves, the convection-dispersion equation which assumes PO₄-P removal under equilibrium condition was used. The following assumptions were applied:
- Isothermal process
- Constant physical properties
- Negligible radial dispersion
- Saturated condition

### TABLE II. EXPERIMENT CONDITIONS FOR COLUMN RUNS

<table>
<thead>
<tr>
<th>Experiment</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface load (dm³ m⁻² day⁻¹)</td>
<td>406</td>
<td>1058</td>
<td>425</td>
<td>1083</td>
</tr>
<tr>
<td>PO₄-P concentration (mg dm⁻³)</td>
<td>11</td>
<td>11</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Pore volume (\( V_p \)) and bed porosity (\( \varepsilon \)) were 0.134 dm³ and 0.446, respectively.
The convection-dispersion equation for one-dimensional steady state flow in the axial direction is as follows (e.g. Miller and Weber [18]):

\[
\frac{\partial C}{\partial t} + \frac{\rho}{\varepsilon} \frac{\partial q}{\partial t} = D_L \frac{\partial^2 C}{\partial Z^2} - u \frac{\partial C}{\partial Z}
\]  \hspace{1cm} (4)

where \( C \) is the solute concentration (mg dm\(^{-3} \)), \( \rho \) is the bulk density of the Filtra P in the column (kg m\(^{-3} \)), \( \varepsilon \) is the porosity, \( q \) is the solid phase concentration (g kg\(^{-1} \)), \( D_L \) is the axial dispersion coefficient (m\(^2 \) s\(^{-1} \)), \( u = Q/A \varepsilon \) is the average pore water velocity (m s\(^{-1} \)), and \( A \) is the cross sectional area of the column (m\(^2 \)). \( Z \) is axial coordinate in the column (cm) and \( t \) is time (day).

The following initial and boundary conditions were used:

\begin{align*}
& C(Z,0) = 0, \hspace{1cm} (5) \\
& -D_L \frac{\partial C}{\partial Z} + uC = uC_0 \text{ for } Z = 0 \text{ and } (6) \\
& \frac{\partial C}{\partial Z} = 0 \text{ for } Z = L. \hspace{1cm} (7)
\end{align*}

In this study the axial dispersion coefficient \( (D_L) \) was estimated from the recommended correlation in the literature. One such an empirical correlation which relates the axial Peclet number to the Schmidt number (effect of temperature) and particle Peclet number is as follows (Delgado [26]):

\[
\frac{1}{Pe_L} = \frac{Pe_m(1 - p^3)}{5} + \frac{Pe_m^2}{25} \left( \frac{p(1 - p)}{5} \right) \left\{ \exp \left[ -\frac{5}{p(1 - p) Pe_m} \right] - 1 \right\} + \frac{1}{\rho \epsilon Pe_m}
\]  \hspace{1cm} (8)

with

\[
p = \frac{0.48}{Sc^{0.15}} + \left( \frac{1}{2} - \frac{0.48}{Sc^{0.15}} \right) \exp \left( -\frac{75 Sc}{Pe_m} \right). \hspace{1cm} (9)
\]

In (8) and (9), \( Pe_L \) is the axial Peclet number, \( Pe_m \) is the particle Peclet number, \( Sc \) is the Schmidt number and \( \tau = \sqrt{2} \) (tortuosity), assuming filter pellets to be spherical [26]. The molecular diffusion coefficient \( (D_m) \) used in the calculation of \( Pe_m \) and \( Sc \) was estimated using the Wilke and Chang equation [27]. The estimated molecular diffusion of PO\(_4\)-P for the actual experimental condition is \( 1.295 \times 10^{-9} \) m\(^2 \) s\(^{-1} \). The effective grain size of \( d_g \) was used as an average pellets diameter in calculation of \( Pe_L \) and \( Pe_m \).

HYDRUS-1D [28] was used to simulate PO\(_4\)-P breakthrough curves. HYDRUS-1D allows simulation of convective-dispersive solute transport coupled with a nonlinear isotherm. The system of equations formed by (2) & (4), together with estimated axial dispersion coefficient by (8) & (9), and with initial and boundary conditions given by (5) - (7), were solved using HYDRUS-1D.

III. RESULTS

Simulation of column dynamics

Table 3 shows from (3) calculated amounts of PO\(_4\)-P removed and number of pore volumes treated at \( C/C_0 = 0.02 \) (breakthrough point for columns C and D), at \( C/C_0 = 0.09 \) (breakthrough point for columns A and B) and \( C/C_0 = 1 \) (saturation point) for columns A, B, C, and D. According to Swedish EPA [3], the maximum allowable phosphorus concentration in the domestic wastewater effluent in areas with environmentally vulnerable recipients is 1 mg dm\(^{-3} \). Therefore, nondimensional breakthrough points for this study (i.e. \( C/C_0 = 0.02 \) and \( C/C_0 = 0.09 \)) were chosen in a way that represent the time that \( C = 1 \) mg dm\(^{-3} \) in the effluent of the columns.

Comparison between the predicted PO\(_4\)-P uptake in the batch experiment by the Langmuir isotherm (3.89 g kg\(^{-1} \) calculated by the Langmuir isotherm at \( C = 11 \) mg dm\(^{-3} \)) and the amounts of PO\(_4\)-P removed in the fixed-bed column experiments at saturation point, \( C/C_0 = 1 \), showed that the batch experiment predicted the amounts of PO\(_4\)-P uptake in column A (4.35 g kg\(^{-1} \)) with approximately 10% deviation. In the case of column B (7.85 g kg\(^{-1} \)), the deviation was almost 50%. Such comparisons were not feasible for columns C and D, due to the fact that the fitted Langmuir isotherm with the batch experiment (Fig. 1b) were in the equilibrium concentration ranges between 0-22.93 mg dm\(^{-3} \).
and moreover no isotherm could be fitted in the concentrations above this range.

Initially, the Langmuir isotherm obtained by the batch experiment was incorporated into the model used to simulate PO₄-P breakthrough curves. The model obtained this way was successfully simulated the PO₄-P breakthrough curve for column A only. The incapacity of the model in representing the breakthrough curve for column B can be justified by the 50% deviation between the predicted and experimental PO₄-P final uptake magnitude.

In order to find the suitable isotherm for simulation of PO₄-P breakthrough curve for column B, the experimental PO₄-P uptake values at saturation point for columns B (7.85 g kg⁻¹) and D (7.37 g kg⁻¹) which had almost the same surface loads, together with experimental data obtained by batch experiment for equilibrium concentrations close to zero (initial concentrations of 0-100 mg dm⁻³) were used to adjust the Langmuir isotherm. Fig. 3 shows the modified Langmuir isotherm.

The Langmuir parameters for this isotherm are: $q_{\text{max}} = 7.86$ g kg⁻¹ and $b = 1.84$ dm³ mg⁻¹. The maximum predicted PO₄-P uptake by this isotherm was almost the same as the PO₄-P uptake value at saturation point for column B. The adjusted Langmuir isotherm is comparable to the dynamic isotherm found by fixed-bed column experimental data only, which is widely used to simulate the column experiments breakthrough curves (see for instance, Pereira et al. [29], da Silva et al. [30]).

![Modified Langmuir isotherm for column B](image)

Fig. 3. Modified Langmuir isotherm for column B

Table III. PO₄-P removed at breakthrough and saturation point for columns A, B, C, and D

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$C/C_p = 0.02$</th>
<th>$C/C_p = 0.09$</th>
<th>$C/C_p = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PO₄-P removed (g kg⁻¹)</td>
<td>Pore Volumes</td>
<td>PO₄-P removed (g kg⁻¹)</td>
</tr>
<tr>
<td>A</td>
<td>-</td>
<td>-</td>
<td>3.27</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>-</td>
<td>6.54</td>
</tr>
<tr>
<td>C</td>
<td>2.30</td>
<td>102.9</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>3.64</td>
<td>166.3</td>
<td>-</td>
</tr>
</tbody>
</table>

The model however was unable to represent the PO₄-P breakthrough curves of columns C and D. Table 4 presents the axial dispersion coefficient ($D_L$), Langmuir isotherm parameters, $q_{\text{max}}$, and $b$ that were used in the simulation of breakthrough curves. Deviations between the predicted and experimental breakpoint are also presented in table 4 for columns A and B.

### IV. DISCUSSIONS

#### A. Mass transport model

The conventional convection-dispersion equation was used in this study to model PO₄-P transport modeling for column experiments. The model accurately predicted the breakthrough points and reasonably reproduced the shape of the experimental breakthrough curves for columns A and B. The model however was unable to represent the PO₄-P breakthrough curves of columns C and D. Table 4 presents the axial dispersion coefficient ($D_L$), Langmuir isotherm parameters, $q_{\text{max}}$, and $b$ that were used in the simulation of breakthrough curves. Deviations between the predicted and experimental breakpoint are also presented in table 4 for columns A and B.
TABLE IV. PARAMETERS USED IN THE SIMULATION OF BREAKTHROUGH CURVES AND ERRORS IN THE PREDICTION OF BREAKTHROUGH TIME FOR COLUMNS A AND B

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$D_v$ (m$^2$s$^{-1}$)</th>
<th>$q_{max}$ (g kg$^{-1}$)</th>
<th>$b$ (dm$^3$ mg$^{-1}$)</th>
<th>Breakthrough pore volumes (C/C$_0$ = 0.09)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Experimental</td>
</tr>
<tr>
<td>A</td>
<td>$2.48 \times 10^{-7}$</td>
<td>4.30</td>
<td>0.84</td>
<td>655.6</td>
</tr>
<tr>
<td>B</td>
<td>$1.67 \times 10^{-7}$</td>
<td>7.86</td>
<td>1.84</td>
<td>1338.5</td>
</tr>
<tr>
<td>C</td>
<td>$2.68 \times 10^{-7}$</td>
<td>4.30</td>
<td>0.84</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>$1.74 \times 10^{-7}$</td>
<td>7.86</td>
<td>1.84</td>
<td>-</td>
</tr>
</tbody>
</table>

Despite all the limitations discussed in literature for using Langmuir isotherm for a general case, in our study when it was fitted to the concentration ranges where precipitation was likely to happen (Fig. 1b) and incorporated into the model, it could successfully simulate the PO$_4$-P breakthrough curve of column A (Fig. 4a). Considering column B, the Langmuir isotherm had to be adjusted in order to predict the breakthrough curve. An alternative route to the one applied here would be to use several columns with the same surface load and varied concentrations in the range of 0-11 mg dm$^{-3}$ as it is normally done in order to find dynamic isotherm. However, it could be expensive and time consuming to do such experiments just to find a Langmuir isotherm for one column. Moreover, equilibrium concentrations for initial concentrations of 0-100 mg dm$^{-3}$ from the batch experiment were close to zero (initial data values in Fig. 3), which shows...
Filtral P effectively remove PO$_4$-P from the solution in these concentrations. Therefore the adjusted Langmuir isotherm (Fig. 3) is actually fitted to experimental data from column experiments B and D which had almost the same surface load and equilibrium concentrations close to zero which can be considered to be zero.

Langmuir isotherm simply could not be fitted for columns C and D in the batch experiment. This can be seen in Fig. 1a, increasing the concentrations caused decreasing the pH of the system and PO$_4$-P uptake amount decreased abruptly which further confirms the fact that precipitation of calcium-phosphate compounds are likely to be the removal mechanism of PO$_4$-P [23]. In order to just test the model, the Langmuir isotherms (Fig. 1b) and (Fig. 3) were used for simulation of PO$_4$-P breakthrough curves of columns C and D, respectively.

B. Column dynamics

Fig. 4 shows both modeled and experimental PO$_4$-P breakthrough curves of column experiments. Comparison between experimental breakthrough curves of columns A-B (Fig. 4a-Fig. 4b) and C-D (Fig. 4c-Fig. 4d) shows that, interestingly, the columns with higher surface loads were treating PO$_4$-P for more pore volumes than the columns with the same influent PO$_4$-P concentrations and lower surface loads. This can be partially explained by looking at the pH curves of column experiments [22], the columns which were treating PO$_4$-P for more pore volumes were also had higher pH, the condition that precipitation of calcium-phosphate were likely to occur. The reason of faster decreasing of pH in the columns with lower surface loads led us to a hypothesis which is beyond the scope of the present study and is discussed on the paper dealing with column experiments [22].

C. Simulation of column dynamics

The model successfully simulated the PO$_4$-P breakthrough points of columns A and B. Column A had almost the same surface load and PO$_4$-P feed concentration as of a four-person household real scale Filtra P column and the breakthrough curve of it was well fitted by the model which used the Langmuir equilibrium isotherm obtained from the batch experiment. The model was unable to represent the PO$_4$-P breakthrough curves of columns C and D. PO$_4$-P breakthrough curves of columns A and B had the typical S-shape and they reached the breakthrough point C/C$_0$ = 0.09, corresponding to C = 1 mg dm$^{-3}$ after several pore volumes.

V. SUMMARY AND CONCLUSIONS

This study investigated the feasibility of modeling PO$_4$-P transport and removal in fixed-bed columns filled with Filtra P by means of the convection-dispersion equation and a Langmuir isotherm. Langmuir model was used to represent the PO$_4$-P uptake in the concentrations where precipitations of calcium-phosphate compounds were likely to occur. In this work, the axial dispersion coefficient was estimated from the correlation given in the literature. Four column experiments were performed at different flow rates and PO$_4$-P feed concentrations. Magnitude of the influent concentrations and surface flow rates were chosen in a way that, the low level almost resemble the condition of a real scale Filtra P column filter, while the high levels represent rather extreme laboratory conditions to accelerate the experiment. The model was further compared by the experimental data obtained from fixed-bed column experiments.

The main results obtained can be summarised as follows:

- The breakthrough curve of column A which had almost the same surface load and PO$_4$-P feed concentration as of a real scale Filtra P column was well fitted by the model which uses the convection-dispersion model and the Langmuir equilibrium isotherm obtained from the batch experiment.

- The same convection-dispersion model incorporated with a modified Langmuir isotherm was also able to fit the breakthrough curve of column B which had low level PO$_4$-P feed concentration and high level influent flow rate.

- Experimental observations however yield that the possibility to combine the convection-dispersion equation with a model accounting for precipitation of calcium-phosphate compounds in the bulk should be examined.

NOTATIONS

- $A$: Cross sectional area of the column, m$^2$
- $b$: Langmuir constant, dm$^3$ mg$^{-1}$
- $C$: PO$_4$-P concentration in the bulk fluid phase, mg dm$^{-3}$
- $C_0$: Initial PO$_4$-P concentration in the bulk fluid phase, mg dm$^{-3}$
- $C_e$: Equilibrium PO$_4$-P concentration in the fluid phase, mg dm$^{-3}$
- $d_{50}$: Hazen’s effective grain size, mm
- $D_L$: Axial hydrodynamic dispersion coefficient, m$^2$s$^{-1}$
- $D_m$: Molecular diffusion coefficient, m$^2$s$^{-1}$
- $L$: Length of the bed, cm
- $m$: Mass of Filtra P in the column, g
- $N$: Number of pore volumes
- $q$: Amount of PO$_4$-P uptake, g of PO$_4$-P per kg of sorbent
- $q_{eq}$: Solid-phase equilibrium PO$_4$-P concentration, g of PO$_4$-P per kg of sorbent
- $q_{max}$: Maximum PO$_4$-P uptake capacity for the Langmuir model, g kg$^{-1}$
- $Q$: Average volumetric flow rate, m$^3$s$^{-1}$
- $t$: Time, day
- $u$: Interstitial velocity, m s$^{-1}$
- $V$: Volume of solution, dm$^3$
- $V_p$: Bed pore volume, dm$^3$
- $Z$: Axial coordinate in the column, cm
- $\rho$: Column void fraction
- $\epsilon$: fixed-bed density, kg m$^{-3}$
- $\tau$: Tortuosity
- $\Phi$: Diameter of the column, cm

Dimensionless groups
Peₐ = Peclet number of particle (= uDm/Dₐ)
Peₐ = Peclet number based on axial dispersion coefficient (= uDₐ)
Sc = Schmidt number (= μ/ρDm)

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